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Sticky Prices and Costly Credit

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Abstract

We develop a theory of money and credit as competing payment instruments, then put it to work in applications. Agents use cash and credit because the former (latter) is subject to the inflation tax (transaction costs). Frictions that make the choice of payment method interesting also imply equilibrium price dispersion. We derive closed-form solutions for money demand, and show how to simultaneously account for the price-change facts, cash-credit shares in micro data, and money-interest correlations in macro data. The effects of inflation on welfare, price dispersion and markups are discussed, as are nonstationary equilibria with dynamics in the price distribution.

1 Introduction

This paper develops a theory of money and credit as competing payment instruments, and puts it to work in applications. This is a classic issue: as Lionel Robbins said in the Introduction to von Mises (1953), "Of all branches of economic science, that part which relates to money and credit has probably the longest history and the most extensive literature." To bring it up to date, we use a New Monetarist approach that tries to take the exchange/payment processes seriously (Section 2 reviews the literature). Both cash and credit are used in the model due to the venerable idea that the former is subject to the inflation tax while the latter involves transaction costs.[1] We consider both fixed and variable transaction costs, which turn out to work rather differently, with a variable cost outperforming a fixed cost in terms of theory and data.

An important ingredient is what Burdett and Judd (1983) call "noisy" search, meaning that sellers post prices, and each buyer sees a random number of them. This leads to a distribution of prices $F(p)$, where any p in a nondegenerate support yields the same profit – intuitively, lower-price sellers earn less per unit but make it up on the volume. We integrate this into the model of money in Lagos and Wright (2005), with alternating centralized and decentralized markets, as is natural, because at its core is an asynchronization of expenditures and receipts crucial for any analysis of money or credit. In the centralized market agents consume, work, adjust their cash balances and settle their accounts. In the decentralized market they trade different goods, as in Burdett-Judd, but with payment frictions: since buyers have no goods or services to offer by way of quid pro quo, they must use cash or credit. Consistent with the conventional wisdom, they tend to use credit for large and cash for small expenditures.

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[1] One needs such a device to get both money and credit into general equilibrium in a nontrivial way. Gu et al. (2016) prove this: if credit conditions are loose, money cannot be valued; if credit is tight, money can be valued but then credit is not essential and changes in credit conditions are neutral. Here transaction costs get around that result.

Costly credit implies a simple demand for money and avoids an indeterminacy that plagues similar models (see below). We also generate endogenous nominal stickiness. To see how, note that sellers post prices in dollars, since this is a monetary model. As the money supply M increases, $F(p)$ shifts so that the real distribution stays the same, but as long as it does not shift too much some firms can keep the same p. Hence prices look sticky, even though sellers can always adjust at no cost. For a seller that sticks to p when M rises, his real price falls, but the probability of a sale increases, and so changing p is simply not profitable. While Head et al. (2012) and others make similar points, we avoid a technical problem in that approach. Also, while their model can match some features of price-change behavior quantitatively, we go further by matching these features plus micro data on payment methods and macro data on money demand.[2]

In another application, we find small effects of inflation on welfare – e.g., eliminating $\pi = 10\%$ inflation is worth only 0.23% of consumption in the baseline setting, where the impact comes mainly from impinging on the cash-credit margin. Even in an extension with endogenous participation, where π affects output directly, the welfare effect is smaller than similar models, as discussed below. We also analyze the impact of π on markups and price dispersion. And we study different specifications for the process by which buyers sample prices. Additionally, we describe nonstationary equilibria, where inflation and deflation arise as self-fulfilling prophecies. For this, we use standard tools, but in terms of substance, we get dynamics in the price distribution, not just the price level.[3] Finally, we deliver closed-form solutions for money demand reminiscent of Baumol-Tobin, but in general equilibrium, not just in decision theory.

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[2] As is standard, by money demand we mean the relationship between real balances and nominal interest rates. Head et al. (2012) have no credit, and hence cannot match the micro data, and do not match money demand at all well. Earlier related work like Caplin and Spulber (1987) or Eden (1994) do not go to the data. So, while we are not the first to think about sticky prices in this way, we try to make a novel quantitative contribution.

[3] Kaplan and Menzio (2016) introduce Burdett-Judd frictions in the product market. In the continuous-time limit of their discrete-time model, they also generate dynamics in an entire price distribution.

Quantitatively, our fixed-cost specification can match standard money demand observations, but not these plus the money-credit shares in the payment data. Our proportional-cost specification can match both. Either specification is consistent with the salient price-change facts, including long durations, large average changes, many small changes, many negative changes, a decreasing hazard, and adjustment behavior that depends on inflation. Although we match these facts reasonably well, the fit is not perfect due to the discipline imposed by other observations; without this discipline – e.g., if we give up on money demand – the model matches price-change data virtually perfectly. However, we think any theory attempting to match the price-change facts should also confront the other facts, since they all pertain to monetary phenomena and all have implications for monetary policy. We try to match the observations simultaneously.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3-4 describe the model and stationary equilibrium. Section 5 discusses calibration. Section 6 presents applications. Section 7 concludes.

2   Literature

There is related work in several areas. New Monetarist papers are surveyed generally in Lagos et al. (2017), but particular models that use Burdett-Judd pricing are Head et al. (2012) and Wang (2016), who embed it in Lagos and Wright (2005), and Head and Kumar (2005) and Head et al. (2010), who embed it in Shi (1997). However, there is a technical problem with indivisible goods and Burdett-Judd price posting in monetary economies: it leads to an indeterminacy (i.e., a continuum) of stationary equilibria.[4] Some papers get around this by using divisible goods, but then another problem arises – what should firms post? Those papers assume

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[4]This comes up in a series of papers spawned by Green and Zhou (1998). See Jean et al. (2010) for citations and more discussion, but here is a simple version of the problem: If all sellers post p then buyers' best response is to bring $m = p$ dollars to the market as long as p is not too high. If all buyers bring m then sellers' best response is $p = m$ as long as m is not too low. Hence, any $p = m$ in some range is an equilibrium.

linear menus, where sellers set p and let buyers choose any q as long as they pay pq; but this may not be a profit maximizing strategy, and that seems like a serious issue. Here, with costly credit, the indeterminacy problem with indivisible goods goes away, so we can avoid the ad hoc restriction to linear menus.

Intuitively, holding more cash reduces the amount of costly credit buyers expect to use, which delivers a well-behaved money demand function and a unique equilibrium with money and credit. While we do not take a stand on whether divisible or indivisible goods are more realistic, indivisibility is an assumption on the physical environment, preferable to a restriction on pricing strategies. Also note the indeterminacy in question concerns stationary equilibria, not dynamic equilibria, which are discussed in Section 6.4. In any case, despite these technical differences, we share with Head et al. (2012) the goal of analyzing pricing without imposing menu costs (e.g., Mankiw 1985), letting sellers only change at exogenous points in time (e.g., Taylor 1980; Calvo 1983), or assuming inattention (e.g., Woodford 2002; Sims 2003). While those devices may be interesting, we want to see how far we can go without them.[5]

On empirical work, Bils and Klenow (2004) find half the prices in BLS data last less than 4.3 months, or 5.5 months excluding sales. Klenow and Kryvtsov (2008) report durations from 6.8 to 10.4 months. Nakamura and Steinsson (2008) report 8 to 11 months. Those papers also find large fractions of small and negative changes, plus evidence of a decreasing hazard. Other work is surveyed by Klenow and Malin (2010), but for convenience we provide a summary of the many studies in Appendix C. One issue for a menu cost approach is that average price changes are fairly big, suggesting high menu costs, but there are also many small changes, suggesting low costs. Midrigan (2011) explains this by firms selling multiple goods, where paying

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[5]In spirit, Caplin and Spulber (1987) and Eden (1994) take a similar approach, but they do not use microfoundations the way we do. Also, Burdett and Menzio (2016) combine search and menu costs, which makes the analysis more difficult, but they are able to show the following: when menu costs become small, their equilibrium converges to the one studied here. Other nonmonetary search models with menu costs include Benabou (1988, 1992a) and Diamond (1993). Burstein and Helwig (2008) is also relevant.

a cost to change one price lets them change the rest for free (see also Vavra 2014).
We account for realistic durations, large average changes, many small and negative
changes, and repricing behavior that depends on inflation without such devices.
We get a decreasing hazard, which is problematic for other models (Nakamura
and Steinsson 2008), and price dispersion at low or no inflation, consistent with
evidence but not other models (Campbell and Eden 2014).[6]

As representative studies, Lucas (2000) and Cooley (1995) discuss the cost of
inflation using money-in-the-utility-function or cash-in-advance models. They find
eliminating an annual inflation of $\pi = 0.10$ is worth around 0.5% of consumption.
Among much other work, we mention Dotsey and Ireland (1996) and Aiyagari et
al. (1998) as related to our approach. In search-and-bargaining models, Lagos et
al. (2017) survey work that gets costs closer to 5.0%. Our findings are smaller,
for reasons explained below. On inflation and price dispersion, empirical findings
are mixed: Parsley (1996) and Debelle and Lamont (1997) find a positive relation;
Reinsdorf (1994) finds a negative relation; Caglayana et al. (2008) find a U-shaped
relation. On markups and inflation, a standard reference is Benabou (1992b), who
reports a small but significant negative relationship. Benabou (1992a) and Head
and Kumar (2005) explain this by inflation increasing dispersion and thus search
effort. Here inflation decreases markups by directly affecting the cash-credit choice.

On money demand, we get exact solutions similar to Baumol (1952), Tobin
(1956), Miller and Orr (1966) and Whalen (1966). The economic intuition is
similar, involving a comparison between the opportunity cost of holding cash and
the cost of tapping financial services. But those papers are partial-equilibrium
analyses, or, more accurately, decision-theoretic analyses of how to manage one's

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[6]In discussions with people in the area, we found more or less agreement that these are the
facts: (1) Prices change slowly, but exact durations vary across studies. (2) The frequency and
size of changes vary across goods. (3) Two sellers changing at the same time do not typiclly pick
the same $\hat{p}$. (4) Many changes are negative. (5) Hazards decline slightly with duration. (6) There
are many small (beow 5%) and many big (above 20%) changes. (7) The frequency and size of
changes, and the fraction of negative changes, vary with inflation. (8) There is price dispersion
even at low inflation. Our model is consistent with all these.

money given that it is the only payment instrument. While such models are still being used to good effect (e.g., Alvarez and Lippi 2014), we like our setup because it is easy to integrate with standard macro, and allows us to investigate general equilibrium issues, like the emergence of inflation as a self-fulfilling prophecy.

On money and credit, one approach follows Lucas and Stokey (1987) by simply assuming some goods require cash and others allow credit. Papers that let individuals choose subject to a cost of credit include Prescott (1987), Freeman and Huffman (1991), Chatterjee and Corbae (1992), Lacker and Schreft (1996) and Freeman and Kydland (2000). See Nosal and Rocheteau (2011) for a general discussion; see Gomis-Porqueras and Sanches (2013), Li and Li (2013), and Lotz and Zhang (2016) for more recent work. There are various interpretations for these transaction costs, including resources used up in record keeping, screening, enforcement, etc. Other interpretations include saying that the cost of credit as a tax that can be avoided by using cash (e.g., Gomis-Porqueras et al. 2014), or that credit requires resources for monitoring (e.g., Wallace 2013; Araujo and Hu 2014). For the exercises in this paper, we prefer to remain agnostic on the exact nature of the costs involved in using credit.

Finally, the paper is related to an extensive nonmonetary literature on Burdett-Judd pricing, including much work in labor following Burdett and Mortensen (1998). Here, as in those models, if firms are homogeneous then theory does not pin down which one charges which p, only the distribution $F(p)$. With heterogeneity, lower-cost firms prefer lower p since they like high volume. Still, for any subset of sellers with the same marginal cost, theory does not pin down which one posts which p. This is relevant for retail, where the marginal cost is the wholesale price. Even if a few retailers get, say, quantity discounts, many others face the same wholesale price, making them homogeneous for our purposes. However, while this bears on our discussion of sticky nominal prices, it not especially important for the other applications.

3   Environment

As in Lagos and Wright (2005), each $t = 1, 2, ...$ has two subperiods: first there is a decentralized market, called BJ for Burdett-Judd; then there is a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms (retailers) with measure 1, and a set of households with measure $\bar{b}$. Agents consume a divisible good x_t and supply labor ℓ_t in AD, while in BJ they consume an indivisible good y_t produced by the firms at unit cost $\gamma \geq 0$. Agents in the BJ market can use credit iff they access at a cost a technology to authenticate identity and record transactions. By incurring this cost, they can get BJ goods in exchange for commitments to deliver d_t dollars in the next AD; otherwise they need cash at the point of sale. We consider both a fixed cost δ and a proportional cost τ. Thus, the transaction cost is $C(d_t) = \delta \mathbf{1}(d_t) + \tau d_t$, where $\mathbf{1}(d_t)$ is an indicator function that is 1 iff $d_t > 0$. The cost is paid by buyers, but not much changes if it is paid by sellers.[7]

Household utility within a period is $U(x_t) + \mu \mathbf{1}(y_t) - \ell_t$, where $U'(x_t) > 0 > U''(x_t)$, $\mu > \gamma + \delta$, and $\mathbf{1}(y_t)$ is an indicator function. Let $\beta = 1/(1 + r)$, $r > 0$, be a discount factor between AD today and BJ tomorrow; any discounting between BJ and AD can be subsumed in the notation with no loss of generality. Let x_t be AD numeraire, and assume it is produced one-for-one with ℓ_t, so the real wage is 1. All agents enter the BJ market for free (later we introduce a cost). Each firm in BJ maximizes profit by posting a price, taking as given the CDF of other prices $F_t(p)$, with support $\mathcal{F}_t$. Every period a household in BJ randomly samples n firms – i.e., sees n independent draws from $F_t(p)$ – with probability α_n. As a benchmark we assume $\alpha_0, \alpha_1, \alpha_2 > 0$ and $\alpha_n = 0 \; \forall n \geq 3$, but this is generalized in Section 4.3.[8]

The money supply per buyer evolves according to $M_{t+1} = (1 + \pi) M_t$, with

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[7]This is similar to the elementary tax-incidence theory, with one interesting caveat: when the cost of credit is paid by sellers, they may want to post different prices for cash and credit.

[8]This paper restricts attention to the case that a_n is exogenously given, although it is known in other applications how to make it endogenous. In Wang (2016), e.g., agents choose α_n, but that model has some important differences – e.g., it has divisible goods and no credit.

changes implemented in AD via lump-sum taxes if $\pi > 0$ or transfers if $\pi < 0$ (although the results are the same if instead the government uses seigniorage to buy AD goods). The AD price of money in numeraire is ϕ_t. In stationary equilibrium, π is the inflation rate, and the nominal interest rate is given by the Fisher equation $1 + i = (1 + \pi)(1 + r)$. As is standard, the money growth rate satisfies $\pi > \beta - 1$, and in stationary equilibrium the Friedman rule corresponds to $\pi \to \beta - 1$. One can of course introduce bonds explicitly, but there is no particular need: simply interpret $1 + i$ as the dollars agents would require in the next AD market to give up a dollar in this AD market, and whether or not such trades occur in equilibrium, they can be priced (as is completely standard in related applications).

3.1 Firm Problem

Let b_t denote the measure of participating households – i.e., tightness – in the BJ market. For now, $b_t = \bar{b}$ because entry is free; later we can have $b_t < \bar{b}$. Assuming $\alpha_0, \alpha_1, \alpha_2 > 0$ and $\alpha_n = 0 \ \forall n \geq 3$, for now, profit for a firm posting p is

$$\Pi_t(p) = b_t \left[\alpha_1 + 2\alpha_2 \hat{F}_t(p_t) \right] (p\phi_t - \gamma), \tag{1}$$

where $\hat{F}_t(p) \equiv 1 - F_t(p)$. Thus, net revenue per unit is $p\phi_t - \gamma$, and the number of units is determined as follows: The probability a buyer contacts this firm and no other is α_1. Then the firm makes a sale for sure. The probability a buyer contacts this firm and another is $2\alpha_2$, as it can happen in two ways – this one first and the other second or vice versa. Then the firm makes a sale iff it beats the other's price, which has probability $\hat{F}_t(p)$. This is all multiplied by b_t to convert buyer probabilities into seller probabilities.

Equilibrium has equal profit $\forall p \in \mathcal{F}_t$. Given this, it is standard to show $F_t(p)$ is continuous and $\mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$ is an interval.[9] Also, $\forall p \in \mathcal{F}_t$ profit from p must

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[9] There cannot be a mass of firms with the same p because any one of them would have a profitable deviation to $p - \varepsilon$, since they lose only ε per unit and make discretely more sales by undercutting others at p. Also, if there were a gap between p_1 and $p_2 > p_1$, a firm posting p_1 can deviate to $p_1 + \varepsilon$ and earn more per unit without losing sales.

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equal profit from $\bar{p}_t$, which is

$$\Pi_t(\bar{p}_t) = b_t \alpha_1 \left(\bar{p}_t \phi_t - \gamma \right), \tag{2}$$

since the highest price firm never beats the competition. Equating (1) to (2) and rearranging, we get a result similar to many other Burdett-Judd applications:

Lemma 1 $\forall p \in \mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$,

$$F_t(p) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\phi_t \bar{p}_t - \phi_t p}{\phi_t p - \gamma}. \tag{3}$$

From this and $F(\underline{p}_t) = 0$ we get

$$\underline{p}_t = \frac{\alpha_1 \phi_t \bar{p}_t + 2\alpha_2 \gamma}{\phi_t \left(\alpha_1 + 2\alpha_2 \right)}. \tag{4}$$

Also, translating from dollars to numeraire, we let $q_t = \phi_t p_t$ and write the real price distribution as

$$G_t(q) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\bar{q}_t - q}{q - \gamma}, \tag{5}$$

with support by $\mathcal{G}_t = [\underline{q}_t, \bar{q}_t]$, and $\hat{G}_t(q_t) \equiv 1 - G_t(q_t)$.

3.2 Household Problem

Consider stationary equilibrium, where real variables are constant, so nominal variables grow at rate π. In real terms, the household's state variable in AD is $A = \phi m - d - C(d) + I$, where ϕm and d are real money balances and real debt from the previous BJ market, $C(d)$ is transaction cost, and I is other income, including net transfers plus profits, assuming firms are owned by the households. All debt is settled in AD, so households start BJ with a clean slate (they could roll over d from one AD market to the next at interest rate r, but since preferences are linear in ℓ there is no point). Hence, the state variable in BJ is real balances, z.

Letting the AD and BJ value functions be $W(A)$ and $V(z)$, we have

$$W(A) = \max_{x, \ell, z} \left\{ U(x) - \ell + \beta V(z) \right\} \text{ st } x = A + \ell - (1 + \pi) z. \tag{6}$$

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Notice the cost of having z next period is $(1 + \pi) z$ in terms of numeraire this period. Eliminating ℓ and letting x^* solve $U'(x^*) = 1$, we have

$$W(A) = A + U(x^*) - x^* + \beta \max_z O_i(z), \qquad (7)$$

where the objective function for the choice of z is $O_i(z) \equiv V(z) - (1 + i) z$, with i given by the Fisher equation. As is standard in similar models, we have (see Appendix A for all non-obvious proofs):

Lemma 2 $W'(A) = 1$ *and the choice of z does not depend on A.*

The BJ value function satisfies

$$V(z) = W(A) + (\alpha_1 + \alpha_2) \left[\mu - \mathbb{E}_H q - \delta \hat{H}(z) - \tau \mathbb{E}_H \max(0, q - z) \right]. \qquad (8)$$

In (8), $\hat{H}(q) \equiv 1 - H(q)$ and $H(q)$ is the CDF of transaction prices,

$$H(q) = \frac{\alpha_1 G(q) + \alpha_2 \left[1 - \hat{G}(q)^2 \right]}{\alpha_1 + \alpha_2}. \qquad (9)$$

Notice $H(q)$ differs from $G(q)$, because a buyer seeing multiple draws of q obviously picks the lowest. Also, notice the costs δ and $\tau(q - z)$ are paid iff $q > z$. Thus, the benefit of higher z is that it reduces the expected cost using credit.

4   Analytic Results

The above discussion characterizes behavior given $\bar{q}$, which will be determined presently. First, we have these definitions:

Definition 1 *A stationary equilibrium is a list $\langle G(q), z \rangle$ such that: given $G(q)$, z solves the household's problem; and given z, $G(q)$ solves the firm's problem with $\bar{q}$ determined as in Lemma 3 below.*

Definition 2 *A nonmonetary equilibrium, or NME, has $z = 0$. A pure monetary*

equilibrium, or PME, has $z \geq \bar{q}$. A mixed monetary equilibrium, or MME, has $0 < z < \bar{q}$, so BJ trade uses cash for $q \leq z$ and credit for $q > z$.

Notice the definitions focus on $G(q)$ and z, but other variables, like x and ℓ, are implicit, as they are not needed in what follows.

The next step is to describe $\bar{q}$. To that end, we have the following result (again see Appendix A):

Lemma 3 *In NME, $z = 0$ and $\bar{q} = (\mu - \delta)/(1 + \tau)$. In MME, $z \in (0, \mu - \delta)$ and $\bar{q} = (\mu - \delta + \tau z)/(1 + \tau)$. In PME, $\bar{q} = z \geq \mu - \delta$.*

Lemma 4 *In MME, $O_i(z)$ is continuous, smooth and strictly concave $\forall z \in (\underline{q}, \bar{q})$, and linear $\forall z \notin (\underline{q}, \bar{q})$.*

Given this, in what follows we study in turn a fixed cost, $\delta > 0$ and $\tau = 0$, then a variable cost, $\tau > 0$ and $\delta = 0$.

4.1 Fixed Cost

Given $\delta > 0$ and $\tau = 0$ and Lemma 4, Figure 1 shows the objective function. Let $\hat{z}_i$ be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. If $O_i^+(\underline{q}) \leq 0$ then $\hat{z}_i = 0$, as in the left panel of Figure 1. If $O_i^+(\underline{q}) > 0$ then we need to check $O_i^-(\bar{q})$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i \in (\underline{q}, \bar{q})$, as in the right panel.

This immediately leads to the following results:

Proposition 1 *In the fixed-cost model with $\alpha_n = 0 \; \forall n \geq 3$: (i) $\exists!$ NME; (ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{i})$; (iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{i}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$; and the thresholds satisfy $\bar{i} \in (\underline{i}, \infty)$,*

$$\underline{i} = \frac{\delta \alpha_1^2}{2\alpha_2 (\mu - \delta - \gamma)} \quad \text{and} \quad \bar{\delta} = \mu - \frac{\gamma (2\alpha_2^2 + 2\alpha_1 \alpha_2)}{2\alpha_2^2 + 2\alpha_1 \alpha_2 - \alpha_1^2}.$$
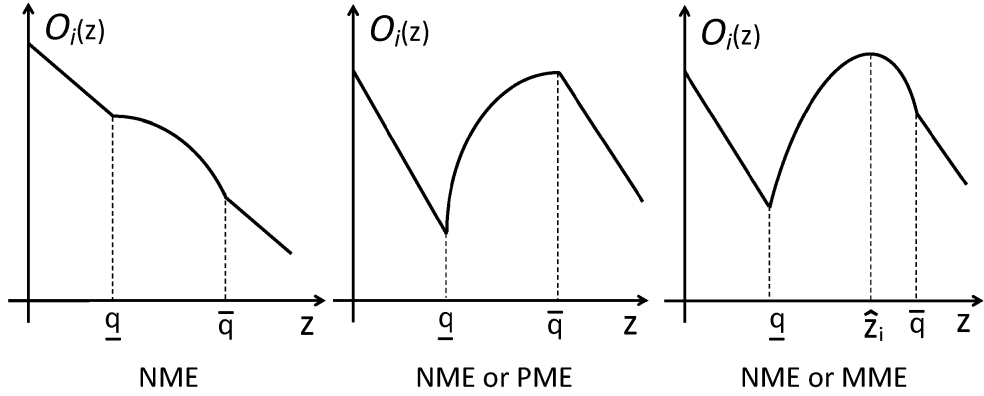
Figure 1: Possible Equilibria with a Fixed Cost

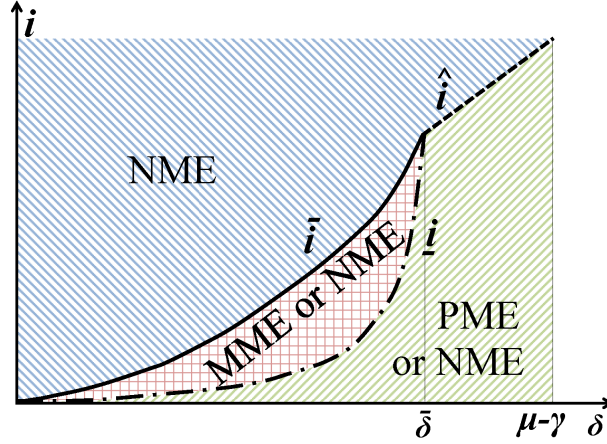Figure 2: Equilibria with a Fixed Cost

As Figure 2 shows, for money (credit) to be used the nominal rate i (transaction cost δ) cannot be too high. Also, note there is a continuum of PME when PME exist, for reasons discussed in fn. 4. A benefit of costly credit is that we get uniqueness of MME, which is our main object of interest. When MME exists, we can insert $G(q)$ into the FOC of (7) and rearrange to get an explicit solution for money demand, i.e., real balances as a function of i,

$$\hat{z}_i = \gamma + \left[\alpha_1^2 \delta \left(\mu - \delta - \gamma \right)^2 / 2\alpha_2 \right]^{1/3} i^{-1/3}. \tag{10}$$

This is reminiscent of famous results by Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966). In those papers, agents sequentially incur

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expenses requiring currency, with a fixed cost of rebalancing z. Their decision rule compares i, the opportunity cost of holding z, with the benefit of reducing the number of financial transactions usually interpreted as trips to the bank. Our agents make at most one transaction before rebalancing z, but its size is random, and they still compare the cost i with the benefit of reducing the use of financial services again loosely interpretable as trips to the bank, although one might say heuristically that our agents go to the bank to get a loan, rather than make a withdrawal. In any case, like the above-mentioned papers, we do not model banking explicitly, but one could, as in, say, Berentsen et al. (2007).

4.2 Variable Cost

It turns out $\tau > 0 = \delta$ is actually easier, and, as shown below, fits the facts better.[10] The price distribution is similar to Section 4.1, and in particular

$$\bar{q} = \frac{\mu + z\tau}{1 + \tau} \text{ and } \underline{q} = \frac{\alpha_1 (\mu + z\tau) + 2\alpha_2 \gamma (1 + \tau)}{(\alpha_1 + 2\alpha_2)(1 + \tau)}.$$

One can check $O_i(z)$ is differentiable everywhere, including $q = \bar{q}$ and $q = \underline{q}$. Hence, as Figure 3 shows, there are only two possible outcomes: if $i > (\alpha_1 + \alpha_2)\tau$ then $\exists!$ NME; and if $i < (\alpha_1 + \alpha_2)\tau$ then $\exists!$ NME and $\exists!$ MME. Thus, PME do not exist, because buyers are always willing to use credit with some probability. This is helpful quantitatively, as it is easier to get a MME for reasonable parameters than in the fixed-cost model (in that version, when δ is moderately high agents abandon credit entirely).

Proposition 2 *In the variable-cost model with $\alpha_n = 0$ $\forall n \geq 3$: (i) $\exists!$ NME iff $\tau \leq \mu/\gamma - 1$; (ii) $\exists!$ MME iff $i < \min\{\tau(\alpha_1 + \alpha_2), i^*\}$; (iii) $\nexists$ PME for $i > 0$; where $i^* = i^*(\tau)$ is the nominal rate that drives buyers' payoff to 0.*

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[10]In particular, it avoids a technical issue with fixed costs that we waited until now to raise: As in any model with non-convexities, agents may want to trade using lotteries. See Berentsen et al. (2002) for details, but the idea is for a seller to post "you get a good for sure if you pay p; if you pay $\tilde{p} < p$ you get a good with probability $P = P(\tilde{p})$." We prefer to avoid this complication, but still cover fixed costs mainly to make contact with the papers discussed in Section 2.
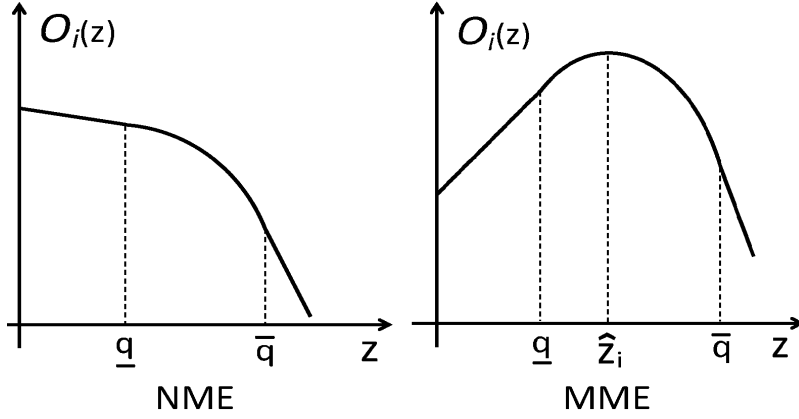
Figure 3: Possible Equilibria with a Variable Cost
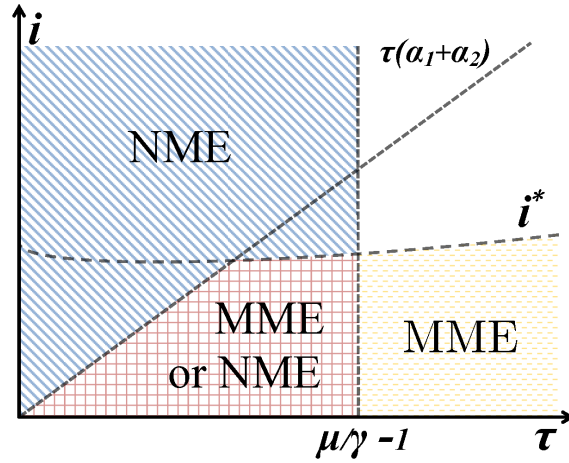
Figure 4: Equilibria with a Variable Cost

As Figure 4 illustrates, MME exists for any value of $\tau > 0$ as long as i is not too big. Also, from (7) we again get a closed-form money demand function,

$$\hat{z}_i = \gamma + \frac{(\mu - \gamma)\left[\tau + (1 + \tau)\sqrt{1 + 4\alpha_2 i/\alpha_1^2\tau}\right]}{1 + 2\tau + 4\alpha_2\left(1 + \tau\right)^2 i/\alpha_1^2\tau}, \tag{11}$$

similar if not the same as the fixed-cost version, with the same interpretation.

14

4.3 Generalized Sampling Distributions

To illustrate the tractability and flexibility of the approach, here we consider alternative specifications for the probability a household randomly samples n prices.[11] To begin, let $N \leq \infty$ be the maximum number of prices that can be sampled. For a firm posting p, profit is

$$\Pi_t(p) = b_t(p\phi_t - \gamma) \sum_{n=1}^{N} \alpha_n n \hat{F}_t(p_t)^{n-1}, \tag{12}$$

while for one posting $\bar{p}_t$, it is again given by (2). From the equal profit condition we get $G_t(q)$ and $H_t(q)$. For households, in the fixed- and variable-cost models, the FOC's required for MME are respectively

$$\sum_{n=1}^{N} \alpha_n \delta H'(z_i) = i \text{ and } \sum_{n=1}^{N} \alpha_n \tau [1 - H(z_i)] = i. \tag{13}$$

Previously in the paper we discuss the case of $N = 2$. Here we show that we can get closed-form solutions with other specifications. Related to Mortensen (2005), consider a Poisson distribution, $\alpha_n = e^{-\eta}\eta^n/n!$, where $\eta = \mathbb{E}n$. We get

$$F_t(p) = 1 - \frac{1}{\eta}[\log(\phi_t\bar{p}_t - \gamma) - \log(\phi_t p - \gamma)].$$

and analogs to Propositions 1 and 2:

Proposition 3 *In the fixed-cost model with a Poisson distribution for n: (i) $\exists!$ NME; (ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{i})$; (iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{i}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$; and the thresholds satisfy $\bar{i} \in (\underline{i}, \infty)$,*

$$\underline{i} = \frac{e^{-\eta}\delta}{\mu - \delta - \gamma} \text{ and } \bar{\delta} = \mu - \frac{(1 - e^{-\eta})\gamma}{1 - 2e^{-\eta}}.$$

Proposition 4 *In the variable-cost model with a Poisson distribution for n: (i)* $\exists!$ *NME iff* $\tau \leq \mu/\gamma - 1$; *(ii)* $\exists!$ *MME iff* $i < \min\{\tau(1 - e^{-\eta}), i^*\}$; *(iii)* $\nexists$ *PME for* $i > 0$; *and* $i^* = i^*(\tau)$ *is the nominal rate that drives buyers' payoff to* 0.

As another example, consider a Logarithmic distribution, $\alpha_n = -\omega^n/n\log(1 - \omega)$, where $\omega \in (0, 1)$. It is easy to derive

$$F_t(p) = 1 - \frac{\phi_t(\bar{p}_t - p)}{\omega(\phi_t\bar{p}_t - \gamma)}.$$

Notice $F_t(p)$ is linear, so p is uniformly distributed. Here are analogs to Propositions 1 and 2:

Proposition 5 *In the fixed-cost model with a Logarithmic distribution for n: (i)* $\exists!$ *NME; (ii)* $\exists!$ *MME iff* $\delta < \bar{\delta}$ *and* $i \in (\underline{i}, \bar{\imath})$; *(iii)* $\exists$ *PME iff either* $\bar{\delta} < \delta < \mu - \gamma$ *and* $i < \hat{\imath}$, *or* $\delta < \bar{\delta}$ *and* $i < \underline{i}$; *and the thresholds satisfy* $\bar{\imath} \in (\underline{i}, \infty)$,

$$\underline{i} = -\frac{\delta}{\mu - \delta - \gamma\log(1 - \omega)} \quad and \quad \bar{\delta} = \mu - \frac{\gamma\log(1 - \omega)}{1 + \log(1 - \omega)}.$$

Proposition 6 *In the variable-cost model with a Logarithmic distribution for n: (i)* $\exists!$ *NME iff* $\tau \leq \mu/\gamma - 1$; *(ii)* $\exists!$ *MME iff* $i < \min\{\tau, i^*\}$; *(iii)* $\nexists$ *PME for* $i > 0$; *and* $i^* = i^*(\tau)$ *is the nominal rate that drives buyers' payoff to* 0.

As in the baseline model, the Poisson case with a fixed or variable cost delivers nice money demand functions,

$$\hat{z}_i = \gamma + \left[e^{-\eta}\delta(\mu - \delta - \gamma)\right]^{\frac{1}{2}} i^{-\frac{1}{2}} \text{ or } \hat{z}_i = \gamma + \frac{(\mu - \gamma)\tau e^{-\eta}}{(1 + \tau)i + \tau e^{-\eta}}. \tag{14}$$

And for the Logarithmic case,

$$\hat{z}_i = \gamma - \frac{\delta}{i\log(1 - \omega)} \text{ and } \hat{z}_i = \gamma + \frac{(\mu - \gamma)(1 - \omega)^{i/\tau}}{1 + \tau - \tau(1 - \omega)^{i/\tau}}. \tag{15}$$

All these specifications entail closed-form solutions for the price distribution and money demand, with similar intuitive interpretations. However, for simplicity, the applications below use $N = 2$.
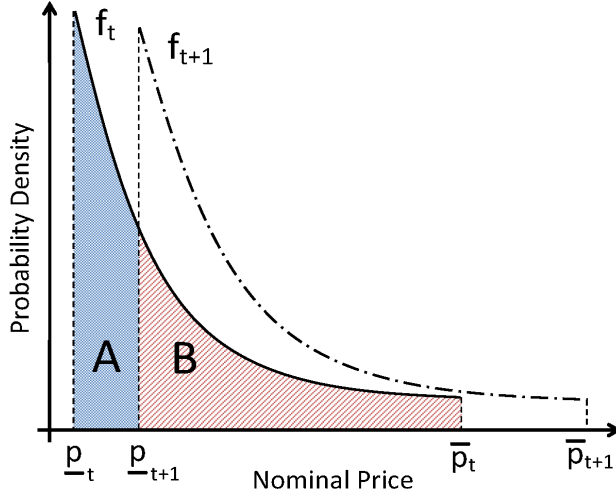
Figure 5: Nominal Price Densities and Inflation

4.4 Repricing Behavior

While this is not the only paper to make the point, and it is not the only point we make, here we sketch the search-based explanation of sticky prices. In the above analysis, $F_t(p)$ is uniquely determined, but an individual firm's p is not. Consider Figure 5, drawn using the calibrated parameters in Section 5.2. With $\pi > 0$, the density F'_{t+1} lies to the right of F'_t. Firms with $p < \underline{p}_{t+1}$ at t (Region A) must reprice at $t+1$, because while p maximized profit at t it no longer does so at $t+1$. But as long as the supports $\mathcal{F}_t$ and at $\mathcal{F}_{t+1}$ overlap, there are firms with $p > \underline{p}_{t+1}$ at t (Region B) that can keep the same p at $t+1$ without reducing profit. They could change, but have no strict incentive to do so.

Consider the repricing strategy in Head et al. (2012): If $p_t \notin \mathcal{F}_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where $\hat{p}$ is a new price; and if $p_t \in \mathcal{F}_{t+1}$ then:

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob. } \sigma \\ \hat{p} & \text{with prob. } 1 - \sigma \end{cases} \tag{16}$$

This defines a *payoff-irrelevant tie-breaking rule*. Very different from Calvo pricing, where firms desperate to change p are simply not allowed, our rule only applies to firms that are indifferent. Also, the calibration below delivers $\sigma = 0.90$, so only

10% of indifferent firms actually change. In any case, for any σ there is a unique symmetric equilibrium where all sellers that change draw $\hat{p}$ from the same repricing distribution, given by:

$$R_{t+1}(p) = \begin{cases} \dfrac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[F_t(p) - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\underline{p}_{t+1}, \bar{p}_t) \\ \dfrac{F_t\left(\frac{p}{1+\pi}\right) - \sigma[1 - F_t(\underline{p}_{t+1})]}{1 - \sigma + \sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\bar{p}_t, \bar{p}_{t+1}] \end{cases} \tag{17}$$

From (17) we can compute repricing statistics from the model and compare them to the facts deemed interesting in the literature. While different values of σ generate different behavior, it is not the case that anything goes – e.g., at high inflation most firms adjust each period – and once we pin down σ there are precise predictions. Hence, while the theory does not impose tight restrictions on individual's behavior, it is still interesting to ask how well it accounts for average repricing behavior. In particular, to the extent that the model is consistent with the facts, it provides a voice of caution about using data to make inferences about Mankiw-style menu costs or Calvo-style arrival rates.

Moreover, while this version of the theory does not impose tight restrictions on individual repricing, only on average repricing, the work of Burdett and Menzio (2018) provides a way around that. In addition to many of our ingredients, they have a menu cost to price changes, which makes the model much harder to solve – and so they work with frictionless payments (transferable utility) rather than money and credit – but they are able to show the following: As the menu costs get small, equilibrium corresponds basically to the outcome we analyze, except firms only change prices when the incentive to do so is strict. That is almost fine for us, although for the quantitative work we prefer to have some firms change when they are indifferent: as mentioned above, and discussed more below, matching the data is facilitated by having around 10% of indifferent firms change. But how can we have any sellers bear a cost to post new prices when they are indifferent

to any $p \in \mathcal{F}$? One answer is to say that they are selling new goods, and hence have no choice but to post a new price. This lets us appeal to the Burdett-Menzio refinement but still have some firms posting new prices, because they are selling new goods, even though they are indifferent to any $p \in \mathcal{F}$.

5 Quantitative Results

In addition to confronting the price-change data, we want to fit the money-credit shares in the payment data and the standard empirical notion of money demand. As in Lucas (2000), that notion is $L_i = \hat{z}_i/Y$, where $Y = x^* + (\alpha_1 + \alpha_2)\mathbb{E}_H q$ is output aggregated over AD and BJ. We use $U(x) = \log(x)$, so $x^* = 1$ (a normalization). The formula for L_i is given in Appendix B, and we target its mean and elasticity in the data. Other key statistics are the average BJ markup $\mathbb{E}_G q/\gamma$, and the aggregate markup across both AD and BJ. These are natural targets because, as is well known in the literature, Burdett-Judd pricing can deliver anything from monopoly to competitive pricing as α_1/α_2 varies, so the markup contains information about this ratio, and then the aggregate markup contains information about the size of AD and BJ.

5.1 Data

We focus on 1988-2004 because our price-change observations are from that period, although information from other periods can be used in calibration. For money, the best available data is the M1J series in Lucas and Nicolini (2015) that adjusts M1 for money-market deposit accounts, similar to the way M1S adjusts for sweeps as discussed in Cynamon et al. (2006). Lucas-Nicolini have an annual series from 1915-2008 and a quarterly series from 1984-2013, and make the case that there is a stable relationship between these and nominal interest rates. We use their quarterly series to correspond better to the price-change sample.[12]

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[12]In these data the average annualized nominal rate is $\mathbb{E}i = 0.041$, which implies $L_{\mathbb{E}i} = 0.277$ and $\eta_{\mathbb{E}i} = -0.116$. The longer annual sample has $\mathbb{E}i = 0.038$, $L_{\mathbb{E}i} = 0.279$, and $\eta_{\mathbb{E}i} = -0.149$;

Markup information comes from the U.S. Census Bureau Annual Retail Trade Report 1992-2008. At the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target for the gross margin is 1.3, which implies a markup of 1.39, consistent with the data analyzed by Stroebel and Vavra (2019). Then we choose a target for the aggregate markup of 1.1, as is typical in macro applications (e.g., Basu and Fernald 1997). Since the BJ markup is 1.39 and the AD markup is 1.0, the BJ market accounts for about 25% of output in the model.

On the shares of money and credit there are various sources. In terms of concept, we interpret monetary transactions to include cash, check and debit card purchases. Here is the rationale: (1) Checks and debit cards use demand deposits that are about as liquid as currency and pay basically the same interest (zero). (2) For our purposes, the interesting feature of credit is that it allows you to pay for BJ goods by working in the *next* AD market, while cash, check and debit purchases all require working in the *previous* AD market, which matters a lot especially because BJ transactions are random. (3) This notion of money in the micro data is consistent with what we use in the macro data. Hence, monetary exchange includes cash, check and debit but not credit cards. Earlier calibrations of monetary models (see Cooley 1995) target 16% for credit purchases, but more information is now available. In grocery-store data, Klee (2008) finds credit cards account for 12% of purchases, but we do not want to focus on just groceries. In Atlanta Fed data discussed by Foster et al. (2019), credit cards account for 23% of purchases. In Bank of Canada data discussed by Arango and Welte (2012), the number is 19%. We target to 20%.[13]

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using these gives similar results. We also tried truncating the data in 2004, to better match the pricing sample, and to avoid the financial crisis; that too gave similar results.

[13] The share of credit in the data is stable over the relevant period, where the bigger changes have been into debit and out of checks, and to some extent, out of currency (Jiang and Shao 2019). Also, we use the credit share by volume, which is better for our purposes than the share by value. As an aside, while the Canadian and American numbers basically agree on shares

For price-change data we mainly use Klenow and Kryvtsov (2008), and benchmark their average duration of 8.6, but alternatives are also considered since there are differences across studies, depending on details. Their average absolute price change is 11.3%, well above average inflation, because there are many negative changes. Since the Klenow-Kryvtsov data are monthly, the model period is a month. Our model-generated money demand data is aggregated to quarterly to line up with Lucas-Nicolini. A month also seems natural to us, as it corresponds to the usual credit card billing period. However, the period does not really matter much for our purposes – a very convenient feature of this class of models.[14]

5.2   Basic Findings

Calibration results are shown in Table 1. Consider first the fixed-cost model, which hits all targets *except* the fraction of credit transactions, because our parameter search is constrained to stay within the region where MME exists. Trying to get 20% BJ credit transactions forces δ into a region where MME does not exist for some values of i in the sample. Hence, with a fixed cost we use the smallest δ consistent with MME at the maximum observed $i = 0.103$, which yields only 11.9% credit transactions. This δ is about 4.7% of the utility parameter μ, which comes primarily from matching average real balances. The value of γ, about one-third of μ, comes primarily from the BJ markup. The probability of sampling one price (two prices) is $\alpha_1 = 0.013$ ($\alpha_2 = 0.081$).

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by volume, they are quite different by value. In discussions with those who collected the data, they could not explain the differences across the data sets in share by value, but this is less of a concern here, given that we calibrate to share by volume.

[14]In case it is not obvious, to see the intuition consider a simple job search model. Let V_0 and $V_1(w)$ be the value of unemployment and employment at wage w, α the arrival rate of jobs, and κ a search cost. Then $rV_0 = \alpha[V_1(w) - V_0] - \kappa$. To change from, e.g., a weekly to a monthly model, we can simply multiply r, α, κ and w by 4 (to a first approximation) without changing payoffs or observables like unemployment or the hourly wage – the only caveat is the constraint $\alpha \leq 1$. Something similar is true here. In particular, for shorter periods, agents get to rebalance z more often, but the lower arrival rates associated with shorter periods imply they hold cash for just as long, on average, before spending it, so the impact of the inflation tax is the same.

	BJ utility	BJ cost	credit cost	$pr(n=1)$	$pr(n=2)$	tie breaker
	μ	γ	δ or τ	α_1	α_2	σ
Fix	8.62	2.91	0.404	0.013	0.081	0.90
Var	5.93	3.14	0.202	0.034	0.048	0.90

Table 1: Baseline Calibration

Alternatively, the variable-cost model hits all targets, including 20% for BJ credit, making it clearly better for our purposes. Now the average transaction cost scaled by BJ utility is $\tau \mathbb{E}_H \max(0, q - z)/\mu = 0.0029$, less than average credit cards fees of around 1.5-2% without counting small fixed costs per transaction. The point is that we do not need big transaction costs. Also notice that α_1 (α_2) is higher (lower) than in the fixed-cost model. A constant across specifications is the tie-breaking parameter $\sigma = 0.90$, implying that indifferent sellers change prices only 10% of the time – or, if one prefers, 10% of sellers are selling new goods and have no choice but to pick a new price, as discussed in Section 4.4.[15]

Figure 6 shows money demand as the solid (dashed) curve with a fixed (variable) cost. The fit is good in both cases, although the curves are different at low values of i. This difference is important for some issues, but not for our applications. The broad conclusion is that a variable-cost model can match well macro money demand data plus the findings in the micro payment studies.

6 Applications

6.1 Sticky Prices

As discussed, the model can in principle generate the appearance of sticky prices. How well can it do quantitatively? Figure 7 shows the Klenow-Kryvtsov data plus the model predictions for the price-change distribution. Both the fixed- and variable-cost versions capture the overall shape of the empirical histogram, but

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[15]While one might consider this in future work, we do not attempt to match any empirical measures of new goods on the market, mainly because the interpretation of 'new' is somewhat loose – e.g., it could be a different brand, a different sized package, or the same good and package at a new location.
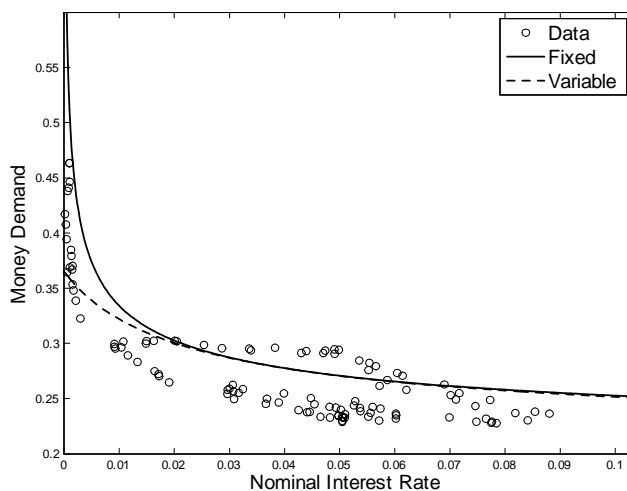
Figure 6: Money Demand for Different Specifications

the fit is not perfect. We now argue, however, that the theory is generally broadly consistent with the facts deemed important in the literature.
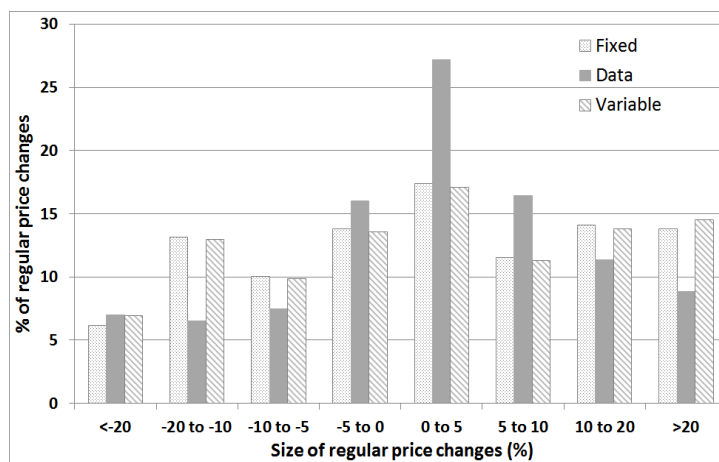
Figure 7: Distribution of Price Changes

The average absolute price change is 11.3% in the data, 11.1% in the model with a fixed cost of credit, and 12.3% with a variable cost, so both are very close to the data. The fraction of small changes (below 5% absolute value) is 44% in the data, 32% with a fixed cost, and 30% with a variable cost, which is off but not dramatically so.[16] The fraction of big changes (above 20% absolute value) is

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[16]This may not be so bad, given that Eichenbaum et al. (2014) find a lower fraction of small

16% in the data, 20% with a fixed cost, and 21% with a variable cost, while the fraction of negative changes is 37% in the data, and 43% in both models. So on these we are slightly off but not too bad. Given the literature says it is not easy to get large average, many small, many big, and many negative adjustments, this seems reasonably good, but not perfect. To be clear, we do *not* calibrate to match any price-change statistics, only to match money demand, payment methods, and markups, although we do set σ to match average duration.
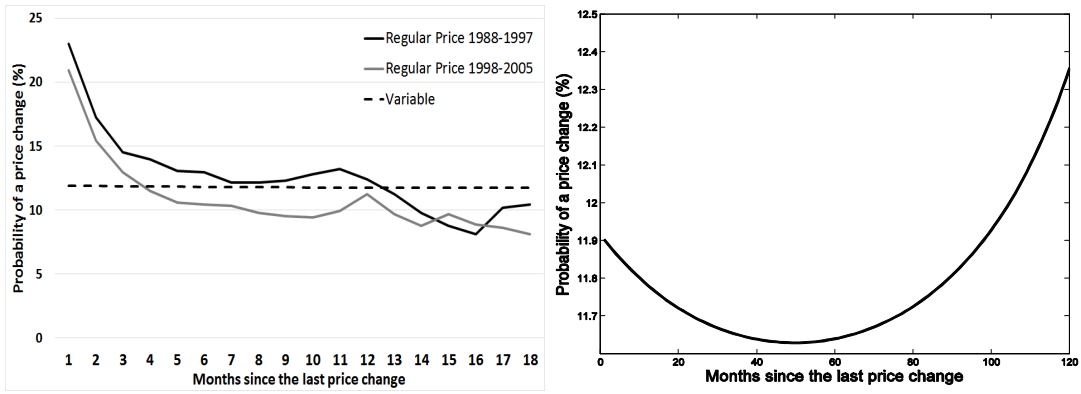

Figure 8: Price Change Hazards

Another observation to consider is the hazard rate, the probability of changing p as a function of the time since the last change. The left panel Figure 8 plots the data from Nakamura and Steinsson (2008) and the right panel shows the prediction from the variable-cost model. We do not generate enough action at low durations, but at least the hazard slopes downward, something they say is hard to get in theory. Now we do not expect to explain every nuance, as there is undoubtedly a lot missing in the model related to the hazard, including experimentation/learning (see, e.g., Bachmann and Moscarini 2012). Still, it is interesting that our hazard decreases for a while, before turning up eventually.

Figures 9 and 10 show the impact of changing duration and inflation in the variable-cost model. The left panel of Figure 9 is for $\sigma \simeq 0$ and an expected

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price changes after they correct for measurement errors.

duration of 1 month; the right is for $\sigma = 0.95$ and an expected duration of 16 months. The latter is particularly interesting because (as Burdett and Menzio 2018 show) the main properties of equilibrium survive the introduction of small menu costs that make sellers change less frequently. Moreover, given the variability of estimates of average duration (see Appendix C), it is worth considering robustness with respect to σ. The left panel of Figure 10 sets π to 0, and the right to 20%, not a robustness check but to see how repricing behavior depends on inflation. As π increases, the fraction of negative adjustments falls, while both the frequency and size increase. This is relevant because it is consistent with the evidence (Klenow and Kryvtsov 2008), and hard to explain with simple Calvo models.

a. Average Duration 1 Month, $\sigma = 1.3 \times e^{-10}$ b. Average Duration 16 Months, $\sigma = 0.95$
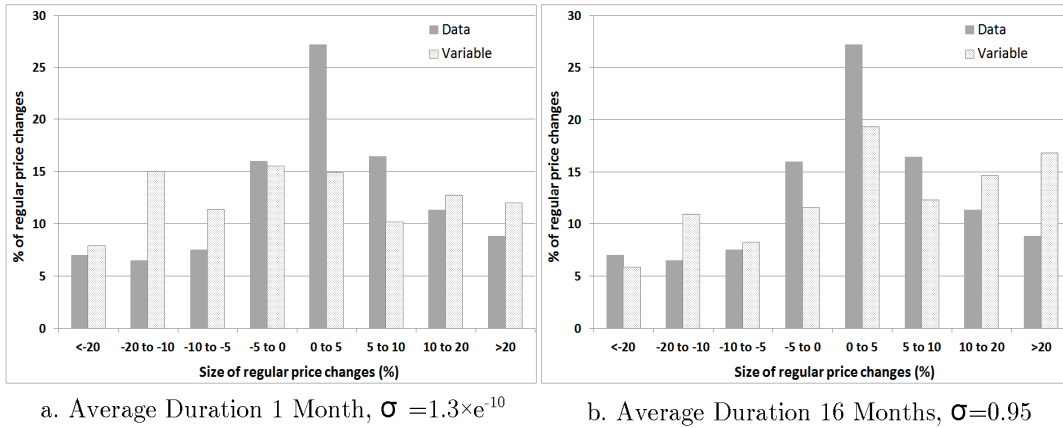
Figure 9: The Effect of Varying Duration

As discussed, previously, Head et al. (2012) also calibrate to quantitatively explain some patterns of price changes in the data. There are two major differences between their exercise and ours. First, the good y in their BJ market is divisible, and picking the curvature of utility over y gives them more flexibility in matching the data. Second, they only target micro price changes while we also use macro money data and micro credit shares to discipline the calibration approach. Figure 11 shows a calibration that gives up on matching money demand. The fit is very good – one might say close to perfect – and certainly at least as good if not better than the result shown in Figure 6 of Head et al. (2012).

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a. 0% Annual Inflation   b. 20% Annual Inflation
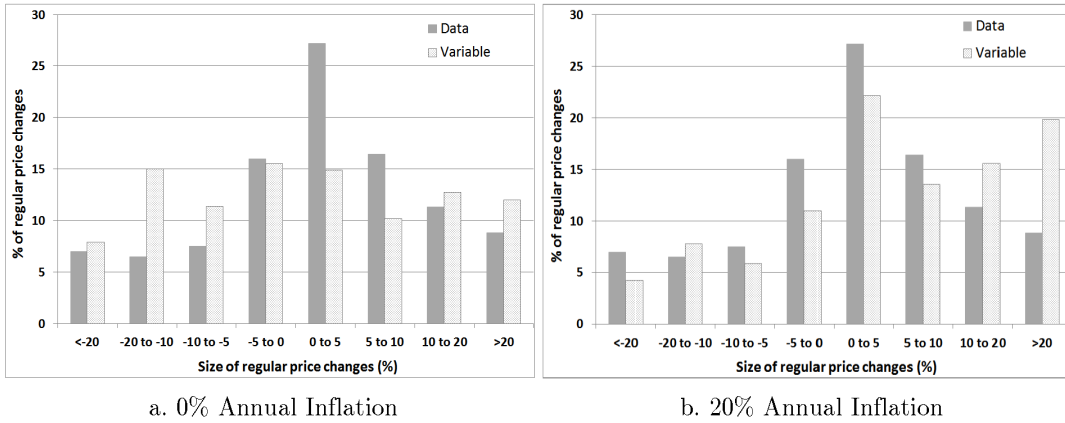
Figure 10: The Effect of Varying Inflation
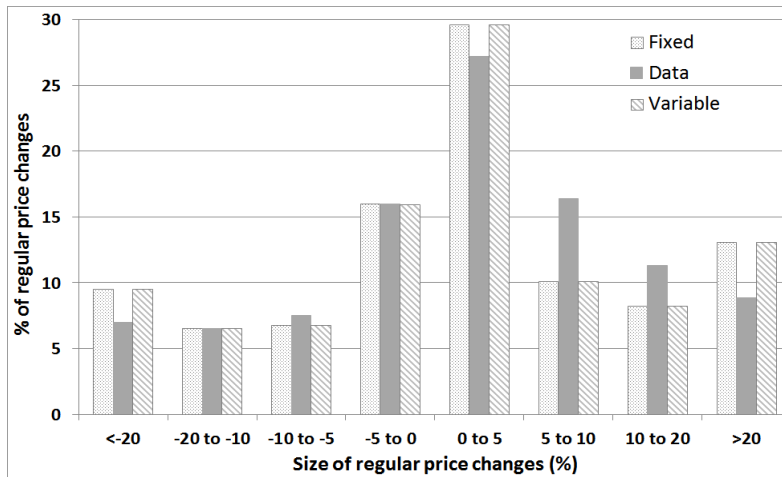
Figure 11: Distribution of Changes Ignoring Money Demand

To summarize, while the fit is not perfect, overall we conclude that there is nothing especially puzzling in the price-change data – this data is pretty much what search theory predicts. This is true even with the discipline imposed by macro and micro observations on money and credit. If we ignore those observations we can do even better, naturally, although it still may be surprising that we do as well as Figure 11 shows. Hence, it is easy to capture sluggish nominal prices quantitatively if we do not impose the discipline of matching other data. While obviously there are other explanations out there, this suggests to us that theories with search frictions should be part of the conversation.

6.2 Inflation in the Baseline Model

A classic issue concerns the welfare cost of inflation. Although this has been analyzed extensively elsewhere, and even though the above model is not the last word on the topic – because it leaves out various potentially important elements to show clearly how even very simple versions can match data on price change behavior – we still pursue the topic briefly to illustrate a few ideas. To begin, let us measure welfare by

$$\Omega \equiv Y - (\alpha_1 + \alpha_2) \left\{ \delta \left[1 - H_\pi (z_\pi) \right] + \tau \int_{z_\pi}^{\bar{q}} (q - z_\pi) \, dH_\pi \right\}, \qquad (18)$$

where $Y = U(x^*) - x^* + (\alpha_1 + \alpha_2)(\mu - \gamma)$ adds the AD and BJ surpluses, while the remaining terms subtract the cost of credit. As is standard, we compute the percent change in consumption that is equivalent to changing π from a given level to some alternative like $\pi = 0$.
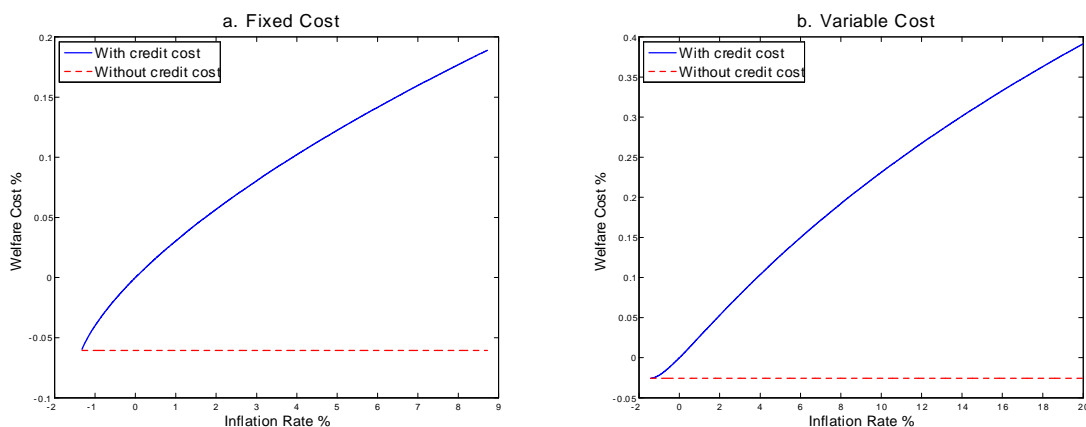
Figure 12: Welfare Effects of Inflation

Figure 12 shows the cost of inflation over the range for which MME exists, from $\pi = \beta - 1$ up to about 9% with a fixed cost (left) or 20% with a variable cost (right).[17] The welfare costs are small: with a variable cost, eliminating 10% annual

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[17]These thresholds are low, but this is not surprising in a representative-agent context. Suppose we introduce heterogeneity across buyers, with some having zero or only very costly access to credit – e.g., the unbanked, who have to deal with loan sharks, pawnshops or payday advances. They would presumably continue to use cash up to higher thresholds.

inflation is worth only 0.23% of consumption, less than estimates in Lucas (2000), and much less than Lagos and Wright (2005). Intuitively, changes in π affect neither the intensive margin of trade, since the good is indivisible, nor the extensive margin, since participation is fixed. Because α_1 and α_2 are exogenous, inflation does not affect the household's search behavior, either. In a similar setup without credit, Wang (2016) allows consumers to choose to sample one or two prices, and gets costs around 10.25% due to the effects on several channels, including search intensity – more on this in Section 6.3.
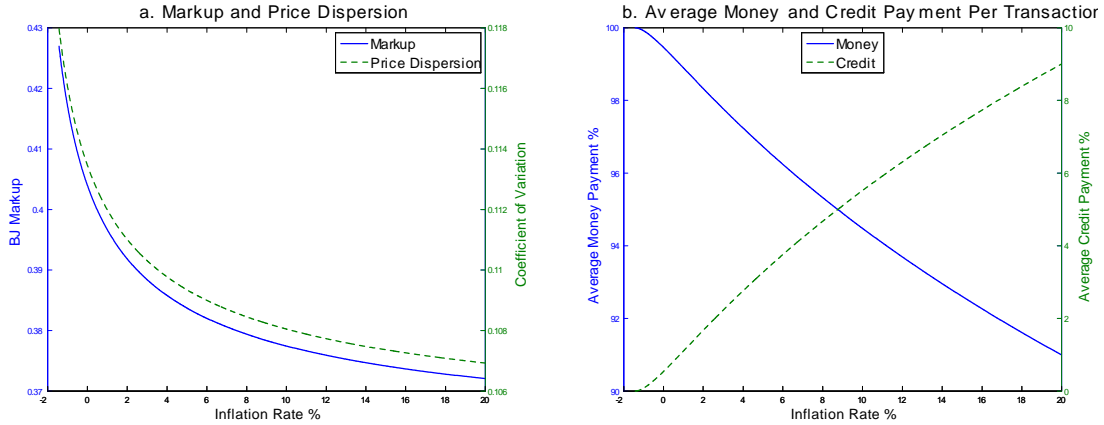
Figure 13: Other Effects of Inflation

Now consider the relationship between inflation, markups and price dispersion. With a fixed cost of credit π does not affect $G(q)$, markups or price dispersion. With a variable cost, one can show $G(q)$ decreases with π in the sense of first-order stochastic dominance. Hence, the average markup and dispersion (coefficient of variation) both decrease with π, as shown in Figure 13. As mentioned in the Introduction, empirical studies find a small but significant negative relationship between markups and π, consistent with the model, while the evidence on dispersion and π is mixed, so it is less clear what one should try to get, but at least this kind of model speaks to the issues.
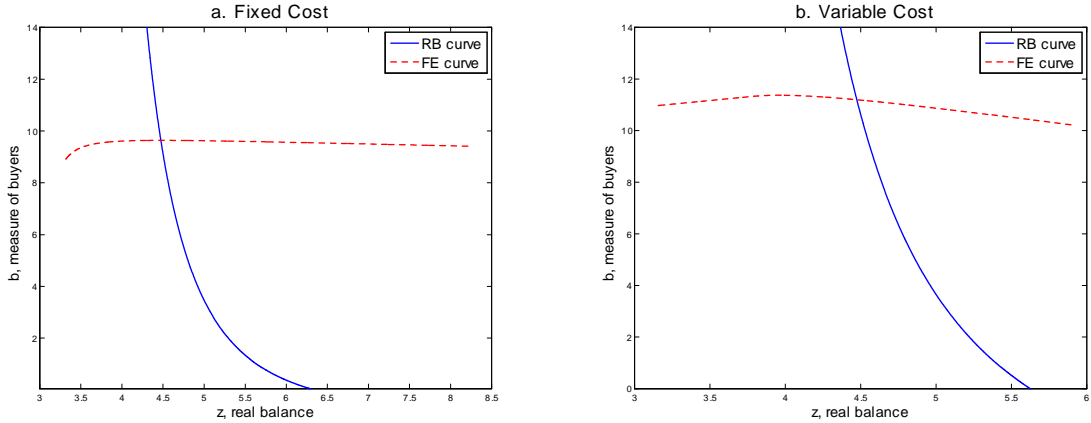
Figure 14: The Real Balance and Free Entry Curves

6.3 Endogenous Participation

We now let buyers choose whether to participate in the BJ market, at cost $k > 0$, to make output depend directly on inflation.[18] Let $W^1(A)$ and $W^0(A)$ be the AD value functions for households that enter and do not enter the next BJ market, respectively, so that $W(A) = \max\{W^1(A), W^0(A)\}$. In equilibrium where some but not all households enter, $W(A) = W^1(A) = W^0(A)$. This simplifies to $\beta\Psi = k$, where Ψ is the expected surplus from participation,

$$\Psi \equiv (\alpha_1 + \alpha_2)\left[\mu - \mathbb{E}_H q - \tau\mathbb{E}_H \max(0, q - z)\right] - i\hat{z}_i. \tag{19}$$

Buyers' arrival rates now depend on tightness, $\alpha_n = \alpha_n(b_t)$. With entry, b adjusts to satisfy (19). An increase in π reduces b, and hence output, although a one-time unanticipated increase in M does not (classical neutrality).

To parameterize the α's, suppose buyers attempt to solicit price quotes and succeed with probability $s = s(b)$, with $s(0) = 1$, $s(\bar{b}) = 0$, $s'(b) < 0$ and $s''(b) > 0$, as is standard. Then suppose that any buyer who succeeds sees one price with probability $1 - \xi$ and sees two with probability ξ. Hence, $\alpha_1(b) = (1 - \xi)s(b)$ and

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[18]Similar monetary models with endogenous entry by buyers include Liu et al. (2011), while those with entry by sellers include Rocheteau and Wright (2005); famous early search models with entry include Diamond (1982).

$\alpha_2(b) = \xi s(b)$. As a special case of the money demand functions derived above, $\hat{z}_i$ now depends on b, as shown in Figure 14 by the RB (real balance) curve. Similarly, $\beta\Psi = k$ is shown as the FE (free entry) curve. The curves intersect at MME. In Figure 14, RB is decreasing and convex while FE is concave, implying a unique MME, from which $F(p)$, $G(q)$ and the rest of the variables follow as usual. One can check that higher π shifts both curves toward the origin, reducing buyer entry and BJ output.
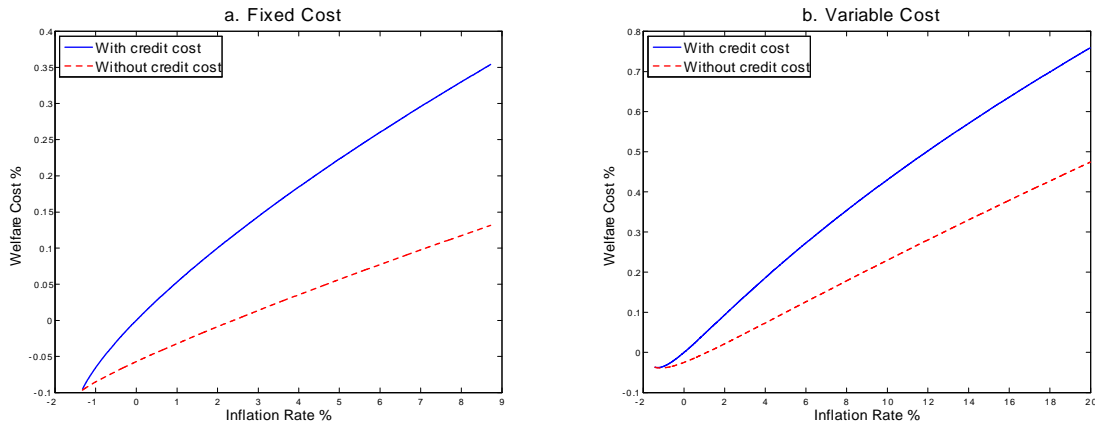

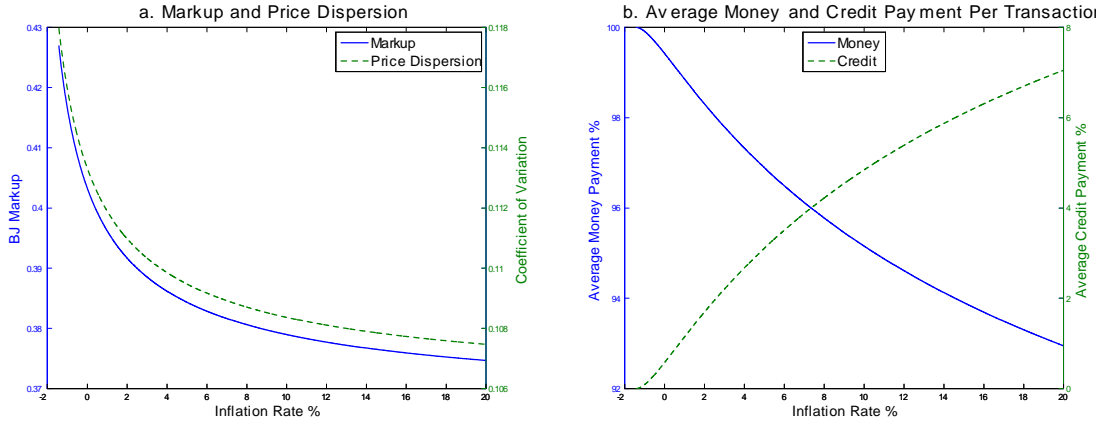Figure 15: Welfare Effects of Inflation with Entry

Figure 16: Other Effects of Inflation with Entry

While our theory is consistent with the appearance of sticky prices, the implications are different from many other models with sticky prices. In those models,

a one-time unanticipated jump in M has real effects. This is because not all firms adjust p, even though they would like to, and hence $F(p)$ does not change enough to keep the same $G(q)$. So prices turn in favor of buyers, making b and output increase. In contrast, in our model, a surprise jump in M affects neither $G(q)$ nor b. A policy advisor seeing only a fraction of sellers adjusting p each period may conclude that a jump in M can have real effects, which is wrong. Although not surprising, it is worth emphasizing that for policy it is not actually enough to say prices are sticky in the data; it is important to know why.

As Figure 15 shows, compared to the benchmark model the cost of inflation approximately doubles, because now an increase in π decreases b. Figure 16 demonstrates how π affects markups, price dispersion, and payment methods in the variable-cost model. Compared to Figure 13, endogenizing participation does not change the impact of inflation on markup or dispersion a lot. In particular, now fewer buyers enter the BJ market at higher π, but since that leads to higher arrival rates for those that enter, their reduction in real balances is attenuated.

6.4 Endogenous Dynamics

Many models of liquidity have nonstationary equilibria.[19] To pursue this, in a general way, assume m has a real flow return ρ: if $\rho > 0$ then m can be interpreted as equity in a 'tree' bearing 'fruit' as a dividend, as in standard finance; if $\rho < 0$ then it can be interpreted as a storage cost, as in some models of commodity money. The novelty here concerns dynamics in the price distribution $F(p)$, not just the price level. While this is also done in Burdett et al. (2017), that model is very special, as individuals are restricted to hold only $m \in \{0, 1\}$.

For simplicity, let us here revert to $k = 0$ so that $b = \bar{b}$, focus only on the model with a variable cost, and keep the asset supply M fixed, to ease notation. The

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[19]See Rocheteau and Wright (2013) and references therein for search models with liquid (fiat as well as real) assets. See Azariadis (1993) for an earlier literature. See Gu et al. (2013) for endogenous dynamics in pure-credit economies.

household's problem is then

$$W(A) = A + U(x^*) - x^* + \beta \max_z O_i(z),$$

where $A = \rho m + \phi m - d - C(d) + I$ includes dividend income ρm, and $O_r(z) = V(z) - (1+r)z$. The Euler equation is

$$\phi_t = \frac{\phi_{t+1} + \rho}{1+r}\left[1 + (\alpha_1 + \alpha_2)\tau\hat{H}(\hat{z})\right], \tag{20}$$

where now we do *not* impose stationarity of ϕ_t. If $\alpha_1 = \alpha_2 = 0$ then (20) is a standard asset-pricing equation, and there is a unique equilibrium with $\phi_t = \rho/r$ $\forall t$, since any other solution to (20) violates the transversality condition (see e.g. Rocheteau and Wright 2013 for a discussion in a related model).

If we do not impose $\alpha_1 = \alpha_2 = 0$, (20) is augmented on the RHS by a liquidity premium capturing the expected reduction in credit costs, $(\alpha_1 + \alpha_2)\tau\hat{H}(\hat{z})$. That dramatically changes the equilibrium set. After some algebra we get

$$\phi_t = \frac{\phi_{t+1} + \rho}{1+r}\left\{1 + \frac{\tau\alpha_1^2}{4\alpha_2}\frac{\left[\mu - (\rho + \phi_{t+1})\right]\left[\mu + (\rho + \phi_{t+1})(1+2\tau) - 2\gamma(1+\tau)\right]}{(1+\tau)^2(\rho + \phi_{t+1} - \gamma)^2}\right\}, \tag{21}$$

giving today's asset price in terms of tomorrow's, $\phi_t = \Phi(\phi_{t+1})$. The left panel of Figure 17 shows $\phi_t = \Phi(\phi_{t+1})$ and the inverse $\phi_{t+1} = \Phi^{-1}(\phi_t)$ for our calibrated parameters, including $\rho = 0$. In this case there is a unique steady state MME at $\phi \approx 4.4$. As typical, the monetary (nonmonetary) steady state is unstable (stable), implying that there are equilibria where $\phi \to 0$, featuring inflation as a self-fulfilling prophecy.

Endogenous cycles exist if we take a small departure from the calibrated parameters. The right panel of Figure 17 makes one change in parameters, reducing α_1 to 0.0001. There is still a unique steady state MME, now with $\phi \approx 3.14$, but textbook methods (e.g., Azariadis 1993) imply the following: Since $\Phi' < -1$ at the monetary steady state, Φ and Φ^{-1} also cross off the 45° line, say at (ϕ_L, ϕ_H)
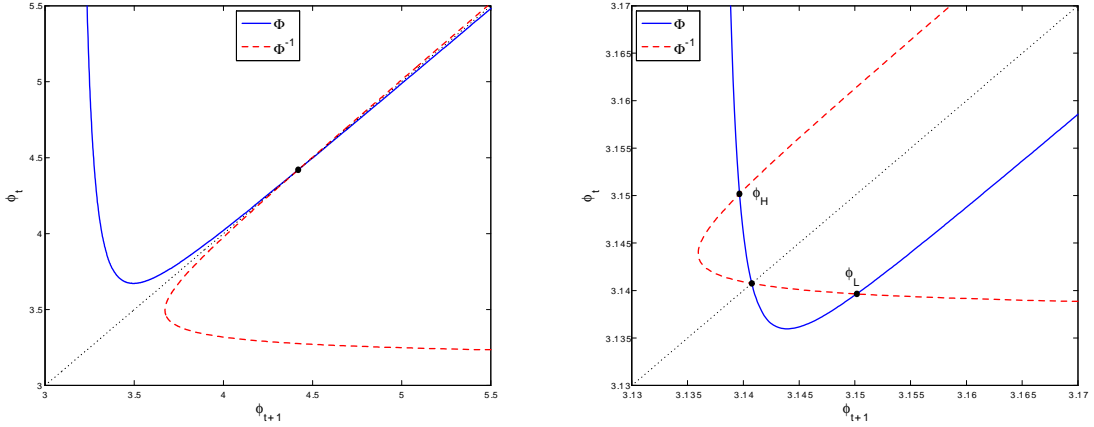
Figure 17: Phaseplane for Dynamic Equilibria

and (ϕ_H, ϕ_L). This is an equilibrium with a cycle of period 2, where ϕ oscillates between ϕ_L and ϕ_H. Heuristically, if $\phi_{t+1} = \phi_L$ is low then liquidity will be scarce at $t + 1$, making buyers want more of the asset at t, and hence making $\phi_t = \phi_H$ high.

While it is not atypical for monetary models to have cyclic equilibria, this intuition is different from OLG models, say, where the results are described in terms of backward-bending labor supply or savings functions. It is also different from other search-based monetary models with divisible goods. In Burdett et al. (2017), e.g., nonlinear utility drives the curvature of Φ and Φ^{-1} and can generate a rich equilibrium dynamics. In this model, while the goods in the decentralized market are indivisible, price posting generates a smooth price distribution in the BJ market, which is the source of the curvature in (21). So the mechanics of the results are quite different across alternative specifications, but it seems a fairly robust idea that economies where liquidity considerations play a role are subject to interesting dynamics.

Also, we reiterate that this economy has fluctuations in the distribution $F(p)$, not just the price level, but the analysis is still easy because $\bar{p}$ is sufficient to pin down $F(p)$. The tractability of the model enables us to discuss changes of key variables over the cycles. In Appendix D, we are able to derive the average price,

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the average credit usage, and the price dispersion explicitly as functions of ϕ_t. We show that as the asset price moves from ϕ_L to ϕ_H, the average posted and transaction price both increase while the share of credit transactions decreases. If ϕ_t is high then liquidity is abundant in the current period, reducing the household's need to use credit, and firms can post higher prices, driving a higher level of price dispersion.
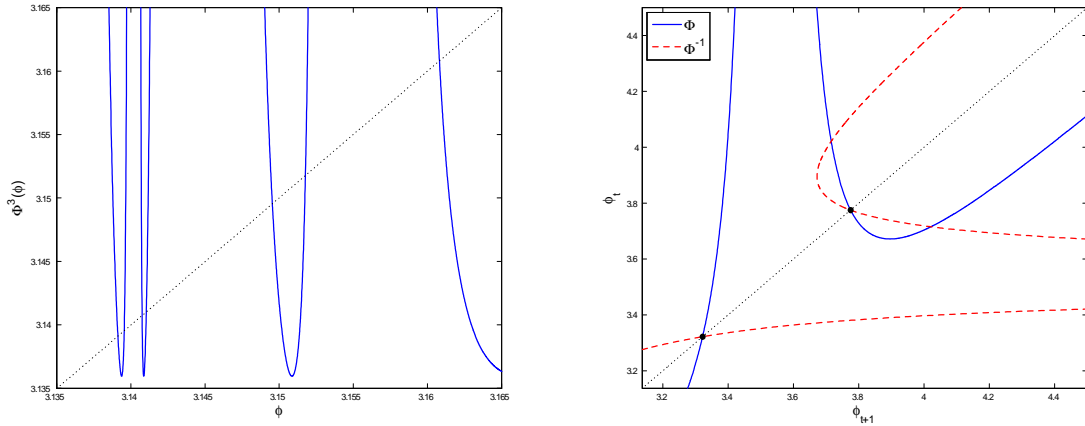
Figure 18: Examples with a 3-Cycle and with Two Steady States

For the same parameters that generate the 2-cycle, the left panel of Figure 18 shows the third iterate $\Phi^3(\phi)$. In addition to the steady state, $\Phi^3(\phi)$ has 6 intersections with the 45° line. This means there exist a pair of 3-cycles. Standard results (again see Azariadis 1993) tell us that the existence of a 3-cycle implies the existence of N-cycles $\forall N$ by the Sarkovskii theorem, as well as chaotic dynamics by the Li-Yorke theorem. So there is a large set of perfect-foresight dynamics, if not for the calibrated parameters, for values that are close. One can also show using standard methods that there are sunspot equilibria, with random fluctuations in ϕ, $F(p)$ and other variables, featuring excess volatility as a self-fulfilling prophecy.[20]

When $\rho = 0$, one might think this dynamic multiplicity arises because there are

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[20]One proof that sunspot equilibria exist, going back to Azariadis and Guesnerie (1986), is to suppose the outcome depends on an extrinsic two-state Markov process, $s \in \{s_1, s_2\}$, where $\varepsilon_s = prob(s_{t+1} \neq s | s_t = s)$. If $\varepsilon_1 = \varepsilon_2 = 1$ this reduces to a 2-cycle, the existence of which we just proved by example. By continuity there are equilibria for $\varepsilon_s < 1$.

two steady states, $\phi > 0$ and $\phi = 0$. However, we can eliminate the equilibrium with $\phi = 0$ by having $\rho > 0$, and as long as ρ is not too big the qualitative results are the same. Heuristically, the dynamic equilibria should not be interpreted as approximating fluctuations across steady states, but around a steady state. Alternatively, we can also set $\rho < 0$, which leads to two steady states, ϕ_1 and $\phi_2 > \phi_1 > 0$, as shown in the right panel of Figure 18, drawn for the same parameters except $\rho = -0.4$. In this case, using other (but still standard) methods, one can again construct sunspot equilibria.[21]

In summary, models with Burdett-Judd price posting, plus money and costly credit, admit cyclic, chaotic and stochastic dynamics, with a price distribution and the use of money/credit varying over time. While the mathematics used to establish these results are standard, the economic ideas are somewhat novel. In particular, we generate dynamics in the price distribution $F(p)$, not just the price level, and while this is also done in Burdett et al. (2017), in that model individuals are restricted to hold only $m \in \{0, 1\}$.

7   Conclusion

This paper has explored money and credit with Burdett-Judd pricing in a Lagos-Wright monetary model. We are not the first to use any of these ingredients, but we pushed harder on some analytics – e.g., the introduction of costly credit is technically useful because it resolves the indeterminacy problem in other models with money and posting – and tried to make a contribution in terms quantitative results. For both fixed and variable transaction costs, and for different assumptions about the way households sample prices, the model delivers exact money

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[21]A method Azariadis (1981) uses in OLG models is this: We seek $(\phi_1, \phi_2, \varepsilon_1, \varepsilon_2)$ such that $\phi_1 = \varepsilon_1 \Phi(\phi_2) + (1 - \varepsilon_1) \Phi(\phi_1)$ and $\phi_2 = \varepsilon_2 \Phi(\phi_1) + (1 - \varepsilon_2) \Phi(\phi_2)$, where $\varepsilon_s \in (0, 1)$ and w.l.o.g. $\phi_2 > \phi_1$. These equations are linear in, and hence easy to solve for, ε_1 and ε_2. Whenever $\Phi'(\phi_s) > 1$ at a steady state ϕ_s, for any ϕ_1 in some range to the left of ϕ_s and any ϕ_2 in some range to the right of ϕ_s, one can check $\varepsilon_1, \varepsilon_2 \in (0, 1)$, which is all we need to have a proper sunspot equilibrium.

demand functions resembling classic results in the literature, but, we think, with better microfoundations. These functions can match macro data, and at least the variable-cost model can match the money-credit shares in micro data.

In one application we showed how the theory can account somewhat well for price-change data even if we impose the discipline of matching other observations, and very well if we do not impose this discipline.[22] In another application we revisited the cost of inflation and considered relationships between inflation, markups, and price dispersion. A final application discussed endogenous dynamics. Other extensions are possible, such as incorporating heterogeneity, or combining menu-cost and search-based monetary models, but must be left for future work.

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[22]As a summary, our model is consistent with all of the stylized facts reported in fn. 6: (1) Empirical price durations vary across studies but are typically quite long. (2) The frequency and size of price changes vary across goods. (3) Two sellers changing p at the same time do not generally pick the same $\hat{p}$. (4) Many changes are negative. (5) Hazards decline at least slightly with duration. (6) There are many small but also many big changes. (7) The frequency and size of price changes, plus the fraction of negative changes, vary with inflation. (8) There is price dispersion even at low inflation. While we did not emphasize (2), it is clear that different values of the preference and cost parameters μ and γ, or arrival rates α_n, as is reasonable for different goods, will affect the price-change behavior. This seems worth further study.

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Appendix A: Proofs

Derivation of (8): The BJ value function can be written

$$V(z) = W(A) + \alpha_1 \int_{\underline{q}}^{z} (\mu - q) \, dG_1(q) + \alpha_1 \int_{z}^{\bar{q}} [\mu - q - \delta - \tau(q - z)] \, dG_1(q)$$

$$+ \alpha_2 \int_{\underline{q}}^{z} (\mu - q) \, dG_2(q) + \alpha_2 \int_{z}^{\bar{q}} [\mu - q - \delta - \tau(q - z)] \, dG_2(q),$$

where $G_n(q) = 1 - \hat{G}(q)^n$ is the CDF of the lowest of n draws from $G(q)$. The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with $q \leq z$, so only cash is used, times the expected surplus, which is simple because $W'(A) = 1$. The third is the probability of meeting a seller with $q > z$, so credit must be used, which adds fixed cost δ and variable cost $\tau(q - z)$. The last two terms are similar except the buyer meets two sellers. The rest is algebra. ∎

Proof of Lemma 3: For part (i), in NME, buyers' BJ surplus is $\Sigma = \mu - q - \delta - \tau q$. Note $\Sigma = 0$ at $q = (\mu - \delta)/(1 + \tau)$, so no buyer pays more than this. If $\bar{q} < (\mu - \delta)/(1 + \tau)$ then the highest price seller has profitable deviation toward $(\mu - \delta)/(1 + \tau)$, which increases profit per unit without affecting sales. Hence $\bar{q} = (\mu - \delta)/(1 + \tau)$.

For part (ii), in MME, for $q > z$, $\Sigma = \mu - q - \delta - \tau(q - z)$. Note $\Sigma = 0$ at $q = (\mu - \delta + \tau z)/(1 + \tau)$, and repeat the argument for NME to show $\bar{q} = (\mu - \delta + \tau z)/(1 + \tau)$. The definition of MME has $z < \bar{q} = (\mu - \delta + \tau z)/(1 + \tau)$, which reduces to $z < \mu - \delta$.

For (iii), in PME, given buyers bring z to BJ they would pay z. Hence $\bar{q} \geq z$, as $\bar{q} < z$ implies the highest price seller has profitable deviation. We also have to be sure there is no profitable deviation to $q > z$, which requires buyers using some credit. The highest such q a buyer would pay solves $\Sigma = \mu - q - \delta - \tau(q - z) = 0$, or $q = (\mu - \delta + \tau z)/(1 + \tau)$. There is no profitable deviation iff $(\mu - \delta + \tau z)/(1 + \tau) \leq z$, which reduces to $z \geq \mu - \delta$. ∎

Proof of Proposition 1: Part (i), for fiat currency $\phi = 0$ is always self-fulfilling, so we can set $G(q)$ according to (5), corresponding to equilibrium in the original BJ model.

For (ii), from Figure 1, MME exists iff three conditions hold: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) is equivalent to $(\alpha_1 + \alpha_2)\delta H^-(\bar{q}) < i$, which holds iff $i > \underline{i}$. Then (b) is equivalent to $(\alpha_1 + \alpha_2)\delta H^+(\underline{q}) > i$, which holds iff $i < \tilde{i}$ where $\tilde{i} = \delta(\alpha_1 + 2\alpha_2)^3/2\alpha_1\alpha_2(\mu - \delta - \gamma) > \underline{i}$. Also, (c) is equivalent to $(\alpha_1 + \alpha_2)\delta H(z_i) - iz_i > (\alpha_1 + \alpha_2)\delta H(0)$, which holds iff $\Delta(i) > 0$ where

$$\Delta(i) = -i\gamma + \frac{\delta(\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^{\frac{2}{3}}\delta^{\frac{1}{3}}\alpha_1^{\frac{2}{3}}\alpha_2^{-\frac{1}{3}}(\mu - \delta - \gamma)^{\frac{2}{3}}(2^{-\frac{1}{3}} + 2^{-\frac{4}{3}}).$$

Notice $\Delta(0) > 0 > \Delta(\tilde{i})$ and $\Delta'(i) < 0$. Thus, $\exists! \, \bar{i}$ such that $\Delta(\bar{i}) = 0$, and $\Delta(i) > 0$ iff $i < \bar{i}$. It remains to verify that $\bar{i} > \underline{i}$, so that (a) and (c) are not mutually exclusive. It can be checked that this is true iff $\delta < \bar{\delta}$. Hence, a MME exists under the stated conditions. It is unique because $\bar{q} = \mu - \delta$, which pins down $G(q)$, and then $\hat{z}_i = \arg\max_{z \in (\underline{q}, \bar{q})} O_i(z)$.

For (iii), from Figure 1, PME exists iff three conditions hold: (a) $O_i^-(\bar{q}) > 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(\bar{q}) > O_i(0)$. Now (a) holds iff $i < \underline{i}$ and (b) holds iff $i < \tilde{i}$. Condition (c) holds iff $i < \hat{i}$. For $\delta > \bar{\delta}$, it can be checked that $\hat{i} < \underline{i}$ and $\underline{i} < \tilde{i}$, so the binding condition is $i < \hat{i}$. For $\delta < \bar{\delta}$, it is easily checked that $\hat{i} > \underline{i}$, and $\underline{i} < \tilde{i}$, so the binding condition is $i < \underline{i}$. ∎

Proof of Proposition 2: Part (i), with fiat currency $\phi = 0$ is always self-fulfilling, so there is a NME iff buyers' payoff from in the BJ market is nonnegative, $(\alpha_1 + \alpha_2)[\mu - (1 + \tau)\mathbb{E}_H q] \geq 0$. Substituting $\mathbb{E}_H q$ into this, after some algebra we can show this holds iff $\tau \leq \mu/\gamma - 1$.

For (ii), from Figure 3, MME exists iff three conditions hold: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $\Psi_M > 0$ where

$$\Psi_M = (\alpha_1 + \alpha_2)[\mu - \mathbb{E}_H q - \tau\mathbb{E}_H \max(0, q - z_i)] - iz_i$$

is buyers' payoff from the BJ market. Now (a) holds automatically since $O_i^-(\bar{q}) = -i$. Then, (b) is equivalent to $(\alpha_1 + \alpha_2)\tau\hat{H}^+(\underline{q}) > i$, which holds iff $i < (\alpha_1 + \alpha_2)\tau$. And (c) is equivalent to

$$\Psi_M = \alpha_2(\mu - \gamma) + \frac{\alpha_1\tau(\mu - z_i)}{1 + \tau} - \frac{\alpha_1^2\tau(\mu - z_i)^2}{4\alpha_2(1 + \tau)^2(z_i - \gamma)} - iz_i = \Psi(z_i) - iz_i > 0.$$

Notice Ψ_M is strictly concave and continuous in i, $\lim_{i\to 0}\Psi_M > 0$, and $\lim_{i\to\infty}\Psi_M < 0$. Hence there exists a unique solution to $\Psi_M = 0$, which defines i^*, so $\Psi_M > 0$ holds $\forall i < i^*$. Hence, a MME exists under the stated conditions. It is unique because $O_i''(z_i) = V''(z_i) < 0$, and then $\hat{z}_i = \arg\max_{z\in(\underline{q},\bar{q})} O_i(z)$.

Finally, for part (iii), from Figure 3 it is clear that there is no PME in the variable-cost model. ∎

Proof of Proposition 3: Substituting α_n into (12) we have

$$\Pi_t(p) = b_t(p\phi_t - \gamma)\eta e^{-\eta}\sum_{n=1}^{\infty}\frac{\left[\eta\hat{F}_t(p_t)\right]^{n-1}}{(n-1)!} = b_t(p\phi_t - \gamma)\eta e^{-\eta F_t(p)},$$

since $e^x = \sum_{i=0}^{\infty}x^i/i!$. As a special case, $\Pi_t(\bar{p}_t) = b_t(\bar{p}_t\phi_t - \gamma)\eta e^{-\eta}$. Equal profit implies

$$F_t(p) = 1 - \frac{1}{\eta}\left[\log(\phi_t\bar{p}_t - \gamma) - \log(\phi_t p - \gamma)\right]$$

$$G_t(q) = 1 - \frac{1}{\eta}\left[\log(\bar{q}_t - \gamma) - \log(q - \gamma)\right],$$

with $\bar{q}$ as in the baseline model and $\underline{q}_t = e^{-\eta}\bar{q}_t + (1 - e^{-\eta})\gamma$. Algebra then yields

$$H_t(q) = \frac{\sum_{n=1}^{\infty}\alpha_n\left[1 - [1 - G_t(q)]^n\right]}{\sum_{n=1}^{\infty}\alpha_n} = \frac{1 - e^{-\eta}(\bar{q}_t - \gamma)/(q - \gamma)}{1 - e^{-\eta}}.$$

In the fixed-cost model, (i) holds as in Proposition 1. For (ii), follow Proposition 1 and check : (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) holds iff $\sum_{n=1}^{\infty}\alpha_n\delta H^-(\bar{q}) < i$ iff $i > \underline{i} = e^{-\eta}\delta/(\mu - \delta - \gamma)$. Then (b) holds iff $\sum_{n=1}^{\infty}\alpha_n\delta H^+(\underline{q}) > i$ iff $i < \tilde{i} = e^{\eta}\delta/(\mu - \delta - \gamma) > \underline{i}$. And (c) holds iff

$\sum_{n=1}^{\infty} \alpha_n \delta H\left(z_i\right) - i z_i > \sum_{n=1}^{\infty} \alpha_n \delta H\left(0\right)$ iff $\Delta\left(i\right) > 0$, where

$$\Delta\left(i\right) = \delta - 2\left[e^{-\eta}\delta\left(\mu - \delta - \gamma\right)i\right]^{\frac{1}{2}} - i\gamma.$$

Given $\Delta\left(0\right) > 0 > \Delta\left(\tilde{\imath}\right)$ and $\Delta'\left(i\right) < 0$, $\exists!\ \bar{\imath}$ such that $\Delta\left(\bar{\imath}\right) = 0$, and $\Delta\left(i\right) > 0$ iff $i < \bar{\imath}$. It remains to verify $\bar{\imath} > \underline{\imath}$, so that (a) and (c) are not mutually exclusive. This is true iff $\delta < \bar{\delta}$, where $\bar{\delta} = \mu - \left(1 - e^{-\eta}\right)\gamma/\left(1 - 2e^{-\eta}\right)$. Hence, MME exists under the stated conditions. The rest of the proof is the same as Proposition 1, except with $\hat{\imath} = \delta(1 - e^{-\eta})/(\mu - \delta)$. ■

Proof of Proposition 4: For (i) we again follow the proof of Proposition 2 and check

$$\Phi_N = \sum_{n=1}^{\infty} \alpha_n \left[\mu - \left(1 + \tau\right)\mathbb{E}_H q\right] \geq 0.$$

After substituting $\mathbb{E}_H q$, we get $\Phi_N = (1 - e^{-\eta} - \eta^{-\eta})[\mu - \gamma(1 + \tau)]$. Thus NME exists iff $\tau \leq \mu/\gamma - 1$.

To prove (ii), we again check: (a) $O_i^-\left(\bar{q}\right) < 0$; (b) $O_i^+\left(\underline{q}\right) > 0$; and (c) $O_i(z_i) > 0$, where

$$O_i(z_i) = \sum_{n=1}^{\infty} \alpha_n \left[\mu - \mathbb{E}_H q - \tau \int_{\hat{z}_i}^{\bar{q}} \left(q - \hat{z}_i\right) dH\right] - i\hat{z}.$$

Now (a) always holds, and (b) holds iff $i < (1 - e^{-\eta})\tau$. For (c), substitute α_n and H and simplify to get

$$
\begin{aligned}
O_i\left(z_i\right) &= \left(1 - e^{-\eta}\right)\mu - \left(1 - e^{-\eta} - \eta e^{-\eta}\right)\gamma - \eta e^{-\eta}\frac{\mu + z_i\tau}{1 + \tau} - \frac{\tau e^{-\eta}\gamma\left(\mu - z_i\right)}{\left(1 + \tau\right)\left(z_i - \gamma\right)} \\
&\quad - \frac{\tau e^{-\eta}\left[\mu - \gamma + \tau\left(z_i - \gamma\right)\right]}{1 + \tau}\log\frac{\mu - \gamma + \tau\left(z_i - \gamma\right)}{\left(1 + \tau\right)\left(z_i - \gamma\right)}.
\end{aligned}
$$

One can show $O_i''\left(z_i\right) < 0$. Since z_i is strictly decreasing in i, $O_i''\left(z_i\right)$ is strictly convex in i on $[0, \infty)$. Moreover, $\lim_{i \to 0} O_i(z_i) > 0$ and $\lim_{i \to \infty} O_i(z_i) < 0$. There is a unique solution to $O_i(z_i) = 0$ and that defines i^*, so $O_i(z_i) > 0\ \forall i < i^*$. Hence, there exists a unique MME iff $i < \min\left\{\tau\left(1 - e^{-\eta}\right), i^*\right\}$.

Finally, as in the proof of Proposition 2, (iii) is true. ■

Proof of Proposition 5: Substituting α_n into (12) we have

$$\Pi_t(p) = b_t \left(p\phi_t - \gamma\right) \sum_{n=1}^{\infty} \left[-\frac{\omega^n}{\log\left(1-\omega\right)} \right] \left[\hat{F}_t\left(p_t\right) \right]^{n-1},$$

and $\Pi_t(\bar{p}_t) = -b_t \left(\bar{p}_t\phi_t - \gamma\right)\omega/\log\left(1-\omega\right)$. Now equal profit implies

$$F_t\left(p\right) = 1 - \frac{\phi_t\left(\bar{p}_t - p\right)}{\omega\left(\phi_t\bar{p}_t - \gamma\right)} \text{ and } G_t\left(q\right) = 1 - \frac{\bar{q}_t - q}{\omega\left(\bar{q}_t - \gamma\right)},$$

with $\bar{q}$ as in the baseline models and $\underline{q}_t = (1-\omega)\bar{q}_t + \omega\gamma$. Also,

$$H_t\left(q\right) = 1 - \frac{\log\left[1 - \omega\left[1 - G_t(q)\right]\right]}{\log\left(1-\omega\right)} = 1 - \frac{\log\left(q - \gamma\right) - \log\left(\bar{q}_t - \gamma\right)}{\log\left(1-\omega\right)},$$

where we used $\sum_{n=1}^{\infty} x^n/n = -\log(1-x)$.

In the fixed-cost model, (i) holds as in Proposition 1. To show (ii), we check: (a) $O_i^-\left(\bar{q}\right) < 0$; (b) $O_i^+\left(\underline{q}\right) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) holds iff $i > \underline{i} = -\delta/[(\mu - \delta - \gamma)\log(1-\omega)]$, and (b) holds iff $i < \tilde{i} = -\delta/[(1-\omega)(\mu - \delta - \gamma)\log(1-\omega)] > \underline{i}$. Then (c) holds iff $\Delta\left(i\right) > 0$, where

$$\Delta\left(i\right) = \delta - \frac{\delta\left[\log\left(\hat{z}_i - \gamma\right) - \log\left(\mu - \delta - \gamma\right)\right]}{\log\left(1-\omega\right)} - i\gamma + \frac{\delta}{\log\left(1-\omega\right)}.$$

It is easy to check $\Delta'\left(i\right) < 0$, $\lim_{i\to 0}\Delta(i) > 0$, and $\exists!\ \bar{\imath}$ such that $\Delta\left(\bar{\imath}\right) = 0$. Hence $\Delta\left(i\right) > 0$ iff $i < \bar{\imath}$. For (a) and (c) to be not mutually exclusive, we check $\bar{\imath} > \underline{i}$. This holds iff $\delta < \bar{\delta}$, where

$$\bar{\delta} = \mu - \frac{\gamma\log\left(1-\omega\right)}{1 + \log\left(1-\omega\right)}.$$

Thus MME exists. Uniqueness follows Proposition 1, as does (iii), except now $\hat{\imath} = \delta/\left(\mu - \delta\right)$. ∎

Proof of Proposition 6: For (i), we check

$$\Phi_N = \left[1 - \frac{\omega}{\log\left(1-\omega\right)} \right] \left[\mu - \gamma\left(1 + \tau\right)\right] \geq 0,$$

which holds iff $\tau \leq \mu/\gamma - 1$. For (ii) we check: (a) $O_i^-\left(\bar{q}\right) < 0$; (b) $O_i^+\left(\underline{q}\right) > 0$; and

(c) $O_i(z_i) > 0$. Now (a) always holds and (b) holds iff $i < \tau$. Then for (c), write

$$
\begin{aligned}
O_i(z_i) \;=\; & \mu - \gamma + \frac{(\omega + \tau)\,\mu - (1+\tau)\omega\gamma + (\omega - 1)\tau\hat{z}_i}{(1+\tau)\log(1-\omega)} \\
& + \frac{\gamma\tau}{\log(1-\omega)}\log\left[\frac{\mu - \gamma + \tau\,(\hat{z}_i - \gamma)}{(1+\tau)\,(\hat{z}_i - \gamma)}\right].
\end{aligned}
$$

Since $\log(1-\omega) < 0$, we have $O_i'(z_i) > 0$. Given $\partial\hat{z}_i/\partial i < 0$, we also have $\partial O_i(z_i)/\partial i < 0$. Define i^* as the solution to $O_i(z_i) = 0$, so $O_i(z_i) > 0$ holds $\forall i < i^*$. Hence, there is a MME iff $i < \min\{\tau, i^*\}$, and it is unique since $O_i''(z_i) < 0$. Finally, (iii) follows Proposition 2. ∎

Appendix B: Calibration Formulae

Consider first the variable-cost model. Inserting $\bar{q}$ and $\underline{q}$, we get

$$G(q) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\mu - q + \tau(\hat{z}_i - q)}{(1+\tau)(q-\gamma)},$$

$$H(q) = 1 - \frac{\alpha_1^2 [\mu - q + \tau(\hat{z}_i - q)][\mu + \tau\hat{z}_i + (q-2\gamma)(1+\tau)]}{4\alpha_2(\alpha_1 + \alpha_2)(1+\tau)^2 (q-\gamma)^2}.$$

The fraction of monetary transactions and the markup are therefore

$$H(\hat{z}_i) = 1 - \frac{\alpha_1^2(\mu - \hat{z}_i)[\mu + \tau\hat{z}_i + (\hat{z}_i - 2\gamma)(1+\tau)]}{4\alpha_2(\alpha_1 + \alpha_2)(1+\tau)^2 (\hat{z}_i - \gamma)^2},$$

$$\frac{\mathbb{E}_G q}{\gamma} = 1 + \frac{\alpha_1(\mu + \tau\hat{z}_i - \gamma + \tau\gamma)\log(1 + 2\alpha_2/\alpha_1)}{2\alpha_2\gamma(1+\tau)},$$

where $\hat{z}_i$ is given in the text. From this we get

$$L_i = \frac{(1+\tau)\hat{z}_i}{\alpha_1(\mu + \hat{z}_i\tau) + (1+\tau)(1+\alpha_2\gamma)},$$

$$\eta_i = \frac{\alpha_1\mu + (1+\tau)(1+\alpha_2\gamma)}{\alpha_1(\mu + \hat{z}_i\tau) + (1+\tau)(1+\alpha_2\gamma)} \frac{\partial\hat{z}_i/\partial i}{\hat{z}_i/i}.$$

Consider next the fixed-cost model. Inserting $\bar{q}$ and $\underline{q}$, we get

$$G(q) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\mu - \delta - q}{q - \gamma},$$

$$H(q) = 1 - \frac{\alpha_1^2(\mu - \delta - q)(\mu - \delta + q - 2\gamma)}{4\alpha_2(\alpha_1 + \alpha_2)(q-\gamma)^2}.$$

The the fraction of monetary transactions and the markup are

$$H(\hat{z}_i) = \frac{[2\alpha_1\alpha_2(\mu - \delta - \gamma)/\delta]^{2/3} i^{2/3} - \alpha_1^2}{4\alpha_2(\alpha_1 + \alpha_2)},$$

$$\frac{\mathbb{E}_G q}{\gamma} = 1 + \frac{\alpha_1(\mu - \delta - \gamma)\log(1 + 2\alpha_2/\alpha_1)}{2\alpha_2\gamma}.$$

From this we get

$$L_i = \frac{\gamma + \left[\alpha_1^2\delta(\mu - \delta - \gamma)^2/2\alpha_2\right]^{1/3} i^{-1/3}}{1 + \alpha_1(\mu - \delta) + \alpha_2\gamma},$$

$$\eta_i = \frac{-1}{3 + 3\gamma\left[\alpha_1^2\delta(\mu - \delta - \gamma)^2/2\alpha_2\right]^{-1/3} i^{1/3}}.$$

Appendix C: Summary of Empirical Findings on Price Changes Statistics

Studies	Data source	Sample period	Empirical evidences [1]							
			Monthly change frequency (%)		Duration (month)		Absolute size of p changes (%)		Fraction of p changes below 5%	Hazard rates
			Mean	Median	Mean	Median	Mean	Median		
Using BLS CPI Research Database										
Bils and Klenow (2004)		1995M1-1997M12	26.1	20.9	...	4.3 (5.5)	...	...	...	...
Klenow and Kryvtsov (2008) [2]		1988M2-2005M1	36.2 (29.9)	27.3 (13.9)	6.8 (8.6)	3.7 (7.2)	14.0 (11.3)	11.5 (9.7)	39.8 (44.3)	Declining hazard
Nakamura and Steinsson (2008)		1988M1-2005M12	23.9-27.7	19.4-21.7	7.5-9.0	4.1-4.6	...	10.7 (8.5)	...	Declining hazard
Klenow and Malin (2010)		1988M1-2009M10	(19.7-23.1)	(10.6-13.0)	(9.0-10.4)	(7.2-8.9)	...	...	...	...
Eichenbaum et. al. (2014) [3]		1988M1-2011M7	22.0 (13.5)	...	6.2 (8.0)	3.4 (6.9)	...	...	40.6 (46)	...
Kehoe and Midrigan (2015) [4]		1988M1-2005M12	22.0 (6.9)	...	4.5 (7.4)	...	...	...	...	...
Using Scanner Data [5]										
Burstein and Hellwig (2007)	Dominick's	1989-1997	41 (26)	...	2.5 (4)	...	13 (11)	...	...	...
Nakamura (2008)	AC Nielsen ScanTrak	2004	43.9 (19.0)	42.7 (17.5)	...	...	...	...	...	...
Midrigan (2011) [6]	Dominick's	1989-1997	34 (2.9)	...	...	...	20 (11)	...	25	...
Eichenbaum et. al. (2011) [6]	A Large U.S. Retailer	2004-2006	43	...	0.6	...	...	...	...	...
Campbell and Eden (2014)	AC Nielsen ERIM	1985-1988	29	...	2.6	...	...	...	...	Declining hazard
Eichenbaum et. al. (2014) [7]	A Large U.S. Retailer	2004	...	...	...	...	...	30	5.2	...
Using Other Data										
Cecchetti (1986)	Newsstand Prices	1953-1979	...	...	72	72	24	25.2	...	...
Carlton (1986)	Stigler-Kindahl Data	1957-1966	...	...	11.7	11.5	3.8	3.9	...	...

(1) For studies that report statistics of both regular (i.e., excl. sales) and posted prices, the figures in parenthese correspond to those of regular prices.

(2) The paper also reported other estimates of durations, with the highest at 13.4 (mean) and 10.6 (median) duration when restricting the sample to 1998-2004 only.

(3) The paper also claimed that many small price changes are due to measurement errors and quality adjustment. Once removing these, the fraction drops to 32.2% for regular price changes and 24.4%

(4) This paper used the same data as Nakamura and Steinsson (2008), but different algorithm to calcualte price changes.

(5) Most scanner data are available on weekly basis, but here only monthly frequencies of price changes are reported, except Midrigan (2011) and Eichenbaum et. al. (2011).

(6) Numbers of price change frequency are weekly frequencies.

(7) The paper also reported statistics calculated using the unit value index (UVI)-based approach; the median change in UVI-based prices is 10 percent and 31.5 percent of the changes are smaller than 5 percent in absolute terms.

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Appendix D: Dynamics

First, we want to derive the average posted price in a nonstationary equilibrium. Given the $F_t(p)$ in (3), we can derive

$$F'_t(p) = \frac{\alpha_1}{2\alpha_2} \frac{\phi_t^2 \bar{p}_t - \phi_t \gamma}{(\phi_t p - \gamma)^2}.$$

Then, the average posted price is

$$\mathbb{E}_{F_t} p = \int_{\phi_t \underline{p}_t}^{\phi_t \bar{p}_t} p F'_t(p) dp = \frac{\frac{\alpha_1}{2\alpha_2} \log\left(1 + \frac{2\alpha_2}{\alpha_1}\right)\left(\frac{\mu + \tau \phi_t}{1+\tau} - \gamma\right) + \gamma}{\phi_t}.$$

It is easy to verify that $\partial \mathbb{E}_{F_t} p / \partial \phi_t > 0$ since $\mu > (1+\tau)\gamma$ under the calibrated parameters. To derive the average transaction price, we define the CDF of nominal transaction price distribution as

$$J_t(p) = \frac{\alpha_1 F_t(p) + \alpha_2 \left\{1 - [1 - F_t(p)]^2\right\}}{\alpha_1 + \alpha_2}$$

and

$$\mathbb{E}_{J_t} p = \frac{\alpha_1(\mu + \tau \phi_t) + \alpha_2 \gamma(1+\tau)}{\phi_t(\alpha_1 + \alpha_2)(1+\tau)}.$$

Similarly, one can check that $\partial \mathbb{E}_{J_t} p / \partial \phi_t > 0$ under all parameter values.

The share of credit transactions is

$$\int_{m_t}^{\bar{p}_t} dJ(p) = \frac{\alpha_1^2}{4\alpha_2(\alpha_1 + \alpha_2)(1+\tau)^2} \frac{\mu - \phi_t}{\phi_t - \gamma} \left[\frac{\mu - \phi_t}{\phi_t - \gamma} + 2(1+\tau)\right].$$

It is decreasing in ϕ_t since $(\mu - \phi_t)/(\phi_t - \gamma)$ is a decreasing function of ϕ_t. Finally, the size of price dispersion is measured in the coefficient of variation ν, defined as the ratio of the standard deviation of F to the mean of F.

$$\begin{aligned}
\nu_t &= \left(\frac{\mathbb{E}_F p^2 - (\mathbb{E}_F p)^2}{(\mathbb{E}_F p)^2}\right)^{\frac{1}{2}} \\
&= \left(\frac{\frac{\alpha_1}{\alpha_1 + 2\alpha_2} - \frac{\alpha_1^2}{4\alpha_2^2} \log^2\left(1 + \frac{2\alpha_2}{\alpha_1}\right)}{\left[\frac{\alpha_1}{2\alpha_2} \log\left(1 + \frac{2\alpha_2}{\alpha_1}\right) + \frac{\gamma(1+\tau)}{\mu + \tau \phi_t - \gamma(1+\tau)}\right]^2}\right)^{\frac{1}{2}},
\end{aligned}$$

After some algebra, one can show that $\partial \nu_t / \partial \phi_t > 0$.