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Simultaneous Innovation and the Cyclicality of R&D

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Abstract

There is ample evidence that R&D investment is mildly pro-cyclical. Whereas the existing literature can explain the positive correlation between investment in R&D and output, the moderate strength of the relationship remains under-explored. This paper develops a stochastic expanding-variety endogenous growth model that accounts for the observed mild pro-cyclicality of R&D. In the model, several firms may simultaneously make the same innovation. Research projects innovated by many firms simultaneously are of higher quality, on average, and contribute relatively more to the expansion of the knowledge stock in the economy. This delivers an endogenous mechanism that breaks the otherwise perfect correlation between R&D and output. A calibration of our model closely matches the cyclical properties of R&D.

Keywords: Simultaneous Innovation, Research and Development, Medium-Term Cycles, Macroeconomic Fluctuations, Endogenous Cycles.

JEL Codes: O30, O40, E32.

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1 Introduction

It is a well-established empirical regularity that R&D investment is mildly pro-cyclical.[1] Depending on the data source, the correlation between the cyclical components of R&D and output is between 0.3 and 0.5.[2] Whereas the existing literature has proposed several mechanisms that can explain the sign of this relationship, its magnitude remains under-explored.[3] In particular, existing theoretical models predict near-perfect correlation between output and R&D, which is at odds with the data.[4]

This paper complements the existing literature by developing an expanding-variety endogenous growth model that can resolve the discrepancy. In our model, there is the possibility that several firms make the same innovation simultaneously. Innovations simultaneously made by many firms contribute more, on average, to the expansion of the knowledge stock in the economy, i.e. they are of higher quality. This mechanism delivers endogenous oscillations in the economy and, as a consequence, mildly pro-cyclical R&D. A calibrated version of our model closely matches the pro-cyclicality and volatility of R&D observed in the data.

The source of technological progress in our model is the invention and adoption of new intermediate varieties. As in Gabrovski (2018) and Kultti *et al.* (2007), the innovation process makes the distinction between potential innovations (ideas) and actual innovations (new varieties). Upon entry into the R&D sector each firm is randomly matched with a particular idea from a pool of feasible research avenues. In particular, the number of firms matched with a given project is a Poisson random variable. This matching technology generates the possibility that some ideas are simultaneously innovated by many firms while others are not innovated at all.[5] This feature allows our model to account for the commonly

[1] See, for example, Griliches (1990), Fatas (2000), Wälde and Woitek (2004), Comin and Gertler (2006), Barlevy (2007), Ouyang (2011), Fabrizio and Tsolmon (2014), and Sedgley *et al.* (2018).

[2] For further details see, for example, Comin and Gertler (2006), Francois and Lloyd-Ellis (2009), and section three of this paper.

[3] See, for example, Fatas (2000), Comin and Gertler (2006), Barlevy (2007), and Francois and Lloyd-Ellis (2009).

[4] This is the case in both papers that develop expanding-variety growth models (see, for example, Comin and Gertler (2006)) and papers that develop Schumpeterian growth models (see, for example, Francois and Lloyd-Ellis (2009)).

[5] For previous growth models that feature simultaneous innovation in the same sector or of the same project see, for example, Corriveau (1998) and Gabrovski (2018).

observed in practice phenomenon of simultaneous innovation.[6] When a firm is matched with an idea and successfully innovates that idea, it applies for a patent over the corresponding variety. If more than one firm apply for the same patent, they each have an equal chance of receiving it. We follow Romer (1990) and Kortum (1997), among others, and assume that knowledge is cumulative — inventing a new variety allows firms to "stand on the shoulders of giants" and gain technological access to a number of new projects.

Each variety is equally productive, but innovations differ in the number of new research avenues they generate, i.e. their "quality". In particular, innovations made simultaneously by many firms lead to more research avenues, on average. This captures the intuition that i) when firms invest more in a given project, it is more likely to be of higher quality; ii) when more firms work simultaneously on the same project, it is more likely that at least one of them will develop a high quality invention.

The main contribution of our paper is the model's ability to reproduce the mild pro-cyclicality of R&D observed in the data. In our model the correlation between output and R&D is 0.43 and R&D is 1.77 times as volatile as output. In the data, the correlation is $0.3 - 0.5$ and the relatively volatility of R&D is $1.79 - 1.9$ times that of output.[7] In our model, innovations made simultaneously by many firm are, on average, of higher quality. This generates a mechanism which leads to endogenous oscillations. As a result, R&D investment is mildly pro-cyclical. In particular, following a positive technology shock both R&D and output converge to their new, higher balanced growth paths (BGP henceforth). Thus, their cyclical components are positively correlated. However, during this transition both series oscillate around their convergent paths in such a way that whenever output overshoots its convergent path, R&D investment undershoots it and vice versa. This reduces the strength of the relationship and leads to mildly pro-cyclical R&D.

To see the intuition behind the endogenous oscillations, suppose that at a given period, τ, the economy features relatively more varieties and relatively less available research av-

[6]For example, on February 14, 1876 Alexander Bell and Elisha Gray applied for a patent over the telephone within hours of each other. This same phenomenon is observed with virtually every major innovation from history, such as the cotton gin, the steam engine, the laser, and the computer (see, for example, Lemley (2011)). Furthermore, instances of simultaneous innovation have also been documented in many cases for non-major innovations as well (see, for example, Cohen and Ishii (2005) and Gabrovski (2018)).

[7]See Comin and Gertler (2006), Francois and Lloyd-Ellis (2009) and section three of this paper.

enues. This scarcity of ideas implies that there will be few innovations made and, as a result, relatively less varieties next period. This relatively low number of varieties at $\tau + 1$ implies lower competition between firms and increases the expected profitability of innovation. Hence, firms have more incentives to enter the R&D sector. This in turn leads to higher congestion in the "market" for ideas (i.e. relatively higher ratio of firms to ideas) and to a higher average number of firms that simultaneously innovate the same idea. Because of this, the average quality of innovations is higher, which leads to more feasible research avenues at period $\tau + 1$. Thus, at period $\tau + 1$ the economy features relatively more ideas and less varieties. As a result, at that period, there is relatively more innovation which leads to more varieties at $\tau + 2$. This then leads to lower expected profits and lower incentives for firms to enter the R&D sector, which ultimately leads to lower mass of ideas at $\tau + 2$. Furthermore, in periods when there are more varieties output is relatively high, whereas R&D investment is relatively low because research avenues are scarce. Conversely, when ideas are relatively abundant R&D investment is high and output is low because such periods feature a relatively lower mass of varieties.

We proceed by introducing the environment and characterizing the equilibrium. Next, we simulate the model and examine its impulse response functions and the oscillations therein. Lastly, we show our model can match key moments in the data and this ability is driven by the presence of endogenous oscillations.

2 The Economy

There are three types of agents in the economy — a final good producer, a unit measure of consumers (households), and a continuum of R&D firms. Time is discrete and infinite. The final good firm employs capital, labor, and intermediate varieties, which it uses to produce a single final good. Consumers supply labor, own the capital stock and the R&D firms, and consume the final good. R&D firms employ labor and engage in innovative activities. Firms that successfully innovate and patent a variety produce that variety.

2.1 Final Good Sector

The final good, Y_t, is produced by a single price taker. The price of the final good is normalized to unity. We follow Comin and Gertler (2006) and endow the firm with the following technology:

$$Y_t = A_t (K_t^{\alpha} L_{P_t}^{1-\alpha})^{1-\sigma} \Big(\int_0^{N_t} X_t^{\lambda}(n) dn \Big)^{\frac{\sigma}{\lambda}}, \quad \alpha, \sigma, \lambda \in (0,1) \tag{1}$$

The firm rents capital, K_t, from households at the rate $r_t + \delta^K$, where δ^K is the depreciation rate of capital and r_t is the households' rate of return. The firm faces a competitive market for labor in production, L_{P_t}, which is hired at the wage w_t. $X_t(n)$ is the amount of a particular variety n employed in production and N_t is the mass of intermediate varieties. The final good firm faces a monopolistically competitive market for these varieties, where a unit of each variety n is bought at the price $P_t(n)$.

We follow the RBC literature and assume the only source of aggregate uncertainty in the model is a productivity shock. In particular, the productivity parameter, A_t, follows an AR(1) process in logs:

$$A_{t+1} = A_t^{\rho} u_{t+1} \tag{2}$$

where $\rho \in (0,1)$ is a persistence parameter and u_{t+1} is a unit mean shock with variance σ_u.

The usual profit maximization of the final good firm implies the following demand functions for labor in production, capital, and intermediate varieties:

$$w_t = (1-\alpha)(1-\sigma)\frac{Y_t}{L_{P_t}} \tag{3}$$

$$r_t = \alpha(1-\sigma)\frac{Y_t}{K_t} - \delta^K \tag{4}$$

$$P_t(n) = \sigma X_t^{\lambda-1} \frac{Y_t}{\int_0^{N_t} X_t^{\lambda}(n) dn} \tag{5}$$

2.2 R&D Sector

The novel featues of our model are contained within the R&D sector. The innovation process consists of three stages and makes the distinction between potential innovations (ideas) and actual innovations (new varieties). At stage one, firms enter the R&D sector at a cost $\eta/(N_t A_t)$ units of labor. The entry cost depends on both the knowledge stock of the economy and the aggregate productivity, A_t. As in Romer (1990), among others, the cost of innovation decreases as the mass of varieties expands, which allows the model to exhibit positive long-run growth. In the spirit of the RBC literature, our model features an economy-wide productivity shock. To this end, we follow Bilbiie *et al.* (2012) and assume that the entry cost is decreasing in A_t. Let μ_t be the mass of R&D entrants and L_{R_t} be the total amount of labor employed in R&D. Then, the economy-wide research production function is given by

$$\mu_t = A_t N_t L_{R_t} \eta^{-1} \tag{6}$$

At stage two, firms are matched with a particular idea from a finite mass ν_t of feasible research avenues. The matching technology is such that the number of firms matched with a particular idea is a random variable that follows a Poisson distribution with a mean equal to the tightness in the market for ideas, $\theta_t = \mu_t/\nu_t$. This allows our model to account for the phenomenon of simultaneou innovation observed in practice. Due to the uncertainty in the matching process some research avenues are innovated by many firms simultaneously while others are not innovated at all, but on average there are θ_t firms working on the same project. The likelihood of simultaneous innovation is captured by the market tightness — a higher θ_t implies the market is relatively more congested and as a result the average number of firms which innovate the same idea is higher. Furthermore, this particular matching technology allows us to capture the oordination frictions in the market for ideas.[8]

Ideas are ex-ante identical. If an idea is invented, it transforms into exactly one new variety. Innovation is uncertain and takes one period to complete. A firm which enters at time t is successful in innovating its project at time $t+1$ with probability p. With probability $1-p$ the firm fails and so it exits the innovation sector. This implies that the number of firms which successfully innovate a particular idea follows a Poisson distribution with mean $p\theta_t$.[9] If a firm successfully innovates an idea, it applies for a patent over the corresponding variety. Each variety is protected by a single patent — in the event that several firms innovate the

[8] For further details see Gabrovski (2018).

[9] See, for example, Kultti *et al.* (2007).

same idea simultaneously, each receives the patent with equal probability.

At stage three, firms that have secured a patent over a variety produce it in a monopolistically competitive market. We normalize the average and marginal costs of production to unity, so profits are given by $\pi_t(n) = (P_t(n) - 1)X_t(n)$. Patents grant perpetual monopoly, but varieties become obsolete with probability δ. Thus, the value of holding a monopoly of a variety n at time t, $V_t(n)$, is given by

$$V_t(n) = E_t \sum_{i=t+1}^{\infty} (1-\delta)^{i-t} d_{it} \pi_i(n) \tag{7}$$

where d_{it} is the stochastic discount factor.

A necessary condition for positive long-term growth in the model is that the mass of ideas, ν_t, grows at a positive rate. We follow Kortum (1997) and Romer (1990), among others, and assume that knowledge is cumulative. In particular, the act of innovating an idea allows firms to "stand on the shoulders of giants" and gain access to new avenues for research. Thus, innovating an idea at time t allows, on average, $M(\theta_t)$ new ideas to enter the pool at time $t+1$. The function $M(\theta_t)$ is assumed to be increasing in the market tightness, θ_t, and $M(\theta_t) > 1$ for all positive θ_t. In our model varieties are equally productive, however, the quality of innovations (as captured by $M(\theta_t)$) is endogenous. The average number of new research projects which enter the pool from the innovation of a single idea, $M(\theta_t)$, depends on the market tightness. This allows our model to capture the intuition that i) the higher is the R&D investment per project, the higher its expected quality; ii) whenever more firms are working on the same project, it is more likely that at least one of them will develop a high quality invention.[10]

To understand the intuition clearly, let us consider an example. Suppose that whenever a firm innovates a particular idea, say a new computer microchip, the number of new research avenues that stem from the innovation is a random variable with distribution $F(q)$. This number would, of course, depend on the technological characteristics of the microchip. The size of the microchip and the temperature at which it runs determines the set of devices

[10] In particular, the average number of firms working on the same project is θ_t and the average R&D investment per project is $\theta_t \eta w_t/(N_t A_t)$.

with which it can be integrated — a relatively large microchip may only be feasibly used in desktop computers, if the chip is smaller it may also be feasibly integrated with laptops, if it is smaller still it may be feasibly integrated with smartphones and/or smartwatches. Then, suppose a new avenue of research, into say smartwatches, becomes available only when at least one firm draws a sufficiently high quality, q. Thus, if exactly m firms successfully innovate the microchip the number of new feasible R&D projects due to the invention of the chip, k, would be the sample maximum, i.e. $k \sim F(k)^m$. Since the number of innovators, m, follows a Poisson distribution with mean $p\theta_t$, then the average number of new ideas which become feasible from the development of a single innovation is given by $M(\theta_t) = \sum_{k=0}^{\infty} k(e^{F(k)p\theta_t} - e^{F(k-1)p\theta_t})e^{-p\theta_t}/(1-e^{-p\theta_t})$. Thus, the higher the expected number of firms that work on the chip, θ_t, (or aalogously the higher the expected R&D effort devoted to the development of the microchip, $\eta\theta_t/(N_t A_t)$) the higher the expected number of new avenues for future research that become available from the innovation of the microchip. In practice, firms may learn from each other's innovations so the number of new projects may exceed the sample maximum. Furthermore, the distribution $F(q)$ is likely to depend on each firm's research intensity. Hence, we keep it general and do not endow $M(\theta_t)$ with a specific functional form.

Once a variety is innovated, it is no longer a potential R&D project, so the corresponding idea is removed from the pool. As a result the average net increase in the stock of research avenues from innovating one new variety is $M(\theta_t)-1$. Furthermore, the matching technology implies that only a fraction $1 - e^{-p\theta_t}$ of ideas are innovated each period. Then, the law of motion for ideas is given by

$$\nu_{t+1} = (1-\delta)\nu_t + (1-\delta)(1-e^{-p\theta_t})(M(\theta_t)-1)\nu_t \tag{8}$$

where we assume varieties can become obsolete before they are innovated, i.e. their corresponding idea can become obsolete.

As each innovated idea is transformed into a new variety, it follows that varieties have the following law of motion

$$N_{t+1} = (1-\delta)N_t + (1-\delta)(1-e^{-p\theta_t})\nu_t \tag{9}$$

2.3 Households

There is a unit measure of infinitely lived identical consumers. They discount the future with a factor β and have the per-period utility function $U(C_t, L_t) = \ln C_t - \chi L_t^{1+1/\phi}/(1+1/\phi)$, where C_t is consumption, L_t is labor hours, ϕ is the Frisch elasticity of labor supply, and χ governs the disutility of labor. Since labor can be devoted to production or R&D, it follows that

$$L_t = L_{R_t} + L_{P_t} \tag{10}$$

Households own capital and have access to a mutual fund that covers all R&D firms. Let a_t denote the amount of shares held by the representative household at the beginning of period t. Firms distribute all profits as dividends, so the total assets of households in the beginning of t are $a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn + (1 + r_t)K_t$. At time t households choose shares a_{t+1} of the mutual fund which covers all R&D firms even though a fraction δ of varieties become obsolete next period. Thus, the household budget constraint is given by:

$$K_{t+1} + a_{t+1} \int_0^{N_{t+1}} V_t(n)dn = (1 + r_t)K_t + a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn + w_t L_t - C_t \tag{11}$$

2.4 Equilibrium

Intermediate good producers maximize per period profits subject to the inverse demand function given by equation (5). This yields $P_t = 1/\lambda$ and

$$X_t = \lambda\sigma \frac{Y_t}{N_t} \tag{12}$$

$$\pi_t = (1-\lambda)\sigma \frac{Y_t}{N_t} \tag{13}$$

$$Y_t = (A_t(\sigma\lambda)^\sigma)^{\frac{1}{1-\sigma}} K_t^\alpha L_{P_t}^{1-\alpha} N_t^{\frac{\sigma(1-\lambda)}{\lambda(1-\sigma)}} \tag{14}$$

Since the production function is symmetric, holding a monopoly over any variety is equally profitable. Furthermore, profits depend on the amount of intermediate varieties, N_t, and on the concavity of the production function. If $\sigma/\lambda > 1$, then the production function exhibits increasing returns to scale and as a result profits are increasing in N_t. If, on the other hand,

$\sigma/\lambda < 1$, then there are decreasing marginal returns to the extra variety and profits are decreasing in N_t.

At stage one of the innovation process, free entry implies that

$$\frac{\eta w_t}{A_t N_t} = \frac{1 - e^{-p\theta_t}}{\theta_t} V_t \tag{15}$$

Each entrant must hire $\eta/(A_t N_t)$ units of labor at the market wage w_t, so the left hand side of (15) captures the cost of engaging in R&D activities. Firms are successful in innovating with probability p. If they do innovate, they receive the patent with probability $\sum_{m=0}^{\infty} Pr($exactly m rivals successfully innovate the same idea$)/(m+1) = \sum_{m=0}^{\infty} e^{-p\theta_t}(p\theta_t)^m/(m+1)! = (1 - e^{-p\theta_t})/(p\theta_t)$. Thus, the right hand side of (15) captures the expected benefit from entering the R&D sector.

The first-order conditions of the representative household yield a standard labor supply condition and the Euler equations:

$$w_t = \chi C_t L_t^{\frac{1}{\phi}} \tag{16}$$

$$\frac{1}{C_t} = \beta E_t \left(\frac{1}{C_{t+1}} (1 + r_{t+1}) \right) \tag{17}$$

$$V_t = (1-\delta)\beta E_t \left(\frac{C_t}{C_{t+1}} (\pi_{t+1} + V_{t+1}) \right) \tag{18}$$

where (18) makes use of the symmetry in varieties and their law of motion. Furthermore, the stochastic discount factor is $d_{it} = (\beta(1-\delta))^i C_t / C_{t+i}$.

Lastly, we can combine the consumer's budget constraint, (11), with the demand for capital, (4), for labor in production, (3), and the free entry condition, (15), to get the law of motion for capital

$$K_{t+1} = (1 - \delta^K) K_t + Y_t - X_t N_t - C_t \tag{19}$$

3 Numerical Results

3.1 Calibration

Because of aggregate uncertainty and positive long-run growth, our economy follows a stochastic BGP. We follow Comin and Gertler (2006) and study a loglinearization of the stochastic BGP around its deterministic counterpart.[11] Following the previous literature (see, for example, Barlevy (2007)), calibrate the model at annual frequency. Hence, the discount rate is set at $\beta = 0.95$. The capital's share of output, α, is 0.33 and its depreciation rate, δ^K, is 0.08. The materials' share of output, σ, is set to 0.5 and the persistence parameter, ρ, to 0.88, as in Comin and Gertler (2006). Set the Frisch elasticity of labor supply, ϕ, to 4 and normalize $L_t = 1$. This yields $\chi = 0.8472$. Following Bilbiie *et al.* (2012), the obsolescence rate of varieties, δ, is set to 0.1. The probability of successfully innovating a research project, p, turns out to be a scaling parameter, so we normalize it to unity.

To pin down the entry cost, η, the gross markup, $1/\lambda$, and the value of $M(\theta_t)$ along its deterministic balanced growth path, $M(\theta)$, we use three balanced growth path restrictions. First, we use data on the fraction of approved patent applications in the U.S. for the period from 1966 to 2011.[12] We match its empirical average of 0.60957 to its model counterpart, $(1-e^{-p\theta})/(p\theta)$. This yields $\theta = 1.0876$. Second, we set the R&D share of output, $\eta w_t \mu_t/(N_t A_t Y_t)$, to the average in the U.S. — 3.1194%.[13] Third, we calibrate the growth rate of output to its empirical counterpart for the period of 1.7546%. This yields $\eta = 0.1879$, $1/\lambda = 1.0841$, and $M(\theta) = 1.4168$.

Lastly, to calibrate the volatility of the technology shock, σ_u, and the elasticity of $M(\theta_t)$ along the balanced growth path, $\varepsilon_{M,\theta}$, we use two second moment conditions. We set $\sigma_u = 0.01074$ to match the standard deviation of per capita non-farm GDP in the data of 2.7279%.[14] To match the standard deviation of per capita patent applications, $p\mu_t$, to its

[11] A detailed description of the deterministic and stochastic BGPs is included in the appendix.

[12] The data on both patents and patent applications is taken from the U.S. Patent and Trademark Office. The data on patent grants is by year of patent applications.

[13] The data is taken from the U.S. Bureau of Economic Analysis. The data on non-farm GDP is in chained 2009 dollars and is taken from NIPA table 1.3.6. The data for R&D expenditures is from NIPA table 5.6.5 and includes software expenditures. To deflate the series for R&D we use the implicit GDP price deflator from NIPA table 1.1.9.

[14] All per capita variables are normalized by the civilian non-institutionalized population. The data on

empirical counterpart of 3.9786%, calibrate $\varepsilon_{M,\theta} = 8.97$.

3.2 Impulse Response Functions

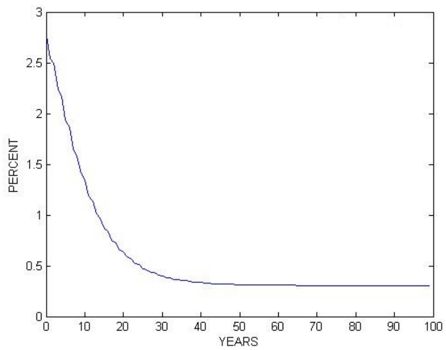
Because the model features endogenous growth, we construct the impulse response functions as percentage deviations from the deterministic balanced growth path of the economy (Figures 1 and 2). Following a positive technology shock, the economy converges to a new, higher BGP. Output, consumption, investment in R&D, labor hours, investment, and profits initially increase and overshoot their new BGPs. In subsequent periods, they gradually converge to the higher BGP. Varieties, on the other hand, converge to the new BGP without initially overshooting it.

As is evident from the impulse response functions, the model exhibits endogenous oscillations. Whereas all variables feature these oscillating behavior, it is more pronounced in the ones that describe the R&D sector. In particular, the pool of ideas, the mass of entrants, the market tightness, and labor hours in research all oscillate around their new BGP following a positive technology shock. Since the tightness, θ_t, and labor hours in research, L_{R_t}, are constant along the BGP, they converge to their old levels. The main reason why the oscillations are more pronounced in the R&D sector is twofold. First, households want to smooth consumption and leisure, so they dislike volatility in total labor hours, output, spending on varieties, and total investment. Thus, the oscillation is least apparent in these variables. Second, the main driver behind the oscillations in the economy is the market for ideas, so variables associated with it exhibit higher magnitude oscillations.

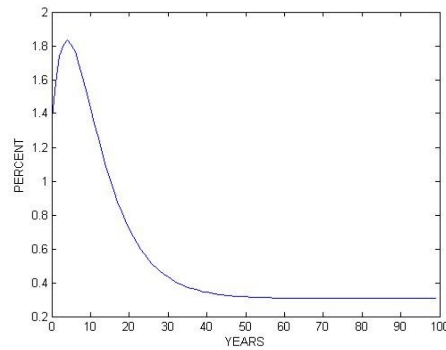
To see the intuition behind why the model exhibits endogenous oscillations, suppose that at period τ there are relatively more varieties, N_τ, and relatively fewer ideas, ν_τ. The scarcity of ideas implies that the number of innovations, $(1-e^{-p\theta_\tau})\nu_\tau$, is relatively low as well. This in turn leads to a low number of varieties the next period, $N_{\tau+1}$. Then, by equation (13), expected profits next period, $E_\tau \pi_{\tau+1}$, are high since there is relatively less competition between intermediate good producers.[15] At the same time, higher mass of varieties, N_τ, decreases the cost of innovation. Both the lower entry cost and higher expected profits

this is taken from the Bureau of Labor Statistics' Employment Situation Release.

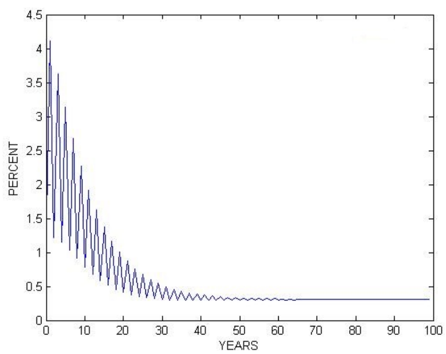
[15]Expected profits are decreasing in the number of varieties next period because in our calibration $\sigma < \lambda$.



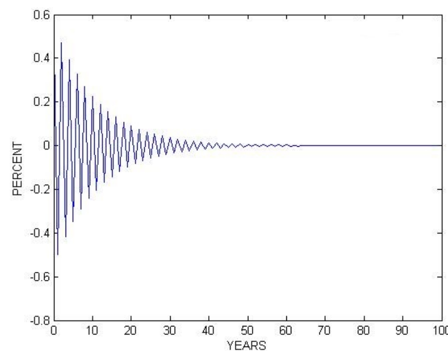
(a) Output



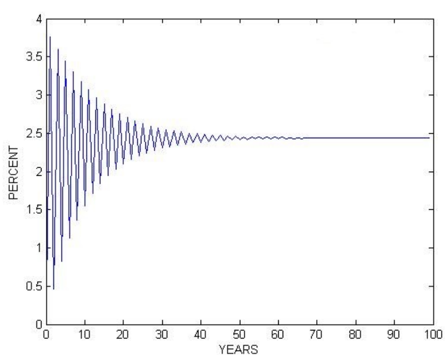
(b) Consumption



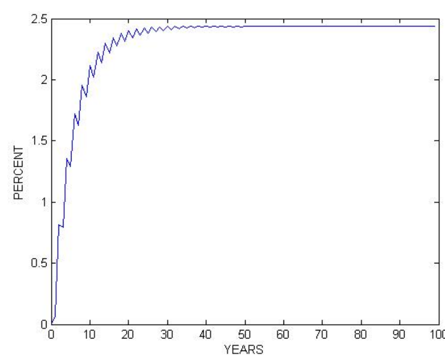
(c) R&D



(d) Market Tightness

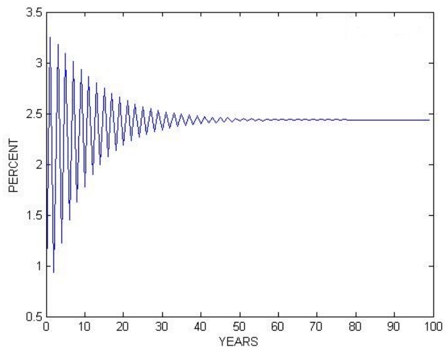


(e) Pool of Ideas

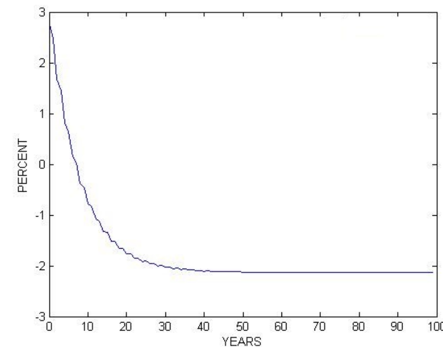


(f) Varieties

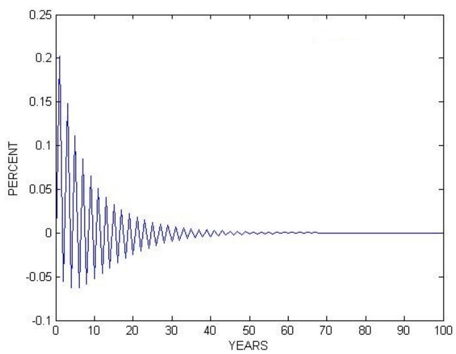
Figure 1: Impulse Response Functions to One Standard Deviation Shock in Productivity



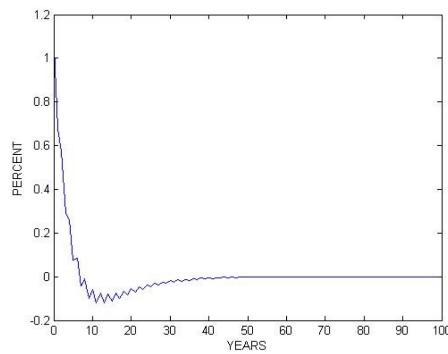
(a) Entry



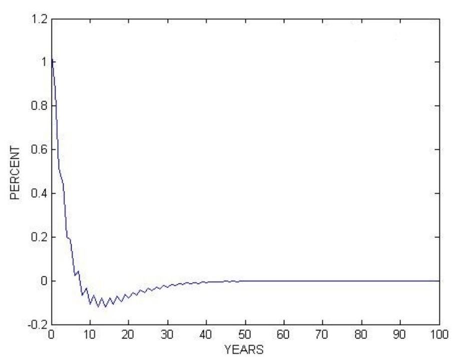
(b) Profits



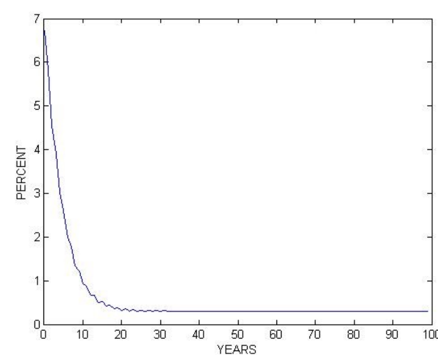
(c) Hours in Research



(d) Hours in Production



(e) Hours



(f) Investment

Figure 2: Impulse Response Functions to One Standard Deviation Shock in Productivity

create incentives for firms to enter the R&D sector. As a consequence the market tightness, θ_τ, increases. The resulting higher congestion implies that more firms, on average, innovate the same idea simultaneously. This leads to a higher average quality of innovation, $M(\theta_\tau)$, and as a consequence to relatively abundant research projects the next period, $\nu_{\tau+1}$. Thus, at $\tau+1$ there are relatively low number of varieties, $N_{\tau+1}$, and relatively high number of ideas, $\nu_{\tau+1}$. Because of high $\nu_{\tau+1}$, the number of innovations at time $\tau+1$ and subsequently the number of varieties $N_{\tau+2}$ are relatively high. This leads to relatively low expected profits, $E_{\tau+1}\pi_{\tau+2}$. Because of lower expected profits and because of the relatively high entry cost at time $\tau+1$ (due to low $N_{\tau+1}$), firms have less of an incentive to innovate. This induces low market tightness, $\theta_{\tau+1}$. Hence, fewer firms innovate the same project simultaneously which results in lower average quality of innovations. Thus, $\nu_{\tau+2}$ is relatively low. At time $\tau+2$ the cycle repeats.

3.3 Cyclicality and Second Moments

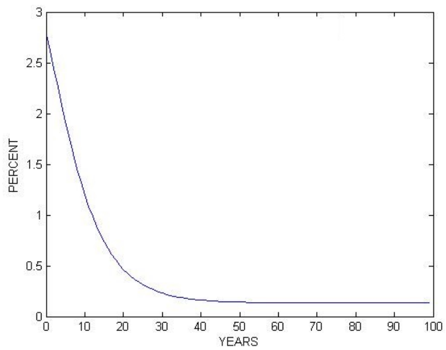
Table 1: Moments for Data and Model

	σ_X/σ_Y			Corr(X,Y)		
Variable X	Data	Benchmark Model	Exogenous Quality	Data	Benchmark Model	Exogenous Quality
R&D	1.79	1.77	0.81	0.43	0.43	0.96
Consumption	0.69	0.54	0.53	0.90	0.93	0.93
Hours	0.70	0.41	0.42	0.83	0.92	0.93
Investment	2.39	2.62	2.67	0.91	0.97	0.97

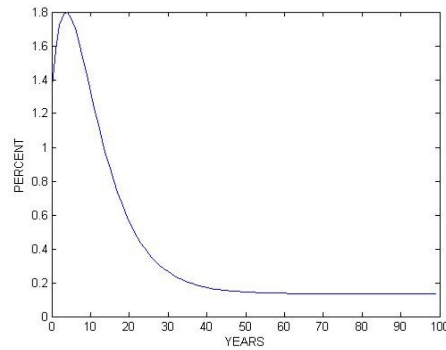
The endogenous oscillations in our model deliver mildly pro-cyclical R&D investment. This is in contrast to the existing literature which predicts near-perfect correlation between output and R&D.[16] Specifically, in periods when varieties are relatively abundant, output is relatively high because the final good firm can employ a wider range of intermediaries in production. These periods also feature a low mass of available research projects. As a result fewer firms engage in R&D.[17] This leads to low aggregate R&D investment. In contrast, during periods with relatively less varieties, output is low. At the same time, ideas are

[16]See, for example, Comin and Gertler (2006) and Francois and Lloyd-Ellis (2009).

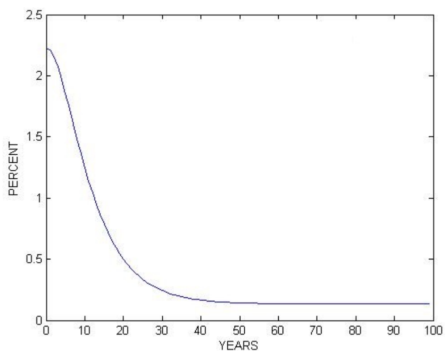
[17]This is true even though such periods feature a relatively high market tightness, θ_t.



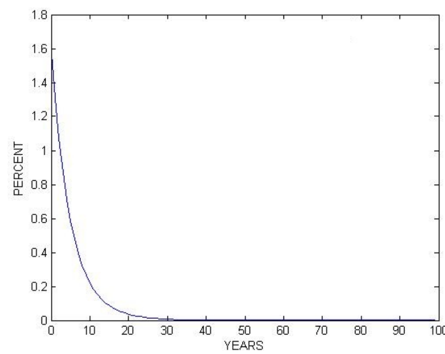
(a) Output



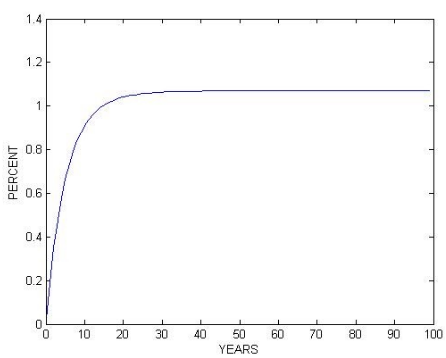
(b) Consumption



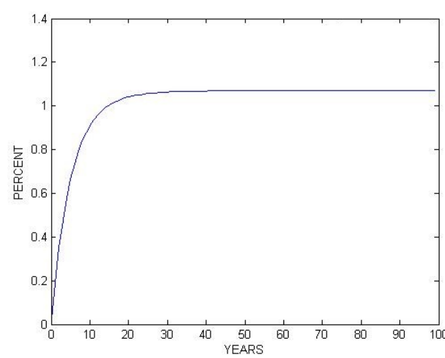
(c) R&D



(d) Market Tightness

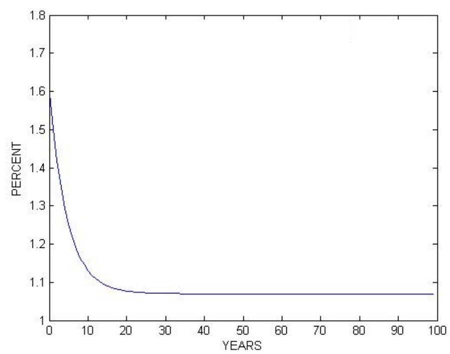


(e) Pool of Ideas

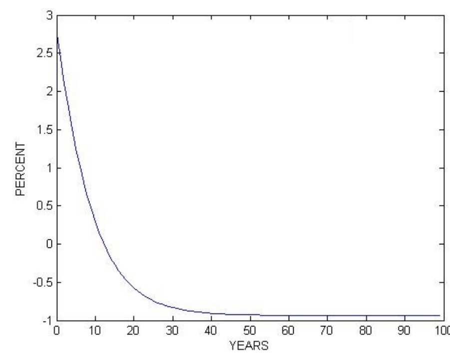


(f) Varieties

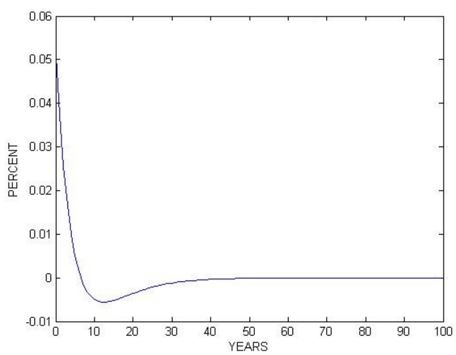
Figure 3: Impulse Response Functions with Exogenous Innovation Quality



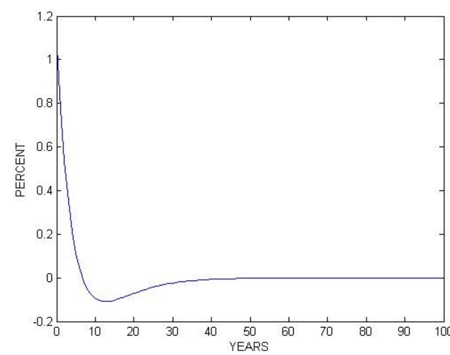
(a) Entry



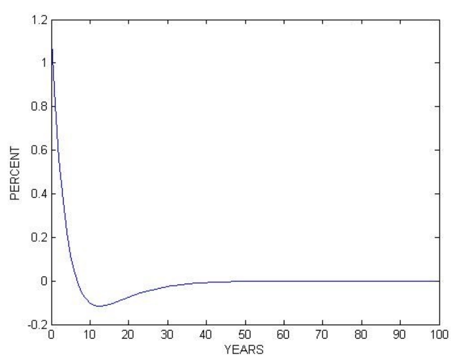
(b) Profits



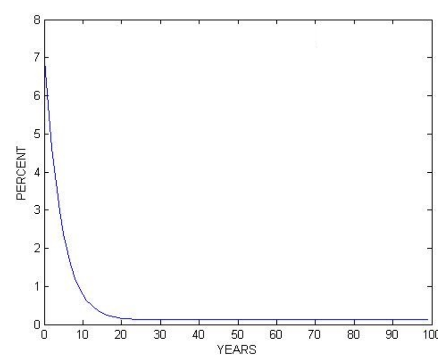
(c) Hours in Research



(d) Hours in Production



(e) Hours



(f) Investment

Figure 4: Impulse Response Functions with Exogenous Innovation Quality

abundant, so entry into R&D, and consequently R&D investment, are high. Thus, although following a positive technology shock both output and R&D investment converge to a new, higher BGP, they oscillate around their convergent paths in such a way that whenever output overshoots its path, R&D undershoots it and vice versa.

Table 1 reports the standard deviations and cyclical properties of the data and our economy.[18] The model matches the empirical moments remarkably well. In particular, it is able to reproduce the mild procyclicality of R&D and its relative volatility. Alternative data sources for R&D yield a correlation with output between 0.3 and 0.5, and relative standard deviation of R&D around 1.9 times that of output.[19] The model does well against these alternative measures as well.

The third and last columns of Table 1 highlight the importance of the endogenous quality in innovation for the model's ability to match the data. These columns report the moments for the model when innovation quality is exogenous, as captured by $M(\theta_t)$ being a constant, i.e. $\varepsilon_{M,\theta} = 0$. In that specification, the economy does not feature endogenous oscillations (see Figures 3 and 4). Hence, the mechanism which induces R&D to overshoot (undershoot) its convergent path whenever output undershoots (overshoots) it is broken. Thus, output and R&D move very closely together and their correlation is almost perfect. Furthermore, the absence of oscillations decreases the relative volatility of R&D by about one half.

4 Conclusion

This paper develops an expanding-variety endogenous growth model that can account for the empirically observed mild pro-cyclicality of R&D investment. In the model, some firms make the same innovation simultaneously. Varieties invented by many firms simultaneously are, on average, of higher quality and so contribute more to the expansion of the knowledge stock in the economy. This mechanism gives rise to endogenous oscillations — periods of relatively scarce research projects and abundant varieties are followed by periods during which research

[18]The data is obtained from the U.S. Bureau of Economic Analysis. Investment in the model corresponds to investment in physical capital and in R&D.

[19]Francois and Lloyd-Ellis (2009) use data from Compustat and find a correlation of 0.5 and relative standard deviation of 1.9. Comin and Gertler (2006) find a correlation of 0.3 and standard deviation of 1.89 using data from the National Science Foundation.

projects are abundant and varieties scarce. Following a positive technology shock both output and R&D converge to a new, higher BGP. Thus, they are positively correlated. Due to the oscillations in the model, however, both variables fluctuate around their convergent paths in such a way that whenever R&D overshoots its path, output undershoots it and vice versa. Thus, R&D is only mildly pro-cyclical.

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5 Appendix

Characterizing the deterministic balanced growth path. Along the deterministic balanced growth path variables grow at the following rates

Proposition 1. *Along the deterministic balanced growth path θ_t, L_{P_t}, L_{R_t}, L_t, r_t are all constant. The growth rates of N_t, ν_t, and μ_t are given by $(1-\delta)(e^{-p\theta} + (1-e^{-p\theta})M(\theta)) - 1$, where θ is the value of θ_t along the deterministic BGP. The growth rates of Y_t, C_t, K_t, w_t are given by $(1+g_N)^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))} - 1$. Moreover, the growth rate of profits is given by $(1+g_Y)/(1+g_N) - 1$.*

Proof. First, it is clear that $g_{LP} = g_{LR} = g_L = 0$. Then, from (17) a constant growth rate of consumption implies that the return on capital, r_t, must be constant. Thus, from the final producer's first order condition with respect to capital, (4), we have that $g_Y = g_K$. Moreover, from the first order condition with respect to labor in production, (3), it follows that $g_w = g_Y$. Next, from the solution for intermediate goods, (12), and the law of motion for capital, (19), it follows that $g_K = 1 - \delta^K + ((1 - \lambda\sigma)Y_t - C_t)/K_t$. As g_K is constant along the BGP, it follows that $g_K = g_C$.

As $g_K = g_Y$ and $g_{LP} = 0$, from the production function in equilibrium, (14), it follows that $g_Y = (1 + g_N)^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))} - 1$. Next, from the solution for profits, (13), it must be the case that $g_\pi = (1 + g_Y)/(1 + g_N) - 1$. Thus, from the Bellman equation for the value of a monopoly, (18), and from the fact that g_C is constant, it follows that $g_\pi = g_V$. Then, free entry, (15), implies that $g_\theta = 0$. Hence, the growth rates of varieties and ideas must equal to each other, $g_\mu = g_\nu$. Thus, the law of motion for varieties, (9), implies that $g_N = g_\nu$. Lastly, the law of motion for ideas, (8), implies that $g_\nu = (1 - \delta)(e^{-p\theta} + (1 - e^{-p\theta})M(\theta)) - 1$.

■

We cannot explicitly solve for all variables along the deterministic balanced growth path. Instead we reduce the model to a system of eleven equations and unknowns in order to provide an implicit solution. Let $x_t := \nu_t/N_t$; $z_t := Y_t/C_t$; $n_t := K_t/C_t$; $\gamma_t := N_t^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))}/K_t^{1-\alpha}$. Omitted time subscripts denote variables along the deterministic balanced growth path.

From the laws of motion for varieties and ideas, (9) and (8), it follows that

$$x = \left(\frac{e^{-p\theta}}{1 - e^{-p\theta}} + M(\theta)\right) - \frac{1}{1 - e^{-p\theta}} \tag{20}$$

From the law of motion for capital, (19), it follows that

$$g_Y = -\delta^K + (1 - \lambda\sigma)\frac{z}{n} - \frac{1}{n} \tag{21}$$

Next, from the Euler equation, (17), and the fact that $g_Y = g_C$ it is immediate to see that

$$g_Y = \beta(1+r) - 1 \tag{22}$$

The equation for demand for capital, (4), gives us

$$r = \alpha(1-\sigma)\frac{z}{n} - \delta^K \tag{23}$$

Next, from the research production function, (6), we have that

$$x\theta = ALR\eta^{-1} \tag{24}$$

Combining the demand for labor in production, (3), and the households' supply decision for labor, (16), we get

$$(1-\alpha)(1-\sigma)z = \chi L^{\frac{1}{\phi}} LP \tag{25}$$

Next, using (18), (15), and (13)

$$\frac{\eta\theta}{A(1-e^{-p\theta})LP} = \frac{(1-\delta)\beta(1-\lambda)\sigma}{(1-\alpha)(1-\sigma)(g_N - (1-\delta)\beta)} \tag{26}$$

From the resource constraint for labor, it is clear that

$$L = LP + LR \tag{27}$$

Next, from the definition of z_t and the production function in equilibrium, (14), one gets

$$z = (A(\sigma\lambda)^{\sigma})^{\frac{1}{1-\sigma}} nLP^{1-\alpha}\gamma \tag{28}$$

Lastly, from proposition 1, it follows that

$$g_y = (1+g_N)^{\sigma(1-\lambda)/(\lambda(1-\sigma)(1-\alpha))} - 1 \tag{29}$$

$$g_N = (1-\delta)(e^{-p\theta} + (1-e^{-p\theta})M(\theta)) - 1 \tag{30}$$

Equations (20) through (30) for a system of eleven equations and eleven unknowns (x, z, γ, n, r, L, LP, LR, θ, g_Y, and g_N) that characterizes the deterministic balanced growth path.

Characterizing the stochastic balanced growth path. The behavior of our economy along its stochastic balanced growth path is described by a system of first order difference equations in x_t, n_t, γ_t, A_t, and θ_t and equilibrium conditions for r_t, z_t, L_t, L_{P_t}, and L_{R_t}.

From the laws of motion for varieties and ideas, (8) and (9), we get the law of motion for x_t

$$x_{t+1} = \frac{(e^{-p\theta_t} + (1 - e^{-p\theta_t})M(\theta_t))x_t}{1 + (1 - e^{-p\theta_t})x_t} \tag{31}$$

Next, from the resource constraint, (19), and the Euler equation, (17), we can derive the law of motion for n_t

$$\beta E_t(n_{t+1}(1 + r_{t+1})) = (1 - \delta^K)n_t + (1 - \lambda\sigma)z_t - 1 \tag{32}$$

To get the law of motion for θ_t we combine (18), (15), (13), (9), and (3) to get

$$\beta E_t\left(z_{t+1}\Big(\frac{(1-\lambda)\sigma}{(1-\alpha)(1-\sigma)} + \frac{\eta\theta_{t+1}}{A_{t+1}L_{P_{t+1}}(1 - e^{-p\theta_{t+1}})}\Big)\right) = \frac{\eta\theta_t(1 + (1 - e^{-p\theta_t})x_t)z_t}{A_t L_{P_t}(1 - e^{-p\theta_t})} \tag{33}$$

From the law of motion for varieties, (9), and capital, (19), we can derive the law of motion for γ_t

$$\gamma_{t+1} = \frac{(1-\delta)^{\frac{\sigma(1-\lambda)}{\lambda(1-\sigma)}}(1 + (1 - e^{-p\theta_t})x_t)^{\frac{\sigma(1-\lambda)}{\lambda(1-\sigma)}} n_t^{1-\alpha}}{((1 - \delta^K)n_t + (1 - \lambda\sigma)z_t - 1)^{1-\alpha}}\gamma_t \tag{34}$$

The last law of motion is simply the evolution of the technology parameter A_t, (2). The equilibrium relationships are derived in the exact same way as the corresponding equations along the deterministic BGP and are as follows.

$$r_t = \alpha(1-\sigma)\frac{z_t}{n_t} - \delta^K \tag{35}$$

$$x_t\theta_t = A_t L_{R_t}\eta^{-1} \tag{36}$$

$$\chi L_t^{\frac{1}{\phi}} L_{P_t} = (1-\alpha)(1-\sigma)z_t \tag{37}$$

$$L_t = L_{P_t} + L_{R_t} \tag{38}$$

$$z_t = (A_t(\sigma\lambda)^{\sigma})^{\frac{1}{1-\sigma}} n_t L_{P_t}^{1-\alpha}\gamma_t \tag{39}$$

These ten equations (31) through (39) along with (2)) form the dynamical system which describes the economy along its stochastic BGP.