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Conflict Over Transnational River Resources: An Applied Game Theoretic Analysis

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Abstract

This paper presents a game theoretic model to analyze transnational river resource conflicts such as the Indus River Basin conflict between Pakistan and India. The model demonstrates that to obtain a share of the river resources from the upstream country downstream, the downstream country must purchase costly armaments to threaten an invasion. Yet in this process, the upstream country has a significant first-mover advantage in extracting the river resource as it can internalize the threat of invasion and prevent it from happening in the non-invasion equilibrium "Peace with Threat of War". When the upstream country does not internalize the threat of invasion from the downstream country, the invasion equilibrium "War" occurs. This paper discusses the implications of each equilibria in the model within the context of the Indus River Basin conflict. This paper also discusses the possibility of Pareto-improving cooperative outcomes by imposing new institutional frameworks.

JEL Codes: F51, F55

Keywords: India, Indus River Basin, International Conflict, Pakistan, Third-Party Arbitration, Transnational River.

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1 Introduction

One of the most serious and persistent conflicts in the post-World War Two international order has been the conflict between India and Pakistan. The two countries have fought four wars against one another and today are both nuclear powers. While the Indo-Pakistani conflict has many causes such as religion and nationalism, controlling natural resources for their expanding populations is also a key structural element that runs beneath the many stated grievances between the two nations. Controlling fresh water is particularly important for both countries; they are competing to control the Indus River Basin, the principal source of fresh water in the South Asian region. Recently, water shortages in Pakistan and rising tensions in both Pakistan and India means that the competition over controlling the Indus River Basin has the possibility of escalating into a full war (Kugelman, 2016).

The focal point of the Indus River Basin conflict is the Kashmir territory, a valley situated between Pakistan and India that has been effectively partitioned between the two states since their independence from the British Empire in 1947. The Kashmir Valley's watershed at the head of the Indus Basin is the principal water supply for large parts of Northern India, all of Pakistan, and parts of Central Asia that would otherwise extremely dry (FAO, 2017). Currently, India is in control of the upstream portions of the Indus River Basin's key tributaries, including the Indus River itself. The result is that India has almost complete upstream control of the Indus River Basin before it flows into Pakistan. This is especially problematic for Pakistan, as the Indus River Basin makes up over 80 percent of Pakistan's total water supply (FAO, 2017). Pakistan has responded to India's upstream control of the Indus Basin with the threat of, and at times, actual use of military force. Throughout the four Indo-Pakistani wars, Pakistan has repeatedly invaded the Kashmir territory in an effort to gain more control of it (Bose, 2003). Today, Pakistan and India still station large numbers of troops in the Kashmir while Pakistan allegedly supports Islamic partisan (or terrorist) groups in the Indian controlled portions of the region (Kugelman, 2016).

The purpose of this paper is to analyze this Indus River Basin conflict from a game theoretic perspective and understand the conflict at a structural level. The paper will do this by constructing a simple game-theoretic model of competition between two countries over a shared transnational resource that includes the potential for armed conflict. This model developed in this paper can also be expanded to model other transnational river conflict across the world including the Mekong River conflict and the Nile River conflict.

This paper will construct a two player game that focuses on the sequential extraction of a river resource with a fixed endowment. The game includes two countries: one country is situated upstream on the transnational river and the other is situated downstream. Both countries extract water resources from the river, but the upstream country extracts its water resources first in a sequential game due to its geographic position. The downstream country has the ability to purchase armaments and invade the upstream country should it want to, inflicting costs both on itself and the upstream country, but in doing so is able to capture more of the river resource for itself. This game-theoretic model is intentionally highly simplified compared to the complex nature of real-world transnational river conflicts such as the Indus conflict, and therefore the analysis the game offers is limited to broad observations and recommendations at the structural level. The recommendations the model offers include insights into how the Indus River Basin conflict can be solved by international institutions and agreements in such a way that benefits both India and Pakistan.

The most significant insight from the analysis of the model is that the armaments purchased by the downstream country, and threat of invasion carried by them, function as a mechanism for reallocating water resources from the upstream country to the downstream country. The armaments are costly, however, which makes them a inefficient mechanism for allocating the water resources between the two countries. This implies that there exists Pareto-improving outcomes where the water resource is allocated cooperatively without the purchase of armaments. The interpretation of this result in the Indus Basin conflict is that Pakistan, as the downstream country in the Indus Basin, is forced to spend heavily on its military and constantly threaten India as a mechanism for securing water resources. This non-cooperative equilibrium leaves both Pakistan and India worse off, yet there is an opportunity to solve the Indus Basin conflict with a Pareto-improving cooperative agreement.

In the next section I will discuss relevant economic literature in the field of international resource competition and how this paper adds to it. Next, in the third section, I will introduce the two country game setup, show the two possible types equilibria, and give a brief discussion of the equilibria and there interpretation in the Indus Conflict. In the fourth section, I will discuss the comparative statics of the game and discuss their relevance in the Indus Basin conflict. In the fifth section I will consider the existence of Pareto-improving cooperative outcomes and potential mechanisms that could be used to achieve those outcomes. I will also discuss how those mechanism could be implemented in the Indus Basin conflict. In the sixth section, I will conclude with a discussion of model extensions and potential applications beyond the Indus Conflict.

2 Literature Review and Contribution

This paper and the model it develops considers transnational river disputes from a game theoretical perspective of resource competition; it draws from and will contribute to the economic literature in that field. The paper principally builds off the model developed by Acemoglu et al. (2012). Acemoglu et al's model considers country level strategic decisions in the context of non-renewable resource extraction with the potential for interstate war. Their model uses the framework of a resource that is completely controlled by one resource-rich country, extracted dynamically over many time periods, and sold to the other resource-poor country which also posses military capabilities to invade the resource-rich country. This paper will build off Acemoglu' et al's game framework of two countries competing over a resource with the military armament and potential invasion components. The major difference that this paper offers is that the resource of interest, a shared transnational river, is geographically differentiated between two countries and sequentially extracted in one time period. Therefore the paper does not consider the dynamics of resource extraction, but focuses on the allocation or resources through power mechanisms including the sequential order of resource extraction derived from geographic positioning. The game developed in this paper, albeit simple, is readily applicable to the Indus River Basin conflict and other transnational river conflicts.

This paper provides theoretical support for the empirical claim made by Caselli et al. (2015) that competition over transnational resources is a significant determinant of interstate war. The authors' hypothesis in Caselli et al. (2015), for which they develop a empirical body of evidence, is that when resource-poor countries are close geographically to another resource-rich country, it increases the incentive for the resource-poor country to go to war to try and capture at least some of the resources from the resource-rich country. This paper investigates this empirical claim by considering a game-theoretic model of resource-based interstate war when there are transnational resources that cross bilateral borders. In the model, the downstream country, potentially a resource-poor country due to the upstream country's river resource extraction, can have a strong incentive to go to war to capture more of the river resource.

Similarly, the model developed in this paper provides additional theoretical support for the claims made by Koubi et al. (2014). The authors survey relevant economic and international relation literature for a meta analysis of the causes of resource conflicts. The authors make the case for resource abundance, rather than scarcity, being a better predictor of interstate conflict. This is due to the potential payoff for the aggressor country being greater when it can capture more resources in war. In this paper's game, when the upstream country extracts a large amount of the river resources, it creates a resource abundance for itself that the downstream may decide to exploit through a war.

The model in this paper incorporates the "guns-and-butter" trade-offs of armament decisions by countries as developed by Powell (1993) and adapted by Acemoglu et al. (2012). Powell creates a model of arms procurement where armament produces positive societal utility for a country, but the armament is also costly: guns versus butter. Powell's model considers arms-race competitions between nations and how they may dynamically spiral out of control to where countries buy many more guns then they otherwise would have wanted, creating an efficiency loss. The model in this paper does not consider traditional arms races like Powell (1993) as only one country can purchase armaments, but instead explores how societies may suffer an efficiency loss due to one country using armament purchases as resource reallocation mechanism.

Finally this paper contributes to the body of theoretical work on the political economy of natural resources and interstate conflict developed by Tornell and Lane (1999), Ross (1999), Caselli (2006), Robinson et al. (2006), Egorov et al. (2009), Yared (2010), and Robinson and Acemoglu (2012).

3 Environment

3.1 Model Setup

Consider a simple, single-period, two country game where Country A and Country B are situated along a transnational river. Country A is geographically positioned upstream of Country B. Both countries extract water resources from the river in quantities denoted by $X_A \geq 0$ and $X_B \geq 0$. The total endowment of river resources that is available to be extracted is given by $E > 0$. For simplicity, Country B extracts all of the remaining river resource endowment that Country A does not extract:

$$X_B = E - X_A$$

Both countries gain utility from the river resources they extract:

$$u_A(X_A, ...)$$

$$u_B(X_B, ...) = u_B(E - X_A, ...)$$

The utility functions satisfy the following assumption:

Assumption 1. *The utility functions are strictly increasing $u_i'(\cdot) > 0$, are concave $u_i''(\cdot) < 0$, and are continuously twice differentiable for both countries $i = A, B$. Both utility functions also meet the following Inada conditions: $lim_{X_i \to 0} u_i'(X_i) = \infty$ and $lim_{X_i \to \infty} u_i'(X_i) = 0$.*

Country B is able to purchase armaments, denoted by $m_B \geq 0$, at a cost given by the function $l(m_B)$. The cost function satisfies the following assumption:

Assumption 2. *The cost function $l(\cdot)$ is strictly increasing $l'(m_B) > 0$ and convex $l''(m_B) > 0$, which indicates that there is an increasing marginal cost of armament for Country B.*

If Country B decides to purchase any amount of armaments greater than zero, it has the ability to invade Country A. If Country B invades, it shrinks Country A's resource extraction to a fraction between zero and one, given by the invasion function $w(m_B) \in (0, 1)$. This models Country B destroying Country A's ability to extract resources upstream with dams and other hydro infrastructure. After an invasion, Country A's resource extraction is limited to $X_A \times w(m_B)$. Country B, by virtue of being downstream and exacting all the remaining resources from the river, gains in its extraction whatever Country A losses in the invasion: $X_B = E - X_A \times w(m_B)$. For simplicity, there is no efficiency loss in this process: Country B steals through the invasion exactly what Country A loses. The invasion function satisfies the following assumption:

Assumption 3. *The invasion function $w(\cdot)$ is decreasing $w'(m_B) < 0$ for $\forall m_B > 0$ and is fully differentiable for all positive values of m_B. Additionally, the $w(\cdot)$ function satisfies the following properties: $w(0) = 1$ and $lim_{m_B \to \infty} \quad w(m_B) = 0$.*

Finally, there are fixed costs of invasion for both countries, denoted $C_A > 0$ and $C_B > 0$ for Country A and Country B respectively. The game will play out in sequential moves as follows:

1. Country B purchases its armaments $m_B \geq 0$ at a cost given by $l(m_B)$.

2. Country A chooses its extraction amount: $X_A \in [0, E]$.

3. Country B chooses to invade Country A or not: $I = \{0, 1\}$ where $I = 1$ is an invasion and applies the given fixed cost to both countries $\{C_A \times I, C_B \times I\}$.

4. Country B extracts all remaining river resources not extracted by Country A including the resources stolen in the invasion.

The game sequence and the endgame payoffs for each country are shown in figure 1 below:

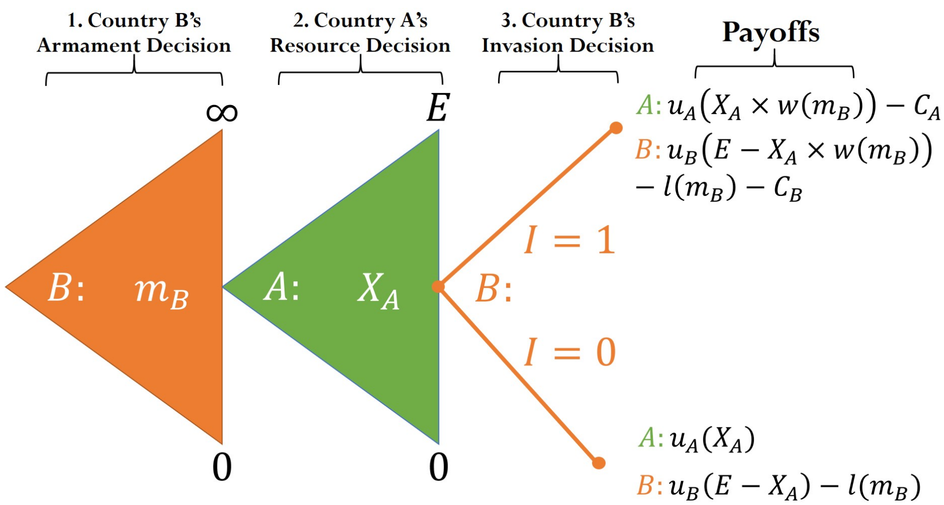

Figure 1: The game sequence and the possible payoffs

Regarding the payoffs shown in figure 1, if Country B decides to not invade Country A, Country A gets the utility from the river resources it extracted and Country B gets the utility from the leftover river resources minus the cost of any armament purchases it made. If Country B decides to invade, Country A loses a fraction of its resource extraction and suffer a fixed cost from the invasion. Country B gains Country A's lost river resources, but Country B pays for the cost of its armament purchases and also suffers a fixed cost of the invasion.

A sub-game perfect Nash equilibrium exists for this game when the functions $u_i(\cdot)$, $w(\cdot)$, $l(\cdot)$ are well defined, common knowledge between players, and meet Assumptions 1-3 shown above. This follows from the theorem developed by Harris (1985) that sub-game perfect Nash equilibria always exist in deterministic continuous games with perfect information. A slight modification is necessary to account for the finite third-stage of the game, but a equilibrium exists for each possible sub-game at the ends nodes of invasion and non-invasion, and therefore the sub-game perfect Nash equilibrium exists. For a full proof of existence of equilibrium, see Appendix A.

3.2 Equilibrium Analysis

There are only two possible types of sub-game perfect Nash equilibrium which occur depending on functional forms and parameter values in the game.[1] There is one possible equilibrium where an invasion occurs, and there is one equilibrium where an invasion does not occur. See figure 2 below for a depiction of the potential sub-game perfect Nash equilibria:

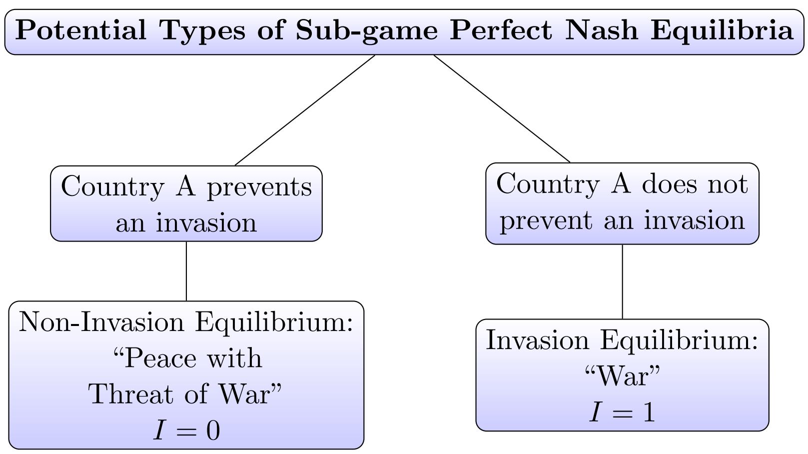

Figure 2: The two possible types of sub-game perfect Nash equilibria.

3.2.1 Non-Invasion Equilibrium: Peace with Threat of War

To solve for the non-invasion equilibrium, assume that Country A is always better off when an invasion does not occur.

Result 1. *In the non-invasion equilibrium, Country A chooses a resource extraction from the function $X_A(m_B; E, C_B)$ which prevents an invasion. Country B chooses an optimal armament $m_B^\star$ that maximizes its utility based on Country A's resource extraction function.*

[1]Assumptions 1-3 and the conditions of $C_A > 0$ and $C_B > 0$ are sufficient to limit this game to only two possible types of equilibria.

I solve the game using backwards induction. Beginning in the third stage of the game, Country A wants to prevent an invasion. Therefore in the second stage Country A makes a resource extraction decision, X_A, such that the payoffs for Country B invading is not higher than the payoffs of not invading. The highest possible X_A that Country A can choose is where Country B becomes indifferent to invading:

$$u_B(E - X_A \times w(m_B)) - l(m_B) - C_B = u_B(E - X_A) - l(m_B)$$

$$u_B(E - X_A \times w(m_B)) - C_B = u_B(E - X_A) \tag{1}$$

When Country A chooses an X_A such that equation (1) holds, the payoffs for Country B invading, the left hand side, are equal to the payoffs of not invading, the right hand side. From equation (1), Country A can derive a resource extraction best response function:

$$X_A(m_B; E, C_B) \tag{2}$$

Where $X_A \in (0, E)$ given Assumptions 1-3 and the conditions placed on the parameters of E and C_B.

This best response function (2) allows Country A to make an optimal resource extraction decision given any set of real, positive inputs including Country B's armament m_B. This resource exaction best response function maximizes Country A's utility by taking as much of the water resources as possible without provoking an invasion.

In the first stage of the game, knowing that Country A will make an optimal resource extraction decision X_A as a response to m_B from that satisfies equation (1), Country B will maximize its utility by purchasing an optimal amount of armament $m_B^\star$ given that it will not invade. Country B's utility maximization problem then becomes:

$$\max_{m_B} u_B(E - X_A(m_B; E, C_B)) - l(m_B) \tag{3}$$

$$F.O.C : -\frac{\partial u_B}{\partial m_B}(E - X_A(m_B; E, C_B))$$

$$\times \frac{\partial X_A}{\partial m_B}(m_B; E, C_B) - \frac{\partial l}{\partial m_B}(m_B) = 0 \tag{4}$$

Assumptions 1 and 2 are sufficient to ensure that Country B's payoff function in equation (3) will be concave. Therefore with specific functional form, the

F.O.C in (4) is necessary and sufficient to solve for a $m_B^\star$ that maximizes Country B's utility.[2]

On the non-invasion equilibrium path, in the first stage of the game Country B purchases an optimal armament of $m_B^\star$ that solves equation (4). In the second stage of the game, County A uses its best response function (2) to choose an optimal extraction amount: $X_A^\star = X_A(m_B^\star; E, C_B)$. Finally, in the third stage of the game, Country B chooses not to invade with $I^\star = 0$. The full strategies and payoffs for the non-invasion equilibrium, labeled "Peace with Threat of War", is given in Table 1. A numerical example is given in Appendix B.1.

Table 1: Potential Types of Equilibria

Equilibrium:	Country A Strategy: X_A	Country B Strategy: m_B, I	Country A Payoff	Country B Payoff
I: Peace with Threat of War	$X_A^\star$ from (2) where $0 < X_A^\star < E$	$m_B^\star$ that solves (4), $I^\star = 0$	$u_A(X_A^\star)$	$u_B(E - X_A^\star) - l(m_B^\star)$
II: War	E	$m_B^{I\star}$ that solves (6), $I^\star = 1$	$u_A(E \times w(m_B^{\star I})) - C_A$	$u_B(E - E \times w(m_B^{\star I})) - l(m_B^{\star I}) - C_B$

3.2.2 Invasion Equilibrium: War

To solve for the invasion equilibrium, assume that Country A is not concerned with preventing an invasion.

Result 2. *In the invasion equilibrium, Country A extracts the full resource endowment $X_A = E$ and Country B chooses an optimal armament $m_B^{I\star}$ that maximizes its utility knowing it will invade with certainty.*

Again, I solve the game using backwards induction. In the third stage of the game, Country A does not want to prevent an invasion. Therefore

[2] $X_A(m_B)$ is negative in m_B, which means the first term of the objective function (3) is increasing in m_B and because $-l(m_B)$ is decreasing in m_B, the objective function (3) has a well defined global maximum within the set of positive real values of m_B.

in the second stage Country A makes a resource extraction decision X_A will maximize its utility given the invasion occurring. Country A choose to extract the full resource endowment $X_A = E$ as extracting anything less will not prevent an invasion and will only give Country A a smaller payoff.

In the first stage of the game, knowing that Country A will take the full resource endowment, Country B will maximize its utility by purchasing an optimal amount of armaments $m_B^{I\star}$ given that it will invade:

$$\max_{m_B^I} u_B(E - E \times w(m_B^I)) - l(m_B^I) - C_B \tag{5}$$

$$F.O.C: -E \times \frac{\partial u_B}{\partial m_B^I}(E - E \times w(m_B^I)) \times \frac{\partial w}{\partial m_B^I}(m_B^I) - \frac{\partial l}{\partial m_B^I}(m_B^I) = 0 \tag{6}$$

Again, Assumptions 1 and 2 are sufficient to ensure that the payoff function in equation (5) will be concave. Therefore with specific functional form, the F.O.C in (6) is necessary and sufficient to solve for a $m_B^{I\star}$ that maximizes Country B's utility given an invasion.

On the invasion equilibrium path, in the first stage of the game Country B purchases an optimal armament of $m_B^{I\star}$ that solves equation (6). In the second stage, Country A extracts E regardless of how many armaments Country B purchased. Finally, in the third stage of the game, Country B chooses to invade, $I^\star = 1$. The full resulting strategies and payoffs for invasion equilibrium, labeled as "War" is included in Table 1. A numerical example is given in appendix B.2.

3.2.3 Sub-game Perfect Nash Equilibrium: Peace or War

Before the game is plays out, Country A compares it's potential payoff within the non-invasion equilibrium where it prevents an invasion and its potential payoff within the invasion equilibrium where it does not prevent an invasion.

Result 3. *In the sub-game perfect Nash equilibrium, Country A chooses the type of equilibrium, either non-invasion or invasion, that is utility maximizing as a sub-game. The game will end in that equilibrium.*

If Country A has a higher payoff in the non-invasion equilibrium:

$$u_A(X_A^\star) \geq u_A(E \times w(m_B^{I\star})) - C_A \tag{7}$$

Country A will play $X_A^\star$ as its equilibrium strategy and the non-invasion equilibrium path will occur. If inequality (7) does not hold and Country A has higher payoff in the invasion equilibrium, County A will play E as its equilibrium strategy and the invasion equilibrium path will occur. Country B, despite being the first mover in the game with its armament purchases, has no control over what type of equilibrium will occur.[3] This is because if Country B believes that Country A will maximize its utility by extracting $X_A^\star$ and preventing an invasion, Country B is better off playing $m_B^\star$ then any other armament. Similarly, if Country B believes that Country A will maximize its utility by extracting E and forcing an invasion, Country B is better off playing $m_B^{I\star}$ than any other armament. Any deviation in armament purchases by Country B off the anticipated equilibrium path will not change the equilibrium and will only decrease Country B's utility. Therefore the sub-game perfect Nash equilibrium includes a sub-game of which anticipated equilibrium type has a higher payoff for Country A. A numerical example is given in appendix B.3.

3.3 Equilibrium Discussion

In this game with perfect information, Country A can always prevent an invasion if it wants to. Result 3 shows that Country A will only allow an invasion to occur if it's payoff is higher by allowing an invasion then by reducing its resource extraction to prevent it. Therefore in this game, while an invasion is technically a move by Country B, Country A has full control over the invasion sub-game with its resource extraction strategy.

From result 1 in the Peace with Threat of War non-invasion equilibrium, the threat of invasion from Country B is what forces Country A to take less than the full resource endowment, allowing more resources to flow downstream to Country B. The problem for Country B, however, is that the threat of invasion is only possible through the purchase of armaments. If Country B does not spend enough on armament, its ability to invade is reduced and Country A can extract more resources. The result is that Country B must choose $m_B^\star \geq 0$ as its armament and must pay $l(m_B^\star) \geq 0$. The armament purchases are costly, but remain a necessity for Country B to obtain any of the river resources. Within the non-invasion equilibrium, the marginal utility of armament for County B, which is interpreted as the gains in allocation of the river resource from additional armament minus the additional cost of that armament, is positive from $m_B = 0$ until $m_B = m_B^\star$, after which it is negative.

Therefore Country B ends up in a poor position where it must "pay-to-play" with its armament purchases to get any non–zero allocation of the

[3] The one exception is the knife's edge case where equation (7) holds with equality. This will case be discussed in the comparative statics section.

transnational river resource. This is because the natural allocation of the resource with no armament purchases, due to the sequential nature of the game, gives the full endowment to Country A. Even if there are limits on how much of the resource Country A can take, Country A still has an extraction advantage over Country B by being the first to extract the resource. Yet Country B's ability to purchase armaments and threaten invasion negatively affects Country A's resource extraction, should it want to prevent an invasion. The threat of invasion could be so severe and costly that Country A extracts very little of the resource in order to prevent it. Therefore there are two sources of power in this game which can secure a higher allocation of resource: being the first-mover as the upstream country and presenting a credible threat of costly invasion as the downstream country. The result from these power dynamics in the game is that the resource gets allocated between the two countries in accordance with each country's respective level of power: measured as fixed geographic advantage as the upstream country and the effectiveness of armament purchases as the downstream country. Yet problem with threat of invasion as mechanism for resource allocation for the downstream country is that it is costly to construct with armament purchases.

The Peace with Threat of War non-invasion equilibrium best represents the current status quo in the Indus Basin conflict between Pakistan and India. The two counties have remained in an uneasy peace since the end of the Kargil War in 1999, yet Kashmir remains the most militarized area in the world with over a million troops from both countries stationed in the valley (Singh, 2016). With its growing population and increasing demand for water resources, India has begun building many large scale dams and hydro projects in the Kashmir that have the capacity to divert large amounts of water from flowing downstream to Pakistan (Khadka, 2016). Pakistan has responded with increasing military funding over the last several years and stationing more troops in the Kashmir (Kugelman, 2016). Pakistan has gone as far as promising that if India significantly diverts water in the Indus Basin, it would be an act of war (Kugelman, 2016). The implication of my analysis is that India is currently attempting to extract as much water in the Indus Basin as it can without provoking a war with Pakistan. Pakistan, on the other side, is using it's nuclear armaments in addition to its conventional forces to try and construct a military threat that will limit India's water extraction.

4 Comparative Statics

In this section I will show how the outcomes of the game change as the game parameters change. There are only three parameter values in the two country game: the total resource endowment, E, the fixed cost of invasion for Country B, C_B, and the fixed cost of invasion for Country A, C_A.[4] In the following subsections I will show how changes in the parameters affect the whether the game ends in non-invasion or invasion equilibrium and how changes to each parameter affects the strategies and outcomes within each type of equilibrium. Additionally in the discussion, I will provide an intuition of how the results of the comparative statics can be interpreted in the context of the Indus Basin conflict.

4.1 Change in Type of Equilibrium

For any given set of game parameters E, C_B and C_A, Country A is either better off in the non-invasion equilibrium or the invasion equilibrium. A knife-edge case occurs when the parameters are such that Country A is indifferent between the two equilibria:

Result 4. *For any given functional form, there exists a set of game parameters that will make Country A indifferent to the non-invasion equilibrium and the invasion equilibrium.*

$$u_A(X_A(m_B^\star; E, C_B)) = u_A(E \times w(m_B^{I\star})) - C_A \tag{8}$$

In equation (8), the payoff for Country A on the non-invasion equilibrium path are equal to the payoff for Country A on the invasion equilibrium path.

To show this, hold E constant at any non-zero or non-infinite value. Certain combinations of C_A and C_B will satisfy equation (8). Rewriting equation (8) with C_A on the left hand side:

$$C_A^I = u_A(E \times w(m_B^\star)) - u_A(X_A(m_B^{I\star}; E, C_B)) \tag{9}$$

By continuity, the value of C_A that makes Country A indifferent to either equilibrium is C_A^I and is a function of the other game parameters and Country B's armament strategy.

[4]Note that the functional forms of $u_i(\cdot)$, $w(\cdot)$, and $l(\cdot)$ also affect the two types of equilibrium and countries' strategy within each equilibrium. I do not consider these changes here.

Assuming E is fixed, C_A^I can be expressed solely as a function of C_B.[5] The function $C_A^I(C_B)$ can then be mapped in figure 3 as the knife edge between the two types of equilibria.

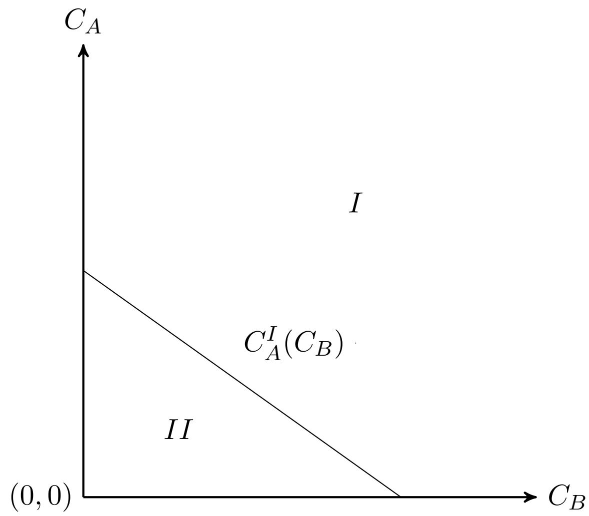

Figure 3: Knife-edge case between the two types of equilibria in the parameter space. I: Non-invasion, II: Invasion

The set of parameters that will make Country A indifferent to either equilibrium are the values of C_A^I that solve (9) for any given C_B. Inversely, the same set of parameters are the values of C_B^I that solve (9) for any given C_A. This is represented by the line shown in figure 3.

On the knife's edge, even when Country A is indifferent between the two types of equilibria, the invasion equilibrium will occur with certainty. This is because if Country B purchases $m_B^\star$ in the first stage of the game, Country A will better off by extracting E and forcing an invasion. Since $m_B^\star < m_B^{I\star}$, Country B will have under-spent on its armament and will be worse off. Therefore Country B purchases $m_B^{I\star}$ in the first stage of the game and the game proceeds on the invasion equilibrium path.

When E is not held constant, depending on functional forms, it can shift the knife's edge case in terms of the two-dimensional parameters space

[5] Country B's optimal non-invasion armament $m_B^\star$ is affected by changes in C_B as the parameter is in equation (4). With implicit differentiation, $m_B^\star$ can be expressed as function of C_B: $m_B^\star(C_B)$

shown in figure 3.[6] Changes to E will have an ambiguous effect on the knife's edge. Consider equation (9):

$$C_A^{I=1} = u_A(E \times w(m_B^\star)) - u_A(X_A^\star(m_B^{I\star}; E, C_B))$$

A increase in E is positive in the first term on the right hand side and negative in the second term, the direction and magnitude of the overall effect on C_A^I will depend on functional forms. Therefore a increase in E is ambiguous in $C_A^I(C_B)$ as the knife's edge between the invasion and non-invasion equilibrium.

4.2 Changes within Equilibrium

4.2.1 Invasion Equilibrium: War

When the changes to the game parameters are such that they do not affect the type of equilibrium that occurs, they can affect strategies and payoffs within the invasion equilibrium.

Result 5. *An increase to C_A such that $C_A < C_A^I$ will decrease Country A's payoff, but will not affect its equilibrium strategy of extracting the full resource endowment. An increase in E will increase Country A's payoff but will not affect its strategy.*

Consider Country A's payoff function within the invasion equilibrium:

$$u_A(E \times w(m_B^{I\star})) - C_A$$

An increase in C_A linearly decreases Country A's payoff, but does not affect its strategy of playing $X_A = E$. An increase in E is positive in Country A's payoff and will be discussed below, but does not affect its strategy. This is because within the invasion equilibrium, changes in the parameters will not make Country A concerned with preventing an invasion, and therefore it will continue to take the full resource endowment in the second stage of the game.

Result 6. *An increase in the resource endowment E increases Country B's optimal armament strategy as well as its payoff.*

[6] When $E = 0$ or $E = \infty$ the game becomes arbitrary. Changing E to either zero or infinity will change the equilibrium, but is an uninteresting case.

Consider the first order condition from equation (6) with a new resource endowment E' where $E' > E$:

$$-E' \times \frac{\partial u_B}{\partial m_B^I}(E' - E' \times w(m_B^I)) \times \frac{\partial w}{\partial m_B^I}(m_B^I) - \frac{\partial l}{\partial m_B^I}(m_B^I) = 0$$

Country B solves equation (6) with the new resource endowment E', which results in a new optimal invasion armament $m_B^{I\star'}$ where $m_B^{I\star'} \geq m_B^{I\star}$. Therefore with the change in E to E' Country B changes its equilibrium strategy from $m_B^{I\star}$ to $m_B^{I\star'}$. An increase in E is positive in Country B's payoff. With its new armament purchases, Country B it will receive the same or greater fraction of the resource endowment that is now expanded. Due to the convexity of the cost function, Country B's gains in resource utility will be greater in magnitude then the increased cost of armament. Country A may lose a greater fraction of the total resource endowment in the invasion due to Country B's increased armament, but will still have a higher payoff as the absolute increase in E will be greater in magnitude then the additional fraction of resources lost by Country A in the invasion.

4.2.2 Non-Invasion Equilibrium: Peace with Threat of War

When the changes to the game parameters are such that they do not affect the type of equilibrium that occurs, they can affect strategies and payoffs within the non-invasion equilibrium. The game parameter C_A is unimportant within the non-invasion equilibrium as long as $C_A > C_A^I$.

Result 7. *An increase in C_B will increase Country A's optimal resource extraction, which leads to an increase in Country B's optimal armament. Country A's payoff will increase and Country B's payoff will decrease.*

Consider equation (1) from which Country A's optimal extraction best response function is derived:

$$u_B(E - X_A \times w(m_B)) - C_B = u_B(E - X_A)$$

As C_B increases, the payoff for Country B invading in response to any given extraction strategy X_A decreases, given on the left hand side of equation (1). This allows Country A to choose a higher X_A and have (1) hold. Therefore an increase from C_B to C'_B changes Country A's extraction best response from $X_A(m_B; E, C_B)$ to $X_A(m_B; E, C'_B)$ where $X_A(m_B; E, C'_B) > X_A(m_B; E, C_B)$.

Country B, in the first stage of the game, solves the first order condition from equation (4) with Country A's new best response function:

$$-\frac{\partial u_B}{\partial m_B}(E - X_A(m_B; E, C'_B)) \times \frac{\partial X_A}{\partial m_B}(m_B; E, C'_B) - \frac{\partial l}{\partial m_B}(m_B) = 0$$

The solution to equation (4) will be a new optimal armament strategy $m_B^{\star'}$ that is greater than the old optimal strategy $m_B^\star$. With C'_B Country B will play $m_B^{\star'}$ as its optimal armament strategy. Country A will then use its best response function to play its optimal extraction strategy considering Country B's new optimal armament: $X_A^{\star'} = X_A(m_B^{\star'}, E, C'_B)$. The increased armament is negative in X_A, but the increased fixed cost of invasion for Country B is positive in X_A. Due to the convexity of the armament cost function $l(m_B)$, the increase in armament will have a smaller effect on $X_A^\star$ then the increase in C_B. This means that $X_A^{\star'} > X_A^\star$ and Country A will have a higher payoff. This also means Country B will have a lower payoff as $E - X_A^{\star'}$ will decrease while $l(m_B^{\star'}) - l(m_B^\star)$ is positive.

Result 8. *An increase in E will increase Country A's optimal resource extraction and will increase Country B's optimal armament. The payoffs for both Country A and Country B will increase.*

From equation (1), an increase in E to E' will change Country A's best response function to $X_A(m_B; E', C_B)$ where $X_A(m_B; E', C_B)$ is greater than $X_A(m_B; E', C_B)$. This is true even as E is both on the left hand side and right hand side of equation (1), because $w(m_B) \in (0, 1)$ ensures that an increase in E will increase the right hand side by a greater magnitude, allowing X_A to increase for equation (1) to hold. Simply put, if the resource endowment expands, Country A will receive at least some of the additional resources.

Country B, in the first stage of the game, solves the first order condition from equation (4) with Country A's new best response function as well as the expanded resource endowment:

$$-\frac{\partial u_B}{\partial m_B}(E' - X_A(m_B; E', C_B)) \times \frac{\partial X_A}{\partial m_B}(m_B; E', C_B) - \frac{\partial l}{\partial m_B}(m_B) = 0$$

The solution to equation (4) will be a new optimal armament strategy $m_B^{\star''}$ where $m_B^{\star''} \geq m_B^\star$. With E', County B will play $m_B^{\star''}$ in the first stage of the game. Country A will then use its best response function to play its new optimal extraction strategy in the second stage: $X_A^{\star''} = X_A(m_B^{\star''}, E', C_B)$. Again, the increased armament is negative in X_A, but the positive effect of

$E' > E$ in X_A will outweigh it in magnitude due to the convexity of $l(m_B)$. Therefore $X_A^{\star''} > X_A^\star$ and Country A will have higher payoff.

This does not mean that Country B will have a lower payoff however, as the expanded endowment no longer means gains by Country A are matched by losses for Country B. Due to the convexity of the cost function $l(m_B)$ along with the concavity of the utility function $u_B(\cdot)$, Country B's armament increase from $m_B^\star$ to $m_B^{\star''}$ will be such that:

$$u_B(E' - X_A^{\star''}) - u_B(E - X_A^\star) > l(m_B^{\star''}) - l(m_B^\star)$$

The gains from the additional armament and expanded resource endowment will outweigh the additional cost of armament. It will never be optimal for Country B to spend more on additional armament then what it gains from the additional resources flowing downstream. Therefore Country B will also have a higher payoff.

4.3 Comparative Statics Discussion

From result 4, when the fixed cost of invasion is sufficiently low for both countries, the combination of fixed costs for both countries will be below the indifference curve and the game ends in the invasion equilibrium of Invasion. Today in the Indus Basin, the fact that both Pakistan and India have nuclear weapons raises the costs of war significantly. In 1947, however, the newly created states of Pakistan and India had far lower costs of war with smaller and purely conventional armies. Bose (2003) shows that when it looked like India would annex the entire Kashmir region and control the entire Indus Basin, Pakistan invaded the Kashmir in an effort to annex it first. Pakistan, as a strategic move, decided it was better to invade and bear the costs of that invasion rather than concede the water resources of the Kashmir, which already at the time were critical to the Pakistani agricultural industry. Ultimately, in part because of its lack of modern armaments, Pakistan was only able to take about a third of the Kashmir in its invasion.

As the cost of invasion rises for both countries, the combination of fixed costs will move onto and beyond the indifference curve in the parameter space and the game will end in the non-invasion equilibrium of Peace with Threat of Invasion. Beginning after the Indo-Pakistani War of 1947, India recognized that completely dominating the water resources in the Kashmir would inevitably lead to more conflict with Pakistan (Kugelman, 2016). In the Peace with Threat of Invasion equilibrium the upstream country reduces its water extraction to prevent conflict. In 1960, Pakistan and India signed the Indus Waters Treaty which put a cap on India's water extraction in the Indus Basin. The Indus Waters Treaty and its extraction caps have become the current focus of the status-quo of the Indus Basin conflict (Kugelman, 2016). Yet as indicated in my analysis of the Peace with Threat of Invasion equilibrium, India honoring the treaty is based largely on Pakistan's military threat.[7] The treaty, however, serves as useful focal point for India to indicate that will not attempt to take as much of the water resources in the Indus Basin as it can, which would provoke a war with Pakistan.

Result 7 states that as the fixed cost of invasion for Country B goes up, it will spend more on armament but have a lower payoff. This models well what currently happening to Pakistan. India's military power, matching its economy, is growing quickly in terms of defense spending and capability (Wirsing, 2016). It is possible in the future that Pakistan will have great difficultly invading India, regardless of what it spends on its own armaments. While Pakistan's nuclear arsenal may be able to impose catastrophic costs on India in the event of war, Pakistan's own cost of invasion would also be so high that the benefit from the water resources it could secure in an invasion would be reduced dramatically. That means India can largely ignore Pakistan's nuclear bluff and extract a high level of water resources in the Indus Basin. In recent years Pakistan has been forced to spend more and more on its military in order to remain competitive with India and present a credible threat of invasion (Wirsing, 2016). The higher the spending on armaments by the downstream country, the more inefficient the armament becomes as mechanism to allocate the water resource compared to a Pareto-improving outcome where the resource is allocated without armament spending.

Result 8 states that as the resource endowment increases, the downstream country will spend more on armament but will also get a higher payoff while the upstream country also gets a higher payoff. According to Briscoe et al. (2006), updating the outdated water infrastructure in the Kashmir could expand the available water resources dramatically and benefit both India as the upstream country and Pakistan as the downstream country. The issue, however, is that in the model the downstream country will spend more on armament to ensure the expanded water resources are not extracted by the upstream country. This is precisely how Pakistan has reacted to major Indian dam projects and infrastructure in the Kashmir. The Indian built dams can increase the total water supply in the Indus Basin, but also give India the capacity to withhold more water from Pakistan (Khadka, 2016). Therefore Pakistan feels it is necessary to threaten

[7] India and Pakistan have gone to war three more times since signing the treaty.

India with its military to make sure the dams are not used to withhold water resources. If the water resource endowment can be expanded and then allocated without armament spending, however, both countries could benefit greatly.

5 Pareto Improving Outcomes

The status-quo of the Indus Basin conflict between Pakistan and India is a non-cooperative equilibrium where Pakistan as the downstream country is forced to spend heavily on its military as mechanism for securing water resources. If Pakistan does not spend heavily on its military, it gives India the chance to take more water resources and leave Pakistan with less. This non-cooperative equilibrium is Pareto inefficient and in the real-world continuously risks war between the two countries. Solving the Indus Basin conflict with a Pareto-improving cooperative solution requires a new mechanism that changes how the water resources are allocated. In this section I will discuss the existence of Pareto-improving outcomes under the non-invasion Peace with Threat of Invasion equilibrium and two possible mechanisms that can be used to achieve those outcomes.

The non-invasion equilibrium is not Pareto-optimal due to the cost of Country B's armament purchases. A simple way to illustrate this is to begin by considering the non-invasion equilibrium strategies and payoffs. Holding Country A's resource extraction constant at $X_A^\star$, Country B forgos its armament purchases of $m_B^\star$ and instead uses a fraction of the money it would have spent on armament, $\alpha \times l(m_B^\star)$, as a transfer payment to Country A and refunds the rest back to itself $(1-\alpha)\times l(m_B^\star)$. This transfer payment and refund will make both countries better off.

The problem with this simple transfer payment, however, is that Country A has no incentive to hold constant its resource extraction once given the transfer. If Country B gets rid of all of its armament purchases in favor of a transfer payment and refund in the first stage of the game, Country A can easily increase its resource extraction from $X_A = X_A^\star$ to $X_A = E$ in the second stage without fear of invasion, making Country B even worse off than before. Therefore Country B will never agree to this transfer payment, even if it has the potential to create a Pareto-improving outcome.

The key to arriving at a Pareto-improving outcome is that Country B must credibly believe that when it disarms, Country A will not take advantage by increasing its water resource extraction. The way that the game is currently constructed, this is not possible. Changing the game with a new mechanism, however, can create the opportunity for Pareto-improving outcomes with the simple transfer payment. Two of these potential changes are infinite repetition and delayed or withheld transfer payment; both will be discussed below.

5.1 Infinite Repetition

Consider a change the in the game from single-period $T = 1$, to infinite periods $T = \infty$. Also assume that inequality (7) holds and Country A is better off in the non-invasion equilibrium. A Pareto-improving equilibrium with a transfer payment is feasible when the discount factor $\delta \in (0, 1)$ is sufficiently high. This follows from the Nash reversion Folk Theorem in infinite repetition as proved by Friedman (1971). In each time period t, Country A has two options: it can accept the transfer payment from Country B and "defect" from the transfer agreement by then taking the full resource endowment, or it can accept the transfer payment and hold to the agreement by maintaining its non-invasion equilibrium resource extraction amount $X_A^\star$. If Country A "defects" in any time period, both countries will move back to the original non-cooperative non-invasion equilibrium for duration of the infinite game. A Pareto-improving outcome is possible when the following inequality holds true for a potential defection in the first period:[8]

$$\sum_{t=1}^{\infty} \delta^t [u_A(X_A^\star) + \alpha \times l(m_B^\star)] \geq [u_A(E) + \alpha \times l(m_B^\star)] + \sum_{t=2}^{\infty} \delta^t [u_A(X_A^\star)]$$

$$\frac{[u_A(X_A^\star) + (\alpha \times l(m_B^\star))]}{1 - \delta} \geq [u_A(E) + (\alpha \times l(m_B^\star))] + \frac{[u_A(X_A^\star)]}{1 - \delta}\delta \tag{10}$$

The payoffs for Country A when holding to the agreement for all time periods is given on the left hand side of inequality (10) and outweigh the payoffs for "defecting" in the first period given on the right hand side of inequality (10). Solving for δ as a function of the parameter α:

$$\delta(\alpha) \geq \frac{-[u_A(X_A^\star) + (\alpha \times l(m_B^\star))] + [u_A(E) + (\alpha \times l(m_B^\star))]}{[u_A(E) + (\alpha \times l(m_B^\star))] - [u_A(X_A^\star)]} \tag{11}$$

As long as δ is large enough that equation (11) holds true, a Pareto-improving outcome is possible with infinite repetition: Country A will never "defect" and will agree to receive the transfer payment while holding its

[8]If equation (10) holds true for a defection in the first period, it will hold true for a defection in any other period as well.

own extraction constant. Equation (11) holds with equality when $\delta = \underline{\delta}(\alpha)$, which is the minimum discount factor necessary for the Pareto-improving outcome as a function of α. If $\delta > \underline{\delta}$, Country B will choose $\underline{\alpha}(\delta)$, which is the lowest possible transfer payment it can make while still having (11) hold with equality. Therefore in the Pareto-improving outcome from infinite repetition, (11) will hold with equality as both Country A and Country B maximize their payoffs.

The issue with infinite repetition as mechanism for arriving at a Pareto-improving outcome, however, is that it makes two key unrealistic assumptions that make it not very useful as a real-world solution to the Indus Basin conflict. First, the discount factor of utility for India is most likely very low; near term payoffs for the extraction of water resource are much more valuable than long term payoffs, especially now as fresh water is currently in high demand in India (Khadka, 2016). If the discount factor is below the threshold value of $\underline{\delta}$, inequality (11) will not hold, as punishment by Pakistan will be ineffective. Second, neither Pakistan nor India make strategic decisions on an infinite time horizon. The Indian federal government, for example, has a election cycle of five years and is unlikely to conduct strategic planning far beyond that. Pakistan's political cycle is even more unpredictable with frequent changes of power from non-democratic mechanisms. Therefore, the time horizon of any agreement is likely to limited. With a small time horizon or a low discount factor, this version of the Folk Theorem no longer applies and Country A and Country B fall back into a non-cooperative inefficient equilibrium.

5.2 Delayed or Withheld Transfer Payment with Third Party Arbitration

The best option for a new mechanism to create a Pareto-improving outcome without relying on unrealistic assumptions is to reserve the payout of the transfer payment to Country A until after the conclusion of the game, and withhold the transfer payment completely if Country A defects. Country B will make the transfer payment in the first stage of the game when it would have purchased armaments, but instead of paying Country A directly, it will give the transfer to a third-party institution. If Country A honors the agreement with Country B and maintains its extraction amount at the non-invasion equilibrium level, the third-party institution will give Country A the transfer payment and Country A's payoff will remain the same. If Country A defects from the agreement by increasing its extraction after Country B does not purchase armaments, then the third-party institution withholds the transfer payment and refunds it completely back to Country B.

Even with the delayed or withheld transfer payment mechanism, it is not guaranteed that Country A will not defect. The transfer payment must be sufficiently large such that it is better for Country A to not defect and receive the payment. The following inequality must hold true:

$$u_A(X_A^\star) + \alpha \times l(m_B^\star) \geq u_A(E) \tag{12}$$

The payoffs for Country A when it does not defect are given on the left hand side and when it does defect on the right hand side. If Country B wants a cooperative outcome, α must be sufficiently large such that inequality (12) holds. If inequality (12) holds, Country A prefers the transfer payment over defecting on the agreement. Country B will attempt to minimize α as much as possible while still having inequality (12) hold.

The presence of neutral institution that can act as an arbitrator between Country A and Country B allows for a Pareto-improving cooperative outcome. This mechanism is a more realistic option for the Indus Basin conflict. To some degree, there has already been neutral arbitration in the Indus Basin. Before the signing of the Indus Waters Treaty in 1960, the World Bank served as a mediator of the negotiations between Pakistan and India (Briscoe et al., 2006). More recently, the World Bank along with the Permanent Court of Arbitration at the Hague have stepped in to resolve disputes over dams in the Kashmir between Pakistan and India (Khadka, 2016). Yet third-party arbitration has been limited overall in the Indus Basin conflict. A new agreement between India and Pakistan that uses transfer payments as a mechanism for cooperation would require a much deeper involvement of a third-party to manage the transfer payments from Pakistan and monitor India's water extraction in the Indus Basin.

An optimistic scenario that uses third-party arbitration to create a Pareto-improving outcome in the Indus Basin conflict is reforming and expanding the Permanent Indus Commission (PIC). The PIC was created as a part of the Indus Waters Treaty, but as a institutional body designed to settle disputes between Pakistan and India has never functioned as anything more than as platform for political grandstanding between both countries (Briscoe et al., 2006). Yet the PIC could be reconstituted as a truly independent body to receive transfer payments from both countries in the form of water consumption fees in the Indus Basin. As Pakistan consumes more water, it would pay much higher fees, constituting a net transfer payment to the PIC. The PIC would then monitor water consumption by both countries in the Indus Basin, and assuming India remains within the constraints of the Indus Waters Treaty, payout the fees in the form of water infrastructure projects in the Indus Basin, with the aim of increasing the total water resource endowment. India would never receive a direct transfer payment, but as a result of the expanded infrastructure, would have the opportunity to extract more water resources in the future. Pakistan would also receive more water resources from the improved infrastructure, and through its transfer payments to the PIC that get passed on indirectly to India, safeguard the extraction limits set by the Indus Waters Treaty. Both countries would get more water resources, prevent a war over the Indus Basin, and limit the purchase of expensive armaments.

6 Conclusion

This paper constructs a simple two player game of sequential transnational resource extraction and potential invasion to analyze the Indus Basin conflict between Pakistan and India. In the game there are two possible types of equilibria, both with different characteristics. The non-invasion "Peace with Threat of War" equilibrium models the status-quo in Indus Basin conflict between Pakistan and India most effectively, but the invasion equilibrium "War" has an historical interpretation. In the non-invasion equilibrium the armament purchases by the downstream country function as mechanism to reallocate resources form the upstream country. This allocation mechanism is costly however, and the result is a Pareto-inefficient non-cooperative equilibrium. As the water resource endowment or the downstream county's fixed of invasion grows, the downstream will purchase additional armaments, further increasing the inefficiency.

The model developed in this paper also shows that if the fixed cost of invasion is low enough for both countries, the invasion equilibrium will occur. Ultimately, it is the upstream country that will make the strategic decision if an invasion occurs with its resource extraction, even as the downstream country is the country with armaments and the ability to invade.

The paper offers potential solutions for Pareto-improving outcomes to the non-cooperative equilibrium. A simple transfer payment under infinite repetition is the best theoretical solution, but carries heavy assumptions that do not necessarily hold in real-world conditions. Another solution is a delayed transfer payment mechanism that is built into a neutral third-party institution. The third-party institution can monitor the upstream country to make sure it honors a cooperative agreement and utilize the transfer payment in an effective manner such as investing in water infrastructure.

The delayed transfer payment with the third-party institution is the most realistic method of solving the Indus Basin conflict.

There are many ways to expand the model developed in this paper that are good avenues for further research. The first is to expand the game from single period to a multiple, but finite, period model. A dynamic game will be able to model the across time strategic interactions of the upstream and downstream country. It can also model the potential benefits of a sustained infrastructure program by the PIC that expands the total resource endowment over time and how it might benefit either country.

Another expansion of the model developed in this paper is to add more countries to the game. Adding a third, fourth, and fifth country situated along a transnational river will complicate the game, but may be useful in modeling other transnational river conflicts such as the Nile conflict between Egypt, Sudan, South Sudan, and Ethiopia or the Mekong River conflict between China, Laos, Cambodia, Thailand, and Vietnam. A multi-country model will allow for interesting strategic actions such as the downstream countries creating a coalition against the upstream country.

I view this paper as a simple contribution in the analysis of transnational resource conflicts as a subset of general resource conflicts. Transnational resource conflicts are already of great importance in the international sphere today and will likely become even more important as the expanding economies of the developing world demand more resources such as fresh water. Further theoretical and empirical study of these transnational resource conflicts is needed. Finding potential solutions to these conflicts is of particular relevance and need for a more peaceful world in the future.

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A Existence of Sub-game Perfect Nash Equilibrium

For simplicity, I will prove existence with sufficient conditions for equilibrium. Assumptions 1-3 are sufficient for the existence of a sub-game perfect Nash equilibrium in the model:

1. The utility functions are strictly increasing $u_i'(\cdot) > 0$, are concave $u_i''(\cdot) < 0$, and are continuously twice differentiable for both countries $i = A, B$.

2. The cost function $l(\cdot)$ is strictly increasing $l'(m_B) > 0$ and convex $l''(m_B) > 0$

3. The $w(\cdot)$ function is decreasing $w'(m_B) < 0$ for $\forall m_B > 0$, is fully differentiable for all positive values of m_B, and $w(0) = 1$ and $lim_{m_B \to \infty}$ $w(m_B) = 0$.

The game is sequential and all functions are common knowledge between both players, therefore this game has perfect information. Harris (1985) proves that sub-game perfect Nash equilibria exist for deterministic continuous games with perfect information when the following conditions are met:

1. For all time periods and players, the pure strategy space is compact: $\forall t \geq 1$ and $1 \leq i \leq N$, S_{ti} is compact.

2. For all time periods and players, the pure strategy space is Hausdorff: $\forall t \geq 1$ and $1 \leq i \leq N$, S_{ti} is Hausdorff.

3. The histories of the game is a closed subset of the pure strategy space: H is a closed subset of S.

4. For all time periods $t \geq 1$, the set of feasible actions given the history A_t is lower hemicontinious.

5. For all players, the payoff functions P_i are continuous.

Assumptions 1-3 are sufficient to satisfy the conditions set by Harris (1985) for sub-game perfect existence. Each action within the sub-game of either invasion or non-invasion, m_B and X_A, are distinct, continuous, and well defined in a sequential order. While the set of possibilities for the action m_B is not compact as it includes infinity, the convexity of $l(m_B)$ along with the concavity of the utility function limits the feasible strategies in the pure strategy space to values of m_B less than infinity. The histories within each sub-game are a closed set as each action has a well defined outcome. The set of feasible actions given the history in the game correspond to well defined outcomes that are nonempty, satisfying hemi-continuity. Finally, the payoff functions $u_i(\cdot)$ are continuous.

Country B's invasion decision I in the last stage of the game is not a continuous strategy. Yet, as shown above, each continuous sub-game where an invasion either occurs or does not occur exists as proven by Harris (1985). Therefore a sub-game perfect Nash equilibrium exists for the overall game.

B Numerical Examples

This appendix includes numerical examples of each equilibrium. The specific functional forms used in all of all of these numerical examples are:

$$u_i(\cdot) = Log(X_i) \quad i = A, B \quad w(m_B) = .94^{m_B} \quad l(m_B) = (1/1000) * (m_B)^2$$

Changes to parameter values will determine when each type of equilibrium occurs.

B.1 Non-Invasion Equilibrium

A numerical example of the non-invasion equilibrium Peace with Threat of War is when the parameters are as follows:

$$C_A = 1, \quad C_B = 1, \quad E = 10$$

I assume that inequality (7) holds and Country A wants to prevent an invasion. I will check that inequality (7) holds with these parameter values in section B.3. From result 1, Country A will choose its extraction X_A in the second stage of the game such that the functional form of equation (1) holds:

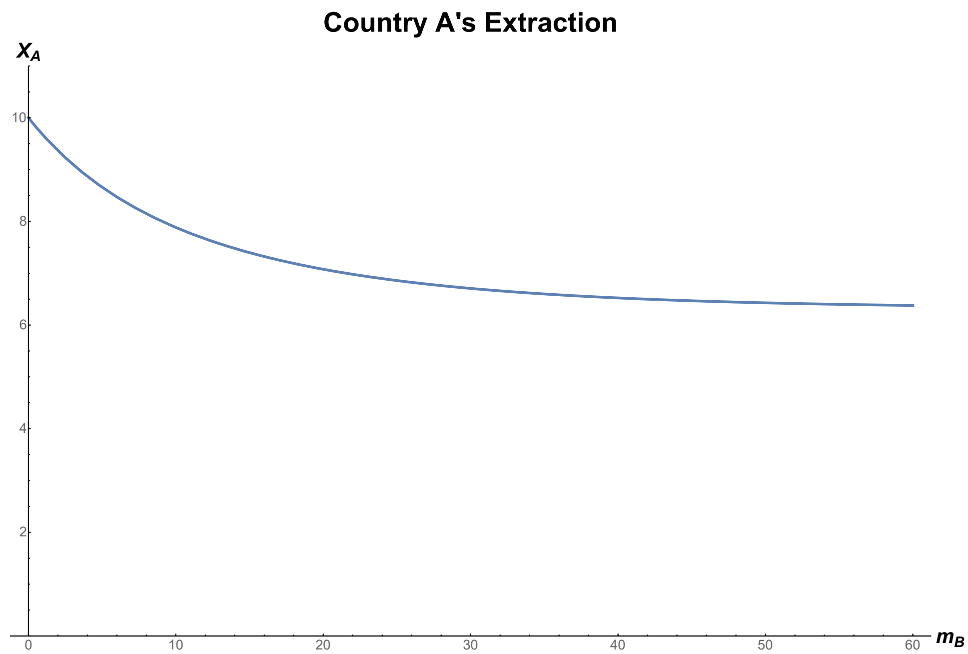
$$Log[E - X_A \times w(m_B)] - C_B = Log[E - X_A]$$

$$X_A(m_B; E, C_B) = \frac{E(-1 + e^{C_B})}{e^{C_B} - .94^{m_B}}$$

This becomes Country A's best response function given inputs. When the parameter values of E and C_B are inserted the function is only dependent on m_B:

$$X_A(m_B) = \frac{10(-1 + e)}{e - .94^{m_B}}$$

Shown below is the graph of Country A's extraction best response as a function of Country B's armament:



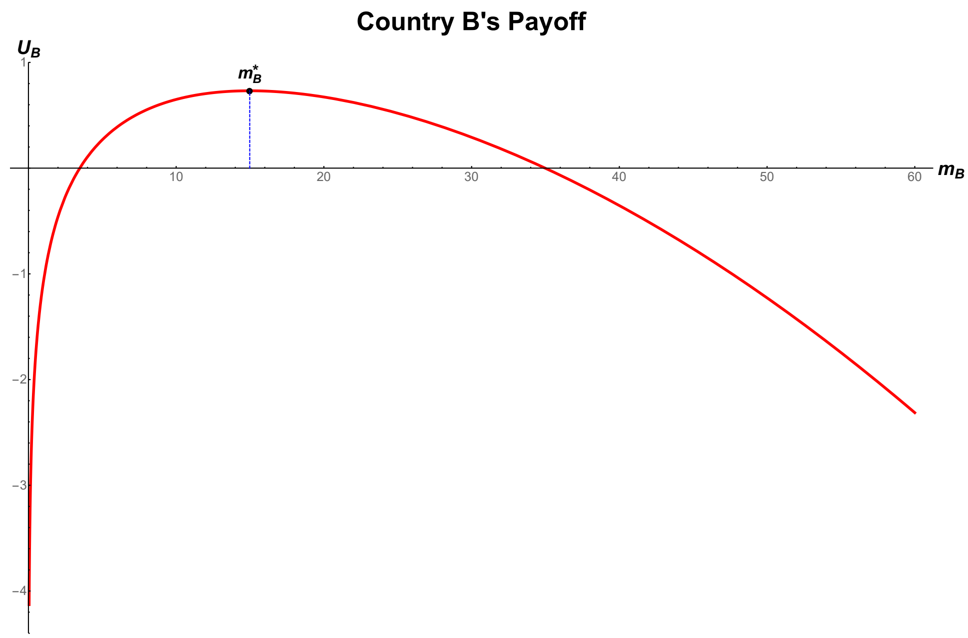
Graph 1: County A's best response extraction as a function of Country B's armament.

Knowing Country A's best response function and that Country A will prevent an invasion, Country B optimizes its armament purchases in equation (3) in the first stage of the game with $m_B^\star$ that solves equation (4):

$$\max_{m_B} Log[10 - \frac{10(-1+e)}{e - .94^{m_B}}] - (1/1000) * m_B^2$$

$$F.O.C: \frac{1.06 \times .94^{m_B}}{(-.94^{m_B}+e)^2(10 - \frac{10(-1+e)}{-.94^{m_B}+e})} - \frac{m_B}{500} = 0$$

The solution to this first order condition is $m_B^\star = 14.98$. This level of armament maximizes Country B's payoff, shown graphically below.



Graph 2: Country B's payoff as a function of its armament given that it will not invade.

Plugging this armament into $X_A(m_B)$ Country A's optimal exaction becomes $X_A^\star = 7.40$. When Country A extracts this much of the water resource, Country B is indifferent to invading and not invading, so Country A can prevent an invasion by extracting $X_A^\star - \epsilon$, which in this case approaches 7.40. The payoff for Country in the non-invasion equilibrium A is:

$$Log[X_A^\star]$$

$$Log[7.40]$$

$$2.01$$

The payoff for Country B in the non-invasion equilibrium is:

$$Log[E - X_A^\star] - (1/1000) * (m_B^\star)^2$$

$$Log[10 - 7.40] - (1/1000) * 14.98^2$$

$$0.73$$

B.2 Non-Invasion Equilibrium: War

A numerical example of the non-invasion equilibrium is when the parameters are as follows:

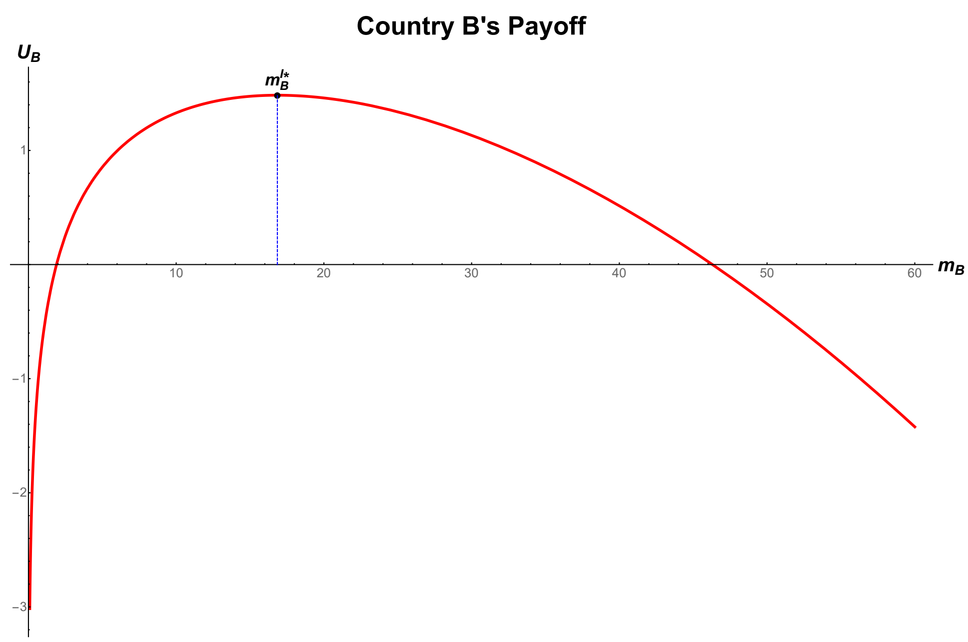
$$C_A = 0.1, \quad C_B = 0.1, \quad E = 10$$

I assume that inequality (7) does not hold and Country A is unconcerned with preventing an invasion. I will check inequality (7) does not hold in section B.3. From result 2, in the second stage of the game, Country A will choose $X_A = E$ regardless of what Country B does, forcing Country B to invade. In the fist stage of the game, Country B will choose an optimal invasion armament $m_B^{I\star}$. Solving the maximization problem from equation (5) with $m_B^{I\star}$ that solves the first order condition (6):

$$\max_{m_B^I} Log[E - E \times .94^{m_B^I}] - (1/1000)(m_B^I)^2 - C_B$$

$$F.O.C: \frac{0.62 \times 0.94^{m_B^I}}{10 - 10 \times 0.94^{m_B^I}} - \frac{m_B^I}{500} = 0$$

The solution to this first order condition is $m_B^{I\star} = 16.85$. This level of armament maximizes Country B's payoff, shown graphically below.



Graph 4: Country B's payoff function as a function of its armament given that it will invade.

With $X_A = E$, the payoff for Country A in the invasion equilibrium is:

$$Log[E \times .94^{m_B^{I\star}}] - C_A$$

$$Log[10 \times .94^{16.85}] - 0.1$$

$$1.16$$

The payoff for Country B in the invasion equilibrium is:

$$Log[E - E \times .94^{m_B^{I\star}}] - (1/1000)(m_B^{I\star})^2 - C_B$$

$$Log[E - E \times .94^16.85] - (1/1000)(16.85)^2 - 0.1$$

$$1.48$$

B.3 Checking for Sub-game Perfect Nash Equilibrium

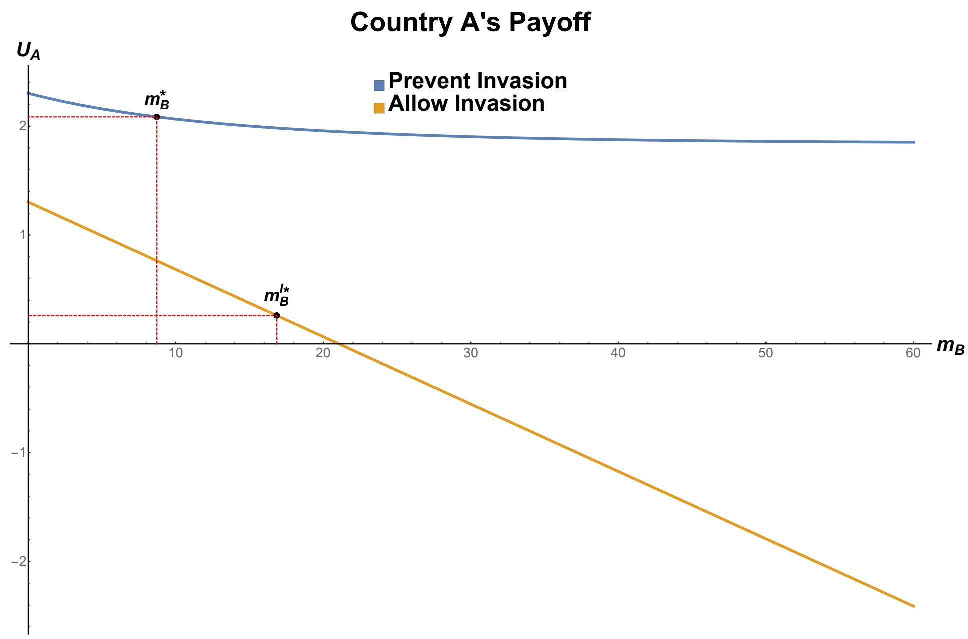
To check that the non-invasion equilibrium is the sub-game perfect Nash equilibrium in section B.1, the functional form of inequality (7) must hold given the parameters from section B.1. Note that $m_B^{I\star}$ is the optimal invasion armament for Country B that solves (6) given the B.1. parameters.

$$Log[X_A^\star] \geq Log[E \times .94^{m_B^{I\star}}] - C_A$$

$$Log[7.40] > Log[10 \times .94^{16.85}] - 1$$

$$2.01 > 0.38$$

Country A prefers the non-invasion equilibrium over the invasion equilibrium. This is shown below graphically with Country A's payoff being higher when it prevents an invasion compared to when it does not prevent an invasion:



Graph 3: Country A's payoff when preventing an invasion and letting an invasion occur.

Country A receives a higher payoff at $m_B^\star = 14.98$ by adjusting its resource extraction to prevent an invasion. Thus Country A chooses to extract $X_A^\star = 7.40$ and with the given parameters in B.1 the sub-game perfect Nash equilibrium is the non-invasion equilibrium.

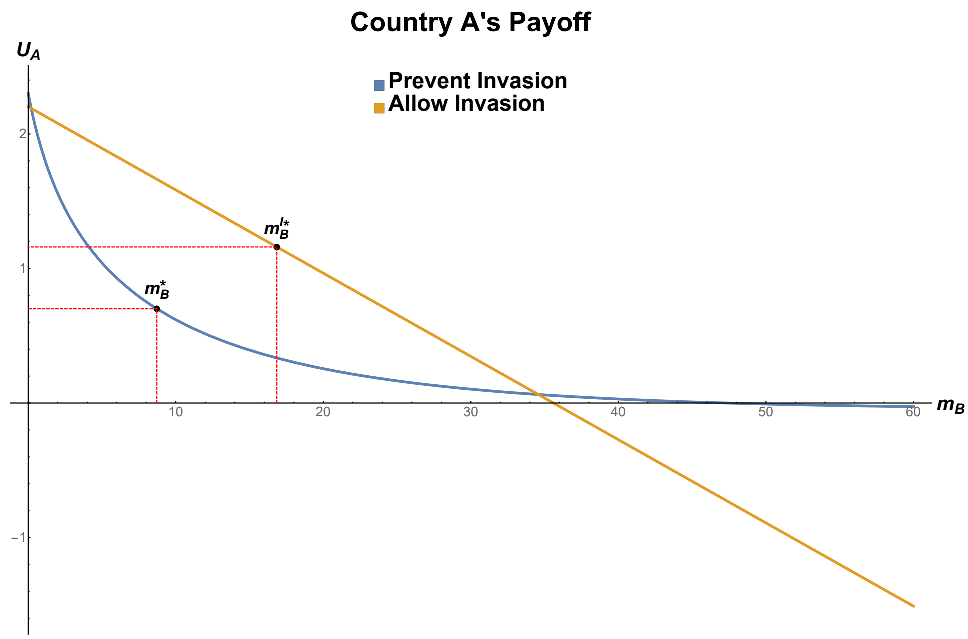
To check that the invasion equilibrium is the sub-game perfect Nash equilibrium in section B.2, the functional form of inequality (7) must not hold given the parameters from section B.2. Note that $m_B^\star$ is the optimal armament for Country B that solves (4) given the B.2 parameters. $X_A^\star$ is the optimal extraction for Country A that solves (1) given $m_B^\star$ and the B.2 parameters:

$$Log[X_A^\star] < Log[E \times .94^{m_B^{I\star}}] - C_A$$

$$Log[1.48] < Log[10 \times .94^{16.85}] - 0.1$$

$$0.70 < 1.16$$

Country A prefers the invasion equilibrium over the non-invasion equilibrium. This is shown below graphically with Country A's payoff:



Graph 5: Country A's payoff if its prevents an invasion and if it allows an invasion.

At $m_B^\star = 8.72$, Country A receives a higher payoff by choosing to extract E which forces Country B to invade. Therefore, knowing that Country A will extract E, Country B will purchase $m_B^{I\star}$ in the first stage of the game, and the game proceeds on the invasion equilibrium path. Thus, with the B.2 set of parameters, the sub-game perfect Nash equilibrium is the invasion equilibrium.