UNIVERSITY of HAWAIʻI® MĀNOA

University of Hawai`i at Mānoa Department of Economics Working Paper Series

Saunders Hall 542, 2424 Maile Way,
Honolulu, HI 96822
Phone: (808) 956 -8496
www.economics.hawaii.edu

Working Paper No. 17-2

Profit-Sharing and Implementation of Efficient Outcomes

By
Ruben Juarez
Kohei Nitta

May 2017

Profit-Sharing and Implementation of Efficient Outcomes*

Ruben Juarez[a] and Kohei Nitta[b]

[a]Department of Economics, University of Hawaii
[b]Department of Economics, Toyo University

May 8, 2017

Abstract

Agents are endowed with time that is invested in different projects that generate profit depending on the allocation of time by the agents. A mechanism divides the profit generated by the projects among agents depending on the allocation of time as well as the amount of profit that every project generates.

We study mechanisms that incentivize agents to contribute their time to the level that generates the maximal aggregate profit at the Nash equilibrium regardless of the production functions (*efficiency*). Our main result is the characterization of all the mechanisms that satisfy efficiency. Furthermore, within this class, a narrow class of mechanisms are monotone in the payoffs of the agents with respect to the addition of agents, time or projects.

Keywords: *Profit-sharing, Efficiency, Implementation.*

JEL classification: C72, D44, D71, D82.

*Financial support from the AFOSR Young Investigator Program under grant FA9550-11-1-0173 is greatly appreciated. We are grateful for helpful comments to Yves Sprumont, Justin Leroux, Katerina Sherstyuk, Gürdal Arslan and seminar participants at the Society for Economics Design Conference in Istanbul and the World Congress of the Econometric Society in Montreal. All remaining errors are our own.

1 Introduction

Profit sharing mechanisms are widely used by companies in order to increase profits. Such mechanisms include direct cash bonuses that are awarded based on their performance, either individually or collectively. Stock options, where employees get a share of the company, and thus are rewarded based on the aggregate profit of the company is a popular mechanism in this class. Although several studies address the effectiveness of profit sharing mechanisms to increase the productivity of a company, empirically and theoretically,[1] little has been said theoretically about the direct relationship between profitability of the company and the rewards to employees. Our paper is the first study in this area, characterizing a large class of mechanisms that align the benefits of the company with the payments of their employees.

We consider a model where agents decide how to allocate their fix endowment of time to different projects that generate profits. Every project could have different production functions that generate profit depending on the agents' time allocation. We focus on the case of asymmetric information, where a planner (such as the owner of the company or manager) does not know the production functions while the employees have perfect information and must assign the payments depending only on the final profit generated by each project and the time allocation of the agents to the different projects.[2] An application of this problem is the division of the end of year surplus in large companies.

The central issue is that although the planner tries to maximize the total profit of the company, he might not know the production functions. Therefore, the planner's goal is to find a mechanism that implements the time allocation which generates the maximum total profit at the Nash equilibrium for any set of production functions. We call this property *efficiency*. [3]

For instance, consider the proportional sharing mechanisms that divides the total profit of each project among the agents in proportion to their time allocation to such projects. This mechanism is not necessarily efficient because agents could have the incentive to put more time in the projects that gives them a larger proportion of time in order to get a larger share of the profits of such projects, whereas the company might produce a larger share when agents invest their time collectively into a single project. To see this, consider the following example. There are three agents named 1, 2 and 3 and three projects $\{12, 13, 123\}$ that generate profit. Each agent is endowed with 1 unit of time. The production functions of such projects are $\alpha(t_1^{12} + t_2^{12})$ for the project 12, $\beta min(t_1^{13}, t_3^{13})$ for the project 13 and $\gamma min(t_1^{123}, t_2^{123}, t_3^{123})$ for the project 123. When $\alpha = 2.5$, $\beta = 3$ and $\gamma = 6$, the efficient time allocation $((t_1^{12}, t_1^{13}, t_1^{123}), (t_2^{12}, t_2^{123}), (t_3^{13}, t_3^{123})) = ((0, 0, 1), (0, 1), (0, 1))$ requires all agents to invest their time into the project 123 and generates a profit equal to 6 units. Under proportional sharing, each agent receives 2 units of the profit each. However, this is not an equilibrium since either agent 1 or 2 have the incentive to allocate all resources to the project 12, where they can receive 2.5 units of profit instead of 2.

[1] Empirical studies of profit sharing in companies include Kruse (1992), Bhargava (1994) and Kraft and Ugarković (2006), while Weitzman and Kruse (1990) and Prendergast (1999) provide surveys of profit sharing in companies. In particular, the stock options mechanisms is widely spread in silicon valley start-ups and has created several millionaire employees, for instance at Google, Facebook and Yahoo.

[2] This asymmetric information is natural in large companies, where the owner (board) of the company sets general profit-sharing policies for employees before the production functions are realized.

[3] Our study considers the case of information asymmetry between the agents and the planner. In particular, we assume that agents have perfect information about the production functions and use this information to make the allocation of their time. On the other hand, the planner does not observe the production function but instead observes the total profit generated by every individual project as well as the individual time contributions of the agents to the different projects. Thus, even though the planner has a disadvantage in the information with respect to the agents, this notion of *efficiency* leads to the first best outcome of the planner, as if the planner had full information.

On the other hand, consider the average profit mechanism, where each agent gets a fixed share of the total profit generated by the company.[4] This mechanism is efficient because if an agent deviates from the equilibrium that generates the maximum total profit (efficient equilibrium), then the total profit of the company will not increase and neither does his payment.

Alternatively, consider the Shapley mechanism, where the profit of every project is distributed equally among the agents who belong to each project irrespective of their time allocation. When the set of feasible projects have the same size, for instance if all the projects have size 2, the Shapley mechanism is also efficient. This is because if an agent deviates from the efficient equilibrium, then the total profit of the company will not increase. Thus, since the profit of the company is just the sum of the profit of each project, the total profit of the projects in which this agent belong to will not increase, and neither does his payment. However, in general the Shapley mechanism is not efficient when the projects have different size.[5]

More generally, consider a mechanism where the payoff of an agent only depends positively on the total (aggregate) profit generated by the projects in which the agent belongs, as well as the time contributions and profit generated by the projects in which the agent does not belong. We call these mechanisms *separable*. Those mechanisms are efficient because if an agent decides to change his time allocation, then his payoff can only influence the total output of his projects. If the total output of his projects decreases, so does the total profit generated by the company; thus, the agent is worse off. The average profit and Shapley mechanisms are particular cases of separable mechanisms.

Our main result is the characterization of all the mechanisms that satisfy efficiency. The class of efficient mechanisms coincide exactly with the class of separable mechanisms (Theorem 1). We also look at the monotonicity of the payments of the agents with respect to the addition of agents, projects or time. Corollary 2 shows that a very narrow class of efficient mechanisms that depend only on the aggregate value of the profit generated by the company (such as the average profit mechanisms) are robust to meet such monotonicity properties. Furthermore, this narrow class of mechanisms are the only mechanisms where the efficient equilibrium is a strong Nash equilibrium, where agents cannot gain by jointly coordinating their time allocation to the different projects.

The rest of the paper is as follows: Section 1.1 surveys the relevant literature. Section 2 describes the model and the main result of the paper. In Section 3, we provide comparative statics and group manipulations to our model. Finally, we conclude in Section 4.

1.1 Related Literature

The concept of implementation of the efficient allocation in a Nash equilibrium has been explored widely in the literature. Maskin and Sjöström (2002) survey full implementation of efficient outcomes in different production functions. However, the literature of implementation in very general economies has typically lead to impossibilities. We contrast with this literature by finding a specific economy where several mechanisms can implement the efficient allocation.

We focus on the case where agents need to contribute their full time allocation and the entire profit is allocated to the agents, therefore the traditional issues of moral hazards are ruled out (e.g., Holmstrom, 1982). This restriction is similar to allocation mechanisms for a fixed divisible

[4]This mechanism can be interpreted as the stock awarding mechanism, in which agents are given a fix share of stocks in the company, and thus their final allocation of profit depends on the aggregate profit generated by the company.

[5]This can be seen in the example above, where projects of size 2 and size 3 exists. Herein, agents 1 and 2 have the incentive to deviate from the efficient equilibrium.

resource (such as a dollar) depending on the report of the agents (e.g., de Clippel, Moulin and Tideman, 2008 and Tideman and Plassmann, 2008).

A closely related work studies cost sharing allocation mechanisms that implement the cost minimizing network. For instance, Juarez and Kumar (2013) focus on implementing the efficient allocation in connection network, where agents should be provided with incentive to select the cost minimizing network. This equilibrium should Pareto dominate all the other equilibriums. Furthermore, the class of mechanisms characterized in Juarez and Kumar (2013) are closely related to the mechanisms characterized in Proposition 2. Other closely related work, Hougaard and Tvede (2012, 2015) characterize truthfully implementing cost minimizing networks by changing the announcement rule. Their main results also lead to characterizing rules related to Proposition 2. In contrast with this literature, the bilateral setting allows us to implement a larger class of rules that have not been explored in the implementation literature.

Our work is the first to introduce the implementation of the efficient time allocation for any set of production functions.

2 The Model

Let $N = \{1, 2, \ldots, n\}$ be the set of agents in an economy. Groups of agents from N can collaborate in projects that generate profit depending on the time allocation of the agents on each project.[6] Formally, let $L \subset 2^N \setminus \{\emptyset\}$ be the set of different projects. For instance, if L contains all subsets of 2 agents, then every group of two agents can collaborate in a project. If $L = 2^N \setminus \{\emptyset\}$ then any potential coalition can collaborate in a project. Let L_i be the set of groups from L that contains agent i and L_{-i} be the set of groups from L that do not contain agent i.

For a given set A and $T \geq 0$, let $\Delta(T, A) = \{x \in \mathbb{R}^A_+ | \sum_{i \in A} x_i = T\}$ be the T-simplex over the set A. Every agent i is endowed with T_i units of time which he can split among the projects in which he belongs to. The set of time allocations of agent i is the set $D_i = \Delta(T_i, L_i)$. Let $\mathbb{D} = \Pi_{i \in N} D_i$ be the set of all time allocations for all agents. For a given time allocation $t \in \mathbb{D}$, the amount t^K_i is interpreted as the amount of time that agent i spends in project K. For this section, we fix the group of agents N, group of projects L and time endowments $T_1, T_2, \ldots, T_n$. Section 3 will look at the possibility of changes with respect in N, L and $T_1, T_2, \ldots, T_n$.

Every project generates profit. Let $\mathbb{F} = \mathbb{R}^L_+$ be the vector of profits for all projects. For a given $F \in \mathbb{F}$, the amount F^K is the profit generated by project $K \in L$.

Definition 1 (Mechanisms). *Fix N, L and $T_1, T_2, \ldots, T_n$. A **mechanism** is a continuous function $\varphi : \mathbb{D} \times \mathbb{F} \to \mathbb{R}^n_+$ such that*

$$\sum_{i=1}^{n} \varphi_i (t, F) = \sum_{K \in L} F^K.$$

The inputs on a mechanism are the different time allocations and profits of every project. The output is a full distribution of the total profit to the agents.

Example 1. A. ***Average profit mechanism***: *the final profit of the entire economy is divided equally among all members. That is, for any $i \in N$,*

$$\varphi_i(t, F) = \frac{1}{n} \sum_{K \in L} F^K$$

[6] For simplicity, we assume that there is no repetition in the projects, although a similar argument can be made when projects repeat.

B. ***Shapley mechanism****: the final profit produced by the project $K \in L$ is shared equally among the agents in K. The total share of every agent is the sum of his shares in the projects. That is, for any $i \in N$,*

$$\varphi_i(t, F) = \sum_{K \in L_i} \frac{F^K}{|K|},$$

where $|K|$ is the number of agents in the project K.

C. ***Proportional sharing mechanism****: the final profit produced by the project K is shared in proportion to the contribution of time of the agents in K. That is, for any $i \in N$,*

$$\varphi_i(t, F) = \sum_{K \in L_i} Pr_i^K(t^K) F^K, \text{ where } Pr_i^K(t^K) = \begin{cases} \frac{t_i^K}{\sum_{j \in K} t_j^K} & \text{if} \quad \sum_{j \in K} t_j^K > 0 \\ 0 & \text{if} \quad \sum_{j \in K} t_j^K = 0 \end{cases}$$

D. ***Generalized Shapley Mechanism****: The share of every agent is a fixed proportion of the profit generated by the projects in which he belongs and the unallocated profit of the projects in which the agent does not belong. Formally, consider any collection of projects $L \subset 2^N \setminus \{\emptyset, N\}$ and positive individual shares $\gamma_1, \gamma_2 \ldots, \gamma_n$ such that for any project $K \in L$, the aggregate shares in this project does not exceed 1, $\sum_{j \in K} \gamma_j \leq 1$. For any individual agent i,*

$$\varphi_i(t, F) = \gamma_i \sum_{K \in L^i} F^K + \sum_{M \in L^{-i}} \frac{\gamma_i}{\sum_{l \in N \setminus M} \gamma_l} (1 - \sum_{j \in M} \gamma_j) F^M.$$

Notice that the average profit, Shapley and generalized Shapley mechanisms are independent of time allocation. On the other hand, the proportional sharing mechanism is dependent on the time allocation. Any convex combination of mechanisms is also a mechanism.

2.1 Efficiency and other desirable properties

The strategy of agent i is an allocation of his time resource between different projects to which he belongs. Let $f = (f^K)_{K \in L}$ be the vector of production functions, where $f^K : \mathbb{R}_+^k \to \mathbb{R}_+$ is a continuous and non-decreasing function on both coordinates. Let $\mathcal{F}$ be the set of vectors of production function.

We study the perfect information non-cooperative game where the strategy of agent i is a function from $\mathcal{F}$ to D_i that assigns to every set of production functions a time allocation for each project. Let $\mathcal{S}_i$ be the set of functions from $\mathcal{F}$ to D_i. The payoff of an agent depends on its own and others' time allocation, production functions and the outcome given by the mechanism.

Definition 2 (Non-cooperative Game G^φ)**.** *Given a mechanism φ, we study the* ***non-cooperative game*** *$G^\varphi = [N, (\mathcal{S}_1, \ldots, \mathcal{S}_n), (\pi_1, \ldots, \pi_n)]$ where*

- *the* ***strategy space*** *of agent i is $\mathcal{S}_i$*

- *the* ***payoff function*** *of agent i at the vector of strategies (S_i, S_{-i}) and production function vector $f \in \mathcal{F}$ is*

$$\pi_i^\varphi(S_i, S_{-i}, f, \varphi) = \varphi_i\left((t_i, t_{-i}), \left[f^K(t^K)\right]_{K \in L}\right), \quad \text{where } t_j = S_j(f) \; \forall j \in N$$

We are interested in Nash equilibrium strategies, where agents have no incentive to deviate, assuming the strategies of the other agents remain fixed. Under a vector of production functions, a set of strategies generates outputs for the different projects. An efficient strategy dominates any other strategy for any set of production functions. We say that a mechanism is efficiency if an efficient strategy can be supported as a Nash equilibrium.

Definition 3 (Nash Equilibrium and Efficiency)**.** • *A strategy profile* $(S_1^*, S_2^*, \ldots, S_n^*)$ *is a* ***Nash equilibrium*** *of the game* G^φ *if for any production function vector* $f \in \mathcal{F}$*,*

$$\pi_i^\varphi(S_i^*, S_{-i}^*, f) \geq \pi_i(S_i, S_{-i}^*, f, \varphi), \quad \forall S_i \in \mathcal{S}_i$$

- *A* ***set of strategies*** $(S_1, S_2, \ldots, S_n)$ ***is efficient*** *if for any* f *and for any other strategy* $\tilde{S}$

$$\sum_{K \in L} f^K(t^K \geq \sum_{K \in L} f^K(\tilde{t}^K), \quad \textit{where } t_i = S_i(f) \textit{ and } \tilde{t}_i = \tilde{S}_i(f)$$

- *A* ***mechanism is efficient*** *if a set of efficient strategies is a Nash equilibrium.*

As discussed in the introduction, the average profit mechanism is efficient. The Shapley and the generalized Shapley mechanisms are efficient for some projects L.

2.2 Separable mechanism and the main result: implementing the efficient time allocation

In this section, we characterize the mechanisms that are efficient. There are several restrictions that efficiency imposes on a mechanism. The first restriction is that the payoff of an agent should depend on the aggregate profit generated by the projects in which he belongs, instead of the profits of individual projects. The second restriction is that the time allocation of an agent should not influence his payoff (but it might influence the payoff of other agents). The separable mechanisms discussed below include these two restrictions.

Definition 4. *A* ***mechanism*** φ ***is separable*** *if there exist functions* $(g_1, g_2, \ldots, g_n)$ *which are non-decreasing in the first coordinate such that*

$$\varphi_i(t, F) = g_i\left(\sum_{K \in L_i} F^K, \times_{B \in L_{-i}}(t^B, F^B)\right) \quad \forall i$$

A mechanism is separable if the payoff of agent i only depends on the total aggregated profit generated by his projects, $\sum_{K \in L_i} F^K$, as well as the profits and time allocations that do not contain agent i, $\times_{B \in L_{-i}}(t^B, F^B)$. The class of separable mechanisms is large. We provide below some examples. Corollary 1 provides the entire class of separable mechanisms under two additional assumptions.

Example 2. *A. The average profit mechanism is a separable mechanism generated by the functions*

$$g_i^{AP}\left(\sum_{k \in L_i} F^K, \times_{B \in L_{-i}}(t^B, F^B)\right) = \frac{\sum_{H \in L} F^H}{n} \quad \forall i$$

B. *Assume that the grand coalition is not feasible, that is $N \notin L$. Consider the mechanism $\varphi_i^*(t, F) = \sum_{B \in L^{-i}} \frac{F^B}{|N \setminus B|}$, for any $i \in N$, where every agent gets paid the average profit of the projects in which he does not belong. The mechanism is separable and generated by the functions,*

$$g_i^* \left(\sum_{K \in L_i} F^K, \times_{B \in L_{-i}} (t^B, F^B) \right) = \sum_{B \in L_{-i}} \frac{F^B}{|N \setminus L_{-i}|} \quad \forall i$$

C. *Shapley is a separable mechanisms only when the set L contains coalitions of the same size c. In this case,*

$$g_i^{Sh} \left(\sum_{K \in L_i} F^K, \times_{B \in L_{-i}} (t^B, F^B) \right) = \frac{1}{c} \sum_{K \in L_i} F^K \quad \forall i$$

D. *The generalized Shapley mechanism is a separable mechanism generated by the functions*

$$g_i^{GSh} \left(\sum_{K \in L_i} F^K, \times_{B \in L_{-i}} (t^B, F^B) \right) = \gamma_i \sum_{K \in L^i} F^K + \sum_{M \in L^{-i}} \frac{\gamma_i}{\sum_{l \in N \setminus M} \gamma_l} (1 - \sum_{j \in M} \gamma_j) F^M \quad \forall i$$

Note that the convex combination of separable mechanisms is also a separable mechanism generated by the convex combination of the g functions. The proportional sharing mechanism is not separable, because the payoff of an agent depends on his allocation of time to different projects.

Theorem 1. *A mechanism is efficient if and only if it is separable.*

The proof is in Appendix A.

We say that a mechanism is **anonymous** if it is independent of the name of the agents. This means that agent i could be replaced with agent j, but the allocations and outputs are the same. We say that a mechanism is **time-independent** if the mechanism only depends on the profit generated by the different projects and not on the time allocated to different projects. The class of efficient and symmetric mechanism is large. We characterize below the class of efficient, symmetric and time-independent mechanisms.

Corollary 1.

- *Consider an integer c such that $0 < c < n$ and let $L^c = \{S \subset N \mid |S| = c\}$ be the set of projects that are of the same size c. A mechanism is efficient, anonymous and time-independent in L^c if and only if it is a convex combination of Shapley and φ^*.*

- *Consider $L^T = \cup_{c \in T} L^c$ for some $T \subseteq \{1 \ldots, n-1\}$. A mechanism φ is efficient, anonymous and time-independent in L^c if and only if φ is a generalized Shapley mechanism with the same weight for every agent. That is, there exists $0 \leq \alpha \leq min_{l \in L} \frac{1}{|l|}$ such that $\varphi_i(F_i, F_{-i}) = \alpha \sum_{S \subset L_i} F^S + \sum_{T \subset L_{-i}} \frac{1 - |T|\alpha}{n - |T|} F^T$.*

- *Consider L such that $N \in L$. A mechanism is symmetric, efficient and time independent if and only if it is the average profit mechanism.*

The implications of this corollary are that whenever agents are substitutes and the grand coalition is not a feasible project, no mechanisms is anonymous and time independent. On the other hand, when the grand coalition is feasible, only the average profit mechanisms meet these properties.

3 Monotonicity and group manipulations

Until now, we have fixed the number of agent $N = \{1, \ldots, n\}$, the time allocations $T = (T_1, \ldots, T_n)$ and the set of projects L. For each problem $[N, T, L]$, the previous section find the class of efficient mechanisms. In this section, we focus on the robustness of such efficient mechanisms with respect to changes in N, L and T. In particular, we focus on the monotonicity of the allocation given a mechanisms with respect to the increase in N, T and L.

Given the problem $[N, T, L]$, we denote by $Eff^{\varphi}[N, T, L]$ the set of efficient Nash equilibria under φ.

Definition 5 (Time-monotonicity)**.** *A efficient mechanism φ is **time-monotonic at** $[N, T, L]$ if for any agent i, time $\tilde{T}_i > T_i$ and any efficient equilibrium $S^* \in Eff^{\varphi}[N, T, L]$, there exists an efficient equilibrium $\tilde{S} \in Eff^{\varphi}[N, (\tilde{T}_i, T_{-i}), L]$ such that $\varphi(S^*(f), f(S^*(f)) \leq \varphi(\tilde{S}(f), f(\tilde{S}(f))$.*

A particular case of time-monotonicity occurs when looking at time monotonicity in the problem $[N, T, L]$ with $T_i = 0$ and $\tilde{T}_i > 0$. We can interpret this case as **agent-monotonicity** of the mechanism, where agent should not be worse-off when new agents join the game.

We say that the vector of production functions $\tilde{f}$ is a **technological improvement** of f if for any project $K \in L$ and time allocation t^K we have that $\tilde{f}^K(t^K) \geq f^K(t^K)$.

The next property requires that technological improvements should not harm any agent, regardless of whether or not they are participating in the project(s) that improved.[7]

Definition 6 (Technology-monotonicity)**.** *An efficient mechanism φ is **technology-monotonic at** $[N, T, L]$ if for any efficient equilibrium $S^* \in Eff^{\varphi}[N, T, L]$, any production functions f and any technological improvement $\tilde{f}$ of f, $\varphi(S^*(f), f(S^*(f)) \leq \varphi(S^*(\tilde{f}), f(S^*(\tilde{f}))$.*

A particular case of technology-monotonicity occurs for a technological improvement from the zero-technology, $f^K(t^K) = 0$ for all t^K, to a non-zero technology $\tilde{f}^K(t^K) > 0$ for some $\tilde{t}^K$. In this case, we can interpret this as **project-monotonicity**, where agents should not get worse-off as more projects are available.

The following property rules out the coordination of time by the agents at an equilibrium. This is capture by using a traditional notion of strong Nash equilibrium.

Definition 7 (Strong Nash equilibrium)**.** *We say that the Nash equilibrium $(S_1^*, \ldots, S_n^*)$ of the game G^{φ} is a **strong Nash equilibrium** if for any group of agents $T \subset N$ and strategies $\tilde{S}_T$ of them, if there exists a production function f such that $\varphi_i(S^*(f), f(S^*(f)) < \varphi_i(\tilde{S}(f), f(\tilde{S}(f))$ for some $i \in T$, then there exists $j \in T$ such that $\varphi_i(S^*(f), f(S^*(f)) > \varphi_i(\tilde{S}(f), f(\tilde{S}(f))$, where $\tilde{S} = (S_T, S^*_{-T})$.*

Corollary 2. *The following three properties are equivalent for the efficient mechanism φ:*

(i) *φ is time-monotonic*

(ii) *φ is technology-monotonic*

(iii) *there exists an efficient Nash equilibrium in the game G^{φ} that is a strong Nash equilibrium*

(iv) *there exists non-decreasing functions: $g_i : \mathbb{R}_+ \to \mathbb{R}_+ \quad$ for $i = 1, 2, \ldots, n$ such that*

$$\sum_i g_i(A) = A \quad \forall A \in \mathbb{R}_+$$

[7] Thomson (2007) defines a very similar concept, which is called "strict resource monotonicity" in allocation problems.

and

$$\varphi(t,F)=\left(g_1\left(\sum_{H\in L}F^H\right),g_2\left(\sum_{H\in L}F^H\right),\ldots,g_n\left(\sum_{H\in L}F^H\right)\right)\quad \forall t \textit{ and } F.$$

The proof of this Corollary is in Appendix A. Note that the average profit mechanism is the only anonymous mechanisms in this class.

4 Conclusion

We have introduced a large class of mechanisms that implement the efficient time allocation. Our main result shows that the class of efficient mechanisms coincide with the class of separable mechanisms, where the payoff of an agent only depends on the total profit generated by his own projects, as well as the time allocations and profits generated by by projects in which the agent does not belong to. This large class of efficient mechanisms is shrunk substantially when more robust monotonicity properties (on time and technology) are imposed. This is also true, when focusing on mechanisms that prevent agent to coordinate their time.

More work needs to be done to understand the sharing of profits in dynamics problems. Juarez, Ko and Xue (2016) have started such study by focusing in the axiomatic division of finite and sequential benefits in companies.

Appendix A. Proofs

Proof of Theorem 1

Proof. First, we show that if a mechanism is separable, then the mechanism is efficient.

Suppose that an agent, say agent i, deviates from an efficient strategy under a separable mechanism. Then, the deviation by agent i does not lead to an increase in total profit of his projects in any production functions due to the definition of efficient strategy. Thus, agent i cannot increase his payoff because g_i is a non-decreasing function in the first coordinate. This is a contradiction.

Next, we show that if a mechanism is efficient, then the mechanism is separable for any set of production functions.

Step 1: We show that if a mechanism is efficient, then the payoff of agent i does not depend on his time allocation. That is,

$$\varphi_i\left(t_i,t_{-i},F\right)=h_i\left(t_{-i},F\right),$$

where t_i is the strategy of agent i and $t_{-i}=(t_1,t_2,\ldots,t_{i-1},t_{i+1},\ldots,t_n)$ is the collection of all agents' strategies but agent i.

First, consider $\bar{k}\in L^i$ and define the production functions as follows:

$$\begin{aligned} f^k(t) &= c^k+\epsilon\left(\sum_{j\in k}t_j^k\right), \quad \text{for } k\neq\bar{k};\\ f^{\bar{k}}(t) &= c^{\bar{k}}+\epsilon\left(2t_2^{\bar{k}}+\sum_{j\in\bar{k}\setminus\{i\}}t_j^{\bar{k}}\right); \end{aligned}$$

where $c^k\in\mathbb{R}_+$ is a constant for each $k\in L$.

By definition of efficient strategy, agent i allocates his full resource to the project $\bar{k}$. That is, $t_i = E_i^{\bar{k}} \in D_i$, where $E_{i,k}^{\bar{k}} = 1$ if $k = \bar{k}$, and $E_{i,k}^{\bar{k}} = 0$ if $k \in L_i \setminus \{\bar{k}\}$.

Then, for all $\tilde{t}_i \in D_i$ we have that

$$\varphi_i \left(E_i^{\bar{k}}, t_{-i}, f(_i^{\bar{k}}, t_{-i}) \right) \geq \varphi_i \left(\tilde{t}_i, t_{-i}, f(\tilde{t}_i, t_{-i}) \right).$$

Let $c = [c^k]_{k \in L}$. As ϵ goes to 0, $f(t) \to c$. Therefore, by continuity of φ we have that

$$\varphi_i \left(E_i^{\bar{k}}, t_{-i}, c \right) \geq \varphi_i(\tilde{t}_i, t_{-i}, c), \quad \text{for any } \tilde{t}_i \in D_i. \tag{1}$$

Alternatively, fix $\tilde{t}_i \in D_i$ and consider the production functions as follows:

$$\begin{aligned} \tilde{f}^k(t^k) &= c^k + \epsilon \left(\min\{\tilde{t}_i^k, t_i^k\} + t_i^k \right), \quad \text{for } k \in L_i; \\ \tilde{f}^k(t^k) &= c^k + \epsilon \left(\sum_{i \in k} t_i^k \right), \quad \text{for } k \in L \setminus L_i. \end{aligned}$$

Notice that the optimal profile for agent i equals $\tilde{t}^i$. Since φ is efficient, we have that

$$\varphi_i \left(\tilde{t}_i, t_{-i}, \tilde{f}\left(\tilde{t}_i, t_{-i}\right) \right) \geq \varphi_i \left(E_i^{\bar{k}}, t_{-i}, \tilde{f}\left(\tilde{t}_i, t_{-i}\right) \right).$$

As ϵ goes to 0, $f(t) \to c$. Therefore, by continuity of φ we have that

$$\varphi_i \left(\tilde{t}_i, t_{-i}, c \right) \geq \varphi_i \left(E_i^{\bar{k}}, t_{-i}, c \right). \tag{2}$$

Hence, by the inequalities (1) and (2),

$$\varphi_i \left(\tilde{t}_i, t_{-i}, c \right) = \varphi_i \left(E_i^{\bar{k}}, t_{-i}, c \right).$$

Thus, the payoff of agent i is independent of his time allocation. Similarly, the payoffs of the other agents are also independent of their own time allocation.

Step 2: We show that the share of agent i depends on the sum of the profits of the projects in which he belongs.

First, consider $\bar{k} \in L_i$ and define the production functions as follows:

$$\begin{aligned} f^{\bar{k}}(t) &= c^{\bar{k}} + t_i^{\bar{k}} + \gamma \sum_{j \in \bar{k}} t_j^{\bar{k}}; \\ f^k(t) &= c^k + \gamma \left(\sum_{j \in k} t_j^k \right), \quad \text{for any } k \in L \setminus \{\bar{k}\} \end{aligned}$$

where $\gamma < 1$. and $c^k \in \mathbb{R}_+$ are arbitrary constants.

By efficiency, for all $t_i \in D_i$ we have that

$$\varphi_i \left(E_i^{\bar{k}}, t_{-i}, f\left(E_i^{\bar{k}}, t_{-i} \right) \right) \geq \varphi_i \left(t_i, t_{-i}, f\left(\bar{t}_i, t_{-i} \right) \right).$$

Therefore, as γ goes to 1,

$$f\left(E_i^{\bar{k}}, t_{-i} \right) \to \left(c^{\bar{k}} + 1 + \sum_{j \in \bar{k}} t_j^{\bar{k}}, \left[c^k + \sum_{j \in k \setminus \{i\}} t_j^k \right]_{k \in L_i \setminus \bar{k}}, \left[c^k + \sum_{j \in k} t_j^k \right]_{k \in L \setminus L_i} \right) = F^*,$$

and

$$f\left(\bar{t}_i, t_{-i}\right) \rightarrow \left[c^k + \sum_{j\in k} t_j^k\right]_{k\in L} = G^*.$$

Thus, by the continuity of φ^i,

$$\varphi_i\left(E_i^{\bar{k}}, t_{-i}, F^*\right) \geq \varphi_i\left(t_i, t_{-i}, G^*\right).$$

Therefore, transferring all the time of i to the project $\bar{k}$ does not decrease the share of agent i.

Alternatively, for a given $t_i = \left(\tilde{t_i^1}, \tilde{t_i^2}, \ldots, \tilde{t_i^n}\right)$, consider the following production functions,

$$\begin{aligned}
\tilde{f}^{\bar{k}}(t) &= c^T + \min\left\{\tilde{t}_i^{\bar{k}}, t_i^{\bar{k}}\right\} + \gamma t_i^1 + t_1^i; \\
\tilde{f}^k(t) &= c^k + \min\left\{\tilde{t}_i^k, t_i^k\right\} + \sum_{j\in k\setminus\{i\}} t_j^k, \quad \text{for any } k \notin L_i \setminus \{\bar{k}\}; \\
\tilde{f}_{jk}(t) &= c^k + \left(\sum_{j\in k} t_i^k\right) \text{ where } k \in L \setminus L_i.
\end{aligned}$$

For $\gamma < 1$, the optimal profile requires $t_i = \left(\tilde{t_i^1}, \tilde{t_i^2}, \ldots, \tilde{t_i^n}\right)$. Comparing this with the suboptimal profile $t_i = (T_i, 0, \ldots, 0)$, and making γ converge to zero, we have that:

$$\varphi_i\left(E_i^{\bar{C}}, t_{-i}, F^*\right) \leq \varphi_i\left(t_i, t_{-i}, G^*\right).$$

Therefore, we have that the payoff of agent i is invariant to the reallocation of profits in the projects that contain agent i.

Hence, by Steps 1 and 2, φ_i depends on the aggregate profit of the projects in which i belongs, as well as others' time allocations and profits only.

Step 3: We show that a mechanism is non-decreasing function on the total output of its own projects.

Consider the following production functions:

$$\begin{aligned}
\tilde{f}^{\bar{k}}(t) &= c^T + (\gamma+\delta)t_i^{\bar{k}} + \gamma \sum_{j\in \bar{k}\setminus\{i\}} t_j^{\bar{k}}; \\
\tilde{f}^k(t) &= c^k + \gamma \sum_{j\in \bar{k}} t_j^k, \quad \text{for any } k \in L \setminus \{\bar{k}\}.
\end{aligned}$$

where $\gamma < 1$ and $\delta > 0$.

Then, at the optimal profile, agent i contributes his full time allocation to the project $\bar{k}$. Therefore, for any arbitrary profile t^i:

$$\varphi_i\left(\sum_{k\in L_i} c^k + (\gamma+\delta)T_i + \gamma \sum_{k\in L_i}\sum_{j\in k\setminus\{i\}} t_j^{\bar{k}}, F_{-i}, t_{-i}\right) \geq \left(\sum_{k\in L_i} c^k + (\delta)t_i^{\bar{k}} + \gamma \sum_{k\in L_i}\sum_{j\in k} t_j^{\bar{k}}, F_{-i}, t_{-i}\right).$$

As γ goes to 0, we have that

$$\varphi_i\left(\sum_{k\in L_i} c^k + \delta T_i, F_{-i}, t_{-i}\right) \geq \left(\sum_{k\in L_i} c^k + \delta t_i^{\bar{k}}, F_{-i}, t_{-i}\right).$$

Therefore, the step follows immediately since $\{c^k\}_{k\in L_i}$ and $t_i^1 \leq T_i$ are arbitrary numbers. $\square$

Proof of Corollary 1

Proof. Consider the mechanism φ that is efficient, symmetric and time-independent.

First, note that by Theorem 1, symmetry and time independence, there exists a function $g : \mathbb{R}^2 \to \mathbb{R}$ such that φ_i can be written as $\varphi_i(F^{il}, F^{ik}, F^{lk}) = g(F^{il}+F^{ik}, F^{lk})$ for any $(F^{il}, F^{ik}, F^{lk}) \in \mathbb{R}^3_+$.

Second, we show that g is linear. That is, there exists constants $\alpha \geq 0$ and $\beta \geq 0$ such that $g(A, B) = \alpha A + \beta B$ for any $A, B \geq 0$.

To see this, note that by efficiency,

$$\varphi_1 + \varphi_2 + \varphi_3 = g(F^{12} + F^{13}, F^{23}) + g(F^{21} + F^{23}, F^{13}) + g(F^{31} + F^{32}, F^{12}) = F^{12} + F^{13} + F^{23}$$

for any $F^{12} \geq 0$, $F^{13} \geq 0$ and $F^{23} \geq 0$.

Consider the profile $(F^{12}, F^{13}, F^{23}) \in \mathbb{R}^3_{++}$ and some small $\epsilon > 0$. Then,

$$g(F^{12} + F^{13}, F^{23}) + g(F^{21} + F^{23}, F^{13}) = g(F^{12} + F^{13} - \epsilon, F^{23} + \epsilon) + g(F^{21} + F^{23} + \epsilon, F^{13} - \epsilon).$$

Let $A = F^{12} + F^{13}$, $B = F^{21} + F^{23}$ and $X = F^{12} + F^{13} + F^{23}$. Then,

$$g(A, X - A) + g(B, X - B) = g(A - \epsilon, X - A + \epsilon) + g(B + \epsilon, X - B - \epsilon).$$

When $A = B$, we have that $G(A, X - A) = \frac{g(A-\epsilon, X-A+\epsilon)+g(A+\epsilon, X-A-\epsilon)}{2}$ for any $X \geq A \geq 0$ and any small $\epsilon > 0$. This implies that $g(A, X - A) = \alpha A + \beta(X - A) + \gamma$ for some constants α, β, and γ. Since $g(0, 0) + g(0, 0) + g(0, 0) = 0$, then $\gamma = 0$. Furthermore, notice that $g(A, X - A) \geq 0$, thus $\alpha \geq 0$ and $\beta \geq 0$.

On the other hand, consider the profile where $F^{12} = F^{13} = F^{23} = C \geq 0$. Then, $3g(2C, C) = 3(\alpha 2C + \beta C) = 3C$. Therefore, $\beta = 1 - 2\alpha$. Since $\alpha \geq 0$ and $\beta \geq 0$, we have that $\frac{1}{2} \geq \alpha \geq 0$.

When $\alpha = 0$ the mechanism φ generates φ^*. When $\alpha = \frac{1}{2}$, the mechanism φ generates Sh. Moreover, when $\alpha = \beta = \frac{1}{3}$, the mechanism φ generates AC. □

Proof of Corollary 2

Proof. Property (iv) ⇒ Property (iii) is clear because under such a function g, every agent allocates their time resources to achieve the maximum (efficient) level of the total profit in the society. In other words, even if some agents form a coalition, they cannot increase their individual payoff because the aggregate profit does not increase.

We prove that Property (iii) ⇒ Property (iv).

Step A1: Show φ_i is independent of time allocations, t_i and t_{-i}.

By Theorem 1, φ is such that

$$\varphi_i(t, F) = \varphi_i\left(\sum_{k \in L_i} F^k, \left(F^b\right)_{b \in L_{-i}}, t_{-i}\right), \quad \forall i.$$

Consider the time-independent production functions:

$$f^k(t^k) = \alpha^k, \quad \forall k \in L \text{ and arbitrary constants } \alpha^k,$$

Suppose that φ_i depends on t_{-i}. Then, one agent, j, can help agent i to receive a higher payoff by changing his time allocation, because the other allocations are constant. This is a violation to the definition of strong efficiency. Therefore, we have

$$\varphi_i(t, F) = \varphi_i\left(\sum_{K \in L_i} F^K, \left(F^B\right)_{B \in L_{-i}}\right), \quad \forall i.$$

Step A2: Show φ_i depends on $\sum_{b \in L_{-i}} F^b$.

Consider the following production functions: $f^k(t_i^K, t_{-i}^K) = t_i^K$ for any $K \in L_i$ and $f^B(t^B) = \alpha^B$ for $B \in L_{-i}$ and some constants α^B. We list all groups in L_{-i} by $(b_1, b_2, \ldots, b_n)$.

Then, the payoff of agent i equals:

$$\begin{aligned}
\varphi_i(t, F) &= \varphi_i\left(\sum_{k \in L_i} F^k, F^{b1}, F^{b2}, F^{b3}, \cdots, F^{bn}\right) \\
&= \varphi_i\left(\sum_{k \in L_i} F^k, F^{b1} + F^{b2}, 0, F^{b3} \cdots, F^{bn}\right) \\
&= \cdots \\
&= \varphi_i\left(\sum_{k \in L_i} F^k, \sum_{b \in L_{-i}} F^b, 0, 0 \cdots, 0\right),
\end{aligned}$$

where at every step, the equality holds by strong efficiency.

Therefore, the payoff of agent i only depends on the sum of its own outputs and the other pairs' outputs. We can apply a similar argument to any other agent. Therefore, we have

$$\varphi_i\left(\sum_{k \in L_i} F^k, \left(F^b\right)_{b \in L_{-i}}\right) = \varphi_i\left(\sum_{k \in L_i} F^k, \sum_{b \in L_{-i}} F^b\right), \quad \forall i.$$

Step A3: Show φ_i depends on $\sum_{h \in L} F^h$.

Consider the following production function:$f^k(t_i^k, t_{-i}^k) = t_i^k + \sum_{j \neq i} t_j^k$. By Theorem 1 and strong efficiency, we have

$$\varphi_i\left(\sum_{K \in L_i} F^K, \sum_{B \in L_{-i}} F^B\right) = \varphi_i\left(\sum_{k \in L_i} F^k + x, \sum_{b \in L_{-i}} F^b - x\right)$$

for any $x < \sum_{b \in L_{-i}} F^b$. Therefore, we have

$$\varphi_i\left(\sum_{k \in L_i} F^k, \sum_{b \in L_{-i}} F^b\right) = \varphi_i\left(\sum_{h \in L} F^h, 0\right) = \varphi_i\left(\sum_{h \in L} F^h\right)$$

Property (iv) $\Rightarrow$ Property (ii) is obvious.

We new show that **Property (ii) $\Rightarrow$ Property (iv).**

Step B1. φ_i is independent of time.

By Theorem 1, the payoff of agent i, depends only on the time allocations of others, the total profit of the projects in which agent i belongs, and the profits of the projects in which agent i does not belong. That is,

$$\varphi_i(t, F) = \varphi_i\left(\sum_{k \in L_i} F^k, \sum_{b \in L_{-i}} F^b, t_{-i}\right), \quad \forall i.$$

Now, we show φ_i is independent of the other's time allocations t_{-i}.
Consider the following constant production functions:

$$f^k(t^k) = \alpha^k, \quad \forall k \in L_i \text{ and arbitrary constants } \alpha^b \text{ for every } b \in L_{-i}.$$

Note that under these production functions, any allocation of time is an efficient Nash equilibrium. Suppose that the technology in the project k has been improved.

$$\tilde{f}^k(t^k) = \alpha^k + \epsilon.$$

Then, by strong monotonicity,

$$\varphi_i\left(\sum_{k\in L_i}\alpha^k+\epsilon, \sum_{b\in L_{-i}}\alpha^b; \tilde{t}_{-i}\right) \geq \varphi_i\left(\sum_{k\in L_i}\alpha^k, \sum_{b\in L_{-i}}\alpha^b; t_{-i}\right), \quad \forall i, \quad t_{-i} \quad \text{and} \quad \tilde{t}_{-i}.$$

By taking the limit as ϵ tends to zero,

$$\varphi_i\left(\sum_{k\in L_i}\alpha^k, \sum_{b\in L_{-i}}\alpha^b; \tilde{t}_{-i}\right) \geq \varphi_i\left(\sum_{k\in L_i}\alpha^k, \sum_{b\in L_{-i}}\alpha^b; t_{-i}\right), \quad \forall i, \quad t_{-i} \quad \text{and} \quad \tilde{t}_{-i}.$$

By exchanging the roles of t_{-i} and $\tilde{t}_{-i}$ we have that

$$\varphi_i\left(\sum_{k\in L_i}\alpha^k+\epsilon, \sum_{b\in L_{-i}}\alpha^b; t_{-i}\right) \geq \varphi_i\left(\sum_{k\in L_i}\alpha^k, \sum_{b\in L_{-i}}\alpha^b; \tilde{t}_{-i}\right), \quad \forall i, \quad t_{-i} \quad \text{and} \quad \tilde{t}_{-i}.$$

Thus,

$$\varphi_i\left(\sum_{k\in L_i}\alpha^k, \sum_{b\in L_{-i}}\alpha^b; \tilde{t}_{-i}\right) = \varphi_i\left(\sum_{k\in L_i}\alpha^k, \sum_{b\in L_{-i}}\alpha^b; t_{-i}\right), \quad \forall i, \quad t_{-i} \quad \text{and} \quad \tilde{t}_{-i}.$$

Step B2. In this step, we show that the payoff of an agent is invariant to transfers of profit from a project in which he does not below to a project in which the agent belongs.

Consider two projects S and T such that $S \cap T \neq \emptyset$ and neither $S \subset T$ nor $T \subset S$. Without loss of generality, assume that $1 \in T \setminus S$, $2 \in S \cap T$ and $3 \in S \setminus T$. Furthermore, consider the following set of production functions:

$$\begin{aligned} f^T(t^T) &= \sum_{i\in T} t_i^T; \\ f^S(t^S) &= \sum_{i\in S} t_i^S; \\ f^K(t^K) &= c^K, \quad \forall K \neq S, T \end{aligned}$$

Note that under efficiency, agent 2 sends any distribution of his time to the projects S and T, while agents 1 and 3 invest their time in the projects T and S respectively.

Consider the following technology improvement of f^T:

$$\begin{aligned} \tilde{f}^T(t^T) &= (1+\epsilon)t_2^T + \sum_{i\in T\setminus\{2\}} t_i^T; \\ f^S(t^S) &= \sum_{i\in S} t_i^S; \\ f^K(t^K) &= c^K, \quad \forall K \neq S, T \end{aligned}$$

We look at the payoff of agent 3. Consider the case when agent 2 transfers the profit from the projects in which agent 3 belongs, to the projects that agent 3 does not belong to. That is, agent 2 transfers some amount t^2 from project S to project T. Formally, by strong monotonicity and efficiency an any $t^2 < T^2$,

$$\varphi_3(\sum_{K\in L^3} F^K, (F^K)_{K\in L^{-3}}) \leq \varphi_3(\sum_{K\in L^3} F^K - t^2, F^T + (1+\epsilon)t^2, (F^K)_{K\in L^{-3}\setminus\{T\}}).$$

At the limit, when ϵ tends to 0, we have that

$$\varphi_3(\sum_{K\in L^3} F^K, (F^K)_{K\in L^{-3}}) \leq \varphi_3(\sum_{K\in L^3} F^K - t^2, F^T + t^2, (F^K)_{K\in L^{-3}\setminus\{T\}}).$$

Alternatively, consider the production functions.

$$\begin{aligned}
\tilde{f}^S(t^T) &= (1+\epsilon)t_2^S + \sum_{i\in S\setminus\{2\}} t_i^S; \\
f^S(t^T) &= \sum_{i\in T} t_i^T; \\
f^K(t^K) &= c^K, \quad \forall K \neq S, T
\end{aligned}$$

By repeating the above argument we have that

$$\varphi_3(\sum_{K\in L^3} F^K, (F^K)_{K\in L^{-3}}) \geq \varphi_3(\sum_{K\in L^3} F^K - (1+\epsilon)t^2, F^T + t^2, (F^K)_{K\in L^{-3}\setminus\{T\}})$$

As ϵ tends to zero, this leads to

$$\varphi_3(\sum_{K\in L^3} F^K, (F^K)_{K\in L^{-3}}) \geq \varphi_3(\sum_{K\in L^3} F^K - t^2, F^T + t^2, (F^K)_{K\in L^{-3}\setminus\{T\}})$$

Hence,

$$\varphi_3(\sum_{K\in L^3} F^K, (F^K)_{K\in L^{-3}}) = \varphi_3(\sum_{K\in L^3} F^K - t^2, F^T + t^2, (F^K)_{K\in L^{-3}\setminus\{T\}})$$

In particular, by repeating the argument for every project that does not contain 3, we have

$$\varphi_3(\sum_{K\in L^3} F^K, (F^K)_{K\in L^{-3}}) = \varphi_3(\sum_{K\in L^3} F^K + F^T, 0, (F^K)_{K\in L^{-3}\setminus\{T\}}) = \varphi_3(\sum_{K\in L} F^K, (0)_{K\in L^{-3}})$$

□

References

Bhargava, Sandeep (1994) "Profit Sharing and the Financial Performance of Companies: Evidence from U.K. Panel Data," *Economic Journal*, Vol. 104, No. 426, pp. 1044–1056.

de Clippel, Geoffroy, Hervé Moulin, and Nicolaus Tideman (2008) "Impartial division of a dollar," *Journal of Economic Theory*, Vol. 139, No. 1, pp. 176–191.

Holmstrom, Bengt (1982) "Moral Hazard in Teams," *Bell Journal of Economics*, Vol. 13, No. 2, pp. 324–340.

Hougaard, Jens Leth and Mich Tvede (2012) "Truth-telling and Nash equilibria in minimum cost spanning tree models," *European Journal of Operational Research*, Vol. 222, No. 3, pp. 566–570.

——— (2015) "Minimum cost connection networks: Truth-telling and implementation," *Journal of Economic Theory*, Vol. 157, pp. 76–99.

Juarez, Ruben and Rajnish Kumar (2013) "Implementing efficient graphs in connection networks," *Economic Theory*, Vol. 54, No. 2, pp. 359–403.

Kraft, Kornelius and Marija Ugarković (2006) "Profit sharing and the financial performance of firms: Evidence from Germany," *Economics Letters*, Vol. 92, No. 3, pp. 333–338.

Kruse, Douglas L. (1992) "Profit Sharing and Productivity: Microeconomic Evidence from the United States," *Economic Journal*, Vol. 102, No. 410, pp. 24–36.

Maskin, Eric and T. Sjöström (2002) *Implementation Theory*, Chap. 5, pp. 237–288, Amsterdam: North Holland.

Prendergast, Canice (1999) "The Provision of Incentives in Firms," *Journal of Economic Literature*, Vol. 37, No. 1, pp. 7–63, March.

Thomson, William (2007) "On the existence of consistent rules to adjudicate conflicting claims: a constructive geometric approach," *Review of Economic Design*, Vol. 11, No. 3, pp. 225–251.

Tideman, T. Nicolaus and Florenz Plassmann (2008) "Paying the partners," *Public Choice*, Vol. 136, pp. 19–37.

Weitzman, Martin and Douglas Kruse (1990) *Profit Sharing and Productivity*, pp. 95–140: Brookings.