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Saunders Hall 542, 2424 Maile Way,
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Competitive Search with Ex-post Opportunism

By
Pere Gomis-Porqueras
Benoît Julien
Liang Wang

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Pere Gomis-Porqueras
Deakin University

Benoît Julien
UNSW Australia

Liang Wang*
University of Hawaii Manoa

June 17, 2016

Abstract

We consider a frictional market where buyers are uncoordinated and sellers cannot commit ex-ante to either a per-unit price or quantity of a divisible good. Sellers then can exploit their local monopoly power by adjusting prices or quantities once the local demand is realized. We find that when sellers can adjust quantities *ex-post*, there exists a unique symmetric equilibrium where the increase in the buyer-seller ratio leads to higher quantities and prices in equilibrium. When sellers post *ex-ante* quantities and adjust prices *ex-post*, a symmetric equilibrium does not exist.

JEL Classification: D40, L10
Keywords: Competitive Search, Price Posting, Quantity Posting

*Corresponding author at: Department of Economics, University of Hawaii Manoa, 2424 Maile Way 542, Honolulu, HI 96822, USA. E-mail address: lwang2@hawaii.edu.

1 Introduction

Search and matching models have been developed and applied in the context of the labor market to capture the underlying frictions in these markets. In particular, in these models the matching technology tries to capture market frictions (see Pissarides (2000), Rogerson et al. (2005)) and search is two-sided so that firms search for workers and unemployed search for jobs.[1] This search framework has also been applied to goods/services markets, and monetary economics (see Nosal and Rocheteau (2010) and Lagos et al. (2016) for an extensive survey). Regardless of the market at hand, the typical assumption in these models is that terms of trade are determined ex-post via a bargaining procedure (generalized Nash, Kalai, or Rubinstein). Recently, however, a large and growing literature, both in labor and monetary economics, has explored the allocative properties of allowing one side of the market to post and commit ex-ante to a trading protocol as a wage (price) or more general trading mechanisms. The competitive search framework captures such departures.[2] The key assumptions in such environments are: (i) sellers are able to fully commit to all posted terms of trade, (ii) none of the agents posting the terms of trade is capable of serving the entire market, (iii) typically goods are indivisible and (iv) each seller serves just one buyer.

In this paper we explore the role of ex-post opportunism on the nature of the equilibrium in a competitive search environment. So far in the literature, the only possibility for sellers to exploit ex-post opportunities is to post auctions, thus allowing for multilateral meetings. Peters and Severinov (1997), Julien et al. (2000) and Albrecht et al. (2014) explore the consequences of posting only reserve prices. Within this spirit, Kim and Kircher (2015) analyze the implications of first-price auctions and second-price auctions, and show that the choice of the trading mechanism is crucial for the existence of equilibrium. Here we explore an alternative procedure to exploit ex-post opportunities. To do so, we consider a competitive search model where sellers are able to produce any continuous quantity at convex cost while serv-

[1] This is essentially the Diamond-Mortensen-Pissarides framework.

[2] See McAfee (1993), Shimer (1996), Moen (1997), Acemoglu and Shimer (1999), Peters (2000), Burdett et al. (2001), and Mortensen and Wright (2002) and Rocheteau and Wright (2005), to name a few.

ing only one buyer at a time.[3] In contrast to the previous literature, sellers only have the ability to partially commit to ex-ante terms of trade (prices or quantities) while choosing the remaining terms of trade unilaterally and ex-post. In particular, we consider sellers posting and committing to a unit price ex-ante, but choosing the quantity/quality margin ex-post after the matching process has taken place. This is reminiscent of Bertrand competition but with endogenous quantity/quality and convex production cost.[4] Many markets have this ex-ante limited commitment feature, especially if one considers the quality margin interpretation of the model. Such type of arrangements are found in labor markets, where vacancies are posted with wage rates but hours worked are left to workers, as in part-time jobs; restaurants with price posting (easily found on websites), but the quality or even quantity is determined ex-post once buyers have ordered; real estate sold before construction; new cars priced and ordered/sold before manufacturing; and essentially, any goods or services that are priced ex-ante but produced upon a match fit our sequence of events.

Under these terms of trade with ex-post opportunism, we show that marginal cost pricing is the unique equilibrium as in the standard perfectly competitive market. We show that the equilibrium can lead to under, over, or efficient production, depending on the aggregate buyer-seller ratio. This is in sharp contrast to the standard competitive search equilibrium with sellers posting both price and quantity, where the resulting equilibrium always generates an efficient allocation.[5]

Finally, in this paper we consider the possibility that sellers post and commit to quantities ex-ante, but determine unilaterally the unit price ex-post. Note that this is reminiscent to a Cournot competition setup. Under this trading protocol, we show that competitive search equilibrium does not exist. In this case, sellers have the incentive to extract all of the buyers' surplus by choosing a unit price equivalent to the Diamond (1972) outcome, that is, price is above marginal cost. These latter terms of trade explored in this paper lead to the Diamond's paradox.

[3]Here we do not have the explicit seller's capacity constraint typically found in the literature.

[4]In pure Bertrand competition models with non-constant marginal cost, Weibull (2006) shows a continuum of price equilibria.

[5]This result holds without fiat money. Rocheteau and Wright (2005) show that under production can occur with money being essential for trade and high inflation.

The paper is organized as follows. Section 2 reviews the competitive search literature with divisible goods. In Section 3 we consider sellers committing to both unit price and quantity. In Section 4, sellers commit to unit price but quantity is chosen ex-post. In Section 5, sellers commit to quantity, but unit price is chosen ex-post. A conclusion follows.

2 Relevant Literature

More recently, in goods market, Geromichalos (2012, 2014), and Godenhielm and Kultti (2015) also allow sellers to produce multiple indivisible units at a convex cost. They also extend the choice of capacity prior to the matching process. Within the labor market, Lester (2010) and Hawkins (2013) allow firms to post multiple indivisible vacancies at a convex cost. However, all these authors assume that *all* terms of trade are posted and committed ex-ante the matching process. Here we consider some ex-post opportunism.

The most closely related paper is that of Peters (1984) who considers a large directed search market where sellers produce a continuous quantity q at convex cost. They also face an exogenous capacity K. Sellers post unit prices, and upon a match, buyers choose the quantity to demand, which is the minimum of q and K. In equilibrium, sellers post a price equal to average cost. Here we differ in that the quantity choice is determined ex-post, rather than demand as in Peters (1984). As a result, the equilibrium is consistent with marginal cost pricing while Peters (1984) finds average cost pricing. In addition, we find over, under, or efficient production is possible, depending on the buyer-seller ratio, while in Peters (1984), there is always over production.

Finally, Godenhielm and Kultti (2014) also assume continuous quantity, but allow sellers to choose capacity q simultaneously, and then prices. Production occurs before the matching process and the authors consider two cases where capacity choices are observed by buyers or not when selecting sellers. The equilibrium price is similar to Burdett et al. (2001), and hence, not tied to marginal or average cost. In contrast here we also consider production on demand.

3 Model

We use the competitive search framework of Moen (1997) with a continuum of uncoordinated buyers and sellers, of measures Θ and 1, respectively. Buyers have preferences $u(q)$ over goods produced by sellers who incur a cost $c(q)$, where $u(\cdot)$ and $c(\cdot)$ satisfy usual assumptions of strict concavity and convexity and $c'''(\cdot) > 0$. Any positive measure of sellers posting the same terms of trade, $\omega = (p, q, \theta) \in \mathbb{R}^3_+$, form a submarket. p denotes the per-unit price, q the quantity or quality, and θ is the corresponding buyer-seller ratio active in that submarket. We define sellers' surplus as $S(p, q) = pq - c(q)$ and buyers' surplus as $B(p, q) = u(q) - pq$. Buyers have access to potentially a continuum of these submarkets since it is costless for sellers to post any terms of trade, and choose which market to enter. All actions are observable to everyone. Within each submarket, buyers and sellers meet according to a matching technology that is homogeneous of degree one, where sellers' (buyers') meeting rate is $\alpha(\theta)$, $(\alpha(\theta)/\theta)$ with $\alpha'(\cdot) > 0$, $\alpha''(\cdot) < 0$, $\alpha(0) = 0$, $\lim_{\theta\to\infty} \alpha(\theta) = 1$, and $\lim_{\theta\to 0} \alpha'(\theta) = 1$.[6]

Given this structure, we construct a competitive search equilibrium by looking for an optimal deviation of a submarket where all sellers post ω and the rest of sellers in other submarkets post ω^c. In equilibrium, buyers and sellers are indifferent across submarkets, $\theta = \Theta$ and $\omega = \omega^c$. This allows us to focus the analysis on one submarket. We first analyze a situation where sellers post ex-ante per-unit prices and quantities. We then explore the situation where sellers do not have as much ex-ante commitment. In particular, we consider a situation where sellers post ex-ante per-unit prices and determine the quantity ex-post. Finally, we characterize the equilibrium when sellers post ex-ante quantities and set the per-unit price ex-post.

3.1 Ex-ante Price and Quantity Posting

In this section, the posted terms of trade are akin to the competitive search section of Rocheteau and Wright (2005). A positive measure of sellers choose the same $\omega = (p, q, \theta)$ to form a submarket while other submarkets post the same $\omega^c = (p^c, q^c, \theta^c)$,

[6] These properties are standard in the matching frameworks of Diamond-Mortensen-Pissarides.

solving

$$\max_{(p,q,\theta)} \alpha(\theta) S(p,q) \tag{1}$$
$$\text{s.t. } \frac{\alpha(\theta)}{\theta} B(p,q) \geq \bar{U} \Leftrightarrow \theta > 0$$
$$\frac{\alpha(\theta)}{\theta} B(p,q) < \bar{U} \Leftrightarrow \theta = 0,$$

where $\bar{U} = \max_{(p^c,q^c)\in\mathbb{R}_+^2} U(p^c, q^c, \Theta) > 0$ is buyers' maximum expected market utility from participating in any other submarkets.[7] In equilibrium, the participation constraint for buyers is always binding as in standard competitive search models. Solving the constraint for pq into $S(p,q)$, it is easy to show that optimality implies an efficient equilibrium quantity $q^* = q^e$ where $u'(q^e) = c'(q^e)$ and an implied per-unit price of

$$p^*(\theta) = \frac{[1 - \varepsilon(\theta)]\, u(q^*) + \varepsilon(\theta)\, c(q^*)}{q^*},$$

where $\varepsilon(\theta) = \theta\alpha'(\theta)/\alpha(\theta)$ is the elasticity of the seller's matching rate.

In a symmetric equilibrium, $\omega = \omega^c$ and $\theta = \Theta$, which implies

$$p^*(\Theta) = [1 - \varepsilon(\Theta)] \frac{u(q^*)}{q^*} + \varepsilon(\Theta) \frac{c(q^*)}{q^*}. \tag{2}$$

The equilibrium price is a convex combination of average utility and cost evaluated at q^*. Notice that if we rewrite the above pricing equation as

$$\varepsilon(\Theta) = \frac{u(q^*) - p^* q^*}{u(q^*) - c(q^*)}, \tag{3}$$

it gives the standard Hosios sharing rule which always holds endogenously with standard competitive search when all terms of trade are committed ex-ante. The buyer's and seller's expected payoffs are $B(p^*, q^*)\alpha(\Theta)/\Theta$ and $S(p^*, q^*)\alpha(\Theta)$, where their surpluses are given by $S(p^*, q^*) = p^*q^* - c(q^*)$ and $B(p^*, q^*) = u(q^*) - p^*q^*$, respectively.

The competitive search equilibrium is always surplus maximizing, but, depending on Θ, it could be that $p^*(\Theta) \lesseqqgtr u'(q^*) = c'(q^*)$. Notice that the equilibrium quantity

[7] This is commonly known as the *market utility* first used by Montgomerry (1991) and McAfee (1993), and subsequently by Moen (1997), Acemoglu and Shimer (1999), and now is standard in competitive search theory.

is invariant in Θ. A sudden inflow of buyers leading to a larger Θ would result in a higher price if $\varepsilon'(\Theta) > 0$ (which holds for many meeting technologies, see Cai et al. (2016)) and no change in quantity traded.

3.2 Ex-ante Price Posting

Prices are posted ex-ante and sellers optimally choose q ex-post. From observed prices, buyers decide which submarket to participate in. If the meeting technology in any submarkets allows multilateral meetings, upon meeting buyers, sellers randomly select a buyer to trade with, and if pairwise meeting technology, trading occurs with that buyer. Then, seller chooses the quantity to produce. Thus, to construct the equilibrium, we need to not only account for the optimal deviation in price, but also the deviating sellers' optimal ex-post reaction to their quantity choice given the ex-ante price.

Since the trading mechanism now has two stages, we solve for equilibrium backward. We first solve for sellers' optimal choice of q given p, and then solve for the competitive search equilibrium choice of $\omega = (p, \theta) \in \mathbb{R}^2_+$.

Consider the *ex-post* problem where deviating sellers take posted p as given, and meet or select a buyer if they meet more than one. They solve

$$\max_q S(p,q) \text{ s.t. } B(p,q) \geq 0.$$

For interior solutions, the optimal quantity $\tilde{q}$ satisfies

$$p = c'(\tilde{q}) \text{ and } u(\tilde{q}) > p\tilde{q},$$

while in the corner solution, the optimal quantity is given by

$$p = \frac{u(\tilde{q})}{\tilde{q}} \text{ and } c'(\tilde{q}) < p.$$

This yields a one-to-one relationship $\tilde{q}(p)$.

Lemma 1 *For any θ, the optimal ex-post choice is given by $p = c'(\tilde{q})$.*

Proof. See Appendix for all the proofs. ■

Taking as given the ex-post optimal choice $\tilde{q}$, we solve for the competitive search equilibrium price. To be consistent with previous notations, let $B(p, \tilde{q}(p)) = u(\tilde{q}(p)) - p\tilde{q}(p) \equiv \tilde{B}(p)$ and $S((p, \tilde{q}(p)) = p\tilde{q}(p) - c(\tilde{q}(p)) \equiv \tilde{S}(p)$.

The positive measure of deviating sellers solve

$$\max_{p,\theta} \alpha(\theta) \tilde{S}(p) \text{ s.t. } \frac{\alpha(\theta)}{\theta} \tilde{B}(p) \geq \bar{U}.$$

It is easy to show that the optimal solution satisfies

$$\frac{1-\varepsilon(\Theta)}{\varepsilon(\Theta)} = -\frac{\tilde{B}'(p)}{\tilde{S}'(p)} \frac{\tilde{S}(p)}{\tilde{B}(p)}, \tag{4}$$

where $\varepsilon(\Theta)$ is as previously defined.

In a symmetric equilibrium, $p = p^c$ and $\theta = \Theta$, which implies $p(\Theta)$. From the above problem, it is a bit more involved to show existence and uniqueness. Fortunately, we can change the problem by substituting for $p = c'(\tilde{q})$ instead and maximize *as if* sellers were choosing $\tilde{q}$ ex-ante.[8] To simplify the notation, let $\tilde{q} = q$ from now on. The problem for sellers then become

$$\max_{q,\theta} \alpha(\theta) S(q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(q) \geq \bar{U},$$

where $B(q) = B(c'(q), q)$ and $S(q) = S(c'(q), q)$. The optimal solution $q(\Theta)$ satisfies

$$\frac{1-\varepsilon(\Theta)}{\varepsilon(\Theta)} = -\frac{B'(q)}{S'(q)} \frac{S(q)}{B(q)}. \tag{5}$$

Interestingly, the above condition could rewrite as

$$\varepsilon(\Theta) = \frac{\eta_s(q)}{\eta_s(q) + \eta_b(q)}$$

where $\eta_s(q) = qS'(q)/S(q)$ and $\eta_b(q) = -qB'(q)/B(q)q$. This looks like the Hosios sharing rule expressed in elasticity because q is determined ex-post. It shows how sellers' optimal ex-post choice of q relates to the impact of that choice on the seller's surplus relative to the overall surplus and the seller's contribution to the matching rate.

[8] Given the nature of the trading mechanism, sellers rationally anticipate that buyers participate in the submarket, knowing that sellers will choose q ex-post to maximize profit given the posted price that attracted buyers in the first place.

Proposition 1 *For any given Θ, there exists a unique symmetric equilibrium where all sellers choose q ex-post such that $p(\Theta) = c'(q(\Theta))$.*

Posting prices ex-ante with quantities determined ex-post always yields marginal cost pricing, but does not always give efficiency.

Proposition 2 *The equilibrium is generically not efficient. Moreover, an increase in Θ leads to higher q and p in equilibrium.*

These results are in sharp contrast to the standard competitive search equilibrium, where sellers post per-unit prices and quantities ex-ante. Efficient quantity q^* is always achieved. With ex-post quantity trading, efficiency is achieved only if Θ happens to lead to $u'(q(\Theta)) = p(\Theta) = c'(q(\Theta))$. The ability to commit ex-ante to all terms of trade is critical to obtain the efficient outcome.

Comparing the allocation of full commitment with ex-ante commitment on price alone, we note that if Θ^c is such that $q(\Theta^c) = q^*$ and

$$p(\Theta^c)q^* = c'(q^*)q^* = [1 - \varepsilon(\Theta^c)]\, u(q^*) + \varepsilon(\Theta^c)\, c(q^*),$$

the full and partial commitment outcomes are equivalent. But this holds only for a very specific value of Θ^c. Since equilibrium (p, q) are both increasing in Θ, for all $\Theta < \Theta^c$, sellers would prefer to deviate by committing to (p, q) instead of just p, while the reverse is true if $\Theta > \Theta^c$.

Partial commitment equilibrium is interesting because it has many applications from restaurant meals to new real estate construction and labor market. The labor market suits this set up particularly well. Assume that a measure v of vacancies are to be matched with a measure u of unemployed, with $\Theta = v/u$. The surplus from a match is $f(h)$ where h is the hours worked by workers upon a match. Let $c(h)$ be the cost for workers implementing h, and wh be the wage revenue paid from the firm to the worker. Consider setting up the competitive search problem as workers competing by posting (w, θ) to attract firms, and choosing h ex-post. This environment fits perfectly the above setup where $q \equiv h$, $u(q) \equiv f(h)$, $c(h) \equiv c(q)$, and $wh \equiv pq$. All results follow. Workers would set wages ex-ante and choose hours ex-post such that $w = c'(h)$. Only a particular value of Θ would result in $f'(h(\Theta)) = w(\Theta) = c'(h(\Theta))$, and so workers would be paid at their marginal product. Otherwise, for other values of Θ, workers could be paid above or below their marginal product. A sudden increase in Θ would lead to increase in both wage and hours worked. Compared to workers posting (w, h, θ), hours in equilibrium would be determined by $f'(h^*) = c'(h^*)$, independent of Θ, with only wage increasing in Θ.

Since Peters (1984) is the most related work, we offer a more detailed comparison. In this model, we assume that the seller chooses q ex-post to maximize ex-post profit subject to an individual rationality constraint for the buyer. This leads to $p = c'(q)$, which gives a quantity supplied by sellers as a function of price that we write as $q = s(p)$. In Peters (1984), it is the buyer who chooses q ex-post to maximize surplus, leading to $u'(q) = p$, or a quantity demanded determined as a function of price, $q = d(p)$, and average cost pricing $p = c(q)/q$. Given the convexity of the cost function, $u'(q) = p = c(q)/q > c'(q)$ always holds in equilibrium of his model. In our model, it is possible that $u'(q) \gtreqless p = c'(q)$ as long as $B(p, q) = u(q) = pq \geq 0$. In Peters (1984), there is always over production compared to the efficient level $u'(q^*) = c'(q^*)$, but in our model, over, under, or efficient production is possible depending on the value of aggregate market tightness Θ. Peters (1984) assumes an exogenous capacity K such that $c'(q) = \infty$, for all $q \geq K$. The ex-post demand by a buyer is $q = \min\{d(p), K\}$. This assumption was made in the spirit of the Edgeworth's model. We do not have such capacity constraint, and introducing it would not change our marginal cost pricing result. Finally, sellers make zero expected profits as in standard Bertrand competition, while in our model, there is marginal cost pricing as in Bertrand, but sellers still have positive profit in equilibrium. Figure 1 illustrates an equilibrium example in our model and Peters (1984).

3.3 Ex-ante Quantity Posting

In this section, we assume that sellers can post a quantity ex-ante and adjust the per-unit price ex-post, after buyers choose which seller to visit. Ex-ante sellers post

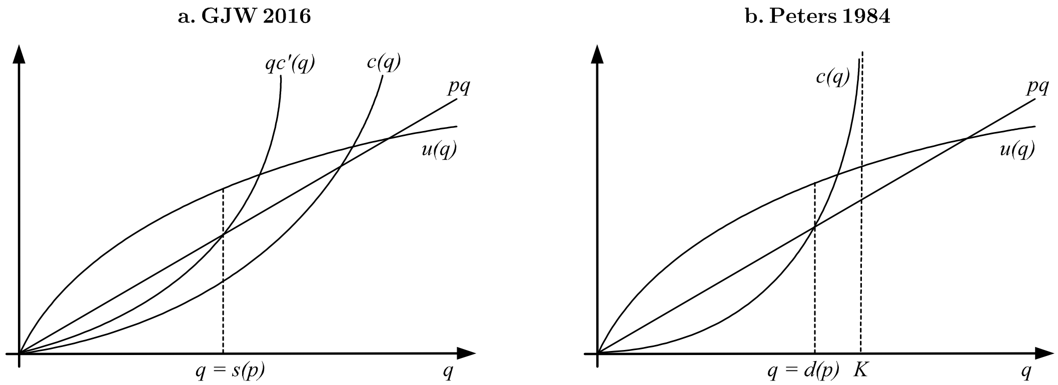

Figure 1: Equilibrium marginal cost pricing in GJW (2016) and average cost pricing in Peters (1984).

$\omega = (q, \theta) \in \mathbb{R}_+^2$ to solve

$$\max_{q,\theta} \alpha(\theta) S(p,q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(p,q) \geq \bar{U},$$

while ex-post, sellers take as given posted quantity and solve

$$\max_{p} S(p,q) \text{ s.t. } B(p,q) \geq 0.$$

Solving the problem backwards, we see that sellers are able to extract all the surplus by pricing $p^* = u(q)/q = g(q)$. Note that the seller's pricing decision does not directly depend on θ.

Sellers take the ex-post pricing rule as given and solve

$$\max_{q,\theta} \alpha(\theta) S(g(q), q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(g(q), q) \geq \bar{U}. \tag{6}$$

From the ex-post pricing decision, it is easy to show that

$$q \frac{\partial p^*}{\partial q} = u'(q) - p^*. \tag{7}$$

Hence, $p^* + q\partial p^*/\partial q - c'(q) = 0$, implying an efficient q^* as $u'(q^*) = c'(q^*)$.

Proposition 3 *A symmetric equilibrium does not exist when sellers post quantities ex-ante.*

When sellers post quantities ex-ante and adjust prices ex-post, they choose to post the efficient quantity to attract buyers, and then extract all the surplus from trade by adjusting prices later. Buyers fully anticipate that the seller's best ex-post choice is to fully extract all of their surplus, which implies that $\bar{U} = 0$, and thus buyers do not participate. If sellers were given the choice of what to post ex-ante, no sellers would want to deviate from posting only q ex-ante. This Cournot type competition suffers from the Diamond's paradox.

4 Conclusion

We consider a frictional market where buyers are uncoordinated and sellers cannot commit to a per-unit price and quantity of a divisible good *ex-ante*. As in Kim and Kircher (2015) that the choice of the trading mechanism is crucial for the existence of equilibrium. In particular, we find that when sellers post ex-ante prices, there exists a unique symmetric equilibrium with marginal cost pricing, and an increase in the buyer-seller ratio leads to higher quantities and prices in equilibrium. When sellers post ex-ante quantities, a symmetric equilibrium does not exist.

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Appendix

Proof of Lemma 1

Define $\bar{q}$ such that $u(\bar{q}) = \bar{q}c'(\bar{q})$ and $\bar{p} = c'(\bar{q})$. We summarize the seller's best ex-post response as follows:

$$p = \begin{cases} c'(\tilde{q}) & \text{for } p \in (0, \bar{p}] \\ u(\tilde{q})/\tilde{q} & \text{for } p \in (\bar{p}, \infty). \end{cases}$$

It is important to highlight that both of these solutions imply a monotone relationship between quantity and price. Notice the following

$$\begin{aligned} p &\leq \bar{p} \Rightarrow \tilde{q}'(p) = 1/c''(\tilde{q}) > 0 \text{ (interior)}, \\ p &> \bar{p} \Rightarrow \tilde{q}'(p) = \frac{\tilde{q}^2}{\tilde{q}u'(\tilde{q}) - u(\tilde{q})} < 0 \text{ (corner)}. \end{aligned}$$

For a given value of Θ, the positive measure of deviating sellers can choose a price that is either in $(0, \bar{p}]$ for an interior solution or in $(\bar{p}, \infty)$ for a corner solution. Define the first possible deviation as $p_1 = c'(\tilde{q}_1)$ and the second possible deviation as $p_2 = u(\tilde{q}_2)/\tilde{q}_2$. It is easy to show that the seller's expected payoff is

$$\pi_1 \equiv \alpha(\Theta)[p_1\tilde{q}_1 - c(\tilde{q}_1)] < \alpha(\Theta)[p_2\tilde{q}_2 - c(\tilde{q}_2)] \equiv \pi_2,$$

while for buyers we have

$$U_1 \equiv \frac{\alpha(\Theta)}{\Theta}[u(\tilde{q}_1) - \tilde{q}_1c'(\tilde{q}_1)] > 0 = \frac{\alpha(\Theta)}{\Theta}[u(\tilde{q}_2) - \tilde{q}_2u(\tilde{q}_2)/\tilde{q}_2] \equiv U_2.$$

It is clear that $\tilde{B}(p) = 0$ holds at a corner solution, and no buyers would participate in a deviating submarket that offers p_2 and $\tilde{q}_2$, as an ex-post profit maximizing response. Buyers fully anticipate that the best ex-post choice of sellers is to fully extract all of their surplus. It must be that any price as part of a competitive search equilibrium is $p \in (0, \bar{p}]$. The optimal ex-post choice is then $p_1 = c'(\tilde{q}_1) \in (0, \bar{p}]$. In other words, $\tilde{q}_1(p_1)$ is the equilibrium anticipated seller's response by buyers. ■

Proof of Proposition 1

An equilibrium has to satisfy equation (5). Let us define the right-hand side of this equation as $\chi(q)$ and let $\bar{q}$ such that $B(\bar{q}) = 0$. It is easy to check $\exists\ \underline{q} > 0$ such that $B'(\underline{q}) = 0$, while $B'(q) < 0$ and $B(q) > 0$ for $q \in (\underline{q}, \bar{q})$. Since $B'(\underline{q}) = 0$, $\lim_{q \to \underline{q}} \chi(q) = 0 < [1 - \varepsilon(\Theta)]/\varepsilon(\Theta)$, and $\lim_{q \to \bar{q}} \chi(q) = \infty$. Therefore, $\exists\ q \in (\underline{q}, \bar{q})$

such that $\chi(q) = [1 - \varepsilon(\Theta)]/\varepsilon(\Theta)$, and equilibrium exists. The equilibrium price $p(\Theta)$ is determined by $p(\Theta) = c'(q(\Theta))$.

To show uniqueness, let $T(q) = B(q) + S(q)$ be the total surplus. Clearly, $T'(q) = B'(q) + S'(q) = u'(q) - c'(q)$ and the equilibrium satisfies

$$\chi(q) = -\frac{[T'(q) - S'(q)] / [T(q) - S(q)]}{S'(q)/S(q)}.$$

We have $B'(q) < 0 \; \forall q \in [\underline{q}, q^*)$, since $B'(\underline{q}) = 0$, and so $0 < T'(q) < S'(q)$ and $\chi(q) > 0$ over this interval. Since $T'(q^*) = 0$, $\chi(q^*) > 0$. For all $q \in (q^*, \bar{q}]$, $T'(q) < 0$ and $\chi(q) > 0$ over this interval. This proves that $\chi(q) > 0$ over $q \in [\underline{q}, \bar{q}]$. Observe that $\chi\left(\underline{q}\right) = 0$ since $B'(\underline{q}) = 0$ and all other components of $\chi(q)$ are positive. Note that $\lim_{q \to \bar{q}} \chi(q) = \infty$ as $T(\bar{q}) - S(\bar{q}) = 0$ and all other components are non-zero. For any strictly concave $u(\cdot)$ and convex $c(\cdot)$, $B'(q) = u'(q) - c'(q) - qc''(q) = T'(q) - qc''(q)$. Now $T'(q)$ is monotonically decreasing in q, while $qc''(q)$ is monotonically increasing in q if $c'''(q) \geq 0$. Hence, $\exists! \; q \in [0, \bar{q}]$ such that $B'(q) = 0$. Call this $q = \underline{q}$ and hence $\exists! \; \underline{q}$ such that $B'(\underline{q}) = 0$. Since $\chi(q) > 0 \; \forall q \in (\underline{q}, \bar{q})$ with $\chi\left(\underline{q}\right) = 0$ and $\chi(\bar{q}) = \infty$, it must be that $\chi'(q) > 0 \; \forall q \in [\underline{q}, \bar{q}]$. Since $\chi(q)$ is monotonically increasing in q over $[\underline{q}, \bar{q}]$, there exists a unique equilibrium $q \in [\underline{q}, \bar{q}]$ for any given Θ. ■

Proof Proposition 2

Given $\varepsilon'(\Theta) < 0$, $[1 - \varepsilon(\Theta)]/\varepsilon(\Theta)$ is increasing in Θ, and $\chi(q)$ is monotonically increasing over $[\underline{q}, \bar{q}]$. Therefore, an increase in Θ leads to higher q and p. ■