University of Hawai`i at Mānoa
Department of Economics
Working Paper Series

Saunders Hall 542, 2424 Maile Way,
Honolulu, HI 96822
Phone: (808) 956 -8496
www.economics.hawaii.edu

Working Paper No. 16-1

Credit, Money and Asset Equilibria with Indivisible Goods

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January 2016

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January 15, 2016

Abstract

In a New Monetarist framework, we study the trade of indivisible goods under credit, divisible money and divisible asset in a frictional market. We show how indivisibility on the goods side, instead of the money or asset side, matters for equilibria. The bargaining solution generates a price that is independent of nominal interest rate, dividend value of the asset, or the number of active buyers carrying the asset for liquidity purposes. To reestablish this link, we consider price posting with competitive search. We derive conditions under which stationary equilibrium exists. With asset and bargaining, we find that for negative dividend value on the asset, multiple equilibria occur. Otherwise, in all possible combinations of liquidity and price mechanisms, including positive dividend value under asset, the equilibrium is unique or generically unique.

JEL: D51, E40. *Keywords:* Nash Bargaining; Competitive Search; Indivisibility; Multiplicity; Uniqueness.

*We thank Begona Dominguez, Aspen Gory, Lucas Herrenbrueck, Pedro Gomis-Porqueras, Ricardo Lagos, Andy MacLennan, Guillaume Rocheteau, and Randall Wright for helpful comments and discussions. We also thanks participants at the Workshop on Money, Banking, Finance and Payments, St-Louis Fed, the WAEI conference, Hawaii, and seminar participants at U. of Wisconsin and U.of Queensland. Han thanks the National Natural Science Foundation of China, No.71373011 for research support.

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1 Introduction

Use of the New Monetarism framework has seen an increasing growth in popularity with its numerous applications in areas such as finance, payment systems, liquidity and monetary analysis.[1] A standard version of the model, based on the Lagos and Wright (2005) monetary framework, assumes that agents trade divisible goods using divisible money as medium of exchange. The literature emerged from the original search based models of Kiyotaki and Wright (1989, 1993) with indivisible goods and money and further from Shi (1995) and Trejos and Wright (1995) using divisible goods and indivisible money.[2] Due to the indivisibility of the medium of exchange these models have degenerate distribution of money. However, with divisible money the distribution of money holdings is endogenous and non-degenerate. One must resort to computation as in Molico (2006) or force a degenerate distribution by assuming quasi-linear utility as in Lagos and Wright (2005) or homothetic preferences.[3]

In this paper we explore the consequences for equilibria of having perishable, indivisible goods that are produced on demand traded with divisible medium of exchange in a decentralized market.[4] Effectively, this is the reverse of the divisible good, indivisible money environment used in the Shi-Trejos-Wright framework. In that framework, agents cannot accumulate more than one unit of money. Here, we assume that buyers only want to consume one unit of the indivisible good but can hold divisible amount of money.

Our view is that all goods have an indivisible component. The indivisibility could be determined by natural aspects, firms packaging strategy or a minimal assembly required to make the good operational. For instance, housing, cars, boats and other durable goods as well as the purchase of a seat on an airplane can be considered indivisible. Although our focus is on modeling perishable goods that are produced on demand our monetary equilibria results are also applicable to durable goods.[5] Examples of indivisible goods that are perishable and produced on demand include many services such as haircuts, three course meals at a restaurant, lawn mowing services or wedding ceremonies. We

[1] Lagos et al. (2015) cite numerous papers using this framework.

[2] In Trejos and Wright (1995), terms of trade are settled by bargaining. With divisible goods and indivisible money, Curtis and Wright (2004) study price posting with random search, while Julien et al. (2008) study auctions and price posting with directed search.

[3] See Wong (2015).

[4] Production on demand is commonly assumed New Monetarism models, few exceptions are Masters (2013), and Anbarci et al. (2015) with divisible goods.

[5] With durable good, one can assume buyers who trade exit the market with perfect replacement of new buyers to maintain the measure of potential buyers constant.

argue that the common assumption of divisible goods can be re-interpreted as allowing buyers to purchase many units of an item at its lowest indivisible denomination. In our model, buyers then are taken to consume only one unit of the good at its lowest indivisible denomination. Our aim is a theoretical exploration of non-convexities including fixed cost of production on the goods side.

We derive and compare equilibrium allocations under different types of liquidity required for purchase of the indivisible good. First, we assume a credit economy in which agents can credibly commit. Sellers can extend credit in the frictional market to be repaid by buyers from consumption goods in the subsequent centralized market. Second, we consider a monetary economy, with money being essential due to imperfect credit. Money holders are subject to an inflation tax due to assumed constant exogenous growth rate of money. Third, we consider an economy where a real asset (e.g. Lucas Tree) is essential and used as a medium of exchange. Unlike the monetary model, the asset is in fixed total supply and it bears an exogenous dividend which could be positive or negative. The reason to consider real assets is that when credit is imperfect, money is not the only object that can serve in the medium of exchange role. Agents holding real assets will often sell them for liquidity purposes to trade in goods markets.[6] In this paper, we assume as an abstraction that agents can pay sellers directly with the real asset.

As in most New Monetarism models, we assume terms of trade determined by generalized Nash bargaining for each of the three liquidity environment. However, we also consider lotteries, and price posted prior to buyers participating in the indivisible goods market using price posting with competitive search.[7] The reason why we study different pricing mechanisms and liquidity environment is that, due to goods' indivisibility, each mechanism and instrument produce different results. With indivisible goods, no adjustment can take place through the intensive margin and the available surplus is fixed. In particular, with money/asset, the bargained price does not depend on nominal interest or

[6] With many of these arrangements the asset is not used as a medium of exchange per se but as collateral. This is not crucially important. In the spirit of Kiyotaki-Moore we can reinterpret Lagos-Wright by saying that a buyer in the DM makes a promise to pay the seller in numeraire in the next CM. If the buyer reneges the seller (or the courts or the mechanism) can take away the asset, just like a house, car or boat can be repossed if one reneges on payments. As in standard in many models, we assume the buyer can only pledge assets he has on hand, and not labor income. Then he uses assets to facilitate deferred settlement just as if he was handing them over for immediate settlement – the equations are the same.

[7] The competitive search framework we use is based on Moen (1997) and Mortensen and Wright (2002). For the use of competitive search in monetary models see Rocheteau and Wright (2005) and Lagos and Rocheteau (2007).

return on the asset, while posted price allows for this dependence.

We show uniqueness of equilibrium in the pure credit economy with an exogenous credit constraint. Because no medium of exchange is needed, and hence no direct cost associated with liquidity, all potential buyers participate in the decentralized market and the solution is akin to the optimal Friedman rule solution in a monetary economy with divisible goods.

The allocations in the money and asset economies differ from those in the credit economy. With bargaining, buyers can commit to bringing the lowest amount of money/asset needed to make sellers indifferent between trading or not trading. This is driven by no adjustment through the intensive margin. The solution is akin to a take-it-or-leave-it offer and buyers can extract the whole surplus. With price posting and with lotteries, sellers are able to extract some of the surplus. With price posting, sellers post the price before buyers enter the decentralized market and with lotteries comes the threat of not delivering the good.

In the monetary economy we show uniqueness or generic uniqueness, as long as the nominal interest rate is not too high. The exogenous nominal rate determines the cost of holding money. Hence, this cost acts as an entry cost into the decentralized market. When the nominal rate is large the congestion induced by the matching technology will reduce the number of buyers in the decentralized market, until the cost is so high that monetary equilibria cease. The threshold for the nominal interest rate differs between the bargaining and the competitive search environment. The reason is that the equilibrium price depends on the number of participating buyers under competitive search while it does not under bargaining. Rocheteau and Wright (2005) also show that there may be multiple equilibria under competitive search. In both models, more participation implies a smaller probability of trade for buyers, but they are compensated by a larger trade surplus. The indivisibility of goods does not eliminate possible multiplicity. In Rocheteau and Wright (2005), buyers may have higher consumption or lower price, whereas in our model only the latter is possible due to indivisible goods.

Under bargaining, we show that for a low range of nominal interest rate, all potential buyers participate in the decentralized market. Money is then superneutral in the sense that it does not affect the real price, equal to sellers' cost, and the number of trades through participation. However, for a higher range of nominal interest rate, not all buyers participate. For high inflation the expected number of trades (hence production) is affected by increase in inflation. Money is then only neutral in this case. We show that using lotteries does not overturn the superneutrality. This differs from monetary models with divisible goods in which money is only neutral. There, inflation affects real balances and quantity exchanged in all trades. We also show that using lotteries does not affect allocations under competitive search and money.

The price determination of the indivisible good in the asset economy is similar to the case with money. With bargaining, the buyer is able to extract the full surplus from trade, while with competitive search the price of the indivisible good is a function of the participation rate and the cost of holding the asset. However, what is important to consider is the dividend value of assets, which can be positive or negative. The dividend value affects the price of the asset as well as the participation of buyers in the decentralized market. Thus, under competitive search, the dividend also has an indirect effect on the price of the indivisible good through participation.

Assets have two functions, store of value and liquidity. Buyers care about both whereas sellers care about store of value only. When the dividend value is low, buyers' liquidity demand drives up asset prices, leading to a lower return, and crowds out sellers' asset holding. When the value of the dividend is higher, buyers' liquidity demand is satisfied and asset is priced fundamentally.

For high enough values of the dividend, all buyers participate in the decentralized market, and we obtain unique equilibrium both for bargaining and competitive search. When the dividend is low, even negative, the congestion effect driven by the matching technology makes the net expected benefit negative if all potential buyers participate. The cost of carrying the asset as medium of exchange is indirectly determined by the liquidity premium on the asset price, which itself depends on the dividend value and the number of active buyers in the decentralized market. In both bargaining and competitive search, a higher number of active buyers means a lower probability of trade and a lower cost of carrying the asset. This generates multiplicity of equilibria. With bargaining, there are exactly two equilibria. One with large number of buyers participating and the equilibrium is stable, and the other with small number of buyers participating but is unstable. The stability concept we use is basic in the sense that a small perturbation would lead to more buyers entering, because net benefit exceeds cost, moving toward the other equilibrium or the reverse toward no participation. However, multiplicity holds on a continuum of dividend values in the bargaining case, whereas it disappears under generic dividend values under competitive search. This differs from the monetary case where the cost of holding money, i.e. nominal interest rate, is exogenous and the pure credit case, with zero direct cost.

The relevant literature on divisible money with indivisible good include Green and Zhou (1998) who consider price posting, but in a random rather than competitive search environment. As is well known, indivisible goods with posting lead to indeterminacy due to strong coordination effect. Jean et al. (2010) reconsider Green and Zhou (1988) using the Lagos and Wright (2005) framework and show that the indeterminacy result remains. We show that the coordination effect disappears with bargaining, and with competitive search. With bargaining, money/asset acts as a commitment to not pay more than what buyer brings. With competitive search, candidates from the continuum of Green and Zhou equilibria can be eliminated by sellers posting attractive terms of trade. Buyers respond to posting because they can direct their search. We show how this work. Galenianos and Kircher (2008) consider a model with terms of trade determined by second price auction and characterize the equilibrium distribution of money holding. Liu, Wang and Wright (2015) consider terms of trade determined as in Burdett and Judd (1983) and focus on money and credit as competing payment instruments. Rabinovich (2015) study commodity money with indivisible good. All of this literature use a random search environment only while we also use competitive (directed) search.

The paper is organized as follows. In Section 2 we describe the general environment. In Section 3 we consider a pure credit economy with an exogenous credit constraint. In Section 4 we study a standard monetary economy. In Section 5 we consider an asset economy followed by discussion. Section 6 includes the study of lotteries and a conclusion follows.

2 Environment

The environment is based on the alternating markets framework of Rocheteau and Wright (2005).[8] Time is discrete and goes on forever. A continuum of buyers and sellers, with

[8]The original alternating markets framework by Lagos and Wright (2005) has agents receiving a preference shock in the CM revealing whether they will be a buyer or a seller in the DM. In Rocheteau and Wright (2005), buyers are always buyers and sellers are always sellers. This framework has been used in, among others, Lagos and Rocheteau (2005) and Berentsen et al (2007). However, as discussed in Lagos and Rocheteau (2005) the difference between the frameworks is immaterial. All our results hold for both frameworks.

measures N and 1, live forever. In each period, all agents participate in two markets which open consecutively. Agents discount between periods with factor $\beta \in (0,1)$, but not across markets within a period, and $r = 1/\beta - 1$ is the discount rate. The first market to open is a decentralized market (DM), and the second is a frictionless centralized market (CM). Both buyers and sellers consume a divisible good in the CM, while only buyers consume a perishable indivisible good in the DM.

Buyers' preferences within a period are separable and given by $U(x_t) - h_t + u\mathbf{1}$, where x_t is CM consumption, h_t is CM labor, u is DM utility from consuming the indivisible good, and $\mathbf{1}$ is an indicator function giving 1 if trade occurs and 0 otherwise. Sellers' preferences are $U(x_t) - h_t - c\mathbf{1}$ with DM good produced at constant cost c. We assume $u > c$. Let x_t be the CM numeraire. We assume that x_t is produced one-to-one from labor h_t.

In this model, trade in the DM implies a price and quantity bundle $(p, q) \in \mathcal{P} \times Q$ where $\mathcal{P} = \{0 \leq p \leq L\}$ and $Q = \{0, 1\}$. L represents the available liquidity in the economy, and $L = D$, an exogenous credit constraint in the credit economy. In the monetary economy, $L = \phi m$ represents the buyer's real money balance, and in the asset economy $L = a$, the real asset holding of buyers.

In the DM, meetings occur according to a general meeting technology which is assumed homogeneous of degree one. Given the buyer-seller ratio n, which is also the measure of participating buyers in the DM, the meeting rate for sellers is $\alpha(n)$, and $\alpha(n)/n$ for buyers. Assume $\alpha' > 0$, $\alpha'' < 0$, $\alpha(0) = 0$, $\lim_{n\to\infty} \alpha(n) = 1$, and $\lim_{n\to 0} \alpha'(n) = 1$. These properties are standard for the meeting process used in the Diamond-Mortensen-Pissarides framework.

3 Credit

Consider an economy in which commitment is feasible. Agents are not anonymous, record keeping and punishment devices are available. In this environment there is no role for money. Rather, sellers in the DM will lend to buyers with the buyers promising to deliver consumption good x_t in the subsequent CM. We assume an exogenous credit constraint, $D > 0$. We consider different trading mechanisms and assume production on demand once a buyer and a seller agree on price.

Buyers in the CM obtain

$$W_t^b(p) = \max_{x,h} \left\{ U(x) - h + \beta V_{t+1}^b \right\} \text{ st } x = h - p, \tag{1}$$

where p is the price of DM good. Buyers participate in the DM if $V_{t+1}^b > 0$. Using the budget constraint to eliminate h and solving for optimal x^* yields $W_t^b(p) = U(x^*) - x^* - p + \beta V_{t+1}^b$. Sellers in the CM get

$$W_t^s(p) = \max_{x,h} \left\{ U(x) - h + \beta V_{t+1}^s \right\} \text{ st } x = h + p,$$

which can be simplified to $W_t^s(p) = U(x^*) - x^* + p + \beta V_{t+1}^s$. Sellers participate in DM if $V_{t+1}^s > 0$. The buyer's payoff in the DM is

$$V_t^b = \frac{\alpha(N)}{N} \left[u + W_t^b(p) \right] + \left[1 - \frac{\alpha(N)}{N} \right] W_t^b(0).$$

A buyer that trades obtains credit p, to be paid in the next CM, and gets utility u from DM consumption. Simplifying the buyer's value function yields

$$V_t^b = W_t^b(0) + \frac{\alpha(N)}{N}(u - p). \tag{2}$$

Similarly, $V_t^s = W_t^s(0) + \alpha(N)(p - c)$ is the DM value function for sellers.

3.1 Bargaining

Upon meeting, a buyer and a seller solve the generalized Nash bargaining problem.[9]

$$\max_p (u - p)^\eta (p - c)^{1-\eta} \text{ st } p \leq D.$$

Proposition 1 *In the model with credit and bargaining, $\exists!$ stationary equilibrium (SE) if $D \geq c$, characterized by*

$$p^B = \begin{cases} \bar{p}^B & \text{if } D > \bar{p}^B \\ D & \text{if } D \leq \bar{p}^B, \end{cases}$$

where $\bar{p}^B = (1-\eta)u + \eta c$.

[9] For Kalai bargaining, the buyer's and the seller's surplus are both proportional to the total surplus from trade. Then, if buyers carry any payment that is at least c, sellers will produce. Hence, buyers choose to bring exactly c, which makes the bargaining problem trivial.

Proof. The stationary equilibrium with credit is fully characterized by the solution to the bargaining problem. Using λ as the multiplier on the credit constraint yields the following Kuhn-Tucker conditions:

$$\begin{aligned} 0 &= (1-\eta)(u-p)^{\eta}(p-c)^{-\eta} - \eta(u-p)^{\eta-1}(p-c)^{1-\eta} - \lambda \\ 0 &= \lambda(D-p) \end{aligned}$$

If $\lambda = 0$, then $p = (1-\eta)u + \eta c \equiv \bar{p}^B$. However, if $\lambda > 0$, then $p = D$. Finally, we need $D \geq c$ to guarantee non-negative surplus for sellers. ■

Note that all buyers are active in the DM under credit since using credit is costless and $(u - p^B)\alpha(N)/N > 0$, for all attainable levels of p^B. As will be demonstrated, introducing money and asset as a medium of exchange in the DM can result in $n < N$ active buyers.

3.2 Competitive Search

We study competitive search equilibrium with price posting. The formulation we use is based on Moen (1997) and in a monetary environment on Rocheteau and Wright (2005). Instead of a single DM, there exist a continuum of submarkets, each identified by masses of sellers posting the same terms of trade.[10] Sellers post DM prices before buyers enter the DM. All sellers commit to their posted prices. After observing all the posted prices, each buyer chooses the one that gives him the maximum surplus. Each seller can only produce for one buyer each period. If a seller is visited by multiple buyers, he chooses one with equal probability. Let n represents the expected queue length for any seller in a submarket offering price p. The meeting rates now depend on queue length induced by price, instead of the aggregate N. As before, the meeting rate for sellers is $\alpha(n)$, and $\alpha(n)/n$ for buyers in the submarket featuring p. By posting a lower price, a seller attracts more buyers and increases his trading probability.

Buyers' payoff in the DM is the same as (2). Buyers choose a submarket that maximizes payoff

$$W_t^b(p) = U(x^*) - x^* - p + \beta \max_{\hat{p},n} \left\{ \frac{\alpha(n)}{n}(u - \hat{p}) + W_{t+1}^b(0) \right\},$$

where $\hat{p}$ represents the price of the seller that the buyer visits in $t+1$. The seller's payoff

[10] In this environment, it is easy to show that terms of trade posted by buyers or by a third party (market maker) yields the same outcomes.

in the DM is

$$V_t^s(p) = W_t^s(0) + \max_{p,n} \alpha(n)(p - c).$$

Let Ω be the equilibrium expected utility of a buyer in the DM. To attract queue length n, sellers must offer price p satisfying $(u - p)\alpha(n)/n = \Omega$. A seller solves

$$\max_{p,n} \alpha(n)(p - c) \text{ st } \frac{\alpha(n)}{n}(u - p) = \Omega,\ p \leq D.$$

Solve for p from the buyers' participation constraint, and substitute into the seller's objective function.

$$\max_{n} \alpha(n)\left[u - c - \frac{n\Omega}{\alpha(n)}\right] \text{ st } u - \frac{n\Omega}{\alpha(n)} \leq D.$$

Proposition 2 *In the model with credit and competitive search, $\exists!$ symmetric SE if $D \geq c$, characterized by*

$$p^C = \begin{cases} \bar{p}^C & \text{if } D > \bar{p}^C \\ D & \text{if } D \leq \bar{p}^C \end{cases}$$

where $\bar{p}^C = [1 - \varepsilon(n)]u + \varepsilon(n)c$, $\varepsilon(n) = \alpha'(n)n/\alpha(n)$ is the elasticity of the matching rate for sellers, and $n = N$.

The proof is similar to Proposition 1. As is standard in related models, this result is identical to the case with bargaining when $\varepsilon(N) = \eta$ (Hosios' condition). Similar to bargaining, $(u - p^C)\alpha(N)/N > 0$ for all p^C meaning that all buyers are active in the DM.

4 Money

Now assume agents in the DM cannot commit, for instance due to anonymity. Therefore, buyers must pay sellers with cash in the DM. Let M_t^s be the money supply per buyer at time t, with $M_t^s = \gamma M_{t-1}^s$ and the growth rate of money, γ, is constant. Changes in M^s occur in the CM via lump-sum transfers (taxes) if $\gamma > 1$ ($\gamma < 1$). Nominal interest rate is given by the Fisher equation $1 + i = \gamma/\beta$ and we assume $\gamma > \beta$. The Friedman rule is the limiting case $i \to 0$. Define ϕ_t as the CM price of money in terms of x_t, and $1/\phi_t$ as the nominal price level. In stationary equilibrium, $\phi_t/\phi_{t+1} = \gamma$. Since there is a cost of carrying money, which may not be covered by the buyer's surplus from DM trade, we allow endogenous participation by buyers and let n denote the ratio of active buyers to sellers in the DM.

Buyers with money holding m in the CM solve

$$W_t^b(m) = \max_{x,h,\hat{m}} \left\{U(x) - h + \beta V_{t+1}^b(\hat{m})\right\} \text{ st } x = \phi_t(m+T) + h - \phi_t \hat{m}, \tag{3}$$

where $\hat{m}$ is the money holding carried to the next DM, and T represents net transfers received from the government. Eliminating h from the budget equation,

$$W_t^b(m) = U(x^*) - x^* + \phi_t(m+T) + \max_{\hat{m}} \left\{\beta V_{t+1}^b(\hat{m}) - \phi_t \hat{m}\right\}, \tag{4}$$

where $U'(x^*) = 1$. Sellers do not bring money into the DM as they do not need to make a purchase. Thus,

$$W_t^s(m) = U(x^*) - x^* + \phi_t m + \beta V_{t+1}^s \tag{5}$$

represents their CM value function.

Buyers' payoff in the DM is

$$V_t^b(m) = \frac{\alpha(n)}{n}\left[u + W_t^b\left(m - \frac{p}{\phi_t}\right)\right] + \left[1 - \frac{\alpha(n)}{n}\right] W_t^b(m), \tag{6}$$

where n represents the number of active buyers in the DM. If a buyer gets to trade, he pays p for the indivisible good and gets u from consumption. Linearity, $\partial W_t^b / \partial m = \phi_t$, allows us to simplify (6) to

$$V_t^b(m) = \frac{\alpha(n)}{n}(u-p) + W_t^b(m).$$

Sellers' payoff is

$$V_t^s = \alpha(n)(p-c) + W_t^s(0).$$

4.1 Bargaining

The generalized Nash bargaining problem is

$$\max_p (u-p)^\eta (p-c)^{1-\eta} \text{ st } p \leq \phi m,\ u - p \geq 0,\ p - c \geq 0. \tag{7}$$

As is standard when $\gamma > \beta$, the feasibility constraint, $p \leq \phi m$, binds and $c \leq \phi m \leq \bar{p}^B$, where $\bar{p}^B = (1-\eta)u + \eta c$ as in Proposition 1. Any negotiated price $p^B \in [c, \bar{p}^B]$ is a potential bargaining solution. Substituting V_{t+1}^b into W_t^b yields the following CM value function

$$W_t^b(m) = U(x^*) - x^* + \phi_t (m+T) + \beta W_{t+1}^b(0) + \max_{\hat{m} \in [\frac{c}{\phi_{t+1}}, \frac{\bar{p}^B}{\phi_{t+1}}]} \beta \left\{ \frac{\alpha(n)}{n} \left(u - \phi_{t+1}\hat{m} \right) - i\phi_{t+1}\hat{m} \right\}. \tag{8}$$

The buyer's surplus from trade decreases in $\hat{m}$. Therefore, his optimal money holding decision in (8) reduces to bringing $\phi_{t+1}\hat{m} = c$. With money and bargaining, a buyer can effectively commit to not paying more than $p^B = \phi_{t+1}\hat{m}$. Bringing $\phi_{t+1}\hat{m} \geq \bar{p}^B$, yields $\eta(u-c)$ as buyer's surplus from trade, which is less than $u-c$, the surplus a buyer gets by bringing exactly $\phi_{t+1}\hat{m} = c$. The solution is akin to buyers making a take-it-or-leave-it offer to sellers in pairwise meetings.

Finally, we need to make sure that the buyer's surplus from trade in the DM covers the cost of carrying money. Define

$$\Phi_n(\phi_{t+1}\hat{m}) = \frac{\alpha(n)}{n} \left(u - \phi_{t+1}\hat{m} \right) - i\phi_{t+1}\hat{m}. \tag{9}$$

The buyer's free entry condition $\Phi_n(c) = 0$ implies

$$i = \frac{\alpha(n)}{n} \frac{(u-c)}{c} = \Psi(n). \tag{10}$$

The matching rate $\alpha(n)/n$ is known to be decreasing in n, thus, $\Psi(n)$ is also decreasing in n. Having fewer active buyers in the DM, reduces congestion externalities in the matching probability and increases the marginal gain of entering the DM. Given i, (10) uniquely determines the measure of active buyers in the DM n^*. Define $\bar{\imath}^{NB} = \Psi(N)$ and $\bar{\imath}^B = (u-c)/c$. We can characterize equilibrium with the following proposition.

Proposition 3 *In the model with money and bargaining: (i) For $i \leq \bar{\imath}^{NB}$, $\exists!$ stationary monetary equilibrium (SME) with $n^* = N$, all buyers are active in the DM; (ii) for $i \in (\bar{\imath}^{NB}, \bar{\imath}^B)$, $\exists!$ SME with $n^* < N$; (iii) for $i \geq \bar{\imath}^B$, $\nexists$ SME.*

Proof. First, $i \leq \bar{\imath}^{NB} = \Psi(N)$ implies $\Phi_N(c) \geq \Phi_N(0)$ and hence (i). For (ii), we need to demonstrate that $\lim_{n \to 0} \Psi(n) = (u-c)/c = \bar{\imath}^B$, which is guaranteed by the assumptions of $\alpha(n)$. Finally, for $i \geq \bar{\imath}^B$, $\Phi_n(c) < \Phi_n(0)$ for all $n > 0$, and the DM is inactive. ■

Unlike Green and Zhou (1998) or Jean et al. (2010), there is no real indeterminacy under bargaining. In those papers, real indeterminacy exists due to coordination failure between buyers and sellers. In the current environment, the unique solution generated by

Nash bargaining works as a coordination device, and both parties know ex ante that p^B will be the price in DM trade. Then, buyers carry just enough money to cover the price.

The real balance in equilibrium only depends on u, c, and does not depend on the bargaining power η or the nominal interest rate i. This is intuitive since the intensive margin of trade cannot adjust. The only gain of bringing money comes from the extensive margin, i.e. to trade or not to trade. In this environment, buyers move first by choosing money balances. Then, buyers can commit to bringing the lowest level of real balances acceptable for trade, that is, the level that makes sellers exactly indifferent between trade or no trade. The nominal interest rate has no effect on the DM real price, buyer's real balances, or the real value of money. For $i \leq \bar{\imath}^{NB}$, all buyers participate in the DM and the total output is not affected by i, either. Therefore, money is superneutral in the model with bargaining for small nominal interest rates.

This result differs from most of the New Monetarist literature, which generally features neutrality of money but real allocations are affected by changes in inflation. The generalized Nash bargaining mechanism determines the buyer's share of surplus according to exogenous bargaining power, which then determines the unique optimal real balance. Monetary variables do not play a role in the determination of real variables, but only affects the price of money ϕ.

When it is costless to carry money to the DM, i.e. $i = 0$, the monetary economy is comparable to the imperfect credit economy in Section 3.1, but with different price in the DM. When $i = 0$, buyers still choose to carry just enough real balance to cover the seller's reservation price c, so as to maximize their surplus from trade. As shown in Proposition 1, the equilibrium price with credit is almost always higher than the seller's reservation price. This is because when facing an exogenous credit constraint, buyers do not have the power to effectively commit to pay c ex ante.

4.2 Competitive Search

The next step is to study the implications for monetary equilibria of using competitive search. The buyer's DM value function is now

$$V_t^b(p, m) = \frac{\alpha(n)}{n}(u - p) + W_t^b(m), \tag{11}$$

where p is the price posted by the buyer's chosen seller. From (4) and (11), buyers' value is

$$W_t^b(m) = U(x^*) - x^* + \phi_t (m + T) + \beta W_{t+1}^b (0) + \max_{\hat{m},p,n} \beta \left\{ \frac{\alpha(n)}{n} (u - p) - i\phi_{t+1}\hat{m} \right\}. \quad (12)$$

Since sellers post p before buyers choose their money holdings, $\phi_{t+1}\hat{m} = p$ as long as $i > 0$. Let Ω again be the equilibrium expected utility of a buyer in the DM. Sellers maximize profit $\pi(n)$,

$$\max_{p,n} \pi(n) = \alpha(n)(p - c) \ \text{ st } \frac{\alpha(n)}{n}(u - p) - ip \geq \Omega, \quad (13)$$

or

$$\max_{n} \pi(n) = \alpha(n) \left[\frac{\alpha(n) u - n\Omega}{\alpha(n) + ni} - c \right]. \quad (14)$$

In equilibrium, the optimal measure of buyers n^* is consistent with the free entry condition

$$\frac{\alpha(n^*)}{n^*} \left(u - p^C\right) - ip^C \geq 0, \quad (15)$$

and p^C is the seller's optimal price

$$p^C = \frac{\alpha(n^*) \{[1 - \varepsilon(n^*)] u + \varepsilon(n^*) c\} + \varepsilon(n^*) n^* ic}{\alpha(n^*) + \varepsilon(n^*) n^* i}. \quad (16)$$

Unlike the case of bargaining, p^C depends on i and n, the measure of active buyers in the market. The real balances buyers bring into the DM are p^C.

We follow Wright (2010) in establishing the existence and uniqueness of monetary equilibrium. Define the aggregate demand of liquidity, $L^d = n^* p^C$, with n^* and p^C both depending on i. Monetary equilibrium is then characterized by the intersection of L^d and the relevant supply curve, which is horizontal at the exogenous policy variable, i. The nominal interest rate is the price of holding liquidity. It determines the equilibrium quantity via L^d, which is characterized by the following lemma.

Lemma 1 *There exist i^{NC} and $\bar{\imath}^C$ with $i^{NC} < \bar{\imath}^C$, such that: (i) for $i < i^{NC}$, $\exists!$ L^d with $n^* = N$ and $dL^d/di < 0$; (ii) for generic $i \in [i^{NC}, \bar{\imath}^C]$, $\exists!$ L^d with $n^* \leq N$ and $dL^d/di < 0$; (iii) for $i > \bar{\imath}^C$, $\nexists$ $n^* > 0$ and L^d is not well-defined.*

Proof. To prove the existence and uniqueness of L^d, it is sufficient to show the existence and uniqueness of $n^* > 0$. Substitute p^C into (15) and we get $\alpha\varepsilon(u - c) i + \alpha^2\varepsilon(u - c)/n^* \geq \alpha[(1 - \varepsilon) u + \varepsilon c] i + \varepsilon n^* c i^2$. Define $h(n^*, i) = \alpha\varepsilon(u - c)i + \alpha^2\varepsilon(u -$

$c)/n^* - \alpha[(1-\varepsilon)u + \varepsilon c]i - \varepsilon n^* c i^2$. Given any $n \in (0, N]$, $h(n,i) = 0$ is a quadratic function in i, which has two real solutions with opposite signs. The positive solution i_+, satisfying $h(n, i_+) = 0$, is an implicit function of n, $i_+(n)$. Let $i_+(0) = \lim_{n \to 0} i_+(n) < \infty$. It is straightforward to show that $i_+(n)$ is continuous on $[0, N]$. Define i^{NC} by $h(N, i^{NC}) = 0$ and $\bar{\imath}^C = \max_{n \in [0,N]} i_+(n)$. For $i < i^{NC}$, $h(N,i) > 0$ hence $n^* = N$. Then $L^d = Np^C(N,i)$ is unique, and $\partial L^d / \partial i = N \partial p^C(N,i)/\partial i < 0$, hence (i). For $i > \bar{\imath}^C$, $h(n^*, i) < 0$ $\forall n^*$, and the free-entry condition does not hold due to $\alpha(n^*)(u - p^C)/n^* - ip^C < 0$, hence (iii).

Regarding (ii), for $i \le \bar{\imath}^C$, $h(n^*, i) = 0$ always holds for some $n^* > 0$, and L^d exists. To show that L^d is generically unique and monotone, consider $L^d = n^* p^C$ and $dL^d/di = \partial L^d/\partial i + (\partial L^d / \partial n^*)(\partial n^* / \partial i)$. Given $h(n^*, i) = 0$, we have $L^d = \alpha(n^*) n^* u / [\alpha(n^*) + in^*]$, hence $\partial L^d/\partial i < 0$ and $\partial L^d / \partial n^* > 0$. Then, it is sufficient to show that n^* is generically unique and $\partial n^*/\partial i < 0$.

We follow Proposition 1 in Gu and Wright (2015) and claim that although there might be multiple n^* which maximize $\pi(n,i)$, n^* is still unique and $\partial n^* / \partial i < 0$ for generic i. To see this, suppose $\pi(n_1^*, i) = \pi(n_2^*, i) = \max_n \pi(n, i)$ and $n_2^* > n_1^*$. Then, n_1^* is the minimum n that maximizes $\pi(n,i)$, and $\pi(n_1^*, i) > \pi(n,i)$, $\forall n < n_1^*$. For $\varepsilon > 0$ small enough, $\pi(n_1^*, i+\varepsilon) > \pi(n, i+\varepsilon)$ also holds for $n < n_1^*$ due to continuity. If $\partial^2 \pi / \partial i \partial n^* < 0$, then $\pi(n_1^*, i+\varepsilon) > \pi(n_2^*, i+\varepsilon)$, and the global maximizer is a unique n in the neighborhood of n_1^*.

Finally, we need to show $\partial^2 \pi / \partial i \partial n^* < 0$. Derive $\partial \pi / \partial n$ from (14),

$$\frac{\partial \pi}{\partial n} = \frac{(\alpha + in)\left[(u-c)\,\alpha' - ic\right] - i\,(1-\varepsilon)\left[(u-c)\,\alpha - inc\right]}{(\alpha + in)^2 / \alpha}.$$

Define $T(i) = (\alpha + in)[(u-c)\alpha' - ic] - i(1-\varepsilon)[(u-c)\alpha - inc]$, and $T'(i) = n[(u-c)\alpha' - ic] - (\alpha + in)c - (1-\varepsilon)[(u-c)\alpha - inc] + inc(1-\varepsilon)$. Since $T_{n=n^*} = 0$, $\partial^2 \pi / \partial i \partial n^* = T'(i)/[(\alpha + in^*)^2/\alpha]$. With $\alpha(u-c) - in^* c > 0$, we have

$$T'_{n=n^*}(i) = \frac{-\left[\alpha\,(u-c) - in^* c\right](1-\varepsilon)\,\alpha - c\,(\alpha + in^*)\,(\alpha + in^* \varepsilon)}{\alpha + in^*} < 0.$$

Therefore, $\partial^2 \pi / \partial i \partial n^* < 0$ holds. Like in Gu and Wright (2015), $\arg\max_n \pi(n,i)$ might have more than one solution for some $i \ge i^{NC}$, but the set of such nominal interest rates has measure zero, hence (ii). ■

Now we are ready to characterize symmetric monetary equilibrium, where all sellers post the same price and buyers visit each seller with the same probability.

Proposition 4 *In the model with money and competitive search: (i) for $i < i^{NC}$, $\exists!$ symmetric SME with $n^* = N$; (ii) for generic $i \in [i^{NC}, \bar{\imath}^C]$, $\exists!$ symmetric SME with $n^* \leq N$ ($<$ if $i > i^{NC}$); (iii) for $i > \bar{\imath}^C$, $\nexists$ SME.*

Proof. First, for $i > \bar{\imath}^C$, $n^* < 0$ as shown in Lemma 1, and there is no monetary equilibrium, hence (iii). For $i < i^{NC}$, L^d is unique and monotonically decreasing in i. Hence, given i, there exists a unique real money holding $\phi_{t+1}\hat{m} = p^C$ and hence a unique SME. Since $h(N, i) > 0$ and $\alpha(N)(u - p^C)/N - ip^C > 0$, we have $n^* = N$, hence (i). Finally, in the proof of Lemma 1, we have also shown the generic uniqueness of L^d and $\partial L^d/\partial i < 0$, for $i \in [i^{NC}, \bar{\imath}^C]$. Therefore, there exists a generically unique real balance $\phi_{t+1}\hat{m}$ and hence symmetric SME with $n^* \leq N$. The inequality becomes strict if $i > i^{NC}$. ■

Note that this environment satisfies all the properties under which Galenianos and Kircher (2012) demonstrate uniqueness of equilibrium when terms of trade of an indivisible good are determined by price posting with directed search. However, multiplicity is possible in our model when the buyer's payoff is zero and they randomize over entry decision. The different result is due to the finite-agent setup in Galenianos and Kircher (2012), which generates the better-reply security of Reny (1999). Given finite agents and zero payoff for buyers, sellers can always post a better terms of trade to increase their trading probability and make buyer's surplus positive. This cannot happen in a model with infinitely many agents, since an individual seller has measure zero and cannot change his own trading probability by posting a different terms of trade.

Similar to Rocheteau and Wright (2005), there may be multiple equilibria under a countable number of i. In both models, a higher n implies a smaller probability of trade for buyers, but they are compensated by a larger trade surplus. The indivisibility of goods does not eliminate possible multiplicity. In Rocheteau and Wright (2005), buyers may have higher consumption or lower price, whereas in our model only the latter is possible due to indivisible goods.

To compare with Nash bargaining, the real DM price and the buyer's real balance under competitive search are always affected by i, and money is not superneutral, while it is still neutral. Therefore, the buyer's surplus from trade under competitive search adjusts endogenously with nominal interest rate, i.e., p^C is decreasing in i. As shown in Lagos and Rocheteau (2005) with divisible goods, higher anticipated inflation gives more bargaining power to buyers. Their results hold under indivisible goods. While under bargaining, the buyer's share of surplus is determined exogenously by the bargaining power, and does not adjust with nominal interest rate. If $i = 0$, holding money is costless. The DM price under competitive search then becomes

$$p^C = [1 - \varepsilon (n^*)] u + \varepsilon (n^*) c,$$

which is the same as the price under pure credit and bargaining with credit, given $\eta = \varepsilon (n)$.

Apart from the finding of equilibrium existence, our results differ quite substantially from those of Jean et al (2010). They consider price posting and random search, and show the existence of a continuum of equilibria, indexed by different real balances. Their result of multiple equilibria is driven by coordination failure of simultaneous moves by buyers and sellers. In order to obtain a unique equilibrium, they impose the assumptions of finite agents and sequential move. With competitive search, we do not need those assumptions to get (generically) unique equilibrium. Buyers can direct their search to the seller who gives the highest expected payoff. Competition among sellers guarantees that a buyer gets Ω from DM trade. Competition on both sides of the market endogenously generates the expected queue length at sellers, which then determines the probability of trade and the share of surplus for both sides. The expected queue length adjusts continuously with the posted price, and the market-clearing price in the DM is uniquely determined when the expected queue length equals the buyer-seller ratio of the entire economy N. This adjustment mechanism does not exist under price posting and random search, leading to Diamond (1982) prices and no monetary equilibria.

5 Asset

We continue to assume that agents in the DM are anonymous and have no commitment. However, instead of using money, buyers in the DM will pay sellers with real assets. The total asset supply is fixed at A^s. Let φ_t be the CM price of real assets in terms of x_t, $A = A^s/N$ be the average amount of assets held by each buyer, and ρ be the dividend of real assets, which can be either positive or negative.

Buyers bring asset holding a into the CM and solve

$$W_t^b (a) = \max_{x,h,\hat{a}} \left\{ U (x) - h + \beta V_{t+1}^b (\hat{a}) \right\} \text{ st } x = (\varphi_t + \rho) a + h - \varphi_t \hat{a}, \tag{17}$$

where $\hat{a}$ is the asset holding carried into the following DM. Substitute h using the budget constraint and solve for optimal x. (17) can be rewritten as

$$W_t^b(a) = U(x^*) - x^* + (\varphi_t + \rho) a + \max_{\hat{a}} \left\{ \beta V_{t+1}^b(\hat{a}) - \varphi_t \hat{a} \right\},$$

where $U'(x^*) = 1$. The CM value function of a seller with asset holding a_s is

$$W_t^s(a_s) = U(x^*) - x^* + (\varphi_t + \rho) a_s + \max_{\hat{a}_s} \left\{ \beta V_{t+1}^s(\hat{a}) - \varphi_t \hat{a}_s \right\}. \tag{18}$$

The buyer's value function in the DM is

$$V_t^b(a) = \frac{\alpha(n)}{n} \left[u + W_t^b \left(a - \frac{p}{\varphi_t + \rho} \right) \right] + \left[1 - \frac{\alpha(n)}{n} \right] W_t^b(a), \tag{19}$$

where p is the price paid by the buyer for the DM good. Using $\partial W_t^b / \partial a = \varphi_t + \rho$, we can simplify (19) to

$$V_t^b(a) = \frac{\alpha(n)}{n} (u - p) + W_t^b(a)$$

Similarly, seller's payoff in the DM is

$$V_t^s(a) = \alpha(n)(p - c) + W_t^s(a) \tag{20}$$

Sellers do not need to hold assets for trading purposes in the DM, but they can still use assets as a store of value. Use (20) to substitute V_{t+1}^s in (18), and we can simplify the seller's decision on $\hat{a}_s$ to the following problem

$$\max_{\hat{a}_s} \left\{ \beta \left(\varphi_{t+1} + \rho \right) \hat{a}_s - \varphi_t \hat{a}_s \right\}. \tag{21}$$

A necessary condition for sellers to hold assets is $\varphi_t = \beta(\varphi_{t+1} + \rho)$, i.e. the asset is priced at its fundamental value.

5.1 Bargaining

The generalized Nash bargaining problem is the same as (7) with a different feasibility constraint $p \leq (\varphi_t + \rho)a$. Substitute V_{t+1}^b into W_t^b and we get the buyer's CM value function.

$$W_t^b(a) = U(x^*) - x^* + (\varphi_t + \rho)a + \beta W_{t+1}^b(0) + \max_{\hat{a}} \left\{ \beta \frac{\alpha(n)}{n}(u - p) + \left[\beta\left(\varphi_{t+1} + \rho\right) - \varphi_t\right]\hat{a} \right\}$$

Since $\beta(\varphi_{t+1} + \rho) \leq \varphi_t$, the feasibility constraint binds and the bargaining solution implies $c \leq (\varphi_t + \rho)\hat{a} \leq (1-\eta)u + \eta c$. Similar to the case where money is the medium of exchange, buyers never carry more real assets than the amount needed to pay for the DM good.

The buyer's decision problem on asset holding can be rewritten as

$$\max_{\frac{c}{\varphi_{t+1}+\rho} \leq \hat{a} \leq \frac{(1-\eta)u+\eta c}{\varphi_{t+1}+\rho}} \left\{ \beta \frac{\alpha(n)}{n} \left[u - \left(\varphi_{t+1} + \rho\right)\hat{a}\right] + \left[\beta\left(\varphi_{t+1} + \rho\right) - \varphi_t\right]\hat{a} \right\}, \tag{22}$$

and the optimal solution is $\hat{a}^*(\varphi_{t+1} + \rho) = c$. Here, $\varphi_{t+1} + \rho$ is the real return on assets measured by CM goods. With bargaining, a buyer can effectively commit to not paying more than the seller's reservation price c, no matter money or asset being the medium of exchange. The buyer's value from participating in the DM is

$$\Phi_n = \beta \left[\frac{\alpha(n)}{n}u + \left(1 - \frac{\alpha(n)}{n}\right)c\right] - \frac{\varphi_t c}{\varphi_{t+1} + \rho}. \tag{23}$$

The measure of DM buyers n^* is determined by the free entry condition, $\Phi_{n^*} \geq 0$.

In the following, we focus on stationary equilibrium. Since the total supply of assets A^s is time-invariant, the asset prices in stationary equilibrium satisfy $\varphi_t = \varphi_{t+1}$. To establish equilibrium existence and uniqueness, we start by characterizing asset prices. There are two cases to consider seeing that assets have two functions, store of value and liquidity. When assets are held for store of value, they are priced fundamentally and both buyers and sellers hold them. When assets are held for liquidity purposes, only buyers will hold assets as they are priced higher than their fundamental value due to a liquidity premium. These two cases are summarized in the following lemma.

Lemma 2 *Given n, the measure of active buyers in the DM: (i) for $\rho \geq (1-\beta)cn/A^s$, $\varphi = \varphi^F = \rho/r$ and $\hat{a} \leq A$; (ii) for $\rho < (1-\beta)cn/A^s$, $\varphi = cn/A^s - \rho > \varphi^F$ and $\hat{a} = A^s/n \geq A$.*

Proof. If assets are priced fundamentally, the price is $\varphi = \varphi^F = \rho/r$. If $\varphi > \varphi^F$, only buyers with measure n hold the assets and $\hat{a} = A^s/n \geq A^s/N = A$. From the solution of (22), the individual buyer's demand for assets is $\hat{a}^* = c/(\varphi + \rho)$. Equating demand with supply yields $\varphi = cn/A^s - \rho$. We need $\rho < (1-\beta)cn/A^s$ to guarantee $cn/A^s - \rho > \varphi^F$. Finally, when $\varphi = \varphi^F$, $\hat{a} = (1-\beta)c/\rho \leq A$, and sellers may hold assets as well. ■

If the dividend of assets ρ is high, a buyer does not need to carry many assets for the DM purchase, and the marginal holder of assets is a seller, who only cares about the store-of-value function. As shown in (21), sellers only hold assets when they are at their fundamental price. In this case, the participation constraint (23) becomes $\Phi_n(\varphi^F) = \beta(u-c)\alpha(n)/n$, which is positive for $n = N$. Hence, when $\rho \geq \rho^F = (1-\beta)cN/A^s$, the liquidity need of all buyers are satisfied and they all participate in the DM. The seller's asset holding is positive if $\rho > \rho^F$.

If ρ is low, the marginal holder of assets is a buyer, who cares about both liquidity and store of value. The liquidity function drives up the asset price to be above its fundamental value, and sellers no longer hold assets. Substitute the asset prices into (23) and the buyer's participation constraint now becomes

$$-\rho \leq \frac{n}{A^s}[\frac{\alpha(n)}{n}\beta(u-c) - (1-\beta)c] = f(n),$$

which can be rewritten as

$$\frac{\alpha(n)}{n}(u-c) - \frac{r\varphi - \rho}{\varphi + \rho}c \geq 0. \tag{24}$$

Define the spread of assets $s = (r\varphi - \rho)/(\varphi + \rho)$. Notice that (24) is similar to (9), the buyer's participation constraint in the monetary economy. We can rewrite the spread as $1 + s = (1+r)\varphi/(\varphi + \rho)$ and $\varphi/(\varphi + \rho)$ is similar to ϕ_{t-1}/ϕ_t, when money is used in the DM. Hence, similar to the nominal interest rate i, the spread s represents the cost of holding assets as the medium of exchange in the DM.

The equilibrium measure of buyers in the DM n^* is determined by $f(n^*) = -\rho$. Define $\rho^{NB} = -f(N)$, and all the buyers participate in the DM if $\rho \geq \rho^{NB}$. Once $\rho < \rho^{NB}$, buyers' participation starts to decrease, and eventually the DM will shut down. We summarize different cases of equilibria in the following proposition.

Proposition 5 *In the model with assets and bargaining: (i) for $\rho \geq \rho^F$, $\exists!$ SE with $\varphi = \varphi^F$ and $n^* = N$; (ii) for $\rho \in [\rho^{NB}, \rho^F)$, $\exists!$ stable SE with $\varphi = \varphi^N > \varphi^F$ and $n^* = N$; (iii) for $\rho \in [\underline{\rho}, \rho^{NB})$, $\exists!$ stable SE with $\varphi = \varphi^{n^*} > \varphi^F$ and $n^* < N$; (iv) for $\rho < \underline{\rho}$, $\nexists$ equilibrium with an active DM.*

Proof. (i) is straightforward from Lemma (2). To establish uniqueness, notice that function f is continuous and satisfies $f(0) = 0$ and $f''(n) < 0$. As shown in Figure 1,

$f(n)$ is represented by the dashed curves below the horizon on the domain of $[0, N]$. For $\rho \geq \rho^F > 0$, there is a unique n^* satisfying $f(n^*) = -\rho$. Define $\underline{\rho} = -\max_{n\in[0,N]} f(n) \leq 0$. For $\rho < \underline{\rho}$, $f(n) < -\rho$ $\forall n > 0$, and the DM shuts down. First, consider the case $\underline{\rho} = 0$ in Figure 1a. Since f is decreasing in n on $[\underline{\rho}, \rho^F)$, there exists a unique equilibrium indexed by n^*. We have shown that for $\rho \in [\rho^{NB}, \rho^F)$, all buyers participate in the DM. Then, the asset price is $\varphi^N = cN/A^s - \rho > \varphi^F$ due to $\rho < \rho^F$. For $\rho \in [\underline{\rho}, \rho^{NB})$, $f(N) < -\rho$ and $n^* < N$. It is then easy to check $\varphi^{n^*} = cn^*/A^s - \rho > \varphi^F$ using $\rho = -f(n^*)$. Second, consider $\underline{\rho} < 0$. For $\rho \in [0, \rho^F)$, as in Figure 1b, $f(n) = -\rho$ still has a unique solution $n^* > 0$, and the above results hold. Then, Figure 1c and 1d show that, for $\rho \in (\underline{\rho}, 0)$, $f(n) = -\rho$ has exactly two solutions, denoted as $0 < n_1^* < n_2^*$. One can easily prove that $n_2^* = N$ for $\rho \geq \rho^{NB}$ and $n_2^* < N$ otherwise. Finally, we want to show that the equilibrium at n_1^* is unstable. For $n < n_1^*$, $f(n) < -\rho$, and buyers want to exit the DM until $n = 0$. For $n > n_1^*$, more buyers want to enter the DM until $n = n_2^*$. Therefore, the equilibrium at n_2^* is stable. ■

Corollary 1 *For $\rho \in (\underline{\rho}, 0)$, $\exists!$ unstable equilibrium with $\varphi = \varphi^{n^*} > \varphi^F$ and $n^* < N$.*

In Figure 1, while the dashed curves below the horizon represent the buyer's participation constraint $f(n)$, the solid curves above the x-axis show how the asset price φ changes with respect to the dividend ρ. When ρ is large enough, assets are priced at the fundamental value and they are not affected by the buyer's participation in the DM. For any dividend between ρ^{NB} and ρ^F, the asset price is above its fundamental price and decreasing in ρ. When ρ becomes small enough, i.e. $\rho < \rho^{NB}$, both dividend and the buyers' participation n^* affect the asset price in opposite directions, and φ is increasing in ρ in stable equilibrium. For $\rho < 0$, the asset prices are still positive due to the liquidity premium in facilitating DM trade, and the two prices correspond to two stationary equilibria.

Notice that when $\rho < 0$, stationary equilibrium is not unique due to a coordination problem. There are two different levels of buyer's participation in the DM, high and low. When the equilibrium participation is high, a larger liquidity demand drives up the asset price φ, which implies a smaller asset spread s, since $\partial s/\partial\varphi < 0$ when $\rho < 0$. Buyers now face a low probability to trade in the DM, but they are compensated by a small cost of holding assets, the medium of exchange in the DM. Similarly, when the equilibrium participation is low, buyers receive a high probability to trade with a large

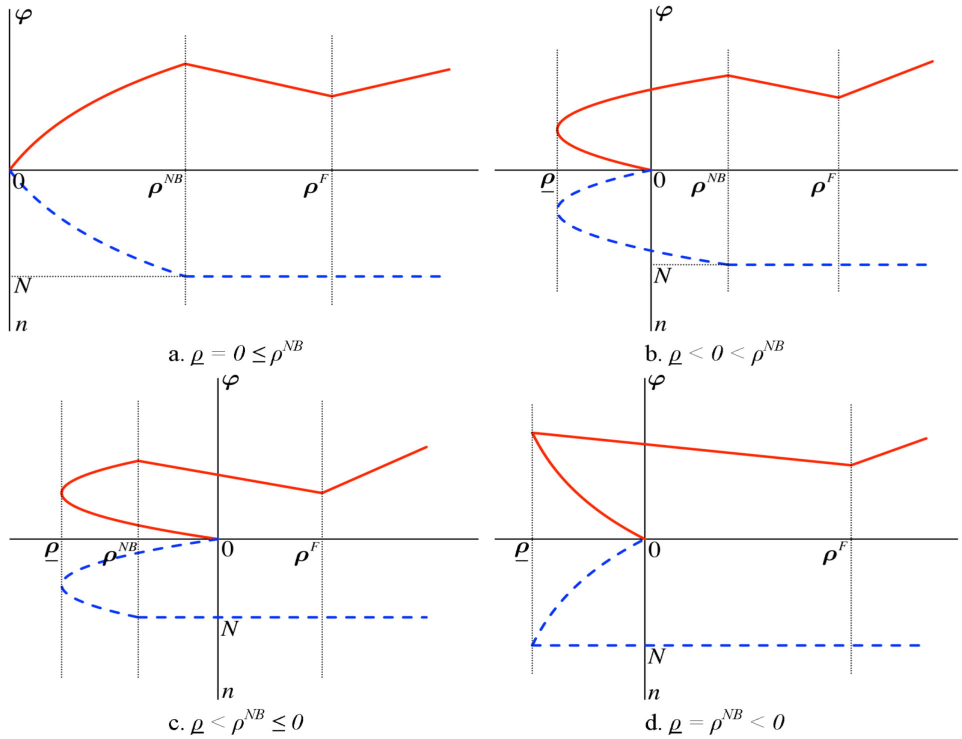

Figure 1: Asset with Bargaining

cost of holding assets. When $\rho \geq 0$, the coordination problem does not exist, since the spread is increasing in asset price, $\partial s/\partial \varphi \geq 0$, and hence equilibrium is unique. Unlike assets, the cost of holding credit or money is exogenously given at zero or i, and it does not depend on the buyer's participation in the DM. Therefore, this coordination effect does not exist in the credit or monetary economy and there is always a unique equilibrium.

When $\rho < 0$, the negative dividend can be interpreted as a storage cost, similar to the stinky fish in Kiyotaki and Wright (1989). When $\rho = 0$, assets are equivalent to money with a constant supply. Unlike money, the cost of holding assets depends on both dividend and the buyer's participation, while i is an exogenous policy choice and leads to unique equilibrium. In the monetary economy, a higher participation in the DM implies a lower probability of trade for buyers. Since the cost of holding money is fixed, instead of adjusting with n like assets, buyers are strictly worse off, and the coordination problem no longer exists. For $\rho > 0$, an equilibrium with assets always exists. However, monetary equilibrium may not exist with deflation, if the surplus from DM trade $(u - c)/c$ is small enough, because the cost of holding money does not adjust with buyers' participation.

When $s = 0$ and carrying assets becomes costless, the asset economy is comparable to the credit economy in Section 3.1. Similar to the findings in the monetary economy, the equilibrium price in the DM is lower with assets than with credit. This is again because buyers can carry just enough assets to cover the seller's production cost, while they cannot make such a commitment with an exogenous credit constraint.

5.2 Competitive Search

Similar to (13), the seller's price posting problem is

$$\max_{p,n} \pi(n) = \alpha(n)(p-c) \text{ st } \frac{\alpha(n)}{n}(u-p) - sp \geq \Omega. \tag{25}$$

Substituting p yields

$$\max_{n} \pi(n) = \alpha(n)\left[\frac{\alpha(n)u - n\Omega}{\alpha(n)+ns} - c\right]. \tag{26}$$

In equilibrium, the optimal queue length of buyers is consistent with the free entry condition

$$\frac{\alpha(n^*)}{n^*}(u-p^C) - sp^C = \Omega \geq 0, \tag{27}$$

and p^C is the seller's optimal price satisfying

$$p^C = \frac{\alpha(n^*)\{[1-\varepsilon(n^*)]u + \varepsilon(n^*)c\} + \varepsilon(n^*)n^* sc}{\alpha(n^*) + \varepsilon(n^*)n^* s},$$

which is a function of s and n.

We study the existence and uniqueness of equilibrium by equating the aggregate demand and supply of liquidity. The aggregate demand of liquidity $L^d = n^* p^C$ is a function of the spread s. Given a one-to-one mapping from the asset price φ to s, the aggregate supply $L^s = (\varphi+\rho)A^s$ is also a function of s. The aggregate demand and supply of liquidity are characterized by the following lemmas.

Lemma 3 *There exist $\bar{s}^C \geq r$ and $s^{NC} \leq \bar{s}^C$, such that: (i) for $s < s^{NC}$, $\exists!$ L^d with $n^* = N$, and $dL^d/ds < 0$; (ii) for generic $s \in [s^{NC}, \bar{s}^C]$, $\exists!$ L^d with $n^* \leq N$ ($<$ if $s > s^{NC}$), and $dL^d/ds < 0$; (iii) for $s > \bar{s}^C$, $\nexists$ $n^* > 0$ and L^d is not well-defined.*

Proof. To prove that L^d is a well-defined function for $s \leq \bar{s}^C$, it is sufficient to show $n^* > 0$ exists and is unique. Substituting p^C into (27) gives $\alpha\varepsilon(u-c)s + \alpha^2\varepsilon(u-c)/n^* = \alpha[(1-\varepsilon)u+\varepsilon c]s + \varepsilon n^* cs^2$. Define $h(n^*, s) = \alpha\varepsilon(u-c)s + \alpha^2\varepsilon(u-c)/n^* - \alpha[(1-\varepsilon)u+\varepsilon c]s - \varepsilon n^* cs^2$.

Given any $n \in (0, N]$, $h(n, s) = 0$ is a quadratic function in s, which has two real solutions with opposite signs. The positive solution s_+, satisfying $h(n, s_+) = 0$, is an implicit function of n, $s_+(n)$. Let $s_+(0) = \lim_{n \to 0} s_+(n) < \infty$, and $s_+(0)$ is continuous on $[0, N]$. Define s^{NC} by $h(N, s^{NC}) = 0$ and $\bar{s}^C = \max_{n \in [0,N]} s_+(n)$. For $s < s^{NC}$, $h(N, s) > 0$ hence $n^* = N$. Then $L^d = Np^C(N, s)$ is unique, and $dL^d/ds = Ndp^C(N, s)/ds < 0$, hence (i). For $s > \bar{s}^C$, $h(n^*, s) < 0$ $\forall n^*$, and the free-entry condition does not hold due to $\alpha(n^*)(u - p^C)/n^* - sp^C < 0$, hence (iii).

Regarding (ii), for $s \leq \bar{s}^C$, $h(n^*, s) = 0$ always holds for some $n^* > 0$, and L^d exists. To show that L^d is generically unique and monotone, consider $L^d = n^* p^C$ and $dL^d/ds = \partial L^d/\partial s + (\partial L^d/\partial n^*)(\partial n^*/\partial s)$. Given $h(n^*, s) = 0$, we have $L^d = \alpha(n^*)n^* u/[\alpha(n^*) + sn^*]$, hence $\partial L^d/\partial s < 0$ and $\partial L^d/\partial n^* > 0$. Then, it is sufficient to show that n^* is generically unique and $\partial n^*/\partial s < 0$.

We follow Proposition 1 in Gu and Wright (2015) and claim that although there might be multiple n^* which maximize $\pi(n, s)$, n^* is still unique and $\partial n^*/\partial s < 0$ for generic s. To see this, suppose $\pi(n_1^*, s) = \pi(n_2^*, s) = \max_n \pi(n, s)$ and $n_2^* > n_1^*$. Then, n_1^* is the minimum n that maximizes $\pi(n, s)$, and $\pi(n_1^*, s) > \pi(n, s)$, $\forall n < n_1^*$. For $\epsilon > 0$ small enough, $\pi(n_1^*, s+\epsilon) > \pi(n, s+\epsilon)$ also holds for $n < n_1^*$ due to continuity. If $\partial^2\pi/\partial s \partial n^* < 0$, then $\pi(n_1^*, s+\epsilon) > \pi(n_2^*, s+\epsilon)$, and the global maximizer is a unique n in the neighborhood of n_1^*.

Next, we need to show $\partial^2\pi/\partial s \partial n^* < 0$. Derive $\partial \pi/\partial n$ from (26),

$$\frac{\partial \pi}{\partial n} = \frac{(\alpha + sn)\left[(u - c)\,\alpha' - sc\right] - s\,(1 - \varepsilon)\left[(u - c)\,\alpha - snc\right]}{(\alpha + sn)^2/\alpha}.$$

Define $T(s) = (\alpha + sn)[(u - c)\alpha' - sc] - s(1 - \varepsilon)[(u - c)\alpha - snc]$, and $T'(s) = n[(u - c)\alpha' - sc] - (\alpha + sn)c - (1 - \varepsilon)[(u - c)\alpha - snc] + snc(1 - \varepsilon)$. Since $T_{n=n^*} = 0$, $\partial^2\pi/\partial s \partial n^* = T'(s)/[(\alpha + sn^*)^2/\alpha]$. With $\alpha(u - c) - sn^* c > 0$, we have

$$T'_{n=n^*}(s) = \frac{-\left[\alpha\,(u - c) - sn^* c\right](1 - \varepsilon)\,\alpha - c\,(\alpha + sn^*)\,(\alpha + sn^*\varepsilon)}{\alpha + sn^*} < 0.$$

Therefore, $\partial^2\pi/\partial s \partial n^* < 0$ holds. Like in Gu and Wright (2015), $\arg\max_n \pi(n, s)$ might have more than one solution for some $s \geq s^{NC}$, but the set of such asset spreads has measure zero, hence (ii).

Finally, we prove $\bar{s}^C \geq r$ by contradiction. Suppose $\bar{s}^C < r$, then for $s_1 = (r\varphi_1 - \rho_1)/(\varphi_1 + \rho_1) \in (\bar{s}^C, r)$, $\rho_1 > 0$ and $n_1^* = 0$. Hence, $\varphi_1 = \varphi_1^F$ and $s_1 = 0$, contradicting

$s_1 > \bar{s}^C > 0$. ■

Recall $s = (r\varphi - \rho)/(\varphi + \rho)$ is the spread of assets and $\partial s/\partial \rho < 0$. As shown in Lemma 3, if the asset dividend is low enough and the cost of holding asset is high enough, the DM will shut down. As long as the DM operates and L^d is well-defined, it is monotonically decreasing in s. The DM participation of buyers varies depending on different values of ρ hence s. Next lemma characterizes the aggregate supply of liquidity.

Lemma 4 *For $\rho < 0$, L^s is convex and $\partial L^s/\partial s < 0$; for $\rho = 0$, L^s is perfectly elastic at $s = r$; for $\rho \in (0, \rho^F)$, L^s is concave and $\partial L^s/\partial s > 0$; for $\rho \geq \rho^F$, L^s is perfectly elastic at $s = 0$.*

Proof. If the asset is priced at the fundamental value and all buyers participate in the DM, $s = 0$ and let $\rho^F = (1 - \beta)p^C_{N,s=0}/A$. If $\rho \geq \rho^F$, the average asset holding $(\varphi + \rho)A^s/n \geq (\varphi^F + \rho)A^s/n \geq \rho^F A/(1 - \beta) = p^C_{N,s=0}$. The liquidity need for assets is satisfied, and the marginal holders of assets only care about the store of value. Hence, $\varphi = \varphi^F$ and $s = 0$. If $\rho = 0$, the cost of holding asset is $s = r$. If $\rho < \rho^F$ and $\rho \neq 0$, substitute s into the liquidity supply and $L^s = (1+r)\rho A^s/(r-s)$, with $\partial L^s/\partial s = (1+r)\rho A^s/(r-s)^2$ and $\partial^2 L^s/\partial s^2 = -2(1 + r)\rho A^s/(r - s)^3$. It is straightforward to check $\partial L^s/\partial s > 0$ and $\partial^2 L^s/\partial s^2 < 0$ for $\rho \in (0, \rho^F)$, and for $\rho < 0$, $\partial L^s/\partial s < 0$ and $\partial^2 L^s/\partial s^2 > 0$. ■

Notice that the spread of assets can be rewritten in two parts, $s = r-(1+r)\rho/(\varphi+\rho)$. If $\rho = 0$ and assets have no dividend return, the second term vanishes and only the discount factor is left. In the following, we first determine s^* by $L^d(s) = L^s(s)$, and then back out asset price and participation in equilibrium.

Proposition 6 *In the model with assets and competitive search, there exist ρ^F, ρ^{NC}, and $\underline{\rho}$, such that: (i) for $\rho \geq \rho^F$, $\exists!$ symmetric SE with $\varphi = \varphi^F$ and $n^* = N$; (ii) for $\rho \in (\rho^{NC}, \rho^F)$, $\exists!$ symmetric SE with $\varphi = \varphi^N > \varphi^F$ and $n^* = N$; (iii) for (generic) $\rho \in [\underline{\rho}, \rho^{NC}]$, $\exists!$ symmetric SE if $\rho > 0$ ($\rho \leq 0$), with $\varphi = \varphi^{n^*} > \varphi^F$ and $n^* \leq N$ ($<$ if $\rho < \rho^{NC}$); (iv) for $\rho < \underline{\rho}$, $\nexists$ equilibrium with an active DM.*

Proof. For $\rho \geq \rho^F$, a downward-sloping L^d and a perfectly elastic L^s ensure the existence and uniqueness of equilibrium s^* with $n^* = N$. For $\rho = 0$, assets become equivalent to money with zero inflation, and the equilibrium existence and uniqueness follows the proof of Proposition 4. For $\rho \in (0, \rho^F)$, L^d is downward-sloping and L^s is upward-sloping, and there exists a unique equilibrium. Lemma 3 shows $\bar{s}^C \geq r$. For $\rho < 0$, if $\bar{s}^C = r$, $\nexists$

non-degenerate equilibrium; if $\bar{s}^C > r$, L^d and L^s may have more than one intersection, hence more than one candidate equilibrium. Given both n^* and p^C being functions of s, we can simplify the seller's maximization problem (25) to

$$\max_s \alpha\left(n\left(s\right)\right)\left[p\left(s\right)-c\right] \text{ st } \frac{\alpha\left(n\left(s\right)\right)}{n\left(s\right)}\left[u-p\left(s\right)\right]-sp\left(s\right)\geq\Omega,$$

a simple (static) decision problem to determine the spread. Given different values of s^* satisfying the first-order condition, some are local minimizers and some are local maximizers. Following theorem 1 in Wright (2010), we can show that under generic ρ, the global maximizer, hence the equilibrium, is unique. Next, we need to show the existence of $\underline{\rho}$. If $\bar{s}^C = r$, $\underline{\rho} = 0$. Consider $\bar{s}^C > r$. $s \leq r$ implies $\rho \geq 0$, and that is case (iii). For $s \in (r, \bar{s}^C)$, $\rho < 0$, $\partial L^s/\partial\rho = (1+r)A^s/(r-s) < 0$, and L^d is constant. Thus, $\exists!$ ρ^* such that $L^s(\rho^*) = L^d$. Define ρ^* as a function of s, and $\underline{\rho} = \min_{s\in[r,\bar{s}^C]}\rho^*(s) < 0$. For $\rho < \underline{\rho}$, $L^s(\rho) > L^d$, and there exists no equilibrium.

For the rest of the proposition on participation and asset prices, first consider $\rho \geq \rho^F$. According to Lemma 4, $s = 0$, which implies $\varphi = \varphi^F$ and $n^* = N$, since the cost of holding assets is zero. Define $\rho^{NC} = (r - s_N)p^C/(1+r)A$. If $\rho \in (\rho^{NC}, \rho^F)$, then $s^{NC} > s > 0$. The buyer's participation constraint is slack, and $(\varphi+\rho)A^s/N = p^C$. Hence, $n^* = N$ and $\varphi = \varphi^N = (1+s)p^C/(1+r)A > \varphi^F$. If $\rho \in [\underline{\rho}, \rho^{NC}]$, the buyer's participation constraint is binding. In this case, $s > 0$ and $(\varphi+\rho)A^s/n^* = p^C$. Therefore, $\varphi = \varphi^{n^*} = n^*(1+s)p^C/N(1+r)A > \varphi^F$. ■

Figure 2 shows the relationship between equilibrium participation n^* and dividend ρ by the dashed curves below the x-axis. Above the horizon, the solid curves represent asset price φ as a function of ρ. As long as the dividend of assets is high enough, all buyers participate in the DM and assets are priced at the fundamental value. On the other hand, if ρ is smaller than the cutoff value ρ^{NC}, only some buyers enter the DM. Since a larger dividend implies a smaller spread s, i.e. a lower cost of holding assets, the buyers' participation is monotonically increasing in ρ. However, the asset price φ may change in a non-monotonic way with respect to ρ. Equating the demand and supply of liquidity, we can get the asset price $\varphi = L^d/A^s - \rho$, which is the difference between the return of holding the asset and its dividend. As ρ gets bigger, the asset return also increases due to a higher demand induced by ρ. Then, the change of the asset price really depends on how much the liquidity demand responds to ρ, which is undetermined under general parameter values.

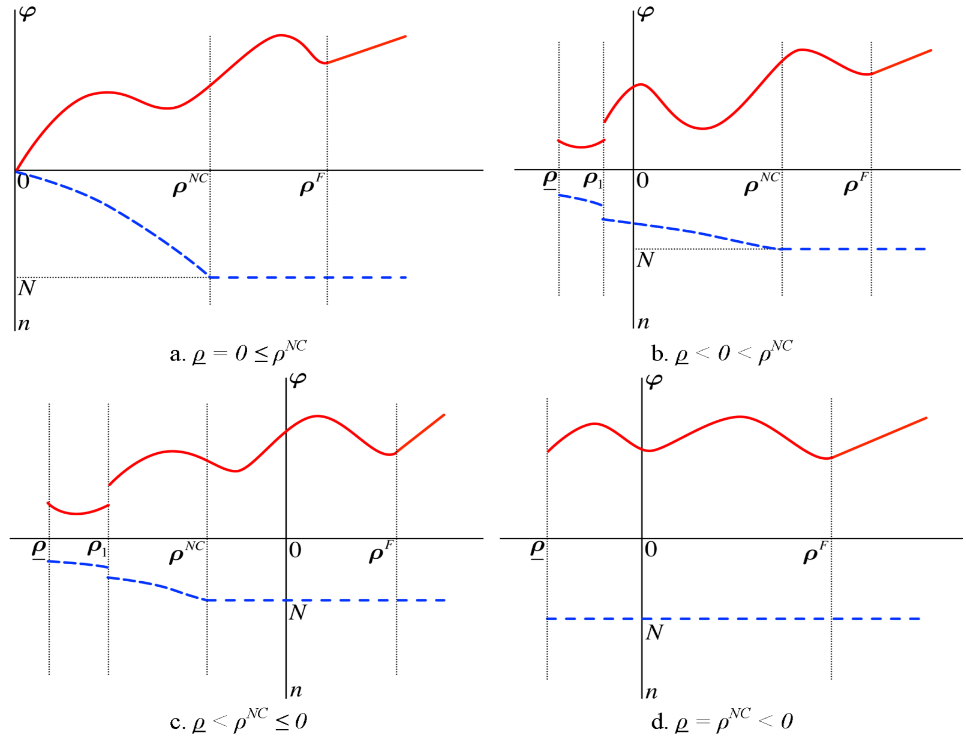

Figure 2: Asset with Competitive Search

Similar to the case with bargaining, multiple equilibria are possible only with $\rho < 0$, due to the same coordination problem. With bargaining, multiplicity is supported by a continuum of ρ. This is because buyers search randomly and the equilibrium price in the bargaining game is the seller's reservation value, independent of the market tightness in the DM. However, with competitive search, the prices posted by sellers direct the buyers' search behavior and serve as a coordination device. As a result, the set of ρ supporting multiple equilibrium is countable and has measure zero, and equilibrium is generically unique.

Compared to the monetary economy with competitive search, Proposition 6 shows equilibrium is unique for $0 < \rho < \rho^{NC}$, i.e. $r > s > s^{NC}$, while with money, there may still exist multiple equilibria for $r > i > i^{NC}$. This is because the cost of holding assets are endogenously determined in equilibrium, while the cost of holding money is an exogenous policy variable. For $i > i^{NC}$, the quantity of liquidity demand for money may not be unique for a countable number of interest rates. Then, for these exogenous i, there are multiple equilibria featuring different real money balances. With assets, the liquidity demand can have multiple values at a countable number of s as well, but the spread is endogenously determined by $L^d = L^s$. According to Lemma 4, L^s is monotonically increasing in s, and there is a unique asset spread given n, the measure of participating buyers. For $\rho \geq 0$, the asset spread is increasing in n. As more buyers enter the DM, they face a higher cost of holding assets and a lower probability of trade. Hence, the coordination problem does not exist and we have a unique equilibrium n^* with a unique asset spread. We can also get equilibrium uniqueness if the cost of holding assets is a constant and does not depend on the buyer's participation, such as $s = r$ when $\rho = 0$. When holding assets is costless, i.e. $s = 0$, and the asset is priced at its fundamental value, the equilibrium has the same price and participation in the DM as the equilibrium with pure credit.

5.3 Discussion

The main message we get from the entire analysis is that indivisibility of goods in the DM matters. Mainly because one looses an intensive margin of adjustment (q) from the standing models. It makes the available surplus fixed if one does not consider or allow for endogenous participation of potential buyers in the DM. Furthermore, different pricing mechanisms yield different equilibrium results, and it matters whether pure credit is used or if buyers need to bring a medium of exchange into the frictional market.

To summarize this effect, we catalog the different cases in the following way. Let $n \leq N$ be the active measure of buyers in the DM. Let $B_L^j(n)$ be buyers' benefit from the DM participation, $L \in \{c, m, a\}$ be the three types of liquidity, credit, money or asset, and $j \in \{b, c\}$ refers to the type of pricing mechanisms, bargaining or competitive search. Furthermore, let p^j be the equilibrium price under the mechanism j.

In the credit economy, we find that buyers participate in the DM if

$$B_c^b(n) = \left(u - p^b\right) \frac{\alpha(n)}{n} \geq 0$$

and

$$B_c^c(n) = (u - p^c(n)) \frac{\alpha(n)}{n} \geq 0.$$

The main difference is in the bargained price being independent of n, but not under competitive search. As long as $B_c^i(N) > 0$, all potential buyers participate in the DM.

In the monetary economy, we find

$$B_m^b(n) = (u - p^b)\frac{\alpha(n)}{n} \geq ip^b$$

and

$$B_m^c(n) = (u - p^c(n,i))\frac{\alpha(n)}{n} \geq ip^c(n,i).$$

Note that the bargained price differs under credit and money. With money, buyers have a first-mover advantage as they choose their money holdings before entering the decentralized market, thus allowing them to extract the total surplus from trade. With credit, buyers are not bringing a resource into the decentralized market. Thus with credit, the buyer's surplus from trade depends on their bargaining power. With competitive search, unlike the case with credit, the price now depends on the nominal interest rate.

Since $\alpha(n)/n$ is decreasing in n, under bargaining, for large enough i, $B_m^b(N) < ip^b$ and not all buyers would participate in the DM. However, with competitive search, we find $p^c(n,i)$ is increasing in n and decreasing in i. Higher i reduces p^c, which increases $B_m^c(n)$, $\forall n$. But we show it also increases $ip^c(n,i)$ the participation cost. This is what generate the potential for multiple equilibria with $n < N$. However, as we show for generic values of i, these possibilities are measure zero. Thus, the monetary equilibrium is also generically unique.

For the case of asset economy we find

$$B_a^b(n) = (u - p^b)\frac{\alpha(n)}{n} \geq sp^b$$

and

$$B_a^c(n) = (u - p^c(n,s))\frac{\alpha(n)}{n} \geq sp^c(n,s),$$

where the spread $s(n,\rho)$ is decreasing in n and increasing in ρ. With bargaining, we find unique n when $\rho > 0$, but when $\rho < 0$, there are two equilibrium n for a continuum value range of ρ. Under competitive search, we show that the asset equilibrium is also generically unique (even when $\rho < 0$).

The above discussion highlights the fact that under bargaining, the equilibrium price does not depend on factors such as nominal interest rate, dividend value of asset, or the measure of active buyers. In addition to indivisible goods loosing an intensive margin of adjustment, it forces an extensive margin of adjustment via buyers' participation.

Under competitive search, the price of the indivisible good reacts to nominal interest rate, dividend value and endogenous participation. The above effects are at the root of the existence, uniqueness or not, of equilibrium under credit, money and asset.

6 Lotteries

Since we consider an environment with indivisible goods, we allow for the use of lotteries in the trading mechanisms. Because it is the good and not the medium of exchange that is indivisible, it has different consequences that are of interest. To do so, we reconsider the three liquidity possibilities and two mechanisms as above. Let $E = \mathcal{P} \times \{0, 1\}$ denote the space of trading events, and $\mathcal{E}$ the Borel σ-algebra. Define a lottery to be a probability measure ω on the measurable space $(E, \mathcal{E})$. We can write $\omega(p, q) = \omega_q(q)\omega_{p|q}(p)$ where $\omega_q(q)$ is the marginal probability measure of q and $\omega_{p|q}(p)$ is the conditional probability measure of p on q. Without loss of generality, as shown in Berentsen et al. (2002), we restrict attention to $\tau = \Pr\{q = 1\}$ and $1 - \tau = \Pr\{q = 0\}$, and $\omega_{p|0}(p) = \omega_{p|1}(p) = 1$, essentially because $\mathcal{P}$ is convex. Randomization is only useful on q because Q is non-convex. Thus, $\tau \in [0, 1]$ is the probability that the good is produced and traded.

6.1 Credit with Lotteries

The buyer's payoff in the DM is

$$V_t^b = \frac{\alpha(N)}{N}\left[\tau u + W_t^b(p)\right] + \left[1 - \frac{\alpha(N)}{N}\right] W_t^b(0). \tag{28}$$

If a buyer gets to trade, he gets credit p, to be paid in the next CM, and gets τu from consumption in the DM. (28) can be rewritten as

$$V_t^b = W_t^b(0) + \frac{\alpha(N)}{N}(\tau u - p).$$

Similarly, the seller's DM value function is

$$V_t^s = W_t^s(0) + \alpha(N)(p - \tau c).$$

First, consider bargaining to be the trading mechanism in the DM. Upon meeting, a buyer and a seller solve the following generalized Nash bargaining problem:

$$\max_{p,\tau} (\tau u - p)^{\eta} (p - \tau c)^{1-\eta} \text{ st } p \leq D,\, \tau \leq 1.$$

Proposition 7 *In the credit model with bargaining and lotteries, $\exists!$ SE if $D \geq c$, characterized by*

$$\left(p^B, \tau^B\right) = \begin{cases} (\bar{p}, 1) & \text{if } D > \bar{p}^B \\ (D, 1) & \text{if } \underline{p}^B \leq D \leq \bar{p}^B \\ \left(D, D/\underline{p}\right) & \text{if } D < \underline{p}^B \end{cases}$$

where $\bar{p}^B = (1-\eta)u + \eta c$ and $\underline{p}^B = uc/(\eta u + (1-\eta)c)$.

Proof. The equilibrium is fully characterized by the solution to the bargaining problem. Using λ_1 and λ_2 for the multipliers on p and τ gives the following Kuhn-Tucker conditions

$$0 = -\eta (\tau u - p)^{\eta-1} (p - \tau c)^{1-\eta} + (1-\eta)(\tau u - p)^{\eta} (p - \tau c)^{-\eta} - \lambda_1 \tag{29}$$

$$0 = \eta u (\tau u - p)^{\eta-1} (p - \tau c)^{1-\eta} - c(1-\eta)(\tau u - p)^{\eta} (p - \tau c)^{-\eta} - \lambda_2 \tag{30}$$

$$0 = \lambda_1 (D - p)$$

$$0 = \lambda_2 (1 - \tau)$$

If $\lambda_1 = 0$, $p^B = (1-\eta)\tau u + \eta \tau c \equiv \tau \bar{p}^B$. Substituting this into (30) implies $\lambda_2 > 0$, and $\tau^B = 1$. We also need $D > \bar{p}^B$ to satisfy $p^B < D$. On the other hand, if $\lambda_2 = 0$, $\tau^B = (\eta u + (1-\eta)c)p/uc$. Substituting this into (29) implies $\lambda_1 > 0$ and $p^B = D$. Then, in order to have $\tau^B < 1$, $D < uc/(\eta u + (1-\eta)c) = \underline{p}^B$. The third case is $\lambda_1 > 0$ and $\lambda_2 > 0$, and $p^B = D$ and $\tau^B = 1$. The conditions we need to guarantee $\lambda_1 > 0$ and $\lambda_2 > 0$ are $\underline{p}^B < D < \bar{p}^B$. Finally, we need $D \geq c$ to guarantee that the seller's surplus from trade is not negative. ■

The expected total surplus from trade is $\tau(u - c)$. As long as the credit constraint is not too tight, i.e. $D \geq \underline{p}^B$, the surplus is maximized at $\tau^B = 1$. Buyer and seller have an additional margin to adjust in the bargaining problem, the lotteries. If the credit constraint is too tight, they can adjust/lower τ to compensate the seller, while they could not do that without lotteries. Lotteries affect the terms of trade only when credit constraint is severe. The solution without lotteries is a subset of the one above forcing $\tau = 1$. Note that all buyers are always active in the DM with credit, since $\alpha(n)\left(\tau^B u - p^B\right)/n > 0$ for all $n \leq N$.

Next, consider competitive search as the DM trading mechanism. To attract queue length n, sellers must post price p to guarantee buyers an expected utility of Ω.

$$\max_{p,\tau,n} \alpha(n)(p-\tau c) \text{ st } \frac{\alpha(n)}{n}(\tau u - p) = \Omega,\ p \leq D,\ \tau \leq 1.$$

Solve for p from the buyer's participation constraint, and substitute into the seller's objective function:

$$\max_{\tau,n} \alpha(n)\left[\tau(u-c) - \frac{n\Omega}{\alpha(n)}\right] \text{ st } \tau u - \frac{n\Omega}{\alpha(n)} \leq D,\ \tau \leq 1.$$

The solution to the problem is characterized by the following lemma.

Proposition 8 *In the credit model with competitive search and lotteries, $\exists!$ symmetric SE if $D \geq c$, characterized by*

$$\left(p^C, \tau^C\right) = \begin{cases} \bar{p}^C, 1 & \text{if } D > \bar{p}^C \\ D, 1 & \text{if } \underline{p}^C \leq D \leq \bar{p}^C \\ D, D/\underline{d}^C & \text{if } D < \underline{p}^C \end{cases}$$

where $\bar{p}^C = (1-\varepsilon(n))u + \varepsilon(n)c$, $\underline{p}^C = uc/(\varepsilon(n)u + (1-\varepsilon(n))c)$, $\varepsilon(n) = \alpha'(n)n/\alpha(n)$ is the elasticity of matching function, and $n = N$.

The proof is similar to Proposition 7, and it is easy to check $\alpha(N)\left(\tau^C u - p^C\right)/N > 0$. When $\varepsilon(N) = \eta$, the equilibrium price is identical to the case with bargaining.

6.2 Money with Lotteries

We consider bargaining first, and the details are essentially the same as before. Given the buyer's real money holding ϕm, the bargaining problem is

$$\max_{p,\tau} (\tau u - p)^{\eta}(p-\tau c)^{1-\eta} \text{ st } p \leq \phi m,\ \tau \leq 1,$$

and we also need to assure that $\tau u \geq p \geq \tau c$.

Lemma 5 *The solution to the bargaining problem is*

$$\left(p^B,\tau^B\right)=\begin{cases}\left(\bar{p}^B,1\right) & \text{if } \phi m>\bar{p}^B\\ \left(\phi m,1\right) & \text{if } \underline{p}^B\leq\phi m\leq\bar{p}^B\\ \left(\phi m,\phi m/\underline{p}^B\right) & \text{if } c\leq\phi m<\underline{p}^B\\ \left(0,0\right) & \text{if } \phi m<c\end{cases}$$

where $\bar{p}^B=(1-\eta)\,u+\eta c$ *and* $\underline{p}^B=uc/(\eta u+(1-\eta)c)$*.*

Proof. Using λ_1 and λ_2 for the multipliers on the money constraint and the lotteries constraint gives the following Kuhn-Tucker conditions.

$$0 = -\eta\left(\tau u-p\right)^{\eta-1}\left(p-\tau c\right)^{1-\eta}+\left(1-\eta\right)\left(\tau u-p\right)^{\eta}\left(p-\tau c\right)^{-\eta}-\lambda_1 \tag{31}$$

$$0 = \eta u\left(\tau u-p\right)^{\eta-1}\left(p-\tau c\right)^{1-\eta}-c\left(1-\eta\right)\left(\tau u-p\right)^{\eta}\left(p-\tau c\right)^{-\eta}-\lambda_2 \tag{32}$$

$$0 = \lambda_1\left(\phi_t m-p\right)$$

$$0 = \lambda_2\left(1-\tau\right)$$

These conditions are very similar to those in the case of credit with lotteries. It is straightforward to check that if $\lambda_1=0$, $p^B=\tau\bar{p}^B$. Substituting this into (32) implies $\lambda_2>0$, and hence $\tau^B=1$. In order to support $\tau^B=1$, the buyer needs to bring enough money to the DM trade, i.e. $\phi m>\bar{p}^B$. On the other hand, if $\lambda_2=0$, $p^B=\tau\underline{p}^B$. Substituting this into (31) implies $\lambda_1>0$ and $p^B=\phi m$. In order to satisfy $\tau^B<1$, we need $\phi m<\underline{p}^B$. If $\lambda_1>0$ and $\lambda_2>0$, $p^B=\phi m$ and $\tau^B=1$. $\lambda_1>0$ implies $\phi m<\bar{p}^B$, and $\lambda_2>0$ implies $\phi m>\underline{p}^B$. Finally, the seller certainly does not trade if the buyer brings $\phi m<c$. ■

Again, we can write the buyer's payoff in the CM as

$$W_t^b(m)=U\left(x^*\right)-x^*+\phi_t\left(m+T\right)+\beta W_{t+1}^b\left(0\right)+\beta\max_{\hat{m}}v\left(\hat{m}\right),$$

where $v\left(\hat{m}\right)=\alpha(n)(\tau^B u-p^B)/n-i\phi_{t+1}\hat{m}$. SME is characterized by the following proposition.

Proposition 9 *In the monetary model with bargaining and lotteries: (i) For* $i\leq i^{NB}$*,* $\exists!$ *SME with* $\phi_{t+1}\hat{m}=\underline{p}^B$*,* $\tau^B=1$ *and* $n^*=N$*; (ii) for* $i\in(i^{NB},\bar{\imath}^B)$*,* $\exists!$ *SME with* $\phi_{t+1}\hat{m}=\underline{p}^B$*,* $\tau^B=1$ *and* $n^*<N$*; (iii) for* $i\geq\bar{\imath}^B$*,* $\nexists$ *SME.*

Proof. First, the buyer does not want to bring $\phi_{t+1}\hat{m}>\bar{p}^B$, since additional money does not affect the surplus from trade. On the other hand, the buyer does not bring $\phi_{t+1}\hat{m}<c$

for no trade. Next, for $\phi_{t+1}\hat{m} \in (\underline{p}^B, \bar{p}^B)$, $v'(\hat{m}) = -\phi_{t+1}\alpha(n)/n - i\phi_{t+1} < 0$, and the buyer chooses $\phi_{t+1}\hat{m} = \underline{p}^B$. For $\phi_{t+1}\hat{m} \in (c, \underline{p}^B)$, $v'(\hat{m}) = \phi_{t+1}\left[\alpha(n)\eta(u-c)/nc - i\right]$, and its sign depends on the value of the nominal interest rate. Since $\alpha(n)\left(u - \underline{p}^B\right)/n - i\underline{p}^B = \underline{p}^B[\alpha(n)\eta(u-c)/nc - i]$, $v'(\hat{m})$ shares the same sign as $\alpha(n)\left(u - \underline{p}^B\right)/n - i\underline{p}^B$. Suppose $v'(\hat{m}) < 0$, then buyers choose $\phi_{t+1}\hat{m} = c$ and $\tau^B = 0$; there is no monetary equilibrium. Suppose $v'(\hat{m}) > 0$, then active buyers of measure n^* in the DM choose $\phi_{t+1}\hat{m} = \underline{p}^B$. The cutoff interest rate satisfying $v'(\hat{m}) = 0$ is $\bar{\imath}^B = \lim_{n\to 0}\alpha(n)(u - \underline{p}^B)/n\underline{p}^B = u - \underline{p}^B/\underline{p}^B = \eta(u-c)/c$. Therefore, if $i < \bar{\imath}^B$, $\exists!$ SME with $\phi_{t+1}\hat{m} = \underline{p}^B$ and $\tau^B = 1$; otherwise, there is no monetary equilibrium. Define another cutoff $i^{NB} = \alpha(N)(u - \underline{p}^B)/N\underline{p}^B = \alpha(N)\eta(u-c)/Nc < \bar{\imath}^B$. If $i \leq i^{NB}$, $n^* = N$; otherwise, $n^* < N$. ■

Notice several things about the monetary equilibrium with bargaining and lotteries. First, the equilibrium real balance $\phi_{t+1}\hat{m} = \underline{p}^B$ and the measure of participating buyers do not decrease with inflation when $i \leq i^{NB}$. Introducing lotteries does not recover conventional results, and money is still superneutral for nominal interest rate being low enough. Whereas for $i \in (i^{NB}, \bar{\imath}^B)$, real balance stays constant but n^* changes with i. Second, lotteries benefit sellers. With lotteries, the seller's surplus from DM trade is $\underline{p}^B - c$, compared to zero surplus from trade without bargaining over lotteries. Because of lotteries, buyers now bring exactly enough money to achieve the maximum expected surplus from trade at $\tau^B = 1$. Third, introducing lotteries makes it harder for a monetary equilibrium to exist. Allowing buyers and sellers to bargain over lotteries in an existing monetary equilibrium without lotteries will cause the equilibrium to collapse. This can be seen from the fact that $\bar{\imath}^B = \eta(u-c)/c$ in the lottery case is smaller than $\bar{\imath}^B = (u-c)/c$ in the case without lotteries. Fourth, with lotteries, the two cutoff values of nominal interest rate increase with the buyer's bargaining power η. As buyers claim more surplus from the DM trade, they can bear a larger cost of holding money. As the buyer's bargaining power decreases, $\bar{\imath}^B$ approaches zero, and monetary equilibrium eventually does not exist. Finally, compared to Berentsen et al. (2002), here the probability τ^B does not change with respect to the buyer's bargaining power or the inflation rate. Introducing lotteries with *indivisible goods* and *divisible money*, the total surplus from trade is affected but not price. However, introducing lotteries with *indivisible money* and *divisible goods*, the total surplus from trade stays the same but price changes according to the value of lotteries in equilibrium.

Next, we turn to competitive search. The seller's price posting problem is

$$\max_{p,\tau,n} \alpha(n)(p-\tau c) \text{ st } \frac{\alpha(n)}{n}(\tau u - p) - ip = \Omega,\ \tau \leq 1.$$

or

$$\max_{\tau,n} \alpha(n)\left[\frac{\alpha(n)\tau u - n\Omega}{\alpha(n)+ni} - \tau c\right] \text{ st } \tau \leq 1.$$

The following proposition shows that sellers do not use lotteries at all in a monetary equilibrium, no matter how many buyers participate in the DM.

Proposition 10 *In the monetary model with competitive search and lotteries: (i) For* $i < i^{NC}$, $\exists!$ *symmetric SME with* $\phi_{t+1}\hat{m} = p^C$, $\tau^C = 1$ *and* $n^* = N$*; (ii) for generic* $i \in [i^{NC}, \bar{\imath}^C]$, $\exists!$ *symmetric SME with* $\phi_{t+1}\hat{m} = p^C$, $\tau^C = 1$ *and* $n^* \leq N$*; (iii) for* $i > \bar{\imath}^C$, $\nexists$ *SME.*

Proof. We only need to check that sellers always choose to post $\tau^C = 1$, and the rest of the proof for existence and uniqueness follows Proposition 4. Using λ for the multiplier on τ, we have the following FOCs:

$$\begin{aligned} 0 &= \varepsilon(n)(p-\tau c) - \frac{\alpha(n)[1-\varepsilon(n)](\tau u - p)}{\alpha(n)+ni}, && (33) \\ 0 &= \tau\left[\frac{\alpha^2(n)u}{\alpha(n)+ni} - \alpha(n)c - \lambda\right], && (34) \\ 0 &= \lambda(1-\tau). \end{aligned}$$

Given the buyer's optimal participation $n = n^*$ and (33), we have

$$p^C = \frac{\alpha(n^*)\{[1-\varepsilon(n^*)]\tau u + \varepsilon(n^*)\tau c\} + \varepsilon(n^*)n^* i\tau c}{\alpha(n^*) + \varepsilon(n^*)n^* i}.$$

Solve for λ from (34), and we need

$$\lambda = \alpha(n^*)(u-c) - cn^*i > 0$$

to get $\tau^C = 1$. Since $p^C/\tau > c\ \forall\tau$, $\alpha(n)(u-c) - cni > \alpha(n)(u - p^C/\tau) - nip^C/\tau \geq 0$. The last inequality is the buyer's participation constraint in the DM, which holds as long as $i \leq \bar{\imath}^C$ and $n^* > 0$. Therefore, sellers always choose to post $\tau^C = 1$ in equilibrium. ■

Proposition 10 shows that, in a monetary equilibrium with competitive search, lotteries are never really used by sellers. With competitive search, sellers have the opportunity to post prices to get the highest possible profits, and they choose to post the terms of trade to guarantee $\tau^C = 1$. In fact, sellers are able to maximize their expected profits even without lotteries, and so with lotteries, equilibrium prices and the cutoff value of the nominal interest rate are the same as before. Things are different with bargaining. The buyer's DM payment with bargaining and lotteries is higher than without lotteries, i.e. $\underline{p}^B > c$. Different payments in the DM lead to different costs of carrying liquidity, and hence different cutoff values of the nominal interest rate to guarantee the existence of monetary equilibrium.

6.3 Asset with Lotteries

Finally, we introduce lotteries to the case where assets are the medium of exchange in the DM. The generalized Nash bargaining problem is

$$\max_{p,\tau} (\tau u - p)^\eta (p - \tau c)^{1-\eta} \text{ st } p \leq (\varphi + \rho) a,\ \tau \leq 1,$$

and we also need to check $\tau u \geq p$ and $p \geq \tau c$.

Lemma 6 *The solution to the bargaining problem is*

$$\left(p^B, \tau^B\right) = \begin{cases} \left(\bar{p}^B, 1\right) & \text{if } (\varphi+\rho)a > \bar{p}^B \\ ((\varphi+\rho)a, 1) & \text{if } \underline{p}^B \leq (\varphi+\rho)a \leq \bar{p}^B \\ \left((\varphi+\rho)a, (\varphi+\rho)a/\underline{p}^B\right) & \text{if } c \leq (\varphi+\rho)a < \underline{p}^B \\ (0,0) & \text{if } (\varphi+\rho)a < c \end{cases}$$

where $\bar{p}^B = (1-\eta)\,u + \eta c$ *and* $\underline{p}^B = uc/(\eta u + (1-\eta)c)$.

Proof. Using λ_1 and λ_2 for the multipliers on the asset constraint and the lotteries constraint gives the following Kuhn-Tucker conditions.

$$\begin{aligned} 0 &= -\eta (\tau u - p)^{\eta-1} (p - \tau c)^{1-\eta} + (1-\eta)(\tau u - p)^\eta (p - \tau c)^{-\eta} - \lambda_1 & (35) \\ 0 &= \eta u (\tau u - p)^{\eta-1} (p - \tau c)^{1-\eta} - c(1-\eta)(\tau u - p)^\eta (p - \tau c)^{-\eta} - \lambda_2 & (36) \\ 0 &= \lambda_1 ((\varphi+\rho)a - p) \\ 0 &= \lambda_2 (1 - \tau) \end{aligned}$$

These conditions are very similar to those in the case of money with lotteries. It is straightforward to check that if $\lambda_1 = 0$, $p = \tau^B \bar{p}^B$. Substituting this into (36) implies

$\lambda_2 > 0$, and hence $\tau^B = 1$. In order to support $\tau^B = 1$, buyer needs to bring enough asset to the DM trade, i.e. $(\varphi + \rho)a > \bar{p}^B$. On the other hand, if $\lambda_2 = 0$, $\tau^B = p^B/\underline{p}^B$. Substituting this into (35) implies $\lambda_1 > 0$ and $p^B = (\varphi + \rho)a$. In order to satisfy $\tau^B < 1$, we need $(\varphi + \rho)a < \underline{p}^B$. If both λ_1 and λ_2 are greater than zero, $p^B = (\varphi + \rho)a$ and $\tau^B = 1$. $\lambda_1 > 0$ implies $(\varphi + \rho)a < \bar{p}^B$, and $\lambda_2 > 0$ implies $(\varphi + \rho)a > \underline{p}^B$. Finally, the seller certainly does not trade if he meets a buyer with $(\varphi + \rho)a < c$. ■

Now the buyer's CM value function is given by

$$W_t^b(a) = U(x^*) - x^* + (\varphi_t + \rho)a + \beta W_{t+1}^b(0) + \beta \max_{\hat{a}} v(\hat{a}),$$

where $v(\hat{a}) = \alpha(n)(\tau^B u - p^B)/n - s(\varphi_{t+1} + \rho)\hat{a}$. The following proposition characterizes equilibria.

Proposition 11 *In the asset model with bargaining and lotteries: (i) for $\rho \geq \rho^F$, $\exists!$ SE with $\varphi = \varphi^F$ and $n^* = N$; (ii) for $\rho \in [\rho^{NB}, \rho^F)$, $\exists!$ stable SE with $\varphi = \varphi^N > \varphi^F$ and $n^* = N$; (iii) for $\rho \in [\underline{\rho}, \rho^{NB})$, $\exists!$ stable SE with $\varphi = \varphi^{n^*} > \varphi^F$ and $n^* < N$; (iv) for $\rho \in (\underline{\rho}, 0)$, $\exists!$ unstable SE; (v) for $\rho < \underline{\rho}$, $\nexists$ equilibrium with an active DM; (vi) $p^B = \underline{p}^B$ and $\tau^B = 1$ hold for (i)-(iii).*

Proof. First, buyers do not want to bring $(\varphi_{t+1} + \rho)\hat{a} > \bar{p}^B$, since additional assets do not affect the surplus from trade. Second, they do not bring $(\varphi_{t+1} + \rho)\hat{a} < c$, for no trade. Next, for $(\varphi_{t+1} + \rho)\hat{a} \in (\underline{p}^B, \bar{p}^B)$, $v'(\hat{a}) = -(\varphi_{t+1} + \rho)[s + \alpha(n)/n] < 0$, and buyers want to choose $(\varphi_{t+1} + \rho)\hat{a} = \underline{p}^B$. For $(\varphi_{t+1} + \rho)\hat{a} \in (c, \underline{p}^B)$, $v'(\hat{a}) = (\varphi_{t+1} + \rho)[\alpha(n)\eta(u - c)/nc - s]$, and the sign of $v'(\hat{a})$ depends on the value of the spread s. Since $\alpha(n)(u - \underline{p}^B)/n - s\underline{p}^B = \underline{p}^B[\alpha(n)\eta(u - c)/nc - s]$, $v'(\hat{a})$ shares the same sign as $\alpha(n)(u - \underline{p}^B)/n - i\underline{p}^B$. Suppose $v'(\hat{a}) < 0$, buyers choose $\tau^B = 0$ and there is no equilibrium with an open DM. If $v'(\hat{a}) > 0$, buyers of measure n in the DM choose $(\varphi_{t+1} + \rho)\hat{a} = \underline{p}^B$. The cutoff spread satisfying $v'(\hat{a}) = 0$ is given by $\alpha(n)(u - \underline{p}^B)/n - s\underline{p}^B = 0$, which is equivalent to the participation constraint $n[\alpha(n)\beta(u - \underline{p}^B)/n - (1-\beta)\underline{p}^B]/A^s \geq -\rho$. Define the LHS of the equation as $g(n)$, which is concave. Let $\underline{\rho} = -\max_n g(n)$, $\rho^F = (1-\beta)\underline{p}^B/A$, and $\rho^{NB} = [(1-\beta)\underline{p}^B - \beta\alpha(N)(u - \underline{p}^B)/N]/A$. For $\rho \geq \underline{\rho}$, all equilibria feature $p^B = \underline{p}^B$ and $\tau^B = 1$. If $\rho \geq \rho^F$, then $\varphi = \varphi^F$; otherwise $\varphi > \varphi^F$. If $\rho \geq \rho^{NB}$, then $n^* = N$; otherwise $n^* < N$. The rest of the proof on equilibrium stability follows directly from Proposition 5. ■

The results here are very similar to the case with money being the medium of exchange. For example, lotteries are not used in equilibrium, since buyers bring enough assets to achieve the maximum expected surplus from trade at $\tau^B = 1$. The DM price $\underline{p}^B$ and the value of lotteries τ^B in equilibrium do not change with respect to ρ. The buyer's asset holding is always just enough to pay for the DM transaction, which is not affected by the spread s. Hence, introducing lotteries does not recover conventional results. Compared to the bargaining model of assets without lotteries, we still get a continuum of multiple equilibria for $\rho \in (\underline{\rho}, 0)$, since the coordination problem continues to exist. Therefore, introducing lotteries does not recover the result of equilibrium uniqueness.

Finally, with competitive search, the price posting problem is

$$\max_{p,\tau,n} \alpha(n)(p - \tau c) \text{ st } \frac{\alpha(n)}{n}(\tau u - p) - sp = \Omega,\ \tau \leq 1,$$

or

$$\max_{n,\tau} \alpha(n)\left[\frac{\alpha(n)\tau u - n\Omega}{\alpha(n) + ns} - \tau c\right] \text{ st } \tau \leq 1.$$

The following proposition characterizes equilibrium, and it is not surprising to see that lotteries are not used in equilibrium, i.e. $\tau^C = 1$. All previous results and intuitions go through.

Proposition 12 *In the asset model with competitive search and lotteries, there exist ρ^F, ρ^{NC}, and $\underline{\rho}$, such that: (i) for $\rho \geq \rho^F$, $\exists!$ symmetric SE with $\varphi = \varphi^F$ and $n^* = N$; (ii) for $\rho \in (\rho^{NC}, \rho^F)$, $\exists!$ symmetric SE with $\varphi = \varphi^N > \varphi^F$ and $n^* = N$; (iii) for (generic) $\rho \in [\underline{\rho}, \rho^{NC}]$, $\exists!$ symmetric SE if $\rho > 0$ ($\rho \leq 0$), with $\varphi = \varphi^{n^*} > \varphi^F$ and $n^* \leq N$; (iv) for $\rho < \underline{\rho}$, $\nexists$ equilibrium with an active DM; (v) $\tau^C = 1$ holds for (i)-(iii).*

Proof. We only need to show $\tau^C = 1$, then the existence and uniqueness follows Proposition 6. Using λ for the multiplier on τ, we get

$$\begin{aligned} 0 &= \varepsilon(n)(p - \tau c) - \frac{\alpha(n)[1 - \varepsilon(n)](\tau u - p)}{\alpha(n) + ns}, && (37) \\ 0 &= \tau\left[\frac{\alpha^2(n)u}{\alpha(n) + ns} - \alpha(n)c - \lambda\right], && (38) \\ 0 &= \lambda(1 - \tau). \end{aligned}$$

In equilibrium

$$p^C = \frac{\alpha(n^*)\{[1-\varepsilon(n^*)]\tau u + \varepsilon(n^*)\tau c\} + \varepsilon(n^*) n^* s\tau c}{\alpha(n^*) + \varepsilon(n^*) n^* s}. \tag{39}$$

If $\lambda > 0$, $\tau^C = 1$, and from (38) we need to show

$$\alpha(n^*)(u-c) - cn^*s > 0.$$

Given $\rho \geq \underline{\rho}$, buyers are active and the participation constraint $\alpha(n^*)(\tau u - p^C)/n^* - sp^C \geq 0$ holds. We can simplify the participation constraint to $\alpha(n^*)(u - p^C/\tau) - n^* sp^C/\tau \geq 0$. Since $p^C/\tau > c \ \forall \tau$, $\lambda > 0$ always holds when buyers are active. ■

7 Conclusion

In this paper, we use a general equilibrium model to study the trade of indivisible goods in frictional markets under different liquidity: credit, money and asset. Indivisibility matters, especially when terms of trade are determined by bargaining and money/asset is used as the medium of exchange. The bargained price gives sellers no surplus and is independent of the nominal interest rate or the dividend on asset. Money is then superneutral as long as the cost of liquidity is low and all buyers participate in the frictional market. Introducing lotteries does not recover conventional results, and money is still superneutral. We then consider price posting with competitive search. The mechanism gives a price that depends on the nominal interest rate under money, the dividend under assets, and for both, on the number of buyers in the decentralized market. Lotteries do not matter under competitive search, but they do under bargaining, as the negotiated price gives sellers a positive surplus.

In the pure credit economy, we show uniqueness of symmetric equilibrium under bargaining and competitive search with price posting. That is because, under credit there is no direct liquidity cost. We also show uniqueness of equilibria under bargaining in the monetary economy due to the exogenous cost of liquidity. Under price posting and competitive search, as in Rocheteau and Wright (2005), there may be multiple equilibria under a countable number of nominal interest rate values. The indivisibility on goods side does not eliminate this possibility, and we get uniqueness for generic values of the nominal interest rate.

In the asset economy, the cost of carrying assets is determined by a spread, a liquidity premium, that depends on the asset's dividend value and the number of buyers in the market with indivisible goods.

Under bargaining, the equilibrium is unique as long as the asset dividend is non-negative. With negative dividend we find two equilibria, with low and high participation. The congestion nature of the matching technology in the indivisible good market, generates a concave net benefit for buyers in the number of active buyers. This leads to a coordination issue with participation and two equilibria.

With competitive search and price posting, we find a unique equilibrium for non-negative dividend values. With negative dividend, we find the equilibrium to be generically unique (the set of dividend values supporting multiple equilibrium is countable and has measure zero). As with bargaining a coordination problem exists, but unlike bargaining, sellers can now post prices to direct the buyers' search behavior. Therefore, using price posting as a coordination device solves the coordination problem present under bargaining and hence the multiple equilibria under generic parameters.

Overall, the consequences of indivisibility on the good side matter and differ from indivisibility on the money side. Lotteries cannot recover conventional results. In particular, lotteries cannot reestablish the link between real balances and anticipated inflation under money and bargaining.

Indivisibility may also affect the bargaining outcome because it isolates the good's price from the nominal interest rate, the dividend value and the number of active buyers using the asset for liquidity purposes. This generates multiplicity of equilibria. Price posting with competitive search re-establish the link and generically produce unique equilibrium. While we have focused on stationary equilibrium, the model can easily be used to study asset price dynamics. We leave this for future research.

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