

Is the Price Elasticity of Money Demand Always Unity?

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Abstract

Including both monetary gold and nonmonetary gold in a standard money-in-utility model, we establish a presumption that the price elasticity of money demand should be less than one under commodity standards. Applying cointegration methods to data of the world, the United Kingdom, and the United States, we find support for the new theory.

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1 Introduction

For much of history, money consisted in large part of monetary metals. In particular, during late nineteenth and early twenty century, many countries adopted gold standards, and gold served as money—either directly as coins held by the public or as claims on bullion held by commercial and central banks. At the same time that large stocks of gold were being held for monetary uses, even larger stocks were being held for nonmonetary uses owing to gold’s superior luster, reflectivity, malleability, conductivity, ductility and resistance to corrosion.¹ In its monetary uses, it was valued only in terms of its ability to purchase consumption goods. As a result, asset holders demanded *real gold balances*; i.e., the nominal stock deflated by an appropriate index of prices for goods. By contrast, in its nonmonetary uses, it was valued in terms of its *physical units*; i.e., its undeflated nominal stock. These dual uses for gold imply that the demand for monetary gold should not be expected to be unit-elastic with respect to the price level, a result that we demonstrate formally in the next section.

Empirical researchers have not noticed that money demand behaves differently under commodity standards from how it behaves under fiat standards. Since the available historical data are dominated by observations from fiat standards, the characteristics of money demand under commodity standards are largely concealed when the researchers use data that span both commodity and fiat standards. More specifically, the hypothesis of long-run price homogeneity is usually not rejected. For example, Allan H. Meltzer (1963) used US data from 1900 to 1958 to estimate money demand functions formulated in both nominal and real terms and found little evidence against price homogeneity. David E. W. Laidler (1971) carried out a similar test using both UK and US data over the period of 1900-1965 and obtained a similar result, notwithstanding a different specification. Milton Friedman and Anna J. Schwartz (1982) also found support for price

¹ According to Kitchin (1931), total world gold production from 1834 to 1889 amounted to 1037 million pounds sterling, among which 49.6 percent were added to the monetary gold stock. Annual data become available beginning only in 1890 and are presented in Figure 1 at the end of the paper. The average fraction of annual gold production that was added to the monetary gold stock rises to 57.7 percent during the 1890-1913 period.

homogeneity for both the United Kingdom and the United States over the period of 1867-1975 using the method of phase averaging to extract the long-run correlation between nominal money demand and the price level. David F. Hendry and Neil R. Ericsson (1991) applied cointegration methods to the annual UK data from Friedman and Schwartz (1982) to confirm price homogeneity. Applying the same approach to annual US data from 1874 to 1975, Ronald MacDonald and Mark P. Taylor (1992) found that price homogeneity cannot be rejected at the 0.05 statistical significance level but can be rejected at the 0.10 level.²

The United Kingdom and United States were on fiat standards for the bulk of the sample periods that these researchers employed.³ For this reason, their findings do not serve as evidence for price homogeneity under commodity standards. In section 2 we present a simple theoretical model that employs the money-in-utility framework augmented by also giving utility to holdings of nonmonetary gold. We demonstrate that under fairly general conditions the price elasticity of money demand should be *less than one* under commodity standards. Section 3 uses three data sets to provide empirical evidence supporting this theoretical prediction. Section 4 draws some final conclusions.

2 Theoretical Model

Consider the problem of a representative household that chooses the paths for its consumption C , its stock of monetary gold M , its stock of nonmonetary gold N and its total real stock of assets A in order to maximize the objective function

$$\underset{\{C_t, M_t, N_t, A_t\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} U\left(C_t, \frac{M_t}{P_t}, N_t\right) dt \quad (1)$$

subject to a sequence of budget constraints of the form

² They also found that the joint hypothesis of price and income homogeneity could be rejected at 0.01 significance level.

³ Specifically, after the onset of World War I in 1914 for both the United States and the United Kingdom and before 1879 for the United States. See Sidney Homer and Richard Sylla (1991). We interpret gold-exchange standards as a kind of fiat standard rather than a full-fledged commodity standard.

$$C_t + \frac{i_t(M_t + N_t)}{P_t} + \dot{A}_t \leq Y_t + \left(i_t - \frac{\dot{P}_t}{P_t} \right) A_t, \quad t \in [0, \infty), \quad (2)$$

and a sequence of borrowing constraints sufficiently tight to rule out Ponzi Schemes but sufficiently loose to never bind. In equations (1) and (2), t is a continuous index of time, P is the price level, i is the nominal interest rate, Y is the household's exogenous real income. We assume that the instantaneous utility function U is increasing, strictly concave and twice-continuously differentiable and satisfies Inada conditions in all of its arguments.

We have chosen to induce a demand for monetary gold by inserting M_t/P_t into the instantaneous utility function. This modeling choice has a long pedigree in monetary economics largely because of its simplicity and its ability to yield sensible demand functions similar to what other modeling choices yield. We could have employed a shopping-time or a cash-in-advance model and obtained similar results.

We have also chosen to induce a demand for nonmonetary gold by also inserting N_t into the instantaneous utility function. This modeling choice also has a long pedigree since nonmonetary gold is simply one type of consumer durable. Even though cars and washing machines as well as jewelry are not demanded for their own sake, their stocks do generate service flows that have many of the same characteristics as nondurable consumer goods and can be plausibly entered as arguments in momentary utility functions.

The first-order conditions for this problem imply that

$$U_m \left(C_t, \frac{M_t}{P_t}, N_t \right) = i_t U_c \left(C_t, \frac{M_t}{P_t}, N_t \right) \quad (3)$$

and

$$U_n \left(C_t, \frac{M_t}{P_t}, N_t \right) = \left(\frac{i_t}{P_t} \right) U_c \left(C_t, \frac{M_t}{P_t}, N_t \right), \quad (4)$$

where we have suppressed the t subscript and appended subscripts c , m and n to the function U in order to indicate derivatives with respect to C , M/P and N . According to equations (3) and (4), households equate the marginal utility of their *real stock* of monetary gold to the marginal utility of the *nominal interest* that they forgo from holding it while they equate the marginal utility of the *physical stock* of nonmonetary gold to the marginal utility of the *real nominal interest* that they forgo from holding it.

Equations (3) and (4) and the implicit function theorem imply that M/P is related to C , i and P by a function of the form

$$\frac{M_t}{P_t} = \Lambda(C_t, i_t, P_t). \quad (5)$$

Totally differentiating equations (3) and (4) and solving for Λ_p , the partial derivative of Λ with respect to $\log P$, yields

$$\Lambda_p = \frac{(i/P)U_c(U_{mn} - iU_{cn})}{D} \quad (6)$$

where

$$D \equiv (U_{mm} - iU_{cm})[U_{mn} - (i/P)U_{cn}] - (U_{mn} - iU_{cn})[U_{mm} - (i/P)U_{cm}]. \quad (7)$$

In equations (6) and (7), we have suppressed arguments of functions and time subscripts. Λ_p generally cannot be signed without making further assumptions about the instantaneous utility function U . However, if one is willing to assume that nonmonetary gold is an Edgeworth substitute for monetary gold ($U_{mn} < 0$) and is a sufficiently weak Edgeworth substitute for C (U_{cn} either nonnegative or not too negative), we then have $\Lambda_p < 0$ on the usual assumption that $D > 0$.⁴ In other words, the monetary stock of gold should be *less than* unit-elastic with respect to the price level.

⁴ $D > 0$ is the necessary condition for a maximum to the household's problem. The assumption that $U_{mn} < 0$ can find support from the fact that gold originally became money for many of the same reasons that it was attractive as a nonmonetary asset. Since it is hard to establish any presumption about the sign of U_{cn} , it is not unreasonable to assume that its effect on Λ_p does not overpower the negative effect of U_{mn} .

The intuition behind this phenomenon is straightforward. Eliminating iU_c between equations (3) and (4) gives us

$$U_m \left(C_t, \frac{M_t}{P_t}, N_t \right) = P_t U_n \left(C_t, \frac{M_t}{P_t}, N_t \right).$$

An increase in the price level raises the relative price of real balances in terms of nonmonetary gold. It is therefore natural to expect gold to shift from monetary to nonmonetary uses. This tendency offsets some part of the usual proportional response of M to P . In terms of the above equation, M/P tends to fall and N tends to rise in response to an increase in P .

3 Empirical Analysis

We present three sets of empirical results for the world, the United Kingdom, and the United States. For each data set, we estimate the log-log demand function

$$\log M_t = \lambda_0 + \lambda_p \log P_t + \lambda_y \log Y_t + u_t \quad (8)$$

for two specifications of the error term u_t . In our data, the nominal interest rate is stationary while $\log M$, $\log P$ and $\log Y$ are cointegrated.^{5,6} We therefore employ the levels specification (8), which excludes the nominal interest rate.⁷ We are primarily interested in the elasticity λ_p , which the theory of the previous section claims is less than one. For this reason, we shall be testing the null hypothesis that it is one against the one-tailed alternative hypothesis that it is less than one. Rejection of the null will support the theory. We also expect the elasticity λ_y to be appreciably larger than zero and will regard our estimates as problematical if our estimates of λ_y are not.

⁵ Using monthly data from Frederick R. Macaulay (1938) on the US commercial paper rate for the period from January 1880 to December 1913, we obtained an augmented Dickey–Fuller test statistics of -7.28 , which is statistically significant at well under the 0.0001 significance level. We take this as evidence that nominal interest rates in the entire gold-standard world were stationary over this sample period. The Dickey-Fuller regression contained an intercept but no time trend.

⁶ The Dickey-Fuller statistics for our measures of $\log M$, $\log P$ and $\log Y$ are -3.06 , $+1.43$ and -2.71 for the world, -2.66 , -1.87 and -2.89 for the United Kingdom and -2.06 , -1.80 and -3.01 for the United States. None of these is statistically significant at conventional levels. We selected the augmentation lags for each Dickey-Fuller regression in order to minimize the Schwarz Information Criterion. Each regression contained both an intercept and a time trend. Table 1 below provides the evidence for cointegration.

⁷ Cointegrating relationships cannot include stationary variables.

We fit equation (8) using Johansen's method of estimating vector error-correction models as well as dynamic ordinary least squares (DOLS) as described by Hamilton (1994, pp. 602-608). In the latter method, the error term u_t is modeled as taking the form

$$u_t = \sum_{j=-q}^q \pi_j \Delta \log P_{t+j} + \sum_{j=-q}^q \eta_j \Delta \log Y_{t+j} + v_t \quad (9)$$

with

$$v_t = \sum_{j=1}^n \phi_j v_{t-j} + e_t, \quad (10)$$

where v_t is an error term; the π_s , η_s and ϕ_s are parameters; and e_t is an independently and identically distributed error term with a zero mean and finite variance. Estimation then proceeds in three steps: First, OLS is applied to the regression

$$\log M_t = \lambda_0 + \lambda_p \log P_t + \lambda_y \log Y_t + \sum_{j=-q}^q \pi_j \Delta \log P_{t+j} + \sum_{j=-q}^q \eta_j \Delta \log Y_{t+j} + v_t \quad (11)$$

in order to obtain estimates of $\{v_t\}$. Second, equation (10) is fitted to these residuals in order to obtain $\{\hat{\phi}_j\}$, the estimates of $\{\phi_j\}$. Third, OLS is applied to the regression

$$\log \tilde{M}_t = \lambda_0 + \lambda_p \log \tilde{P}_t + \lambda_y \log \tilde{Y}_t + \sum_{j=-q}^q \pi_j \Delta \log \tilde{P}_{t+j} + \sum_{j=-q}^q \eta_j \Delta \log \tilde{Y}_{t+j} + e_t, \quad (12)$$

to obtain consistent estimates of the λ s and their standard errors. In equation (12),

$$\log \tilde{M}_t \equiv \log M_t - \sum_{j=1}^n \hat{\phi}_j \log M_{t-j},$$

$$\log \tilde{P}_t \equiv \log P_t - \sum_{j=1}^n \hat{\phi}_j \log P_{t-j}$$

$$\log \tilde{Y}_t \equiv \log Y_t - \sum_{j=1}^n \hat{\phi}_j \log Y_{t-j}.$$

In our first data set, M is the world stock of monetary gold, P is the world price level, and Y is world output. The world monetary gold stock comes from Warren and Pearson (1935). Our measure of world output is the sum of Angus Maddison's (1995) estimates of real GDP for France, Germany, Italy, United Kingdom, and United States.⁸ Our measure of the world price level is the real-GDP-weighted average of these countries' price

⁸ These series are directly addible because they are expressed in common base-year units; i.e., 1990 Geary-Khamis dollars.

levels.^{9,10} The series are annual and span the period 1880-1913, the heyday of the International Gold Standard.

Table 1: Johansen Tests for Cointegration of $\log M$, $\log P$ and $\log Y$, 1880-1913

Economy	Hypothesized Number of Cointegrating Vectors	Eigenvalues	Trace Statistic	Maximum Eigenvalue Statistic
World	0	0.565	42.5 ^a	28.4 ^a
	≤ 1	0.322	14.2	13.2
	≤ 2	0.028	0.9	3.8
UK	0	0.505	30.4 ^b	22.5 ^b
	≤ 1	0.218	8.0	14.1
	≤ 2	0.002	0.1	3.8
US	0	0.701	51.4 ^a	38.6 ^a
	≤ 1	0.318	12.8	14.1
	≤ 2	0.015	0.5	3.8

Notes. The specification for the world includes two lags, and those for the United Kingdom and the United States include one lag. Each specification was estimated using EViews, assuming trend in the series but not in the cointegrating relationships. The superscripts a and b indicate statistical significance at the 0.01 and 0.05 levels.

The first three rows of Table 1 report statistics for testing whether $\log M$, $\log P$ and $\log Y$ are cointegrated for the data described in the previous paragraph. These statistics indicate the existence of exactly one cointegrating vector for the world as a whole.

The first two rows of Table 2 report two sets of estimates for λ_p and λ_y , one based on Johansen's method and the other on DOLS. The former estimate of λ_p is less than one though not statistically significantly so. The latter estimate, however, is significantly less

⁹ The price levels are the GDP or NNP deflators for Germany, Italy and United Kingdom, the GNP deflator for the United States, and a cost of living index for France. The data come from Mitchell (1980) except for those for the United States, which come from Gordon (1986).

¹⁰ Weighting with real GDP is equivalent to calculating an implicit deflator. For example, suppose that country j 's output and price level in period t are Y_{jt} and P_{jt} . Then

$$P_t = \sum_j \left(Y_{jt} / \sum_k Y_{kt} \right) P_{jt} = \sum_j P_{jt} Y_{jt} / \sum_k Y_{kt},$$

which is just the ratio of the sum of nominal GDPs to the sum of the real GDPs.

than one at the 0.01 level. Both estimates of λ_y are sensible and precisely estimated, significantly positive and close to one.

Table 2: Money Demand under the Gold Standard, 1880-1913

Location and Method	λ_p	λ_y
World, Johansen	0.877 (0.149)	0.969 (0.049)
World, DOLS	0.728 ^a (0.091)	1.072 (0.028)
United Kingdom, Johansen	0.642 ^b (0.127)	0.997 (0.026)
United Kingdom, DOLS	0.537 ^b (0.229)	0.992 (0.059)
United States, Johansen	0.309 ^a (0.137)	1.560 (0.044)
United States, DOLS	0.603 ^a (0.189)	1.539 (0.066)

Notes. The figures in parentheses are standard errors. The superscripts a and b indicate statistical significance at the 0.01 and 0.05 levels. The vector error-correction model had two lags for the world and one for the United Kingdom and the United States. In the DOLS regressions for the world, $q = n = 2$ and ϕ_1 and ϕ_2 were estimated to be 0.979 (0.180) and -0.550 (0.178). In the DOLS regression for the United Kingdom, $q = n = 1$ and ϕ_1 was estimated to be 0.609 (0.155). In the DOLS regressions for the United States, $q = n = 1$ and ϕ_1 was estimated to be 0.653 (0.123). The lag lengths were chosen on the basis of pretests.

Under the gold standard, deposits and banknotes were nearly perfect substitutes for monetary gold. As a result, the demand for the stock of money should have the properties that the theory in the previous section identified for monetary gold. We should therefore find that equation (8) with $\lambda_p < 1$ characterizes the demand for money in countries that were on the gold standard.

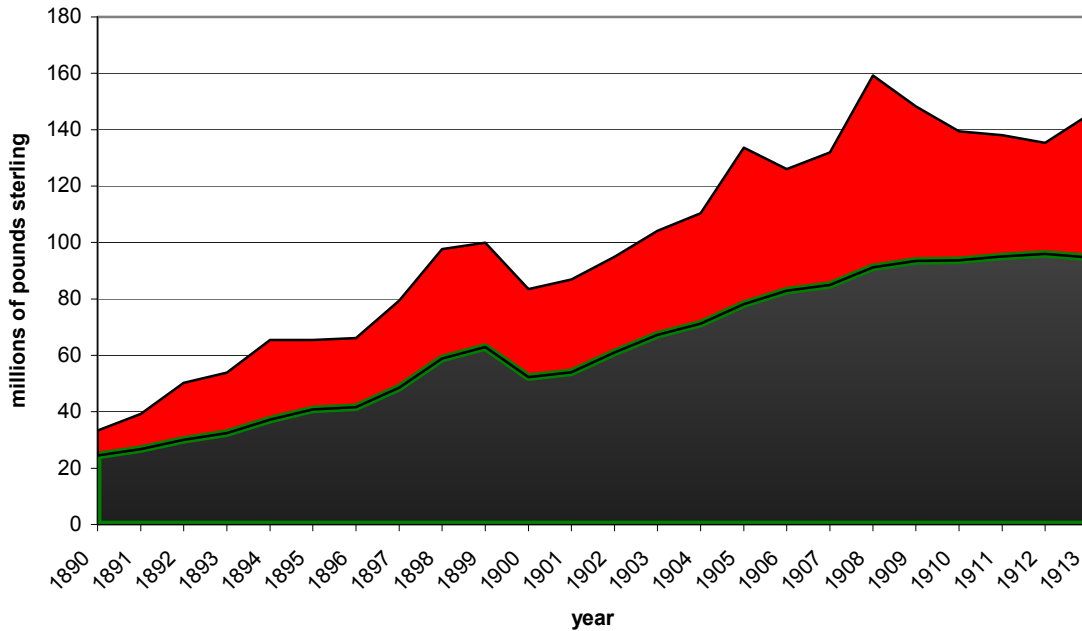
M , P and Y in our second and third data sets come from Tables 4.8 and 4.9 of Friedman and Schwartz (1982) and are the M2 money supplies, the implicit deflators and real net national products for the United Kingdom and the United States. The last six rows of Table 1 report statistics for testing whether $\log M$, $\log P$ and $\log Y$ are cointegrated. The statistics indicate the existence of one cointegrating vector for both countries.

We used both Johansen’s method and DOLS to estimate λ_p and λ_y for both the United Kingdom and United States, and the last four columns of Table 2 report our estimates. In both cases the price elasticities are significantly less than one, and the income elasticities take on plausible and highly significant values.

4 Conclusion

We have established a theoretical presumption that the price elasticity of money demand is less than unity under commodity monetary standards. Using data from the heyday of the International Gold Standard, we have also provided evidence supporting the theory.

Figure 1: World Gold Production and Addition to Monetary Stock



Source: Table I of Kitchin (1931).

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