

**COLLUSION VIA SIGNALING IN MULTIPLE
OBJECT AUCTIONS WITH COMPLEMENTARITIES:
AN EXPERIMENTAL TEST**

by

Anthony M. Kwasnica and Katerina Sherstyuk

Working Paper No. 01-2
February 2001

Collusion via Signaling in Multiple Object Auctions with Complementarities: An Experimental Test*

Anthony M. Kwasnica[†] and Katerina Sherstyuk[‡]

February 19, 2001

Abstract

We experimentally study bidder collusion in open ascending auctions for multiple objects. The project is based on the theoretical results by Brusco and Lopomo (1999), who give theoretical support for the following claims: (1) simultaneous ascending bid auctions can be vulnerable to collusion in the multi-object case; (2) The sole presence of complementarities does not hinder collusion; (3) Collusion is a “low numbers” phenomenon. We focus on a simultaneous ascending auction for two objects. Several experimental treatments are considered: markets with low numbers (2 bidders) and high numbers (5 bidders), no complementarities (additive values) and complementarities (superadditive values). Experimental results are largely consistent with the theory. Collusion is often observed in two-person markets with or without complementarities. Previous experience under the same treatment greatly facilitates bidder collusion. There is no evidence of collusion in five-person markets. We further study collusive strategies adopted by bidders in two-person markets. While most strategies make extensive use of signaling, in the presence of complementarities, bidders use collusive strategies that are supported only by repeated play.

*Financial support by the Australian Research Council and the Krumrine Endowment is gratefully acknowledged. We would like to thank Pino Lopomo for his comments, John Ledyard for his support, Anil Roopnarine and David Porter for development of the experimental software, and the Caltech Social Science Experimental Laboratory and the Economic Science Laboratory at the University of Arizona for their hospitality.

[†]Department of Management Science and Information Systems, Pennsylvania State University, 339 Beam Building, University Park, PA 16802. Email: kwasnica@psu.edu

[‡]Department of Economics, University of Melbourne, and Department of Economics, University of Hawaii at Manoa, 2424 Maile Way, Honolulu, HI 96822. Email: katyas@hawaii.edu

1 Introduction

This study presents an experimental examination of bidder collusion in open ascending auctions for multiple objects. Auctions are an increasingly popular institution on the Internet for consumer to consumer (ebay and Amazon), business to consumer (OnSale), and business to business (FreeMarkets) transactions. Auctions are also now being utilized for government procurement and privatization programs in many countries. The value of the objects being auctioned off has made the possibility of collusion even more daunting; the potential losses in revenue and efficiency are tremendous.

In many of these auctions, multiple heterogeneous items are for sale simultaneously. In auctions of these sort, such as auctions for timber, automobiles, or bandwidth, there has been the suggestion that bidders can profit by splitting the markets. This allows each bidder to act as “local” monopsonist for a particular object driving the price they must pay down. However, assuming that communication between the bidders is limited to the bidding, any such splitting of the markets must be coordinated by signaling in the early stages of the auction.

The Federal Communications Commission spectrum auctions brought to the forefront the possibility of collusion of this sort (Milgrom, 1998; Cramton and Schwartz, 2000). The recent FCC spectrum auctions employed a simultaneous multi-object ascending auction format. Cramton and Schwartz (2000) report that firms bidding for similar licenses used signaling and bidding at low prices to tacitly coordinate on license allocations across markets; the deviations from tacit agreements were punished with retaliating bids. There is evidence that bidders sometimes used the financially inconsequential portion of the bid (the last three digits) in order to signal their identity or to indicate a market that bidder will retaliate against in the event the current bid is raised. In fact, in 1997, the FCC fined Mercury PCS \$650,000 for “placing trailing numbers at the end of its bids that disclosed its bidding strategy in a . . . manner that specifically invited collusive behavior” (FCC 1997).

Collusion in auction markets has been studied by both economic theorists (Milgrom, 1987; Graham and Marshall, 1987; McAfee and McMillan, 1992; Brusco and Lopomo, 1999) and experimentalists (Isaac, Ramey and Williams, 1984; Isaac and Walker, 1985; Kwasnica, 2000; Sherstyuk, 1999). Experimental studies indicate that collusion can be quite effective in posted-offer and sealed bid markets, provided that the sellers (buyers) are allowed to communicate between periods (Isaac, Ramey and Williams, 1984; Isaac and Walker, 1985; Kwasnica, 2000). The growing empirical literature presents evidence of bidder collusion in both sealed bid auctions, such as auctions for state highway construction contracts and school milk markets (Feinstein, Block and Nold, 1985; Porter and Zona, 1993 and 1999) and oral ascending bid auctions, such as forest service timber sales (Baldwin,

Marshall and Richards, 1997).

Brusco and Lopomo (1999) (BL) provide a theoretical foundation for the existence of collusion via signaling in multiple object auctions. They demonstrate that there exists a perfect Bayes Nash equilibrium where the bidders capture a larger portion of the surplus than under the “competitive” bidding equilibrium. In the collusive equilibrium, bidders use the multi-object simultaneous ascending nature of the auction to signal preferences over objects and allocate the objects among bidders at low bids; the equilibrium is enforced by the threat of reverting to competitive bidding if deviation occurs. Such equilibria exist in situations where bidders’ valuations for two objects are either independent or there exists a large complementarity for the purchase of both objects. BL also show that collusion is a low numbers phenomenon; the collusive outcome of the auctions differs less from the competitive outcome as the number of bidders increase. However, the equilibrium suggested by BL is extremely complex and requires common beliefs about contingent punishments making it unlikely that bidders would be able to implement these strategies in reality; this is especially true when compared to the relative simplicity of the competitive equilibrium. There may also exist many other collusive equilibria displaying similar features. Unfortunately, it is difficult to study this phenomenon empirically due to the lack of observability of bidders’ valuations; empirical tests have a difficult time distinguishing between collusive behavior and low valuations. Therefore, a natural avenue for investigation appears to be the experimental laboratory. Recent experimental evidence suggests that collusion in multiple object auctions might be successful. Kwasnica (2000) found that bidders were able to collude in all 10 experiments of a multi-object sealed bid auction. These bidders used quite sophisticated strategies in order to determine the winning bidders. In his setting, the only Bayes Nash equilibrium is the competitive equilibrium (Maskin and Riley, 1996); therefore, cooperative equilibria are obtained by allowing for preplay communication. The equilibria suggested by BL are non-cooperative in nature; it might be reasonable to expect that they could be achieved without communication. Sherstyuk (1999) provides some insight into the possibility of collusion in ascending price auctions without communication. When a single market ascending auction is amended to allow bidders to place tie bids, collusive Nash equilibria can be supported through the use of trigger strategies. She reports that collusion is often observed in both common and private value experimental ascending auctions of this sort. The collusive outcomes described by BL add a degree of complexity for the bidders by requiring signaling in addition to trigger strategies.

In this study, we explore the possibility of collusion in multi-object ascending price auctions without communication. We use laboratory experiments to examine the following questions.

1. Do bidders collude in ascending multi-object auctions?
2. If so, what sort of collusive strategies are used? Are they consistent with the theoretical predictions of BL?
3. How does the inclusion of complementarities affect the success of collusion?

The main purpose of this paper is to discover whether the economic intuitions provided by BL can be supported in the experimental laboratory. Our results qualitatively support the BL predictions. Bidders can collude in auctions for multiple objects both with and without complementarities. As predicted, collusion is only observed in small groups (size of 2). On the other hand, we show that collusive strategies often differ markedly from those suggested by BL. In addition, previous experience with the auction institution appears to be salient in enabling collusion between bidders.

The remainder of the paper is organized as follows. In section 2, we state the theoretical predictions on collusion and illustrate them using simple examples. Section 3 describes the experimental design. Results are presented in section 4. In section 5 we examine individual behavior. We conclude and discuss issues for future consideration in section 6

2 Theoretical predictions

The theoretical framework, as given by BL, is the following. Let there be two objects, A and B , and the set N of bidders, $i = 1, \dots, n$. The institution is the simultaneous ascending bid auction, in which each object is auctioned off in a separate market via an ascending bid auction run simultaneously for both objects. Let a_i be bidder i 's value for object A , and b_i be bidder i 's value for object B . Bidder i 's value for the package AB is given by

$$u_i(AB) = a_i + b_i + k_i,$$

where k_i is the additive complementarity term. It is assumed that values (a_i, b_i, k_i) are drawn independently across bidders from the same probability distribution with support $[0, 100]^2 \times K$, where K is either $\{0\}$ (no complementarity) or an interval $[\underline{k}, \bar{k}]$ with $\underline{k} > 100$ (large complementarity). Finally, a_i and b_i have identical marginal distributions, denoted by f and F respectively. Under these conditions, BL show the following:

1. With no complementarity, the Separate English Auction strategy profile (SEA: bid up to your value on each object independently of the other object) forms a perfect Bayesian equilibrium (PBE) in the simultaneous ascending bid auction. The resulting allocation is efficient and the prices are equal to the second highest values for each object. We will refer to the corresponding outcome as the SEA competitive outcome.

2. With large complementarities, $\underline{k} > 100$, there exists a (competitive) PBE with the following outcome: the two objects are allocated to the bidder with the highest value for the package, at a price equal to the second highest valuation for the package; the allocation is always efficient. We will refer to the corresponding price as the Vickrey price for the package, and the corresponding outcome – the Vickrey competitive outcome.
3. If there are two bidders, $n = 2$, then under certain restrictions on the distribution of values F and the distribution over K , collusive outcomes (each bidder buys only one object at a price below the CE price) can be supported as PBE in the simultaneous ascending bid auction. These equilibria are sustained using the threat to revert to competitive play if players deviate from their collusive strategies. The above mentioned restrictions hold, in particular, if F is uniform, and if K is degenerate on some value k , with $k = 0$, or $k > 100$, i.e., the complementarity is common and known across bidders. We will refer to the corresponding outcomes as BL collusive outcomes.
4. Collusion is a low numbers phenomenon. If $n > 2$ and $k = 0$ (no complementarity), the prices under the collusive BL outcome differs less from the competitive outcome. BL collusive strategies prescribe bidding competitively until only two bidders are left bidding; then strategies similar to the $n = 2$ case are employed. With a positive complementarity and $n > 2$, collusive equilibria are not characterized.

Example 1 Let there be $n = 2$ bidders, with $a_1 = 96$, $b_1 = 72$, $a_2 = 6$, $b_2 = 54$. If $k = 0$ (no complementarity), then the SEA competitive outcome is $p_a = 6$, $p_b = 54$, with both items allocated to bidder 1. The BL collusive strategy prescribes each bidder to signal their most preferred item by bidding on it first. They stop bidding if no one else bids on this item. Hence, the BL collusive strategy in this case yields the following outcome: item A is allocated to bidder 1 at $p_a = 1$, and item B is allocated to bidder 2 at $p_b = 1$.¹ Note that the resulting allocation is inefficient.

If $k = 101$ (large complementarity), then the Vickrey competitive outcome is $p_a + p_b = 6 + 54 + 101 = 161$, with both items allocated to bidder 1. The BL collusive outcome coincides with the one described above (for $k = 0$) in this case.

Example 2 Let there be $n = 2$ bidders, with $a_1 = 38$, $b_1 = 8$, $a_2 = 36$, $b_2 = 29$. If $k = 0$ (no complementarity), then the SEA competitive outcome is $p_a = 36$, with item A allocated to bidder 1, and $p_b = 8$, with item B allocated to bidder 2. The BL collusive

¹We assume that a bid of 1 is the minimum allowable bid as well as the minimum bid increment for future bids.

strategy prescribes each bidder to first bid on their most preferred item, and, if both bidders bid on the same item, keep bidding on it until one of the bidders switches to the other item; then stop. In this case, both bidders will start bidding on item A , until its price reaches 8, at which point bidder 2 will switch to item B . The rationale for bidder 2's switch is that he would prefer to win B at a price of 1 and a potential profit of 28 than to raise the bidding on A to 9 for a maximum potential profit of 27. Once the markets have been split, the bidders discontinue bidding. Hence, the BL collusive strategy yields the following outcome: item A is allocated to bidder 1 at $p_a = 8$, and item B is allocated to bidder 2 at $p_b = 1$.

If $k = 101$ (large complementarity), then the Vickrey competitive outcome is $p_a + p_b = 38 + 8 + 101 = 147$, with both items allocated to bidder 2. If colluding bidders anticipate that the common complementarity term will be competed away under the Vickrey competitive outcome, then the BL collusive strategy and the corresponding outcome are the same as in the no complementarity case:² item A is allocated to bidder 1 at $p_a = 8$, and item B is allocated to bidder 2 at $p_b = 1$.

It is useful to add a number of additional observations to the above predictions. First, it is easy to note that competitive PBE outcomes characterized by BL (items 1 and 2 above) have a close correspondence to competitive equilibria (CE) in the neoclassical sense.³ Second, BL do not consider the case of a moderate complementarity, where $0 < k_i < 100$. In general, both the “competitive” and collusive equilibria are hard to characterize in this case. We note, however, that in the special case of common complementarity, $K = \{k\}$, and $n = 2$ bidders, for any $k > 0$, there exists a competitive equilibrium price and allocation pair, with the resulting allocation being efficient. Specifically, let us call a CE price p a *minimal* CE price if for any other CE price \tilde{p} , $p_a + p_b \leq \tilde{p}_a + \tilde{p}_b$. Then we can state the following (see Sherstyuk, 2000, for proofs and details):

Observation 1 *Suppose there are two bidders, $n = 2$, and the complementarity term is common, $k \geq 0$. Then for any $k \geq 0$, the set of competitive equilibrium prices and allocations is non-empty, and any CE allocation is efficient. The minimal CE prices have the following characteristic:*

²In this paper we focus on the “high collusion” equilibria of BL, which maximize bidders’ expected surplus among all possible collusive equilibria

³A price $p = (p_a, p_b)$ is a competitive equilibrium price if, given p , there is an allocation of objects to bidders $\mu : \{A, B\} \rightarrow N$ such that each bidder gets a package in their demand set, i.e., there is no excess demand. Such price and allocation pair (p, μ) is called a competitive equilibrium. Given that bidders’ values for objects are non-negative, $a_i, b_i \geq 0$, the equilibrium also requires no excess supply, i.e., both objects are allocated to bidders.

- Suppose that allocating both items to one bidder, or packaging, is efficient:

$$a_i + b_i + k \geq \max\{a_i + b_j, a_j + b_i, a_j + b_j + k\},$$

for some $i, j \in N$, $i \neq j$. Then the minimal CE prices correspond to the Vickrey price for the package. That is, they satisfy the following constraint:

$$p_a + p_b = a_j + b_j + k. \quad (1)$$

- Suppose that splitting of items between the bidders is efficient:

$$a_i + b_j \geq \max\{a_j + b_i, a_i + b_i + k, a_j + b_j + k\},$$

for some $i, j \in N$, $i \neq j$. Then the minimal CE prices satisfy the following constraint:

$$p_a + p_b = a_j + b_i + 2k. \quad (2)$$

Further, for the case of large complementarities, the strategies described by BL to support the Vickrey equilibrium are quite complex. Our numerical simulations reveal that if the complementarity is common, unsophisticated “honest” bidding (bid on the object or the package that maximize one’s payoff at current prices) leads to CE outcomes in simultaneous ascending bid auction, with the prices being the minimal equilibrium prices. This gives us additional grounds to expect that if the bidders behave competitively in the laboratory setting, the outcomes will converge to the CE predictions for any value of the common complementarity term.

Examples 1 and 2 (continued) Let $k = 50$ (moderate complementarity). Then for the bidder values given in examples 1 and 2 above, the CE outcome is the following. Example 1: $p_a = 6 + 50 = 56$, $p_b = 54$, thus $p_a + p_b = 6 + 54 + 50 = 110$ (the Vickrey price), with both items allocated to bidder 1. Example 2: $p_a = 38$, $p_b = 8 + 50 = 58$, thus $p_a + p_b = 38 + 8 + 50 = 96$ (the Vickrey price), with both items allocated to bidder 2. It is straightforward to check that honest bidding leads to the competitive outcomes for the cases $k = 0$, $k = 50$ and $k = 101$, in both examples.

Our final theoretical consideration concerns the choice of collusive strategies. BL strategies support collusive outcomes as equilibria in a one shot auction game. Other theoretical predictions (Milgrom, 1987) often rely on repeated nature of bidder interactions to support collusive equilibria in auctions. If bidders interact with each other repeatedly and view each auction as part of an infinitely repeated game, then possibilities for bidder

collusion are much richer than discussed by BL. In particular, collusion can be sustained at minimal prices, and bidders may split markets not only within periods, but also across periods. The latter strategy allows bidders to capture the complementarity term, which cannot be captured under collusion considered by BL, or under any market splitting within a period. To summarize:

Observation 2 *The following collusive outcomes can be supported as Nash equilibria in the infinitely repeated auction game, provided that the bidders are patient enough:*

1. *(Minimal bid) The two items are allocated to two bidders, chosen at random in every period, at the minimal (seller reservation) prices.*
2. *(Bid rotation) Bidders take turns across periods in buying both items at the minimal prices, thus capturing the complementarity term.*

For examples 1 and 2 above, these predictions say that the items will be allocated at minimal prices $p_a = p_b = 1$, to either different bidders (minimal bid), or to the same bidder (bid rotation).

In the case of a strong common complementarity ($k > 100$), bidders will prefer the bid rotation outcome to the BL collusive strategy since it allows them to capture the extra payoff. However, when there is no complementarity (or k is small), the BL strategy will yield higher expected payoffs for the bidders. The following observations follow easily from proposition 3 in BL.⁴

Observation 3 *Suppose there are two bidders, $n = 2$.*

- *If $k = 0$, then the expected value to the bidders of the BL strategy is strictly greater than the expected value from the minimal bid and bid rotation strategies.*
- *If there is a common complementarity term $k > 100$, then the expected value to the bidders of the bid rotation strategy is greater than the expected value of the BL and minimal bid strategies.*

Since collusive strategies discussed by BL are rather complex and require bidders to form consistent beliefs about each others' behavior, we allow repeated play in our experiments. We are therefore interested in studying whether bidders, if they collude, use strategies suggested by BL, or, in the case of a positive complementarity, they adopt payoff-superior collusive strategies that rely on the repeated nature of the auctions. The

⁴BL, revised draft, April 2000.

fact that the BL strategy is (ex-ante) superior in the no complementarity case allows us to examine the motivations of experimental subjects. If bidders are motivated by the repeated game setting, then we might expect rotation or minimal bid in all complementarity conditions. If bidders do not find the repeated setting compelling, we might expect the BL strategy in all cases. On the other hand, if bidders primary motivations are coordination on Pareto superior equilibria, then we would expect bidders to switch from a BL strategy with no complementarity to rotation in the presence of a positive complementarity.

3 Experimental design

Groups of subjects participated in a series (up to 25) of computerized ascending auctions for two fictitious objects labeled *A* and *B*. The group composition stayed the same throughout the session. Within an auction period, each object was sold in a separate auction run simultaneously for both objects. Bidders were free to place, at any time, as many bids as they desired as long as the bid was at least as great as the reservation price (equal to one experimental dollar), and the bid was strictly greater than previous bids on that object. The auction period ended when no new bids had been placed for a number of seconds.⁵ Each object was then allocated to the bidder with the highest bid for that object.

Bidders' valuations for the objects were integers between 1 and 100. Valuations were independently drawn from the discrete uniform distribution for each period. In some sessions bidders faced a complementarity for the two objects: In addition to their randomly drawn values for each object, bidders earned an "extra payoff" if they were the highest bidder on both objects. This complementarity term was common to all bidders and announced at the beginning of the experiment.⁶

Three treatment variables were considered: market (group) size, complementarity, and experience. To test the effect of market size on the incidence of collusion, we conducted experimental auctions with two and five bidders. Previous experimental evidence, such as Van Huyck, Battalio and Beil (1990), suggests that groups of size two can more easily coordinate on Pareto dominant Nash equilibria than can larger groups (who almost always fail). Thus, a larger group size may lead to less collusion for two reasons: less theoretical possibilities for collusion, as discussed by BL, and coordination failure, as observed by Van Huyck et al.

To examine the effect of complementarities, we conducted sessions with either no com-

⁵The number of seconds that were required to elapse before the end of the auction varied slightly across experiments due to computer conditions. Thirty seconds was the typical requirement in five person experiments and forty seconds was typical in two person experiments.

⁶See appendix for the experimental instructions.

plementarity, $k = 0$, a moderate complementarity, $k = 50$, or a strong complementarity, $k = 101$. BL describe collusive equilibria for both the no complementarity and strong complementarity case. However, the increased complexity of positive complementarity environments may lead to less collusion due to subjects' cognitive abilities. On the other hand, the repeated auction setting provides comparable possibilities for bidder collusion in all complementarity treatments.

Finally, we considered previous experience with the auction institution as a treatment variable. Bidders were either inexperienced indicating that they had not previously participated in any of these auctions, or they were experienced indicating that they had participated in one previous session. In a number of early experimental sessions, we conducted experiments with groups of subjects who had participated in an earlier session under some treatment ("mixed" experience). In later sessions, experienced subjects were asked to participate in an identical auction institution in terms of the group size variable ("sorted" experience).⁷ Given the sophisticated nature of collusive equilibria, we expected that experience may be necessary for the subjects to successfully coordinate on these equilibria. Table 1 lists the number of experiments completed under each treatment variable combination.

TABLES 1, 2 AROUND HERE

A total of 40 experimental sessions were completed using students, mostly undergraduates, at the University of Arizona, California Institute of Technology, University of Hawaii, University of Melbourne, and Pennsylvania State University. Up to five 2-person markets, or up to three 5-person markets were run independently in each session. Depending upon the speed at which the auctions progressed, subjects completed between 6 and 25 auction periods in a session. For inexperienced subjects, an average of 16.6 and 13.4 periods were completed in the 2-person and 5-person markets respectively; the experienced sessions averaged 22.6 and 17.5 periods. A total of 121 separate markets were observed. Table 2 provides a break down of experimental sessions by subject pool.

4 Results

The data from the experimental sessions are summarized in tables 3-5 and figures 1-5. Table 3 summarizes experimental sessions by treatment and subject pool, and classifies experimental outcomes according to criteria to be described below. Tables 4-5 present descriptive statistics on market prices, efficiencies, and bidder gains, pooled by treatment.

⁷In most late sessions, experienced subjects participated in an identical auction institution in terms of both group size and complementarity variable.

Figures 1-5 give examples of market price dynamics by treatment. For expositional convenience, the prices we report are the sums of prices for both objects.⁸ Market efficiency, reported in table 5, is defined in the usual way, as the ratio between the social surplus realized and the maximal social surplus attainable:

$$\text{Efficiency} = \frac{SS_{actual}}{SS_{max}}.$$

Social surplus is measured as the sum of the values of the winning bidder(s):

$$SS_{actual} = \sum_{i=1}^n a_i x_{ai} + b_i x_{bi} + k x_{ai} x_{bi}$$

where i is the bidder index, $i = 1, \dots, n$, a_i and b_i are bidder i 's valuations for objects A and B respectively, k is the common complementarity term, and x_{ji} , $j = a, b$, are the allocation coefficients, with $x_{ji} = 1$ if and only if object j is allocated to bidder i , and $x_{ji} = 0$ otherwise. The maximal social surplus (SS_{max}) is the allocation which maximizes the sum of the values across all bidders. Relative bidder gains, also reported in table 5, are defined as the ratio between the realized and the maximal bidder gains (where the latter is equal to the maximal social surplus):

$$\text{Relative Gains} = \frac{SS_{actual} - (\sum_{i=1}^n p_a x_{ai} + p_b x_{bi})}{SS_{max}},$$

where p_a and p_b are prices at which the objects are sold. Relative gains can be considered a measure of collusive effectiveness. The greatest profits a collusive group can hope to obtain is by obtaining the efficient outcome and paying the auctioneer the minimal prices.

TABLES 3-5 AND FIGURES 1-5 AROUND HERE

We compare the actual market characteristics with the following theoretical predictions, discussed in section 2:

- **SEA competitive** outcome is the only competitive prediction for the 2N and 5N treatment, but it may also have some predictive power for markets with complementarities if bidders do not fully take the complementarity term into account.

Notation: “SEA” in the tables, “priceAB” in the figures.

- **“Vickrey” competitive equilibrium** outcome is the CE prediction for the positive complementarity treatments; we use this term to denote the corresponding CE prediction for both $k = 50$ and $k = 101$ cases.⁹

Notation: “Vick” or “Vprice” in the tables, “Vprice” in the figures.

⁸Although each item was sold in a separate market, the market outcomes rarely differed across objects; for example, in no case the bidders allocated one object to one bidder at a “collusive” price, and the second object to another bidder at a “competitive” price.

⁹For the moderate complementarity case, $k < 100$, depending on bidder value draws, competitive equilibrium outcomes may involve either allocating both objects to the same bidder, in which case the

- **BL collusive equilibrium prediction** is only characterized for 2N, 2Y101, and 5N treatments.

Notation: “BL” in the tables, “collsAB” in the figures.¹⁰

- **Minimal bid** outcome allocates the objects randomly between bidders at the minimal price in every period.

Notation: “MinBid” in the tables, “minbid” in the figures.

- **Bid rotation** outcome allocates both objects to the same bidder at the minimal prices. The winning bidder varies across periods.

Notation: “Rotation” in the tables; prices coincide with “minbid” in the figures.

Based on average prices in each market, we classified market outcomes into competitive and collusive categories in the following way:

- For the 2N and 5N (no complementarity) treatments, the markets were classified as:
 - COMPETITIVE, if the average market price was no lower than 15% below the SEA competitive equilibrium prediction;
 - SEMI-COLLUSIVE, if the average market price was between 50% and 85% of the SEA competitive equilibrium prediction;
 - COLLUSIVE, if the average market price was below 50% of the SEA competitive equilibrium prediction.
- For the 2Y and 5Y (complementarity) treatments, the markets were classified as:
 - COMPETITIVE-SEA, if the average market price was within 15% of the SEA competitive prediction;
 - COMPETITIVE-MIXED, if the average market price was more than 15% above the SEA competitive prediction, but more than 15% below the Vickrey competitive prediction;
 - COMPETITIVE-VICKREY, if the average market price was within 15% of the Vickrey competitive prediction;

CE price is the “true” Vickrey price, or splitting the objects among bidders, in which case the CE price is below the “true” Vickrey price (see equations 1, 2). For the value draws used in the experiment, with $k = 50$, the CE price differed from the Vickrey price in at most 2 out of 25 periods in each market. For the sake of convenience, we therefore use the term “Vickrey outcome” to denote the CE outcome in all treatments with complementarities.

¹⁰For presentation clarity, the collusive BL prices are displayed in the figures for the 2N treatment only (figure 1). In the 2Y101 treatment, it is obvious from figure 3 that the BL prediction is out-performed by one of the alternative predictions for all markets displayed. In the 5N treatment, the collusive BL prices coincide with the SEA competitive prices in 23 out of 25 periods.

- SEMI-COLLUSIVE, if the average market price was between 50% and 85% of the SEA competitive prediction, or if the actual prices were close to the minimal bid in at least 35% of the periods (see market 3-211 in figure 2 for example);
- COLLUSIVE, if the average market price was below 50% of the SEA competitive equilibrium prediction.

Classification results are given in table 3. These results are robust to variations in threshold price levels used to distinguish between categories. More generally, we will call a market NON-COMPETITIVE if the average market price is 15% below the SEA competitive prediction, or lower; we will call a market COMPETITIVE otherwise.

Based on the data in the tables and the figures, we obtain the following results.

Result 1 (Collusion in small size markets) *There was a significant amount of collusion in 2N and 2Y50 markets. Thus, collusion does occur in small markets, and the presence of a common moderate complementarity does not hinder collusion.*

Support: Tables 3-5 and figures 1-2. From table 3, in the 2N treatment with inexperienced subjects, 9 out of 30 independent 2-person markets (30%) were non-competitive (collusive or semi-collusive). For experienced subjects, 12 out of 18 markets (55%) were collusive. From table 4, the prices, on average, were more than 30% below the SEA price in both 2N experienced treatments; bidder gains were 61.39% as compared to 48.7% of the SEA prediction in 2N experienced (mixed) treatment, and 63.21% as compared to 49.34% for the SEA prediction in the 2N experienced (sorted) treatment (table 5). Collusion was observed in 3 out of 4 subject pools (MEL, CIT and PSU) where the 2N treatment was tested.

In the 2Y50 treatment, out of 16 independent markets with inexperienced subjects, 5 were collusive, and one was semi-collusive; overall, 37.5% of markets were non-competitive. On average the actual prices were 1.88% less than the SEA prediction. The variance on this difference was 58.74 percentage points indicating significant heterogeneity in prices across markets (table 4). Collusion was observed in both subject pools (MEL and CIT) where the 2Y50 treatment was tested. \square

We next consider the effects of treatment variables: market size, experience and complementarity, on the incidence of collusion. Table 6 presents the results of logit estimation of the probability of a market being competitive (i.e., average market price exceeding 85% of the SEA competitive prediction) as a function of indicator variables for market size (“large”=0 if $N = 2$, and “large”=1 if $N = 5$), subject experience, moderate and large complementarity, and subject pools.

TABLE 6 AROUND HERE

Result 2 (Collusion with large numbers) *Collusion is a small numbers phenomenon: no collusion was observed in 5-person markets.*

Support: Tables 3-5, 6 and figures 4-5. All markets in 5N and 5Y treatments are classified as competitive (table 3). The mean market price in the 5N treatment is 1.63% percent above the SEA competitive prediction, with a standard deviation of only 7.5 percentage points; market efficiency is at 98.25% (with a standard deviation of 1.93% only), and relative bidder gains are at 15.27%, as compared to the SEA prediction of 18.84%. In 5Y50 and 5Y101 treatments, the average market prices, market efficiencies and bidder gains are all in the range between the SEA and Vickrey competitive predictions, and a distance from any of the collusive predictions (tables 3-5). The results of logit analysis (table 6) indicate that the market size has a highly positive and statistically significant effect on market competitiveness. \square

Result 3 (Effect of experience) *Experience in the same size market (“sorted”) increases the incidence of collusion in 2-person markets. Experience in any size market (“mixed”) does not always increase the incidence of collusion. That is, experience is market-size specific.*

Support: Tables 3 and 6. In 2N markets at CIT, the percentage of non-competitive (collusive and semi-collusive) markets increased from 50% among inexperienced subjects (3 out of 6 markets) to 80% among subjects experienced in the same size market (4 out of 5 markets). In 2N markets at PSU, the percentage of non-competitive markets increased from 10% (1 out of 10 markets) for inexperienced subjects to 60% (3 out of 5 markets) for subjects experienced in the same size market. In 2Y101 markets at PSU, the percentage of collusive markets increased from zero among inexperienced subjects (in 9 independent markets) to 28.6% among subjects experienced in the same size market (2 out of 7 markets). Table 6 indicates that experience in the same size market (“sorted”) has a strong negative effect on market competitiveness, whereas the effect of experience in any size market is statistically insignificant. \square

Result 4 (Collusion with large complementarities) *The presence of a large complementarity was detrimental for collusion: there was very little collusion in 2-person markets with large complementarities.*

Support: Tables 3 and 6. Table 6 indicates that the effect of large complementarity, $k = 101$, on market competitiveness is positive and significant at the 5% level. No collusion

was observed in any of 17 independent markets in 2Y101 treatment with inexperienced subjects, at either PSU or UH. With experienced subjects (sorted), only 2 out of 7 markets were collusive at PSU, and none of 4 markets were collusive at UH. The overall percentage of collusive markets in the 2Y101 experienced (sorted) treatment was only 18.2%, which is below the percentage of collusive markets in experienced (sorted) markets in either 2N or 2Y50 treatments. \square

The data presented in tables 3 and 6 also indicate that there was a significant amount of heterogeneity in collusive tendencies across subject pools; specifically, MEL and CIT subjects were more likely to be collusive than PSU and UH subjects. Yet, it is clear from the statistical analysis (table 6) that the lower incidence of collusion in the 2Y101 markets as compared to 2N markets cannot be attributed solely to the subject pool effects and is due to the large complementarity term itself.

The last observation concerns the comparative performance of SEA and Vickrey competitive predictions in markets with complementarities where average prices reached the competitive levels (85% of the SEA price or above). Table 7 presents the results of an ordinary least squares regression of the average per market price on the SEA competitive predictions, Vickrey competitive predictions for markets with complementarities, and a dummy variable for experienced subjects. (We allow for differentiated effects of these variables in markets without complementarities and markets with moderate and large complementarities. The variable for market size proved insignificant and was therefore excluded from the regression.) The following result suggests that discovering the Vickrey competitive outcome was not a trivial task for the subjects; in this sense the treatments with complementarities presented an extra level of difficulty for subjects as compared to the corresponding treatments without complementarities.

TABLE 7 AROUND HERE

Result 5 (SEA and Vickrey competitive predictions with complementarities)

Among 2Y and 5Y inexperienced markets classified as competitive, “competitive-mixed” prediction explains the overall data better than the Vickrey competitive prediction. The share of competitive markets converging to Vickrey outcomes increased with experience.

Support: Tables 3-5, 7, figures 2, 3 and 5. The regression results in table 7 indicate that, on average, the prices in competitive markets with complementarities were half-way between the SEA and the Vickrey predictions; experience in markets with large complementarities significantly increased the price levels. From table 3, in the 2Y50 treatment, only 2 out of 10 competitive markets (20%) are classified as Vickrey-competitive, while the other

8 markets (80%) have average prices between the SEA and Vickrey predictions. In the 2Y101 treatment, 5 out of 17 competitive markets (29.4%) are Vickrey, while the other 12 markets (70.6%) are competitive-mixed. In 2Y101 experienced (sorted) treatment, the share of Vickrey markets among competitive markets increases to 6 out of 9 (66.7%), while the other 3 markets (33.3%) are competitive-mixed. In the 5Y101 treatment, the share of Vickrey markets is 2 out of 5 (40%) for inexperienced subjects, and 3 out of 4 (75%) for experienced subjects; the other markets were competitive-mixed. See also support for results 2 regarding prices, efficiencies and bidder gains in 5Y treatments. \square

Given the large amount of 2Y competitive markets where the prices did not reach the Vickrey level, a natural question is whether these lower prices were due to subjects' bounded rationality and their inability to fully realize the role of the complementarity term, or their attempts to suppress price competition in order to achieve higher bidder gains. From table 5, observe, for example, that in 2Y101 treatment, the SEA outcome yields average bidder gains of approximately 50%, whereas the Vickrey outcome yields average gains of only 19% of the maximum. The actual gains for inexperienced subjects were half-way between the two predictions, at the 34.35% level. Given the above evidence that markets with large complementarities were rarerly successful in achieving collusive price levels (result 4), and that deviations from the Vickrey outcome towards the SEA outcome were observed also in 5Y treatments, where competitive forces were strong, we conclude that most of the price deviation from the Vickrey prediction should be attributed to bidder bounded rationality rather than successful efforts to suppress price competition.¹¹

5 Individual behavior and bidder collusive strategies

The pooled data is generally consistent with the BL theory: collusion does occur in small size markets with multiple objects. The next step is to take a closer look at individual level data in order to understand the strategies employed by the bidders. Is the BL prediction similar to the experimental observations? Do other possible strategies such as minimal bid and rotation do well? By looking at individual behavior, we may be able to identify greater incidence of attempts to collude; there may be markets where some bidders attempted to collude but were thwarted by the competitive behavior of other bidders in their market.

For the purposes of this analysis we separate markets into two categories: COLLUSIVE (average market price is less than 50% of the SEA prediction; 21 markets total), or NON-COLLUSIVE (all other markets). Such separation allows us to focus on the power of

¹¹From casual observations of bidder behavior during the experiments, we know that some bidders had difficulties realizing that they should bid above their separate item valuations in treatments with complementarities, both in 5-person and 2-person markets.

alternative collusive predictions in the markets which have been pre-selected as collusive on the basis of low average market prices (see table 3).

We begin by looking at the qualitative predictions of models of tacit collusion in multi-object ascending auctions. Observations of real world auctions and the BL theory predict:

1. Signaling of preferred markets in early bids, and
2. Retaliation in the event of deviation from the collusive strategy.

Both strategies are observed in the data. A bidder is considered to be signaling in a given period if one of the following conditions is met:

1. They bid on their highest valued object first,
2. They bid on both objects at the same time but placed a strictly higher bid on their highest valued object, or
3. They had the same valuation for both objects.¹²

While signaling may not be sufficient for the development of collusive behavior, it is a necessary part of the strategies described by BL and a common claim in actual auction markets.

TABLE 8 AROUND HERE

Result 6 (Signaling in markets with no complementarities) *There was a significant amount of signaling in 2 and 5-person markets without complementarities. In 2N markets, more signaling was observed in markets classified as collusive.*

Support: Table 8. The mean proportions of initial bids that can be classified as signaling are listed in table 8. Since a bidder who is randomly selecting an initial object to bid on will appear to be signaling half of the time, we compare the mean signaling proportion under each treatment to 50%. In the 2N experienced markets and 5N experienced and inexperienced markets, the level of signaling is significantly greater than 50%. In 2N and 5N treatments, 59 out of 151 (39%) subjects placed signaling bids at least 75% of the time. We also find that the level of signaling appears related to collusive outcomes in the no complementarity treatment; 20 out of 26 (77%) subjects in 2N markets that were classified as collusive placed signaling bids at least 75% of the time. A chi-squared test for difference in the proportion of signaling in collusive and non-collusive groups is significant

¹²When a bidder has identical valuations across markets, it is impossible to reject the possibility that the bidder is signaling.

at any reasonable level of significance. In both the 2N inexperienced and 2N experienced treatments, the mean level of signaling when collusion was observed was significantly greater than under the competitive outcomes.¹³ \square

Signaling is only half the story. When collusive agreements are broken, bidders must be willing to punish deviant bidders by reverting to the SEA strategy. Unfortunately, it is difficult to distinguish retaliatory moves to the SEA equilibrium from purely competitive bidding. However, a few clues into the willingness of some bidders to punish defectors is provided in the data.

Result 7 (Overbidding) *Some bidders are willing to bid above their values. The persistence (and growth) of this behavior amongst experienced bidders suggests that bidders are punishing non-collusive behavior.*

Support: In 2N and 5N treatments, 29 out of 154 (19%) bidders placed bids in two or more periods that were above their valuations for an object. In the complementarity treatments, 33 out of 156 (21%) bidders placed at least two combined bids that exceeded their valuations with the added payoff. While some of these bids might be attributed to mistakes, the amount of overbidding actually increased with experience. In the no complementarity case, 19 out of 49 (39%) experienced bidders overbid at least twice and, in the complementarity case, 16 out of 52 (31%) of experienced bidders overbid. A chi-squared test for difference in proportion of overbidding in experienced and inexperienced groups is significant at the 10% level for both treatments. \square

While the BL strategies do not propose bidding at a loss in order to punish non-collusive bidders, we observe this at times. Previous experimental investigations of ascending auctions (without complementarities) found little evidence of overbidding. In a one shot auction, any such strategy is not rational. The repeated nature of our auction might rationalize this strategy. It seems to us that the only reasonable explanation for the persistence of such behavior must be attempts to punish deviant behavior in order to encourage cooperation in future auction periods. This evidence suggests that at least some bidders attempt to implement collusive strategies that utilize signaling with the threat to revert to SEA (or worse) strategies if bidding does not stop at a low level.

While table 8 indicates that there is significant signaling in no complementarity treatments, the particularly low levels of observed signaling in some of the complementarity treatments (2Y101 experienced) suggests that a further analysis of the actual strategies

¹³In the 2N inexperienced case, the difference of means is significantly different at the 5% level, and, in the experienced treatment the difference between the means is significant at any reasonable level of significance.

utilized is needed. Tables 9-11 examine how well various strategies fit the data. We first compare the observed final allocations in each period with four possible strategies:

1. Rotation – one bidder is allocated both objects.
2. Split – each bidder is allocated one object (consistent with minimal bid and BL).
3. BL – the bidder predicted by the BL collusive outcome is allocated each object.
4. Efficient – the objects are allocated to the bidders required to obtain the SS_{max} (usually consistent with competitive models).

Table 9 reports the proportion of periods that are consistent with each strategy for the 21 markets classified as collusive; table 10 pools all the experimental data across treatments. These different strategies also predict various prices for the objects. A stricter standard is to require that the allocation match those predicted by the strategy and that the prices be “close.” Since bidders often started with bids that were somewhat greater than the minimal bid or placed bids in increments greater than the minimum required increment (1 experimental dollar), we classified a price realization as being consistent with a particular strategy if the sum of the winning bids on the two objects were within 10 experimental dollars of the prediction. We compare the five theoretical predictions identified in section 4 to the observed allocations and prices. The mean proportion of periods consistent with these strategies are listed in table 11.

TABLE 9 AROUND HERE

TABLE 10 AROUND HERE

TABLE 11 AROUND HERE

It is convenient to consider markets with complementarities first.

Result 8 (Bidder behavior in markets with complementarities) *Among 2Y markets classified as collusive, bid rotation dominates all other descriptions of bidder behavior.*

Support: Tables 9-11. In 7 out of 8 2Y50 and 2Y101 experiments classified as collusive, at least 70% of the observed allocations have one bidder winning both objects. This is strong evidence against the BL and minimal bid strategies, which require splitting of the markets in all periods. However, the Vickrey competitive outcome would also predict winning both objects in all 2Y101 periods and the vast majority of the 2Y50 periods. When the price

information is also considered (table 11), none of the data is consistent with the Vickrey prices. The proportion of the data consistent with rotation also drops when the price information is added, but rotation remains the strategy that is consistent with the data the greatest proportion of the time. \square

It is clear that in 2Y experiments bidders are able to use the repeated nature of the experimental design in order to select a more profitable strategy. The rotation strategy enables the bidders to capture the complementarity term that would be lost under any spitting arrangement such as BL. However, in 2N experiments, bid rotation no longer has this advantage; the BL strategy yields higher expected payoffs.

Result 9 (Bidder behavior in markets with no complementarity) *Among 2N markets classified as collusive, the allocation of objects is often consistent with the BL strategy.*

Support: Tables 9-11. In all 13 2N observations classified as collusive, bidders are more likely to split markets; 11 out of 13 observations split the markets 80% or more of the periods. While this is a strong rejection of bid rotation, minimal bid, BL, and other collusive strategies are possibly consistent with these allocations. Since any period in which different bidders win is classified as consistent with the split prediction but BL requires a particular split of the markets, it is not surprising that more allocations are consistent with the split classification. If bidders were actually utilizing a random minimal bid strategy, we would expect that half of the time the split classification will be consistent with the BL strategy as well. In 12 out of 13 observations, the proportion of splitting observations that are consistent with the BL strategy as well is far greater than half. The introduction of prices drives a wedge between the minimal bid and BL strategies; minimal bid always predicts the minimum bid level, but BL predicts higher bids in the case of conflict. Not surprisingly, the performance of both strategies decline with the inclusion of prices. The relative performance of each of the two market splitting strategies does not change markedly. Pooling across collusive 2N experiments, the null hypothesis that the mean difference of observed prices and the BL prices is zero cannot be rejected at a 5% level; the same hypothesis can be rejected for the mean difference between observed prices and minimal bid prices. In all experiments, the average price is considerably higher than the minimal bid prediction. \square

The relative strength of the BL strategy in the 2N treatment suggests that bidders, when they successfully collude, are strategically splitting markets. Given the obviously higher coincidence of signaling and collusion (Result 6), this result is not that surprising. We speculate that it is this ability to increase expected profits that enables successful tacit collusion. If bidders are splitting markets or following a rotation scheme in these

experiments, the temptation to occasionally deviate from the collusive strategy can be extremely tempting at times. For example, suppose the bidders have coordinated on a strategy of bidder 1 always winning market A and 2 always winning market B. Inevitably there will be valuation draws where 1's value for A is small but large for B. This would obviously create a tremendous incentive for 1 to break the collusive arrangement in order to participate in the auction for B for "just this one period." However, if he does, the fragile collusive equilibrium they had reached would be in jeopardy. A BL-type strategy minimizes this threat since bidders are more likely to be allocated the object they have a high value on. When combined with result 8 that bidders utilize a rotation strategy in positive complementarity experiments, these results suggest that bidders will coordinate on Pareto improving strategies.

6 Conclusions

The analysis of the pooled level data allows us to reach a number of conclusions. In accordance with recent empirical observations and the BL theory, we found that collusion occurs in experimental auctions for multiple objects only when the number of bidders is small. The presence of a common moderate complementarity does not eliminate collusion. The incidence of collusion increases with bidder experience in small size markets.

A closer examination of the individual data provides insights into the behavior supporting the collusive observations. Especially in the no complementarity treatments, signaling and retaliatory bidding are recognized by bidders as tools to support collusive play. Thus, outcomes of these auctions, when classified as collusive, often match the BL model quite well. However, when there is a positive complementarity, there is the added concern of "leaving money on the table" in the form of an uncaptured complementarity term. Successful collusive bidders appear to avoid this by utilizing a bid rotation strategy (giving both objects to the same bidder).

These results bear close resemblance to those in the experimental equilibrium selection literature; small groups of experimental subjects are capable of coordinating on Pareto superior Nash equilibria. Here we demonstrate that small groups of bidders are able to coordinate on Pareto improving perfect Bayes Nash equilibria in an environment where the strategies are significantly more complex than those previously examined.

There are a number of results that are inconsistent with theories of collusion in these auctions. First, the presence of a large complementarity appears to make collusion less likely. The BL market split is still a PBE with $k = 101$ and rotation is a more profitable collusive option. What makes a large complementarity substantially different than a moderate one? Is the desire to capture the large common term creating increased

competitiveness? Second, experienced bidders are willing to bid above their one period valuations. While such behavior clearly appears related to the attempts to enforce collusive strategies, we are unaware of non-cooperative theory, repeated or otherwise, that rationalizes this behavior. Are such extreme actions also observed in real auctions?

While the experimental, empirical and theoretical literature on collusion in auctions is growing, these results suggest two future avenues of research. First, given enough time, some groups manage to collude while others do not. The information on why some groups are successful must be contained in the dynamics of the bidding process. Was the collusive outcomes the results of well planned behavior by a few insightful bidders, or was it the result of some fortuitous event? Would all groups end up colluding if given enough time? Second, the simultaneous ascending bid auction is one particular institution for the sale of multiple objects; other institutions might be more or less susceptible to collusion. For example, in a first-price sealed bid auction, the BL strategy is no longer an equilibrium. Would collusion be observed experimentally? While Kwasnica (2000) tells us that we should expect collusion when communication is allowed, we are not aware of any studies that look for the formation of tacit collusion. Increased experimental and theoretical work along these lines could provide the potential auctioneer with a thorough understanding of the opportunities for collusion when implementing different auction formats. An understanding of how collusive strategies are manifested in the lab may also allow the auctioneer to recognize activities by real bidders that are more likely to be collusive in nature.

Appendix

Experiment Instructions

Introduction

You are about to participate in an experiment in the economics of market decision making in which you will earn money based on the decisions you make. All earnings you make are yours to keep and will be paid to you IN CASH at the end of the experiment. During the experiment all units of account will be in experimental dollars. Upon concluding the experiment the amount of experimental dollars you earn will be converted into dollars at the conversion rate of 0.015 dollars per experimental dollar. Your earnings plus a lump sum amount of \$5 dollars will be paid to you in private.

Do not communicate with the other participants except according to the specific rules of the experiment. If you have a question, feel free to raise your hand. I will come over to you and answer your question in private.

In this experiment you are going to participate in a market in which you will be buying units of fictitious assets. At the beginning of the experiment, you will be assigned to a market with ONE other participant(s). You will not be told which of the other participants are in your market. *What happens in your market has no effect on the participants in other markets and vice versa.*

From this point forward, you will be referred to by your bidder number. You are bidder number _____ in this experiment.

Resale Values and Earnings

Trading in your market will occur in a sequence of independent market days or trading periods. Two assets, A and B, will be for sale in the market in each period. During each market period, you are free to purchase from the computer a unit of each of the two assets if you want. The value to you of any decisions you might make will depend on your “resale values” for the assets which will be assigned to you at the beginning of each trading period. Resale values may differ among individuals. *You are not to reveal your resale values to anyone. It is your own private information.*

If you purchase an asset, your earnings from the asset purchase, which are yours to keep, are equal to the difference between your resale value for that asset and the price you paid for the asset. That is:

$$\text{YOUR EARNINGS} = \text{RESALE VALUE} - \text{PURCHASE PRICE}.$$

Suppose for example that you buy asset A and that your resale value is 64 for this asset. If you pay 30 for the asset then your earnings are

$$\text{EARNINGS FROM THE ASSET} = 64 - 30 = 34 \text{ experimental dollars.}$$

You can calculate your earnings on your accounting sheet at the end of each period.

Your total earnings in any period are given by the sum of your earnings for each asset plus an **extra payoff** if you buy both assets. In this experiment, the extra payoff of 101 experimental dollars will be received by any bidder whenever that bidder purchases both assets A and B. Suppose, for example, that you purchased asset A for earnings of 50 and asset B for earnings of 80. Then your total earnings in that period would be

$$\text{TOTAL EARNINGS} = 50 + 80 + \text{extra payoff}.$$

Remember, if you purchase a unit of a particular asset, you must use the resale value for that asset. Your earnings from the asset are zero if you do not buy the asset in this period.

Market Organization

The market is organized as follows. At the beginning of each trading period, you will be informed about your resale values for the assets, and the minimal (reservation) prices for which each of the two assets can be sold for. Buyers may then submit bids by entering bids into the computer. Any bidder is free at any time during the period to place a bid to buy one unit of either asset at a specified price. Any bid at least as high as the minimal price is allowed. Each subsequent bid in the period must be *higher* than the existing bids for that asset. For example, if the current highest bid is 31 for asset A, you must bid more than 31 for asset A. As long as the period is open, you are free to make as many bids as you like. The period closes when no new bids have been placed for 40 seconds.

After the period is closed, the assets are sold to the bidders with the highest bids for each asset, provided that these bids are at least as high as the seller's reservation prices. No assets will be sold if all bids are below the seller's reservation prices.

Example 1 Suppose that, in a given period, the seller's reservation price is 60 for each asset. If the following bids are entered:

buyer 2 bids 62 on A

buyer 1 bids 61 on B

buyer 1 bids 72 on A

buyer 2 bids 77 on B

Then asset A is sold to buyer 1 for 72 experimental dollars, and asset B is sold to buyer 2 for 77 experimental dollars.

After the asset allocations and prices are announced, you are required to record your earnings on your record sheet. There will be a 40 second pause between period to allow you to record your earnings. This will continue for a number of periods. At the end of the experiment, you will be asked to calculate your total profits and record them on the record sheet enclosed.

Submitting Bids and the Bidbook

When the period is open, on your screen you will see a button called, 'Bidbook'. If you press that button, a new window will appear titled, 'Player Bid Page'. Your bidbook may look something like this:

Asset	Price	Minimal Price	Quantity	Value
A		1	1	52
B		1	1	13

Your resale values for the assets in this period are displayed in the "Value" column. The "Minimal price" column shows the seller's minimal (reservation) prices for assets. In

the hypothetical example above, the seller's reservation price is 1 for both assets, and resale values are: 52 for asset A, and 13 for asset B. This indicates that you would receive a resale value of 52 for asset A if you place the highest bid in that market. Likewise, your value for asset B would be 13. These numbers may change from period to period.

You may submit a bid by typing a bid price for either asset in the 'Price' column, and then selecting the 'Submit' button. Notice that you can place bids for assets A and B either individually or simultaneously. If you place a bid for both A and B at the same time, the computer will treat them just as if you had placed them in two separate bids. If your bid is valid, it will appear in the bottom portion of main ('Asset Market Experiments') window. This window displays current bids in the period. Your bids are indicated in blue while the bids of others are colored black. You may view previous periods' results by selecting the 'Result' button on your main screen.

Determination of Resale Values

For each buyer the resale value for each asset in each period will be between 1 and 100. For each of the two assets, each number from 1 to 100 has equal chance of appearing. It is as if each number between 1 and 100 is stamped on a single ball and placed in an urn. A draw from the urn determines the resale value of an asset for an individual. The ball is replaced and a second draw determines the resale value for another participant. The procedure is then repeated to determine the values of the second asset. The resale values each period are determined the same way. The following is a table in which the probability of getting a value in a certain range is listed: (It is for your reference)

Range of Resale Value	Probability of a value in this range
1-10	10%
1-20	20%
1-30	30%
1-40	40%
1-50	50%
1-60	60%
1-70	70%
1-80	80%
1-90	90%
1-100	100%

Are there any questions?

Exercise 1 Suppose buyers' resale values lie between 1 and 100 experimental dollars. Suppose, in a given period, that the seller's reservation price for the assets is 30 experimental dollars each. The buyers' resale values are: 70 experimental dollars (buyer 1 asset A), 40 experimental dollars (buyer 1 asset B), 45 experimental dollars (buyer 2 asset A), 55 experimental dollars (buyer 2 asset B). Suppose the extra payoff for buying both assets is 10 experimental dollars for either buyer. The end-of-the period bids are the following:

buyer 2 bids 29 on A

buyer 1 bids 30 on B

buyer 2 bids 42 on B

(1) Which assets are sold? _____

(2) What did buyer 1 buy? _____ At what price(s)? _____

What is his (her) profit? _____

(3) What did buyer 2 buy? _____ At what price(s)? _____

What is his (her) profit? _____

Exercise 2 Suppose buyers' resale values, the seller's reservation prices, and the extra payoff are as given in exercise 1, but the end-of-the period bids are the following:

buyer 2 bids 29 on A

buyer 1 bids 30 on B

buyer 2 bids 42 on B

buyer 2 bids 30 on A

(1) Which assets are sold? _____

(2) What did buyer 1 buy? _____ At what price(s)? _____

What is his (her) profit? _____

(3) What did buyer 2 buy? _____ At what price(s)? _____

What is his (her) profit? _____

Period zero will be practice. You will receive no earnings for this period. If you have a question, please raise your hand and I will come by to answer your question.

Are there any questions?

References

- [1] Baldwin, L., R. Marshall and J.-F. Richard, 1997. Bidder collusion at forest service timber sales. *Journal of Political Economy*, 105(4), 657-699.
- [2] Brusco, S., and G. Lopomo, 2001, Collusion via signaling in simultaneous ascending bid auctions with multiple objects and complementarities, NYU working paper EC-99-05.
- [3] Cramton, P. and J.A. Schwartz, 2000. Collusive bidding: lessons from the FCC spectrum auctions. *Journal of Regulatory Economics*, forthcoming.
- [4] Federal Communications Commission, 1997. Notice of Apparent Liability for Forfeiture. FCC 97-388.
- [5] Feinstein, J., M. Block and F. Nold, 1985. Asymmetric information and collusive behavior in auction markets. *American Economic Review*, 75(3), 441-460.
- [6] Graham, D.A. and R.C. Marshall, 1987. Collusive bidder behavior at single-object second price and English auctions. *Journal of Political Economy*, 95, 1217-1239.
- [7] Isaac, M.R., Ramey, V. and A.W. Williams, 1984. The effects of market organization on conspiracies in restraint of trade. *Journal of Economic Behavior and Organization*, 5, 191-222.
- [8] Isaac, M.R. and J. Walker, 1985. Information and conspiracy in sealed bid auctions. *Journal of Economic Behavior and Organization*, 6, 139-159.
- [9] Kwasnica, A.M., 2000. The choice of cooperative strategies in sealed bid auctions. *Journal of Economic Behavior and Organization* 42(3), 323-346.
- [10] Maskin, E., and John Riley, 1996, Uniqueness in Sealed High bid Auctions, unpublished manuscript.
- [11] McAfee, R.P. and J. McMillan, 1992. Bidding rings. *American Economic Review*, 92(3), 579-599.
- [12] Milgrom, P., 1987, Auction theory, In: T. Bewley, (Ed.), *Advances of Economic Theory*, Fifth World Congress. Cambridge University Press, pp. 1-32.
- [13] Milgrom, P., 1998. Game theory and spectrum auctions. *European Economic Review*, 42, 771-778.

- [14] Porter, R.H and J.D. Zona, 1993. Detection of bid rigging in procurement auctions. *Journal of Political Economy*, 101(3), 518-538.
- [15] Porter, R.H and J.D. Zona, 1999. Ohio school milk markets: an analysis of bidding. *Rand Journal of Economics*, 30(2), 263-288.
- [16] Sherstyuk, K., 1999. Collusion without conspiracy: an experimental study of one-sided auctions. *Experimental Economics*, 2(1), 59-75.
- [17] Sherstyuk, K., 2000. A note on honest bidding in a multiunit auction with additive common complementarity. Mimeo, University of Hawaii.
- [18] Van Huyck, J., R. Battalio, and R. Beil, 1990, Tacit coordination games, strategic uncertainty and coordination failure, *American Economic Review* 80, 234-248.

No. of bidders	Complemen- tarity	Notation	No. of independent markets	
			Inexperienced Subjects	Experienced Subjects
2	None	2N	30	18
2	50	2Y50	16	6
2	101	2Y101	17	11
5	None	5N	9	2
5	50	5Y50	3	0
5	101	5Y101	5	4

Table 1: Number of experimental markets by treatment

University	Notation	No. Sessions	No. Markets
Arizona	UArizona	4	9
Caltech	CIT	5	18
Hawaii	UH	6	19
Melbourne	MEL	12	39
Penn State	PSU	13	36

Table 2: Session completed at each university

Treatment	Subject pool	No. of sessions	Session codes	No. of indep. markets	No. of periods	Exchange rate	Outcomes
2N	Mel	2	101, 102	10	6 -- 15	0.04	5 competitive, 3 semi-collusive, 2 collusive
2N	CIT	1	201	5	20	0.02	3 competitive, 1 semi-collusive, 1 collusive
2N	PennState	2*	301, 303	10	16 -- 25	0.02	9 competitive, 1 semi-collusive
2N	UH	1	501	5	22	0.02	4 competitive, 1 semi-collusive
2N exp (mixed)	Mel	2	103, 104	6	17 -- 25	0.03	3 competitive, 3 collusive
2N exp (sorted)	CIT	1	204	5	16 -- 25	0.02	1 competitive, 4 collusive
2N exp (sorted)	PSU	1	304	5	25	0.02	2 competitive, 3 collusive
2N exp (sorted)	UH	1	502	2	25	0.02	2 competitive
2Y50	Mel	2	111, 112	10	9 -- 10	0.04	1 compet-SEA, 4 compet-mixed, 2 compet-Vick, 3 collusive
2Y50	CIT	2	211, 212(1)	6	24 -- 25	0.015	3 compet-mixed, 1 semi-collusive, 2 collusive
2Y50 exp (mixed)	Mel	1	113	5	12 -- 25	0.015	4 compet-mixed, 1 compet-Vick
2Y50 exp (sorted)	CIT	1	212(2)	1	25	0.015	1 collusive
2Y101	PennState	3	321, 322, 325	9	17 -- 25	0.015	5 compet-mixed, 4 compet-Vick
2Y101	UH	3	521, 522, 524	8	15 -- 18	0.015	7 compet-mixed, 1 compet-Vick
2Y101 exp (sorted)	PennState	3	323, 324, 326	7	25	0.015	2 compet-mixed, 3 compet-Vick, 2 collusive
2Y101 exp (sorted)	UH	2	523, 524	4	12 -- 25	0.015	1 compet-mixed, 3 compet-Vick
5N	Mel	2	151, 152	4	6 -- 10	0.15*	4 competitive
5N	CIT	1	251	1	18	0.15	1 competitive
5N	PennState	3	351, 352, 353	4	25	0.15	4 competitive
5N exp (mixed)	Mel	1	153	1	15	0.15	1 competitive
5N exp (sorted)	PennState	1	354	1	22	0.15	1 competitive
5Y50	Mel	2	161, 162	3	7 -- 12	0.15	1 compet-mixed, 2 compet-Vick
5Y101	U Arizona	2	471, 472	5	9 -- 12	0.15	3 compet-mixed, 2 compet-Vick
5Y101 exp (sorted)	U Arizona	2	473, 474	4	10 -- 21	0.15	1 compet-mixed, 3 compet-Vick

* one session, exp #302, is excluded due to computer problems

"**exp (mixed)**" stands for a subject who has earlier been through at least 6 periods under SOME treatment;

"**exp (sorted)**" stands for a subject who has earlier been through at least 6 periods in the same size market

Table 3: Summary of experimental sessions

Markets	No. of mkts		Price AB, francs					% deviation from SEA	% deviation from Vprice
			actual	SEA	Vprice	BL	MinBid		
<u>2N markets</u>									
Inexperienced subjects	30	mean	58.95	64.52	n/a	14.28	2	-7.98	n/a
		stddv	16.58	8.44	n/a	2.5	0	24.39	n/a
Experienced, mixed	6	mean	46.61	67.91	n/a	13.57	2	-31.78	n/a
		stddv	29.73	5.74	n/a	3.2	0	43.18	n/a
Experienced, sorted	12	mean	39.64	66.15	n/a	13.7	2	-38.12	n/a
		stddv	32.63	4.42	n/a	2.14	0	53.28	n/a
<u>2Y50 markets</u>									
Inexperienced subjects	16	mean	62.72	64.07	120.51	n/a	2	-1.88	-47.48
		stddv	41.25	11.17	15.52	n/a	0	58.74	33.47
Experienced, mixed*	6	mean	75.51	64.97	124.72	n/a	2	15.74	-39.58
		stddv	34.73	5.39	8.61	n/a	0	50.62	27.05
<u>2Y101 markets</u>									
Inexperienced subjects	17	mean	123.61	66.61	177.06	14.525	2	86.01	-30.17
		stddv	36.78	5.87	5.59	2.098	0	53.27	20.41
Experienced, sorted	11	mean	130.71	66	177.06	13.092	2	98.34	-26.35
		stddv	68.68	3.63	3.9	2.0616	0	103.88	38.52
<u>5N markets</u>									
Inexperienced subjects	9	mean	133.48	131.21	n/a	129.43	2	1.63	n/a
		stddv	12.49	4.22	n/a	2.59	0	7.5	n/a
Experienced, all**	2	mean	134.99	131.78	n/a	128.86	2	2.44	n/a
		stddv	1.15	0.13	n/a	0.76	0	0.76	n/a
<u>5Y50 markets</u>									
Inexperienced subjects	3	mean	155.87	129.42	167.28	n/a	2	20.43	-6.83
		stddv	9.86	2.31	3.71	n/a	0	6.83	5.21
<u>5Y101 markets</u>									
Inexperienced subjects	5	mean	170.96	128.05	217.39	n/a	2	33.69	-21.15
		stddv	19.99	2.12	5.66	n/a	0	17.19	10.93
Experienced, sorted	4	mean	210.23	133.28	219.73	n/a	2	58.01	-4.27
		stddv	25.02	3.87	2.89	n/a	0	20.76	11.79

Theoretical predictions differ slightly across sessions since they are based on the actual bidder values drawn

* One market was sorted; the other 5 markets were mixed.

** One market was sorted, and one was mixed.

Table 4: Descriptive statistics of market prices by treatment

			Efficiency, percent						Relative gains, percent					
Treatment	No. mkts		Actual	Vick	SEA	BL	MinBid	Rotation	Actual	Vick	SEA	BL	MinBid	Rotation
<u>2N treatment</u>														
Inexperienced subjects	30	mean	94.63	n/a	100	92.74	74.18	74.18	47.49	n/a	49.71	80.94	73.71	73.71
		stddev	4.99	n/a	0	2.04	6.41	6.41	12.29	n/a	5.91	2.72	6.49	6.49
Experienced, mixed	6	mean	97.28	n/a	100	93.08	75.27	75.27	61.39	n/a	48.7	82.3	74.83	74.83
		stddev	3.14	n/a	0	1.93	6.54	6.54	22.08	n/a	6.23	2.63	6.64	6.64
Experienced, sorted	12	mean	93.74	n/a	100	92.66	75.04	75.04	63.21	n/a	49.34	81.21	74.61	74.61
		stddev	4.48	n/a	0	2.16	4.07	4.07	22.89	n/a	3.45	2.78	4.15	4.15
<u>2Y50 treatment</u>														
Inexperienced subjects	16	mean	91.17	100	89.26	n/a	54.7	87.21	55.37	25.91	53.05	n/a	54.09	87.05
		stddev	4.49	0	1.91	n/a	5.89	4.95	21.63	3.77	5.83	n/a	5.96	5
Experienced, mixed	6	mean	93.31	100	89.14	n/a	54.37	88.53	49.05	25.66	52.4	n/a	53.79	88.4
		stddev	3.03	0	0.92	n/a	6.02	4.34	20.1	4.63	3.42	n/a	6.1	4.38
<u>2Y101 treatment</u>														
Inexperienced subjects	17	mean	89.74	100	79.57	53.68	43.49	90.61	34.35	19.14	50.71	47.14	42.95	90.52
		stddev	6.78	0	1.47	2.02	3.44	3.05	14.82	2.07	3.68	2.34	3.47	3.08
Experienced, sorted	11	mean	94.34	100	80.01	53.9	42.71	91.62	35.68	19.87	51.77	48.05	42.17	91.55
		stddev	5.34	0	0.92	1.73	2.11	2.32	26.76	2.05	1.52	1.5	2.13	2.35
<u>5N treatment</u>														
Inexperienced subjects	9	mean	98.25	n/a	100	100	59.99	59.99	15.27	n/a	18.84	20.02	59.47	59.47
		stddev	1.93	n/a	0	0	0.46	0.46	5.62	n/a	1.11	1.65	0.47	0.47
Experienced, all	2	mean	99.64	n/a	100	100	59.72	59.72	16.78	n/a	19.19	21.16	59.21	59.21
		stddev	0.16	n/a	0	0	0.16	0.16	1.78	n/a	0.85	1.24	0.16	0.16
<u>5Y50 treatment</u>														
Inexperienced subjects	3	mean	96.18	100	85.46	n/a	47.51	73.06	17.9	15.64	20.91	n/a	46.97	72.79
		stddev	3.52	0	0.11	n/a	0	0.47	0.65	0.33	0.01	n/a	0.02	0.47
<u>5Y101 treatment</u>														
Inexperienced subjects	5	mean	87.65	100	73.63	n/a	37.64	78.53	18.77	12.61	22.78	n/a	37.13	78.36
		stddev	8.61	0	3.26	n/a	0.24	1.07	4.78	0.93	4.31	n/a	0.24	1.08
Experienced, sorted	4	mean	95.76	100	71.92	n/a	38.43	78.53	13.07	13.42	19.97	n/a	37.93	78.36
		stddev	3.48	0	1.72	n/a	0.83	0.54	7.9	0.53	2.83	n/a	0.84	0.54

* Theoretical predictions differ slightly across sessions since they are based on the actual bidder values drawn

Table 5: Efficiency and bidder gains by treatment

Dependent variable: competitive market			
Independent Variable	Estimated Coefficient	Standard Error	t-Statistic
one	-0.14	0.54	-0.26
large market	2.62*	1.14	2.29
experience-mixed	0.73	0.83	0.88
experience-sorted	-1.86**	0.75	-2.48
$k = 50$	1.04	0.64	1.62
$k = 101$	1.89*	0.94	2.01
CIT	0.19	0.74	0.25
PSU	1.75*	0.83	2.12
UH	3.09**	1.29	2.39
log likelihood		-47.845	
number of observations		120	
percent correctly predicted		80.83	

Table 6: Logit estimation of the probability of market being competitive. * – significant at 5% level; ** – significant at 2% level.

Dependent variable: average market price				
Independent Variable	Estimated Coefficient	Standard Error	<i>t</i> -Statistic	
one	2.16	8.75	0.25	
SEA price, all markets	1.01***	0.10	10.26	
SEA price, markets with $k > 0$	-0.49**	0.20	-2.41	
Vprice, markets with $k = 50$	0.46***	0.13	3.39	
Vprice, markets with $k = 101$	0.50***	0.10	5.22	
Experience, all markets	4.76	7.09	0.67	
Experience, markets with $k = 50$	-9.36	12.30	-0.76	
Experience, markets with $k = 101$	26.79***	9.87	2.71	
Number of Observations	92			
Corrected R-squared	0.83			

Table 7: Ordinary least squares regression of average market prices in competitive markets on the SEA and Vickrey predictions. ** – significant at 2% level, *** – significant at 1% level.

Treatment	No of Individuals	Outcome	Average	
2N inexperienced	60	all	mean	0.51
			stddv	0.27
	54	non-collusive	mean	0.49
			stddv	0.27
2N experienced	6	collusive	mean	0.69*
			stddv	0.22
	36	all	mean	0.63*
			stddv	0.29
2Y50 inexperienced	16	non-collusive	mean	0.40
			stddv	0.25
	20	collusive	mean	0.82*
			stddv	0.15
2Y50 experienced	32	all	mean	0.42
			stddv	0.25
	22	non-collusive	mean	0.43
			stddv	0.24
2Y101 inexperienced	10	collusive	mean	0.41
			stddv	0.28
	12	all	mean	0.42
			stddv	0.28
2Y101 experienced	10	non-collusive	mean	0.36
			stddv	0.25
	2	collusive	mean	0.73
			stddv	0.21
5N inexperienced	34	all non-coll	mean	0.47
			stddv	0.30
5N experienced	22	all	mean	0.27*
			stddv	0.23
	18	non-collusive	mean	0.26*
			stddv	0.22
5Y50 inexperienced	4	collusive	mean	0.30
			stddv	0.28
	45	all non-coll	mean	0.68*
			stddv	0.19
5Y101 inexperienced	10	all non-coll	mean	0.67*
			stddv	0.20
	15	all non-coll	mean	0.67*
			stddv	0.22
5Y101 experienced	21	all non-coll	mean	0.58
			stddv	0.21
	18	all non-coll	mean	0.49
			stddv	0.31

Table 8: Proportion of initial bids that are consistent with signaling. * – significantly different than 0.5 at the 5% level.

Experiment	Market	Treatment	Periods	Rotation	Split	BL	Efficient
102	2	2N inexp	15	0.13	0.87	0.40	0.27
102	5	2N inexp	15	0.33	0.67	0.60	0.27
201	2	2N inexp	20	0.00	1.00	0.90	0.50
103	1	2N exp	25	0.40	0.60	0.56	0.88
103	4	2N exp	25	0.12	0.88	0.80	0.60
104	1	2N exp	25	0.12	0.88	0.84	0.52
304	1	2N exp	25	0.00	1.00	0.68	0.40
304	4	2N exp	25	0.12	0.88	0.68	0.68
304	5	2N exp	25	0.00	1.00	0.84	0.36
204	1	2N exp	25	0.00	1.00	0.92	0.44
204	2	2N exp	25	0.20	0.80	0.72	0.60
204	4	2N exp	25	0.12	0.88	0.80	0.44
204	5	2N exp	25	0.00	1.00	0.92	0.36
112	3	2Y50 inexp	10	0.70	0.30	0.20	0.60
112	4	2Y50 inexp	10	1.00	0.00	0.00	0.50
112	5	2Y50 inexp	10	0.80	0.20	0.10	0.60
211	1	2Y50 inexp	24	0.83	0.17	0.17	0.54
211	2	2Y50 inexp	25	0.48	0.52	0.32	0.44
212	2	2Y50 exp	25	1.00	0.00	0.00	0.56
323	3	2Y101 exp	25	0.96	0.04	0.04	0.64
326	4	2Y101 exp	25	1.00	0.00	0.00	0.96

Table 9: Collusive Outcomes: Proportion of allocations consistent with various strategies.

Treatment	No of Markets	Outcome		Rotation	Split	BL	Efficient
2N inexperienced	30	all	mean	0.47	0.53	0.5	0.7
			stddv	0.19	0.19	0.16	0.19
	27	non-collusive	mean	0.5	0.5	0.49	0.8
			stddv	0.15	0.15	0.14	0.15
	3	collusive	mean	0.16	0.84	0.63	0.34
			stddv	0.17	0.17	0.25	0.13
2N experienced	18	all	mean	0.29	0.71	0.63	0.67
			stddv	0.23	0.23	0.19	0.23
	8	non-collusive	mean	0.53	0.47	0.45	0.85
			stddv	0.04	0.04	0.04	0.16
	10	collusive	mean	0.11	0.89	0.78	0.53
			stddv	0.13	0.13	0.12	0.17
2Y50 inexperienced	16	all	mean	0.79	0.2	0.17	0.6
			stddv	0.16	0.16	0.14	0.13
	11	non-collusive	mean	0.8	0.19	0.17	0.63
			stddv	0.16	0.16	0.16	0.14
	5	collusive	mean	0.76	0.23	0.15	0.53
			stddv	0.19	0.19	0.11	0.06
2Y50 experienced	6	all	mean	0.81	0.18	0.18	0.63
			stddv	0.11	0.11	0.11	0.1
	5	non-collusive	mean	0.77	0.22	0.22	0.65
			stddv	0.11	0.11	0.11	0.1
	1	collusive	mean	1	0	0	0.56
			stddv	0.11	0.11	0.11	0.1
2Y101 inexperienced	17	all non-coll	mean	0.81	0.18	0.16	0.62
			stddv	0.17	0.17	0.16	0.12
2Y101 experienced	11	all	mean	0.94	0.05	0.04	0.74
			stddv	0.05	0.05	0.05	0.19
	9	non-collusive	mean	0.93	0.06	0.05	0.73
			stddv	0.06	0.06	0.06	0.2
	2	collusive	mean	0.98	0.02	0.02	0.8
			stddv	0.02	0.02	0.02	0.22

Table 10: Mean proportion of allocations consistent with models.

	No of markets							
Treatment	Markets	Outcome		Rotation	Split	BL	SEA	Vick
2N inexperienced	30	all	mean	0.05	0.09	0.08	0.41	NA
			stddv	0.09	0.20	0.14	0.24	NA
	27	non-collusive	mean	0.05	0.04	0.04	0.45	NA
			stddv	0.09	0.08	0.07	0.22	NA
	3	collusive	mean	0.04	0.58	0.42	0.08	NA
			stddv	0.08	0.33	0.17	0.04	NA
2N experienced	18	all	mean	0.03	0.34	0.24	0.32	NA
			stddv	0.08	0.36	0.25	0.31	NA
	8	non-collusive	mean	0.05	0.01	0.01	0.60	NA
			stddv	0.11	0.03	0.02	0.24	NA
	10	collusive	mean	0.02	0.60	0.42	0.09	NA
			stddv	0.04	0.28	0.19	0.09	NA
2Y50 inexperienced	16	all	mean	0.23	0.06	0.04	0.13	0.12
			stddv	0.31	0.11	0.06	0.14	0.17
	11	non-collusive	mean	0.09	0.02	0.02	0.17	0.18
			stddv	0.12	0.05	0.04	0.15	0.18
	5	collusive	mean	0.54	0.16	0.08	0.04	0.00
			stddv	0.38	0.15	0.08	0.04	0.00
2Y50 experienced	6	all	mean	0.13	0.00	0.00	0.13	0.12
			stddv	0.27	0.00	0.00	0.15	0.09
	5	non-collusive	mean	0.02	0.00	0.00	0.16	0.14
			stddv	0.04	0.00	0.00	0.16	0.08
	1	collusive	mean	0.68	0.00	0.00	0.00	0.00
			stddv	0.68	0.00	0.00	0.00	0.00
2Y101 inexperienced	17	all non-coll	mean	0.06	0.01	0.02	0.08	0.18
			stddv	0.13	0.04	0.05	0.13	0.19
2Y101 experienced	11	all	mean	0.20	0.03	0.02	0.02	0.35
			stddv	0.35	0.06	0.05	0.06	0.29
	9	non-collusive	mean	0.05	0.02	0.02	0.02	0.43
			stddv	0.07	0.05	0.05	0.07	0.26
	2	collusive	mean	0.90	0.06	0.04	0.02	0.00
			stddv	0.03	0.08	0.06	0.03	0.00

Table 11: Mean proportion of allocations and prices consistent with different models.

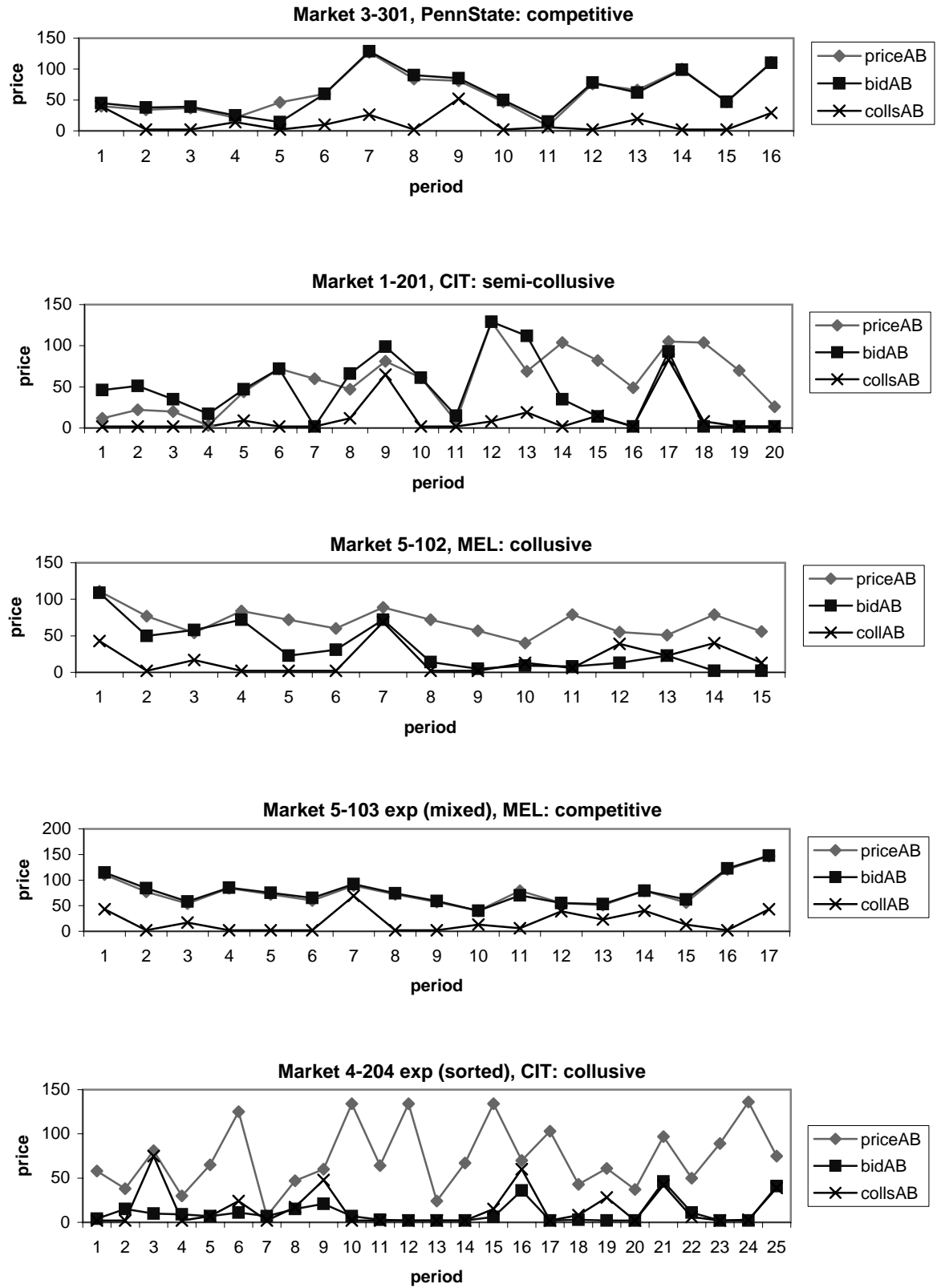


Figure 1: Examples of market price dynamics in 2N treatment.

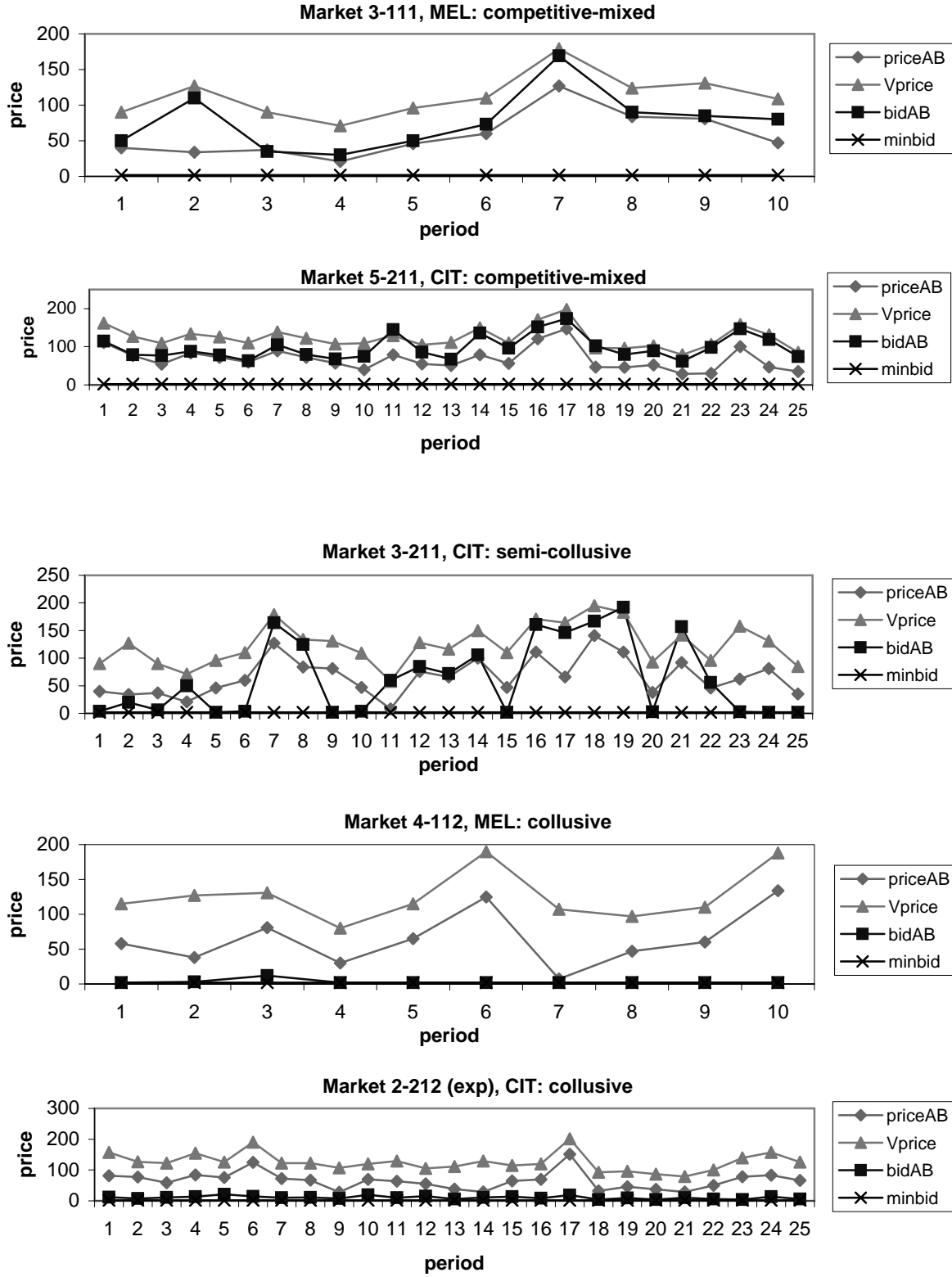


Figure 2: Examples of market price dynamics in 2Y, $k=50$ treatment.

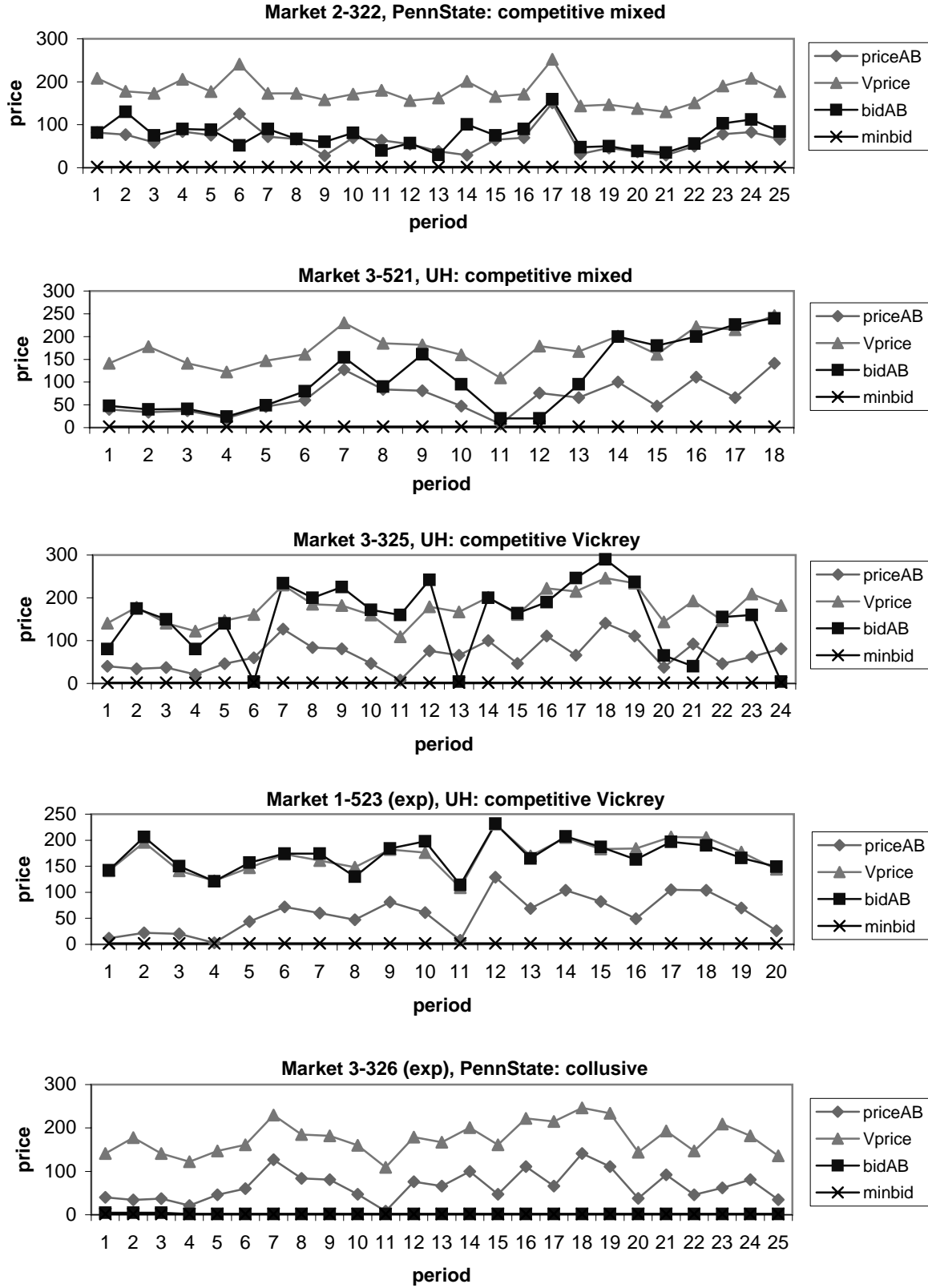


Figure 3: Examples of market price dynamics in 2Y, k=101 treatment.

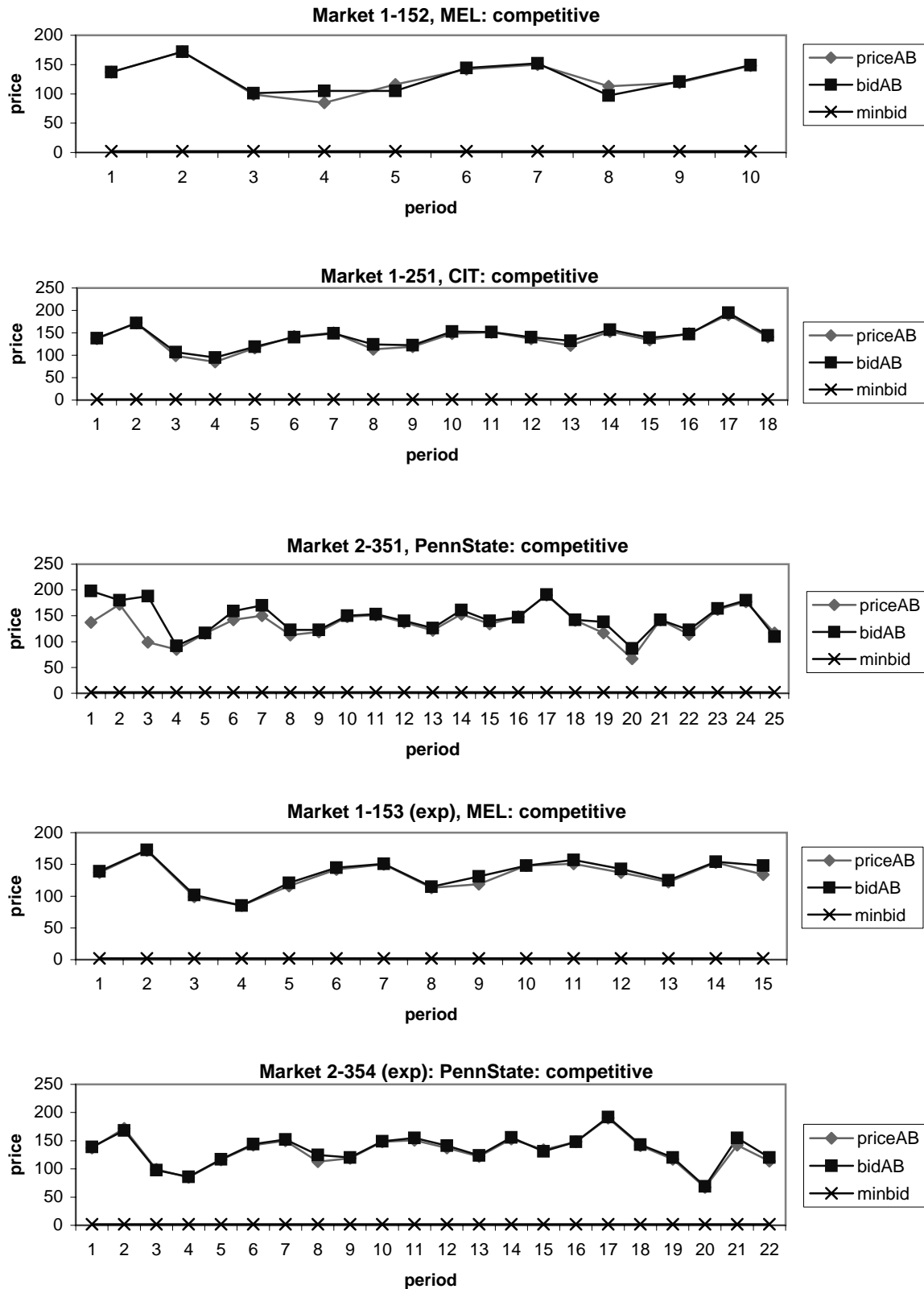


Figure 4: Examples of market price dynamics in 5N treatment.

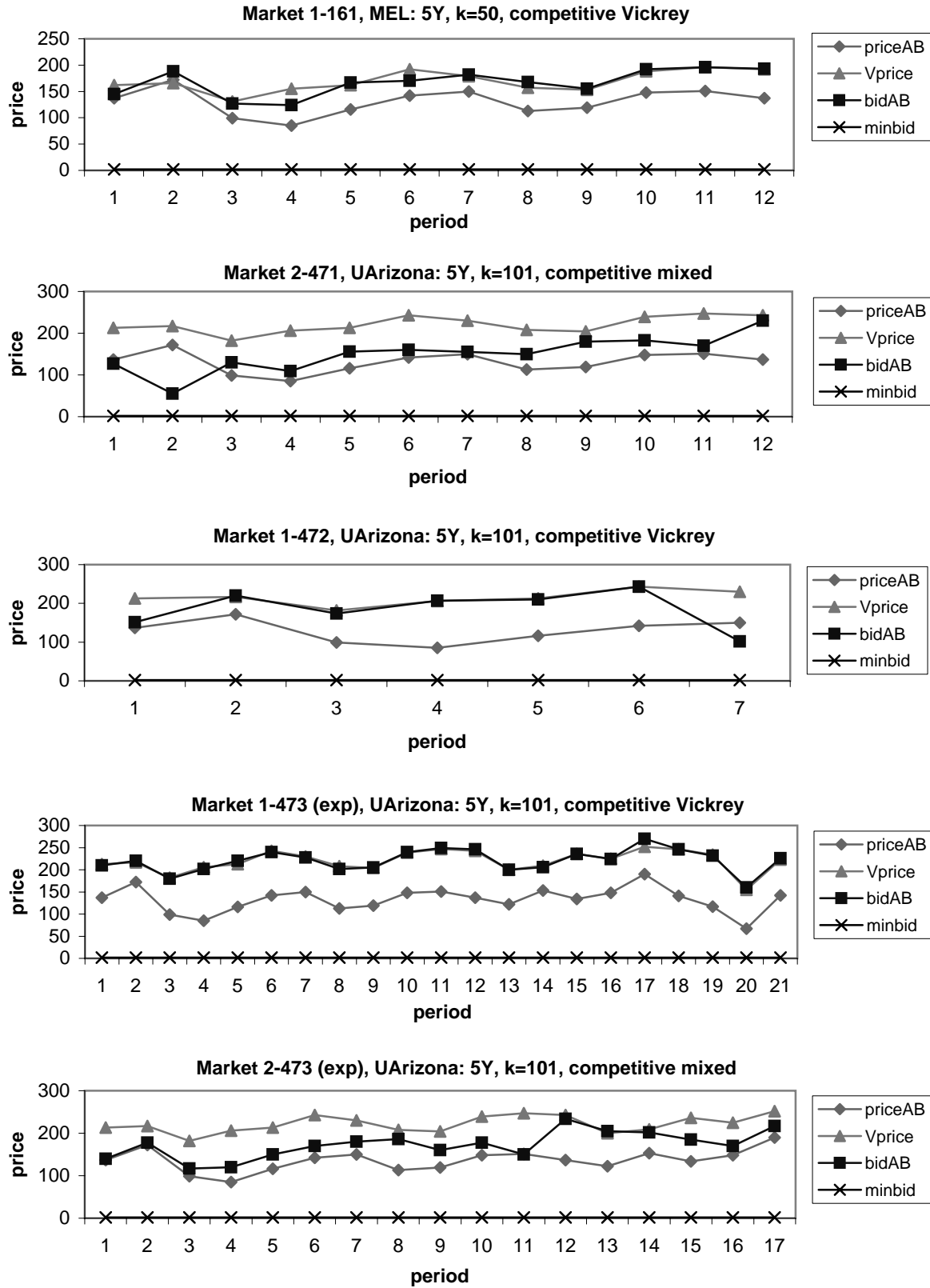


Figure 5: Examples of market price dynamics in 5Y treatments.