

**Asymmetric Information, Capital and Ownership  
Structures and Corporate Income Taxation**

by

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**Abstract**

This paper develops a signalling model to investigate a firm's optimal financial response to corporate income taxation under informational asymmetries. The model obtains informationally constrained efficient equilibria in which a firm's debt level and inside equity position jointly serve as a single separating signal. Separating equilibria are characterized differently depending upon the tax obligation and the relative profitability of a high-quality firm. When the quality difference between firms is relatively large, a high-quality firm shows a unique optimal capital structure, in which the debt-equity ratio is increased as the tax rate rises but is reduced as the firm's profitability increases.

**Keywords and JEL Codes**

Corporate Income Taxation(**H25**), Capital Structure(**G32**),  
Asymmetric Information(**D82**).

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## I. Introduction

Although there have been numerous studies on the financial behavior of firms, a few strands of the literature on capital structure still have loose ends, and one of them is with regard to the consequences of the corporate income tax on a firm's capital structure in the existence of asymmetric information.

The Modigliani-Miller Irrelevance Theorem (MM Theorem) stands firm under conditions of complete information and competitive markets without taxes. The MM Theorem serves as a basis of the assumption, which has been used in many tax studies including the traditional general equilibrium analysis, that firms do not alter their financial structure and all new projects are financed by equity issue. This assumption is unrealistic for obvious reasons such as the tax shield provided by the corporate income tax, but a more satisfactory method to model endogenous capital structure in a tax study has yet to be developed.

On the other hand, the capital structure literature is abundant with attempts to explore the consequences of market imperfections caused by uncertainty or asymmetric information. For example, the pecking order theory was reinforced by Myers and Majluf [1984], the effect of agency costs was highlighted by Jensen and Meckling [1976], and signalling models became "a familiar approach to the determination of financial structure" (Ross [1987]).

A variety of signalling vehicles have been employed in signalling models, such as debt level in Ross [1977], inside equity position in Leland and Pyle [1977], underpricing in Rock [1986] and dividends in John and Williams [1985]. In particular, Leland and Pyle [1977] and Ross [1977] obtain a unique separating equilibrium in which the MM Theorem is held to be valid in the sense that the value of a firm is identical to that in the first best equilibrium under perfect information. Combining their signal vehicles into

one and considering a corporate income tax system, this paper intends to examine a firm's optimal capital structure and its response to tax changes in a market with informational asymmetries. More precisely, both debt level and inside equity position of a firm are the means of passing project risk on to outside investors and hence serve as a signal for the firm's true quality which is private information. We first characterize the equilibrium capital structure of a firm without taxes, and then, employing the corporate income tax system in DeAngelo and Masulis [1980], investigate the relationship between the capital structure and exogenous factors including the tax rate.

Although there have been many studies on the effects of corporate income taxation on firms' financial decisions under uncertainty, only a few of them have considered asymmetric information. For example, John and Williams [1985] and Bernheim [1991] offer explanations of the dividend puzzle using signalling with taxable dividends, and MacKie-Mason [1990] incorporates various signalling costs in an empirical investigation of tax consequences. In contrast, this paper highlights the risk sharing induced by the incentive compatibility principle of signalling with financial structures.

A firm's inside equity position signals the quality of the firm by showing the extent of entrepreneur's willingness to bear the risk of his own firm, and has been used in previous signalling models such as Leland and Pyle [1977], Grinblatt and Hwang [1989], Bajaj, Chan and Dasgupta [1998], and Cheong [1998]. Of these models, the first three do not consider the corporate income tax and limited liability of equity holders, and the last does not include debt financing by firms. Furthermore, they neglect the informational content of debt. On the other hand, the debt-signalling model in Ross [1977], in which a firm's debt level serves as a signal by the firms' manager facing the incentive compensation schedule, ignores the tax shield effect as well as other aspects of a firm's financial structure. In contrast, this paper considers both debt level and inside equity position of a firm which are jointly decided and both carry essential information for outsider investors' evaluation

of a firm.

The idea of joint determination of debt level and inside equity position is not new. It is, in fact, central in Jensen and Meckling's [1976] theory of agency costs and is empirically supported according to Crutchley and Hansen [1989].<sup>1</sup> It is also modelled as a manager's optimal behavior in the corporate control contest by Stulz [1988] and Israel [1992]. Unlike these studies, entrepreneurial risk aversion creates the basic motivation for their voluntary transfer of information through signalling in our model.

Among the many equilibria obtained in our signalling model, we focus on the Pareto-dominant separating equilibrium which is informationally-constrained efficient and supports only profitable projects as in the case of complete information. It is, however, characterized differently with and without the corporate income tax. Without the corporate income tax, the optimal capital structures of both the high-quality and low-quality firms are not unique and the high-quality firms' inside equity position and debt level are, respectively, bounded from below and bounded from above unlike those of the low-quality firms. Other things being equal, the high-quality firms hold more inside equity than the low-quality firms with the same debt level and issue less debt than the low-quality firms with the same inside equity position.

Under the corporate income tax system, we find two different types of separating equilibria: one allowing for tax exhaustion for all firms and the other requiring the high-quality firms to pay taxes. Intuitively, the first type of equilibrium prevails when the high-quality firms are not much better than the low-quality firms, but the second type emerges when the quality differences are so large that low-quality firms have a strong incentive to pretend to be high-quality firms. In the second type of separating equilibrium, high-quality firms show unique financial behavior and their debt-equity ratio increases

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<sup>1</sup> There are also empirical findings against the hypothesis of joint determination. See, for example, Chaplinsky and Niehaus [1993].

with the tax rate but decreases with their profitability. The latter result is particularly significant since it is consistent with the common empirical findings that leverage ratios are inversely related to profitability, which contradicts the prediction made by previous signalling models, such as Leland and Pyle [1977] and Ross [1977]. In this sense, this paper extends the usefulness of the signalling approach and enriches our understanding of firms' financial behavior under informational asymmetries. Another notable result from our model is that, unlike other exogenous elements of the model, the tax rate cannot serve as a determining factor of the type of equilibrium.

This paper is organized as follows. Section II describes the model. Sections III and IV are the main body of the paper, discussing the separating equilibria without the corporate income tax system and then with the tax system. Section IV also includes the results from comparative statics, and the concluding comments are shared in Section V.

## **II. Model**

We consider the financial market with adverse selection due to private information on the true quality of firms. Entrepreneurs set up firms to execute their own risky projects. They are risk averse, and seek external financing as a means of passing on project risk. We assume project risk is all idiosyncratic so that it can be freely absorbed in a well-diversified portfolio of an outside investor. Therefore, if the quality of firms are known publicly, all entrepreneurs can effectively cash their projects, and neither entrepreneurs nor outside investors will bear any more than a negligible amount of risk. However, such behavior is not possible when the information on firms' quality is not shared by outside investors. Outside investors know the population distribution of firms of different quality, but cannot tell one from the other. In order to overcome such informational asymmetries, entrepreneurs (or firms) send signals to outside investors. As in Leland and Pyle [1977], Grinblatt and Hwang [1989], Bajaj, Chan and Dasgupta [1998], and Cheong [1998], the

previous studies with similar settings, an entrepreneur signals the quality of his own firm by announcing his inside equity position which indicates the extent of his willingness to bear the firm's risk. Our model, however, departs from the earlier models by considering debt as an additional means of signalling. In our model, a firm issues both equity and debt, both of which carry information on the quality of the firm and, thereby, complement each other as signalling vehicles. Along with inside equity position, the level of risky debt signals the entrepreneur's willingness to bear the firm's risk, revealing the unused debt capacity which otherwise could have been used to reduce his risk-bearing. Under a corporate income tax system with debt deduction, the unused debt capacity also shows the entrepreneur's willingness to sacrifice the benefit of the tax shield since not being able to issue as much debt as possible is a costly burden on the entrepreneur. Our model explicitly considers these dual effects of debt and, in this sense, extends the analysis of Ross [1977] on the role of debt without taxes.

There are two periods in our model. In the first period, entrepreneurs set up their own firms, issue equity and debt as they announce their inside equity positions, and execute their projects using the cash-flow from the equity and debt sale and from their initial wealth (denoted by  $w_0$ ). All remaining wealth is invested in a risk-free security which earns a fixed rate of return (denoted by  $r$ ). In the second and final period, project outcome is known. Debt is first paid back (in the face value), the corporate tax is paid next, and then the rest is divided among the equity holders. Firms are dismantled as their projects become useless after being used once, and all consumption occurs in this period.

For simplicity, we assume there are only two possible outcomes from a project, and denote them as  $y_1$  and  $y_2$ , where  $0 \leq y_1 < K < y_2$  and  $K$  is the fixed capital outlay. We further assume that there are only two types of projects, so that they are conveniently identified by the probability of the better outcome (denoted by  $P_2$  while  $P_1$  denotes the

probability of the worse outcome).<sup>2</sup> We call a firm/project/entrepreneur with the higher  $P_2$  a *high type* and one with the lower  $P_2$  a *low type*.

Following DeAngelo and Masulis [1980], we consider the corporate income tax system that allows expensing and debt payment deduction but not loss offsets. Under this tax system, a firm has a positive tax obligation only if it truly earns economic profits after debt payment; that is, only if its project output exceeds the total of the debt deduction and the opportunity cost of capital expense. Since the debt payment is deductible from the tax base, firms want to borrow as much as possible. We assume, however, there is a fixed ceiling to debt level, which is set at  $y_2$ .<sup>3</sup> On the other hand, debt below  $y_1$  is risk free and, hence, it does not carry any information about the firm's quality. In order to have debt as a meaningful signalling vehicle, we assume all debt is risky and, therefore, it follows that  $y_1 \leq D \leq y_2$ .

Under the expensing scheme, the capital outlay is immediately written off, and the tax base only includes economic profits. If the tax system comes with the depreciation scheme, the tax base would include not only economic profits but also the opportunity cost of the capital outlay which could be earned untaxed elsewhere. This kind of distortion, which exists in reality, is ignored in this model but its consideration would not affect our main results. Due to the proportional nature of the corporate income tax, the risk held inside a firm is partly shared by the government in the sense of Domar and Musgrave [1944]. Although it is not as strong as in a model with the full loss offset provision, the

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<sup>2</sup> Such assumption of two types is often made in the literature and increasing the number of types of projects will not detract from the main story as long as types are discrete. Cheong [1998] illustrates how a separating equilibrium obtained with two types can be extended to a three-type case. If types are continuous, our model will be modified in such a way that  $P_2$  is a function of signals, thereby serving as an evaluation schedule for outside investors.

<sup>3</sup> The debt ceiling of  $y_2$  may seem arbitrarily high; however, setting it at values lower than  $y_2$  would not affect the main results in the paper as long as it is exogenously given in the model.

risk-sharing effect in our model illustrates the discrepancy between private risk-taking and social risk-taking.<sup>4</sup> We assume the details of the tax system are publicly known and taken as given by entrepreneurs and outside investors.

### III. No Tax Case

Without the corporate income tax, borrowing does not create the advantage of tax shields and, therefore, debt and equity serve equally as means of spreading project risk. A separating equilibrium in the financial market incorporates the incentive of a high-type entrepreneur to distinguish himself from low-type firms and that of a low-type entrepreneur to pretend to be a high type. A separating equilibrium is characterized by each type's signal and outside investors' evaluation schedule that is consistent with the firms' beliefs on which their signals are based. As is often the case with a signalling game, we find a multiple of separating equilibria in our model and, among them, we focus only on the Pareto-dominant separating equilibrium in which a low-type firm obtains his first-best utility.<sup>5</sup> Consequently, the equilibrium behavior of a low type is independent of the informational asymmetries while a high type's equilibrium behavior is affected substantially.

First, it is noted that the combination of inside equity position and debt level that allows a low type to have his first best utility is not unique. An entrepreneur can completely avoid bearing any risk by holding zero inside equity or issuing the maximum debt of  $y_2$  and, therefore, it must be the choice of a low type but not a high type in the separating equilibrium. The equilibrium choice of a high type is obtained as the solution to the high

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<sup>4</sup> See, for example, Chapter 4 in Atkinson and Stiglitz [1980] for further discussion on the tax effect on risk-taking.

<sup>5</sup> This equilibrium corresponds to the no-waste equilibrium in Milgrom and Roberts [1982] and is usually chosen as the unique equilibrium. For example, Cheong[1998] applied Cho-Kreps' [1987] Intuitive Criterion to select this equilibrium, whereas Leland and Pyle[1976], Ross[1977] and Grinblatt and Hwang [1989] selected this equilibrium in consideration of the low type's interest.

type's expected utility maximization problem given by

$$\max_{\alpha^H, D^H} P_1^H u((1+r)B^H) + P_2^H u((1+r)B^H + \alpha^H(y_2 - D^H))$$

subject to

$$u((1+r)B^L) = P_1^L u((1+r)B^H) + P_2^L u((1+r)B^H + \alpha^H(y_2 - D^H)) \quad (1)$$

where

$$B^H = (1 - \alpha^H)V_E^H + V_D^H + (w_0 - K)$$

$$V_E^H = \frac{P_2^H(y_2 - D^H)}{1+r}$$

$$V_D^H = \frac{P_1^H y_1 + P_2^H D^H}{1+r}$$

$$B^L = V_E^L + V_D^L + (w_0 - K)$$

$$V_E^L = \frac{P_2^L(y_2 - D^H)}{1+r}$$

$$V_D^L = \frac{P_1^L y_1 + P_2^L D^H}{1+r}$$

and  $P_j^i$  denotes the probability of outcome  $y_j$  ( $j = 1, 2$ ) of a type  $i$  project ( $i = H$  (high) or  $L$  (low)) and  $B^i$  denotes the wealth of a type- $i$  entrepreneur at the end of the first period and  $V_E^i$  denotes the value of equity of a type- $i$  firm and  $V_D^i$  denotes the value of debt of a type- $i$  firm. Equation (1) is the incentive compatibility constraint, of which the left-hand side is the first-best utility level of a low-type entrepreneur and the right-hand side is his expected utility when he successfully imitates a high type's signal ( $\alpha^H$  and  $D^H$ ), so that he is mistaken as a high type by outside investors.

One can easily see that, in this case, the MM Theorem holds regardless of the firm's type since for any  $D$ ,

$$V_D + V_E = \frac{P_1 y_1 + P_2 y_2}{1+r}.$$

This result is basically due to the use of expected value pricing without bankruptcy costs and is also obtained by previous signalling models. However, it is worthwhile to note that

in our model equity-holders of a firm including the entrepreneur himself bear only a limited liability up to the amount of their equity investment in the case that the firm defaults. In contrast, models of the Leland and Pyle type do not distinguish corporate debt from personal debt, such that an entrepreneur assumes unlimited liability for debt payment.

From the first order conditions of the high type's optimization, we obtain the following result:

**Proposition 1.** *The equilibrium inside equity position ( $\alpha^H$ ) and debt level ( $D^H$ ) of a high-type firm is not unique and is, in fact, any combination of  $\alpha^H$  and  $D^H$  satisfying  $\alpha^H(y_2 - D^H) = X$ , where  $0 \leq \alpha^H \leq 1$ ,  $0 \leq D^H \leq y_2$ ,<sup>6</sup> and  $X$  is the unique solution to*

$$\begin{aligned} & u(P_1^L y_1 + P_2^L y_2 + (1+r)(w_0 - K)) \\ & = P_1^L u(-P_2^H X + P_1^H y_1 + P_2^H y_2 + (1+r)(w_0 - K)) \\ & \quad + P_2^L u(P_1^H X + P_1^H y_1 + P_2^H y_2 + (1+r)(w_0 - K)) \end{aligned} \quad (2)$$

The signalling equilibrium is characterized as follows: a high-type entrepreneur announces his choice of  $\alpha^H$  and  $D^H$  that satisfies Equation (2). As for a low-type entrepreneur, he obtains the first-best utility level by choosing one of the numerous combinations of  $\alpha^L$  and  $D^L$  satisfying either  $\alpha^L = 0$  or  $D^L = y_2$ . Observing signals from firms, outside investors compute  $\alpha(y_2 - D)$  for each firm, and consider a firm as a high type only if the number is equal to  $X$  obtained from Equation (2). Such an evaluation rule for outside investors is consistent with every entrepreneur's expectation and signalling behavior and, thereby, constitutes the separating equilibrium.

The signalling equilibrium is not efficient in the first-best sense since not all of the risk in the market is dissipated but some remains to be borne by high-type firms. The extent to which high-type entrepreneurs suffer from their risk-bearing measures the cost of signalling.

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<sup>6</sup> As obvious from Equation (2), these constraints on  $\alpha^H$  and  $D^H$  are not binding, and the lowest upper bounds and greatest lower bounds can be easily computed.

From the social viewpoint, the signalling cost is pure resource waste but is unavoidable in the presence of the market imperfection caused by informational asymmetries. In this sense, the equilibrium is informationally-constrained efficient.

Proposition 1 also states that the equilibrium signal of a high-type firm is not unique but it gives the same value to  $\alpha^H(y_2 - D^H)$ , which combines the unused portions of the two risk-spreading vehicles of equity and debt. One immediate implication of the proposition is that a high-type firm's inside equity position is inversely related to its debt level, other things being equal. Otherwise identical firms may have different financial structures and, therefore, firms cannot be distinguished solely on the basis of their outstanding debt level. A firm with a high debt level could be as good as a firm with a lower debt level if it retains an appropriate share of inside equity. This is notable considering that previous studies on capital structure with a focus on debt-equity ratios have often failed to offer a satisfactory explanation of the diversity of the ratios observed in inter-industry cross-sectional data.

The fundamental reason why the high type's optimal capital structure is not determined uniquely is that debt and equity issues are equally effective in passing out risk when there are no taxes. In fact, the rate of the trade-off between holding more inside equity and issuing more debt from a given capital structure is identical for both types of firms when low types are perceived as high types by outside investors. The following proposition shows that the marginal rate of substitution between inside equity and debt does not depend upon the firm type.

**Proposition 2.** *The indifference curves of a high type and a successfully-imitating low type are both upward-sloping and concave in the  $(\alpha, D)$  plane and, furthermore, their slopes are independent of firm type.*

Therefore, a successfully-imitating low type's indifference map is identical to that of a true high type's although each indifference curve is associated with different utility level

for each type. Figure 1 illustrates a few indifference curves derived from an exponential utility function.<sup>6</sup> The figure also shows the indifference curve that serves as the incentive compatible constraint for a high type, on which any combination of debt level and inside equity position, is a separating signal for a high type.

## IV. Tax Consequences

### 1. Separating Equilibrium

Under the corporate income tax system, a firm pays no taxes except when  $y_2$  is realized and  $D$  is chosen so that  $y_2 - D \geq (1+r)K$ . In this case, the firm first makes debt payment of  $D$ , then tax payment of  $t[y_2 - D - (1+r)K]$ , and finally the rest,  $y_2 - D - t[y_2 - D - (1+r)K]$ , is paid out to equity holders. If the project outcome is  $y_1$ , the firm defaults, and the debt holders claim the whole amount of  $y_1$ . If the project outcome is  $y_2$  but it holds that  $y_2 - D \leq (1+r)K$ , the debt is paid back first and the rest is all distributed to equity holders. Therefore, no matter whether  $y_1$  or  $y_2$  is realized, the total dividend available for equity holders is (weakly) decreasing in the debt level and, other things being equal, only firms with relatively lower debt levels pay taxes.

We construct the Pareto-dominating separating equilibrium as done in the previous section without taxes. First, the value of debt is not affected by the existence of taxes and is evaluated by the same formula,

$$V_D^i = \frac{P_1^i y_1 + P_2^i D^i}{1+r}, \quad i = H, L.$$

The value of equity, however, depends upon the amount of tax obligation which in turn depends upon the level of debt. Similarly, the residual output available for each entrepreneur

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<sup>6</sup> Figure 1 is drawn for the case in which  $U$  is an exponential utility function with the absolute risk aversion coefficient being 0.1, that is,  $u(C) = -e^{-0.1C}$ . The parameters are specified as follows:  $y_1 = 0, y_2 = 40, P_1^H = 0.2, P_2^H = 0.8, P_1^L = P_2^L = 0.5, K = 20, r = 0.05$  and  $w_0 = 10$ .

at the end of the second period is dependent on the tax obligation and the level of debt. It follows that a high type's expected utility in the separating equilibrium is given by

$$Eu^H = P_1^H u(\Gamma_1^\tau) + P_2^H u(\Gamma_2^\tau), \text{ where}$$

if  $0 < D^H < y_2 - (1+r)K$ , then

$$\begin{aligned} \Gamma_1^\tau &= P_1^H y_1 + P_2^H D^H + (1+r)(w_0 - K) \\ &\quad + (1 - \alpha^H) P_2^H ((1 - \tau)(y_2 - D^H) + \tau(1+r)K) \end{aligned}$$

$$\Gamma_2^\tau = \Gamma_1^\tau + \alpha^H ((1 - \tau)(y_2 - D^H) + \tau(1+r)K)$$

if  $y_2 - (1+r)K < D^H < y_2$ , then

$$\begin{aligned} \Gamma_1^\tau &= P_1^H y_1 + P_2^H D^H + (1+r)(w_0 - K) \\ &\quad + (1 - \alpha^H) P_2^H (y_2 - D^H) \end{aligned}$$

$$\Gamma_2^\tau = \Gamma_1 + \alpha^H (y_2 - D^H),$$

where  $\Gamma_i^\tau$  is the consumption of an entrepreneur perceived as a high type, that is, either a true high type or a successfully-imitating low type in state  $i$ , and  $\Gamma_i^\tau$  differs depending upon the level of  $D^H$ . The utility function above decreases in  $\alpha$  but increases in  $D$  and is piecewise-continuous. For the range of  $D^H$  greater than  $y_2 - (1+r)K$ , the function is identical to what was previously derived without taxes and, therefore, the indifference curves are concave with the high type's indifference curves having the same curvature as those of a successfully-imitating low type. When  $D^H < y_2 - (1+r)K$ , the indifference curves are still upward-sloping but are distinguished by firm type, and we obtain the single-crossing property between the types. <sup>7</sup>

**Proposition 3.** *For the range of  $D$  satisfying  $D < y_2 - (1+r)K$ , other things being equal,*

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<sup>7</sup> In general, the single-crossing property is not necessary for the existence of separating equilibria but it significantly simplifies the process of finding them. Bernheim [1991] provides an example of constructing signalling equilibria in a model which violates the single-crossing property.

the indifference curves of a high type are flatter than those of a successfully-imitating low type.

This single-crossing property is simply due to a low type needing to be compensated with a larger increase in  $D$  than a high type would need for the same marginal increase in  $\alpha$ , while remaining at the same utility level. As illustrated in Figures 2 and 3,<sup>8</sup> indifference curves are one-piece curves with the single-crossing property only if they lie entirely below the line of  $D = y_2 - (1 + r)K$ ; otherwise, they are two-piece, piece-wise continuous curves with the single crossing property holding only for the debt level below  $y_2 - (1 + r)K$ .<sup>9</sup> Since the indifference curve of a low type associated with his first-best utility level serves as the incentive compatibility constraint for a high type, the separating equilibrium differs depending upon whether that indifference curve is a single-piece or two-piece curve.

In the separating equilibrium, a high type's expected utility,  $Eu^H$ , is maximized subject to the following incentive compatibility constraint:

$$u(P_1^L y_1 + P_2^L y_2 + (1 + r)(w_0 - K)) = P_1^L u(\Gamma_1^\tau) + P_2^L u(\Gamma_2^\tau) . \quad (3)$$

Using the properties of the indifference curves of both types, the separating equilibrium is established by the following proposition.

**Proposition 4.** *In the separating equilibrium, a low-type firm's optimal choice is any combination of  $\alpha^L$  and  $D^L$  such that either (i)  $\alpha^L = 0$  and  $D^L > y_2 - (1 + r)K$  or (ii)  $\alpha^L > 0$  and  $D^L = y_2$ . As for the optimal choice of a high-type firm, first consider  $D^*$  satisfying*

$$u(P_1^L y_1 + P_2^L y_2 + (1 + r)(w_0 - K)) = P_1^L u(P_1^H y_1 + P_2^H D^* + (1 + r)(w_0 - K))$$

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<sup>8</sup> Figures 2 and 3 are drawn using the utility function:  $u(C) = -e^{-0.1C}$ . The parameters for Figure 2 are as follows:  $y_1 = 0, y_2 = 40, P_1^H = 0.1, P_2^H = 0.9, P_1^L = 0.2, P_2^L = 0.8, K = 20, r = 0.05, w_0 = 10$  and  $\tau = 0.35$ . The same parameters are used for Figure 3 except that  $P_1^L = P_2^L = 0.5$ .

<sup>9</sup> An indifference curve cannot stay entirely above the demarcation,  $D = y_2 - (1 + r)K$ , since it must not cut the vertical axis ( $\alpha = 0$ ) in the range.

$$+ P_2^L u(P_1^H y_1 + P_2^H D^* + (1+r)(w_0 - K) + (1-\tau)(y_2 - D^*) + \tau(1+r)K). \quad (4)$$

If  $D^*$  is lower than or equal to  $y_2 - (1+r)K$ , a high-type firm holds all equity inside ( $\alpha^H = 1$ ) and  $D^*$  is its optimal debt level. If  $D^*$  is higher than  $y_2 - (1+r)K$ , the firm may choose any combination of  $\alpha^H$  and  $D^H$  satisfying both  $D^H \geq y_2 - (1+r)K$  and  $\alpha^H(y_2 - D^H) = X$ , where  $X$  is determined by Equation (2).

In equilibrium, a low-type entrepreneur neither bears risk nor pays taxes. The only difference from his behavior in the no-tax equilibrium is that he issues enough debt to exhaust his tax base. His debt level is not uniquely determined and hence his debt-equity ratio is indefinite. If he chooses the debt level of  $y_2$ , his equity has no market value and, therefore, the debt-equity ratio is not well defined.

In Proposition 4,  $D^*$  is the maximum debt level that a tax-paying high-type firm can issue without violating the incentive compatibility constraint. Other things being equal, it is easy to see that it increases in the true quality of a low-type project. If it is higher than  $y_2 - (1+r)K$ , the tax base is exhausted and a high-type firm does not have any tax obligation. Then no firm pays corporate income taxes and the equilibrium with taxes is identical to the equilibrium without taxes except that the ranges of the debt level for each type and the inside equity position of the high type are narrower. In other words, there exists a continuum of separating signals for a high type as illustrated in Figure 2. It should be noted, however, that for a given  $\alpha$ ,  $D^H$  is lower than  $D^L$  and, for a given  $D$ ,  $\alpha^H$  is higher than  $\alpha^L$ . Therefore, types are not revealed by either debt level or inside equity position alone but by both. For example, a high-type firm may borrow more than a low-type firm but then it has to hold more inside equity.

In this type of equilibrium, a high type's maximum debt level (denoted by  $D^{**}$ ) and his minimum inside equity position (denoted by  $\alpha^{**}$ ) are respectively obtained from the

following equations:

$$\begin{aligned}
u(P_1^L y_1 + P_2^L y_2 + (1+r)(w_0 - K)) &= P_1^L u(P_1^H y_1 + P_2^H D^{**} + (1+r)(w_0 - K)) \\
&+ P_2^L u(P_1^H y_1 + P_2^H D^{**} + (1+r)(w_0 - K) + y_2 - D^{**}).
\end{aligned} \tag{5}$$

$$\begin{aligned}
u(P_1^L y_1 + P_2^L y_2 + (1+r)(w_0 - K)) &= P_1^L u(P_1^H y_1 + P_2^H (y_2 - (1+r)K) \\
&+ (1+r)(w_0 - K) + (1 - \alpha^{**})P_2^H (1+r)K) \\
&+ P_2^L u(P_1^H y_1 + P_2^H (y_2 - (1+r)K) + (1+r)(w_0 - K) \\
&+ (1 - \alpha^{**})P_2^H (1+r)K) + \alpha^{**}(1+r)K).
\end{aligned} \tag{6}$$

Equation (5) is derived from the incentive compatibility constraint, Equation (3), with  $D^{**}$  replacing  $D^H$  and  $\alpha^H$  being the maximum value, 1. Similarly, Equation (6) is obtained from the same equation by replacing  $\alpha^H$  by  $\alpha^{**}$  and  $D^H$  being the minimum value,  $y_2 - (1+r)K$ . It is clear from these equations that the corporate income tax rate does not affect any of equilibrium characteristics although it distorts the curvature of indifference curves where  $D^H$  is below  $y_2 - (1+r)K$ .

The separating equilibrium is characterized differently if  $D^*$  is lower than  $y_2 - (1+r)K$ . In this case, a high-type firm pays corporate income taxes in the amount of  $P_2^H t[y_2 - D - (1+r)K]$  and its optimal choices of  $\alpha^H$  and  $D^H$  are uniquely 1 and  $D^*$ , respectively. Figure 3 illustrates this type of unique separating equilibrium. Intuitively, this is the case in which a high-type project is far more profitable than a low type, so that a low type's potential gain from successfully pretending to be a high type is very high. To block low types' mimicking behavior, a high type has to bear the burden of a greater signalling cost, that is, he not only has to hold all equity inside but also must leave part of the output unshielded from the corporate income tax. In this case, the signalling cost borne by a

high type includes the reduced tax shield as well as the forced risk-bearing and tends to increase as the quality gap between types increases.

There is, however, a limit to the increased signalling cost caused by increases in the quality gap. Suppose a low-type project has a negative value. It will be not undertaken neither in the first-best equilibrium nor in the separating equilibrium in which the true value is revealed. In this case, a combination of  $\alpha^H$  and  $D^H$  just sufficient to separate from an imaginary low type with zero value is also sufficient to block a low type with any negative value. Only high types will be seen in the market and their signalling costs and hence the overall equilibrium characteristics are not affected by the existence of unprofitable low-type firms. This result reflects the constrained efficiency of the signalling equilibrium.

## 2. Comparative Statics

In the previous sub-section, we constructed the two different types of separating equilibria: the first in which no firms pay corporate income taxes and the second in which only high types do. Since the financial structure of a low-type firm in equilibrium is rather trivially determined, here we focus on the financial structure of a high-type firm. First, we consider the equilibrium when  $D^*$  is less than or equal to  $y_2 - (1 + r)K$ . In this case, the separating signal of a high type is uniquely determined and we can then conduct comparative static analysis. We define the debt-equity ratio of a high type (denoted by  $l^H$ ) as the ratio of the book value of outstanding debt ( $D^*$ ) and the market value of equity (denoted by  $V_E$ ), that is

$$l^H = \frac{D^*}{V_E}$$

where

$$V_E = \frac{P_2^H(1 - \tau)(y_2 - D^*) + \tau(1 + r)K}{1 + r}$$

and  $D^*$  is obtained from Equation (4). We further assume that the probability of project failure does not exceed the corporate tax rate, that is,  $P_1^H < \tau$ . While it seems an empirical

issue whether this assumption is realistic, it is particularly useful since it allows for a definite sign of the partial derivative of the expected utility of a successfully-imitating low type,  $EU^L$ , with respect to  $D^*$ , given by

$$\frac{\partial EU^L}{\partial D^*} = P_2^H (P_1^L u'(\Gamma_1^*) + P_2^L u'(\Gamma_2^*)) - P_2^L (1 - \tau) u'(\Gamma_2^*) \quad (7)$$

where  $\Gamma_1^*$  and  $\Gamma_2^*$  are respectively identical to  $\Gamma_1^\tau$  and  $\Gamma_2^\tau$  except  $\alpha^H$  is set to 1. Under the given assumption, it is easily seen that  $\frac{\partial EU^L}{\partial D^*} < 0$ . A careful observation shows that the first term on the right-hand side of Equation (7) is due to an increase in the expected value of debt and the second term is due to a decrease of the expected value of dividend, resulting from a small increase in  $D^*$ . Therefore, it is suggested that  $D^*$  must be inversely related to any exogenous variable that has a positive instant effect on the low type's expected utility since his equilibrium utility level must remain fixed at the first best level. Further investigation obtains the following results on the debt-equity ratio of a high-type firm.

**Proposition 5.** *If  $y_1 < D^* < y_2 - (1+r)K$  and  $P_1^H < \tau$ , then (i)  $\frac{dl^H}{dP_2^H} < 0$ ; (ii)  $\frac{dl^H}{d\tau} > 0$ .*

The first result in the proposition is that the debt-equity ratio and profitability of a high-type firm are inversely related. As discussed earlier, the more profitable a high-type firm becomes, the larger the potential gain that a low type obtains from successfully pretending to be a high type; therefore, a high type needs to send a stronger separating signal. In other words, the pressure on a high-type firm to keep its debt capacity unused gets stronger, and its debt-equity ratio has to be lowered. This result is consistent with a salient empirical finding that more profitable firms borrow relatively less. Such empirical result is found in Kester [1986], Titman and Wessels [1988], Baskin [1989], Chaplinsky and Niehaus [1993] and many others but the result cannot be compromised with the static trade-off theory or, more importantly, previous signalling models such as in Leland and Pyle [1977] and Ross [1977].<sup>10</sup> In this sense, our model highlights the usefulness of the

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<sup>10</sup> For example, Megginson [1997] writes, "While this model [of Ross'] and the other

signalling approach and thereby contributes to a better understanding of a firm's financial behavior under informational asymmetries.

In the same vein, one might speculate that  $P_1^L$  would affect  $l^H$  in a similar way, thinking that an increase in  $P_1^L$  also means a increase in the profitability of a high type relative to a low type. However, it is not an accurate conjecture and the effect of  $P_1^L$  on  $l^H$  turns out uncertain as shown by the following relations.

$$\frac{1}{1+r} \frac{dl^H}{dP_1^L} = \frac{1}{\Delta^2} (P_2^H (1-\tau)y_2 + \tau(1+r)K) \frac{dD^*}{dP_1^L},$$

where

$$\frac{dD^*}{dP_1^L} = \frac{(y_1 - y_2)U'(\Gamma_0) + (u'(\Gamma_2^*) - u'(\Gamma_1^*))}{P_2^H (P_1^L u'(\Gamma_1^*) + P_2^L u'(\Gamma_2^*)) - (1-\tau)P_2^L u'(\Gamma_2^*)}$$

and  $\Delta$  is the numerator of  $V_E$  and  $\Gamma_0 = P_1^L y_1 + P_2^L y_2 + (1+r)(w_0 - K)$ . Although denominator of  $\frac{dD^*}{dP_1^L}$  is positive under the assumption that  $P_1^H < \tau$ , the sign of the numerator is not determined since the first term is negative while the second is positive. Intuitively, those terms respectively represent the two conflicting forces in action when  $P_1^L$  increases: (i) a decrease in the reservation utility of a low type, which gives a stronger incentive to mimic a high type, and (ii) a decrease in his utility obtained when he successfully pretends. When the first tendency dominates the second, high types are forced to bear a greater signalling cost. Figures 2 and 3 together illustrate such a case in which the equilibrium type is switched as a result of an increase in  $P_1^L$ . However, there is also the possibility that the second tendency outweighs the first, allowing high types to raise their debt-equity ratio in their favor.

The second result in the proposition states that the debt-equity ratio of a high-type firm increases with the corporate income tax rate. In a sense, it is straightforward since the tax only affects the profitability of a high-type firm but not of a low-type firm and a tax

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signaling models that follow, are intuitively attractive, observed capital structure patterns suggest that they are poor predictors of actual behavior.”

increase makes a high-type firm less profitable. It is, however, an interesting comparison with what was actually observed immediately after the Tax Reform Act 1986, which led to a small reduction in the effective corporate income tax rate.<sup>11</sup> According to Gordon and MacKie-Mason [1990], the actual increase in average debt-value ratios was substantially smaller than the forecast made on the basis of the increased tax advantage of debt over equity. Their forecast was made using the empirical model of MacKie-Mason [1990], which includes estimation variables such as ownership dilution, initial discount and dividend payout but not the joint signal of inside equity position and debt. In contrast, our model concentrates on the informational content of inside equity position and debt and predicts that a decrease in tax rate would result in a decrease, rather than an increase, in the debt-equity ratio. Although the two models are not perfect by themselves, it is suggested that a comprehensive approach incorporating the two might be able to provide a better explanation of what happened in reality.

When  $D^*$  is greater than or equal to  $y_2 - (1 + r)K$ , we obtain a different type of equilibrium in which there is a continuum of separating signals for a high type. A high-type firm does not pay corporate income taxes and, thus, the equilibrium financial structure is independent of the tax rate. In this case, comparative static analysis is not possible, and we instead examine how the ranges of a high type's debt level and inside equity position are affected by  $P_2^H$ .

**Proposition 6.** *If  $y_1 < y_2 - (1 + r)K < D^*$ , (i)  $\frac{dD^{**}}{dP_2^H} < 0$ ; and (ii)  $\frac{d\alpha^{**}}{dP_2^H} > 0$ .*

The sufficient condition in Proposition 6 requires that the project output in the good state must exceed that in the bad state by more than the opportunity cost of capital outlay. It does not seem to have an intuitive meaning; however, it is more likely to be satisfied as

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<sup>11</sup> Gordon and Mackie-Mason [1990] report that the fall in the effective tax rate was small (from 0.318 to 0.289) relative to the cut in the statutory rates (from 0.46 to 0.34) due to the simultaneous changes in depreciation and investment tax credits.

$y_1$  approaches 0, other things being equal.

The above proposition implies that a high type's indifference curve associated with the equilibrium utility level shifts toward the corner of  $\alpha^H$  being 1 and  $D^H$  being  $y_2 - (1+r)K$ , as his project becomes more profitable relative to a low type. The range of his equilibrium signals shrinks and, therefore, he has to lower the debt level if he wants to keep the current inside position; alternatively, he must increase his inside equity position in order to keep the current debt level. It is also possible that the current debt level or inside equity position may not be even feasible. Such a tendency is, in fact, consistent with the result in Proposition 5 that a high type's signalling cost rises with the relative profitability of his project, although the exact change in his signal cannot be predicted here.

Due to the non-uniqueness of a high type's equilibrium signals, the relationship between his relative profitability and debt-equity ratio is not uniformly determined. However, it is worthwhile to repeat that it is the combination of debt level and inside equity position and not the independent value of each that enables outside investors to identify firm types correctly.

Propositions 5 and 6 provide a comparison of entrepreneurs' reaction to (small) changes in exogenous variables for each type of equilibrium. As discussed earlier, a change in an exogenous factor can even lead to a switch of equilibrium types. It should be noted, however, that type of equilibrium is determined independent of the corporate income tax rate and cannot be affected by a tax change of any scale although a tax increase does tend to increase  $D^*$ .<sup>12</sup>

**Proposition 7.** *If  $D^* < y_2 - (1+r)K$  and  $P_1^H < \tau$ , (i)  $D^*$  rises with  $\tau$  but (ii) cannot rise above  $y_2 - (1+r)K$ .*

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<sup>12</sup> In fact, equilibrium type may have been equivalently identified on the basis of the value of  $D^{**}$  in Equation (5) instead of using  $D^*$  in Equation (4). In that case, it is obvious that the tax rate does not affect the type of equilibrium since Equation (5) does not include  $\tau$ .

The implication of Proposition 7 is interesting in that the government can raise the tax rate without worrying about completely losing the tax base. As the tax rate increases, the binding constraint on a high type entrepreneur's financial decision weakens so that he can borrow more and increase the tax shield; however, there is a limit to this beneficial effect of the tax and he will never achieve tax exhaustion.

## V. Concluding Remarks

Developing a simple yet useful signalling model in which inside equity position and debt level of a firm jointly serve as a signalling vehicle, this paper examines the determination of a firm's financial structure under asymmetric information regarding the true quality of a firm. Unlike other signalling models, our model explicitly considers the limited liability of equity holders and the corporate income tax with debt deduction and obtains a set of interesting results.

The Pareto-dominant separating equilibrium is characterized differently with and without the corporate income tax although only profitable projects are financed in either case as if there were no adverse selection problem. Without the corporate income tax, the firms' equilibrium capital structure is not unique and the high-quality firms' inside equity position and debt level are, respectively, bounded from below and bounded from above. Other things being equal, the high-quality firms hold more inside equity than the low-quality firms with the same debt level and issue less debt than the low-quality firms with the same inside equity position. Under the corporate income tax system, there are two different types of separating equilibria. In one type of equilibrium, all firms achieve tax exhaustion and their equilibrium capital structure is not unique. Roughly speaking, this type of equilibrium prevails when the high-quality firms are not too superior to the low-quality firms. When the firms' quality difference is relatively large, the other type of equilibrium emerges and the unique optimal capital structure is obtained by the high-

quality firms. A high-quality firm lowers its debt-equity ratio as its profitability increases but raises the ratio as the corporate income tax rate rises. However, the tax rate, unlike other exogenous elements of the model, can never be a determining factor of the type of equilibrium. In contrast with other signalling models, the overall results from our model are consistent with empirical findings and complement existing ideas on a firm's financial behavior. In this sense, our model highlights the usefulness of the signalling approach and, thereby, contributes to a better understanding of a firm's financial behavior under informational asymmetries.

Our model may be extended in many meaningful ways. First, it may be further developed into a tax analysis model in which one can investigate the incidence and welfare cost of the corporate income tax. Endogenizing the capital outlay, increasing the number of production periods and adding other income taxes will make the model more realistic in this direction. On the other hand, one can further explore the capital structure literature specific to firms' financial behavior under asymmetric information. One may include bankruptcy costs in the model, deal with the added dimension of the entrepreneurial moral hazard problem, or consider a continuum of entrepreneur types instead of discrete types.

Figure 1. Indifference Curves of Both Firm Types without Taxes

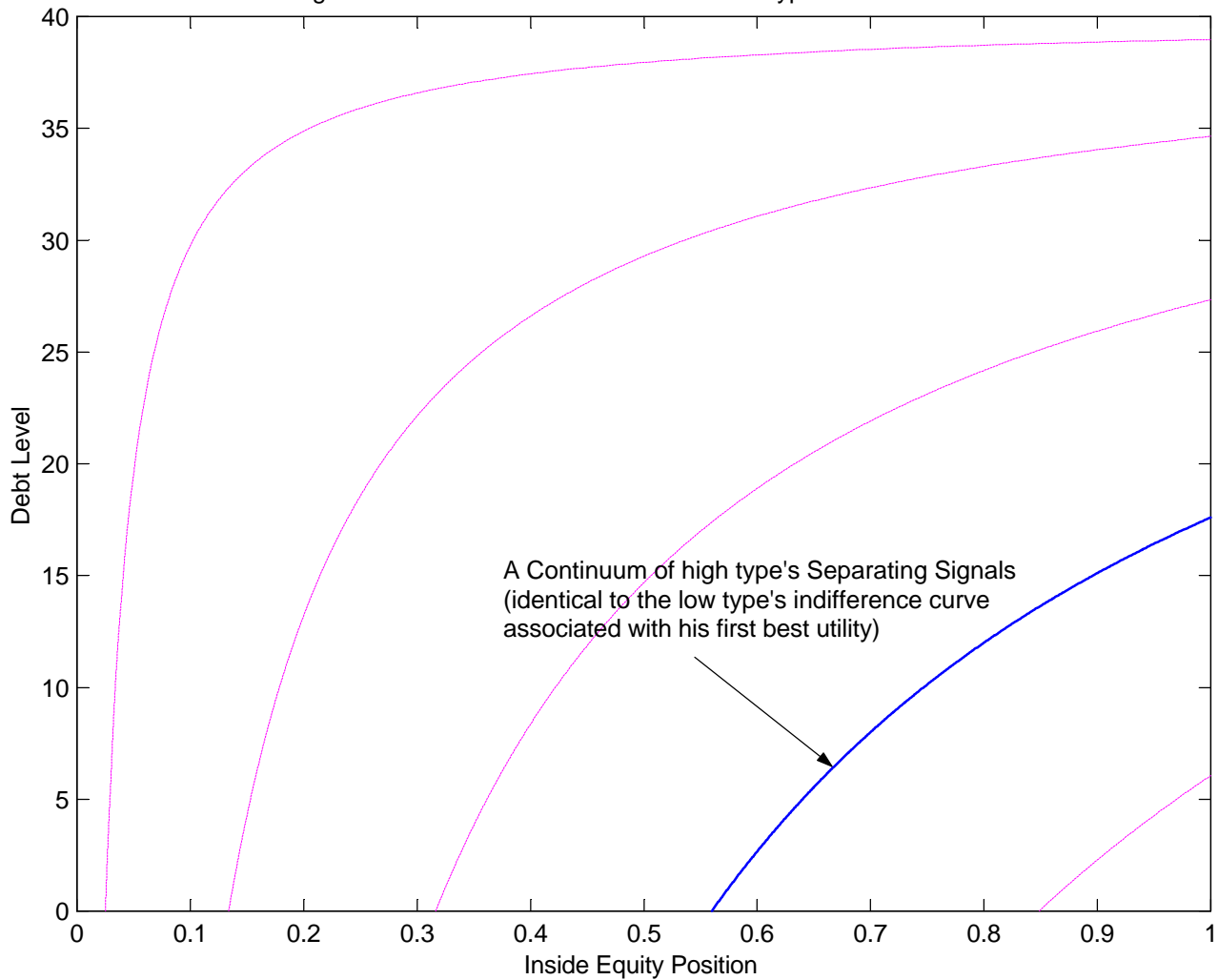


Figure 2. Equilibrium with Non-unique Separating Signals for a High Type under Tax System

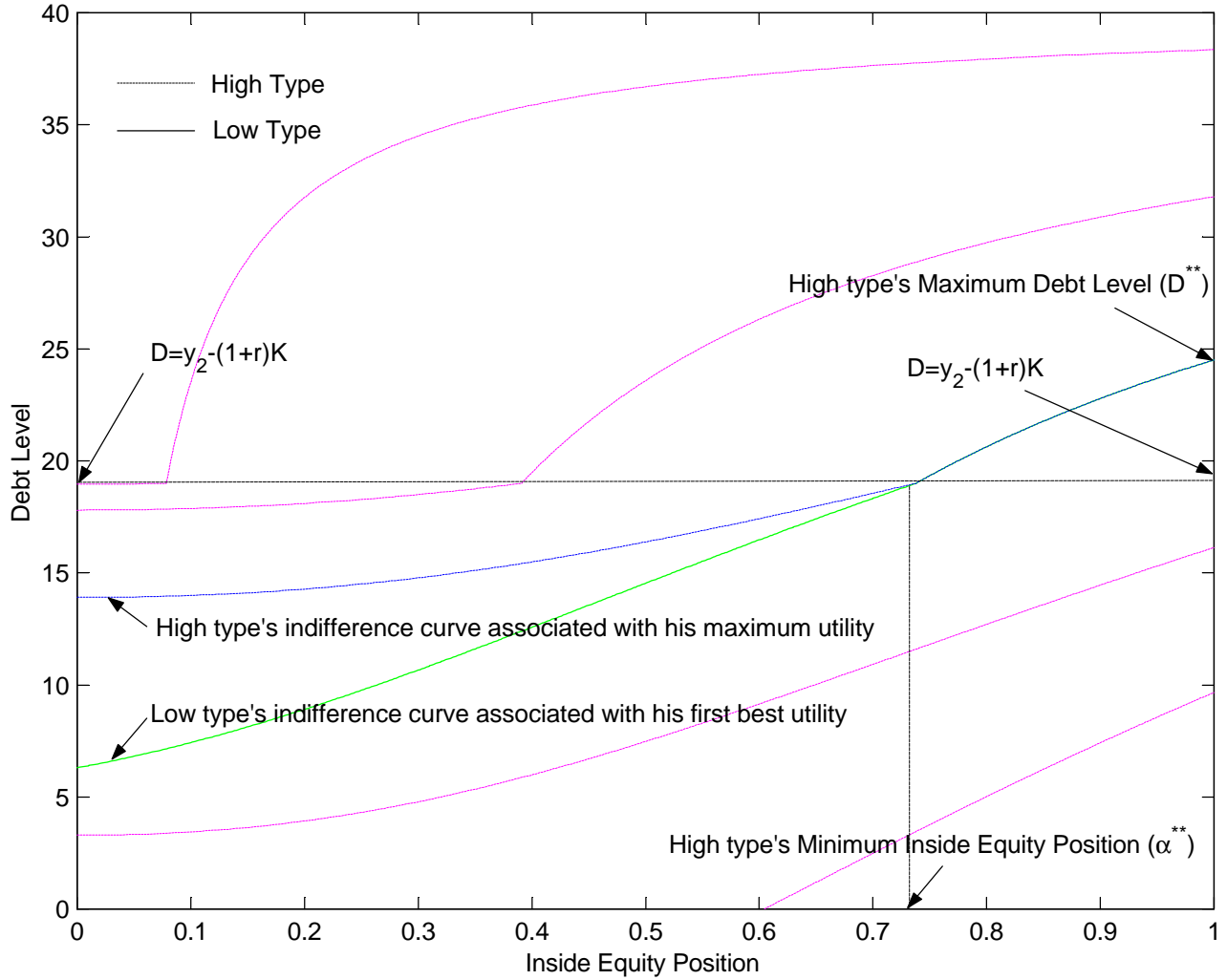
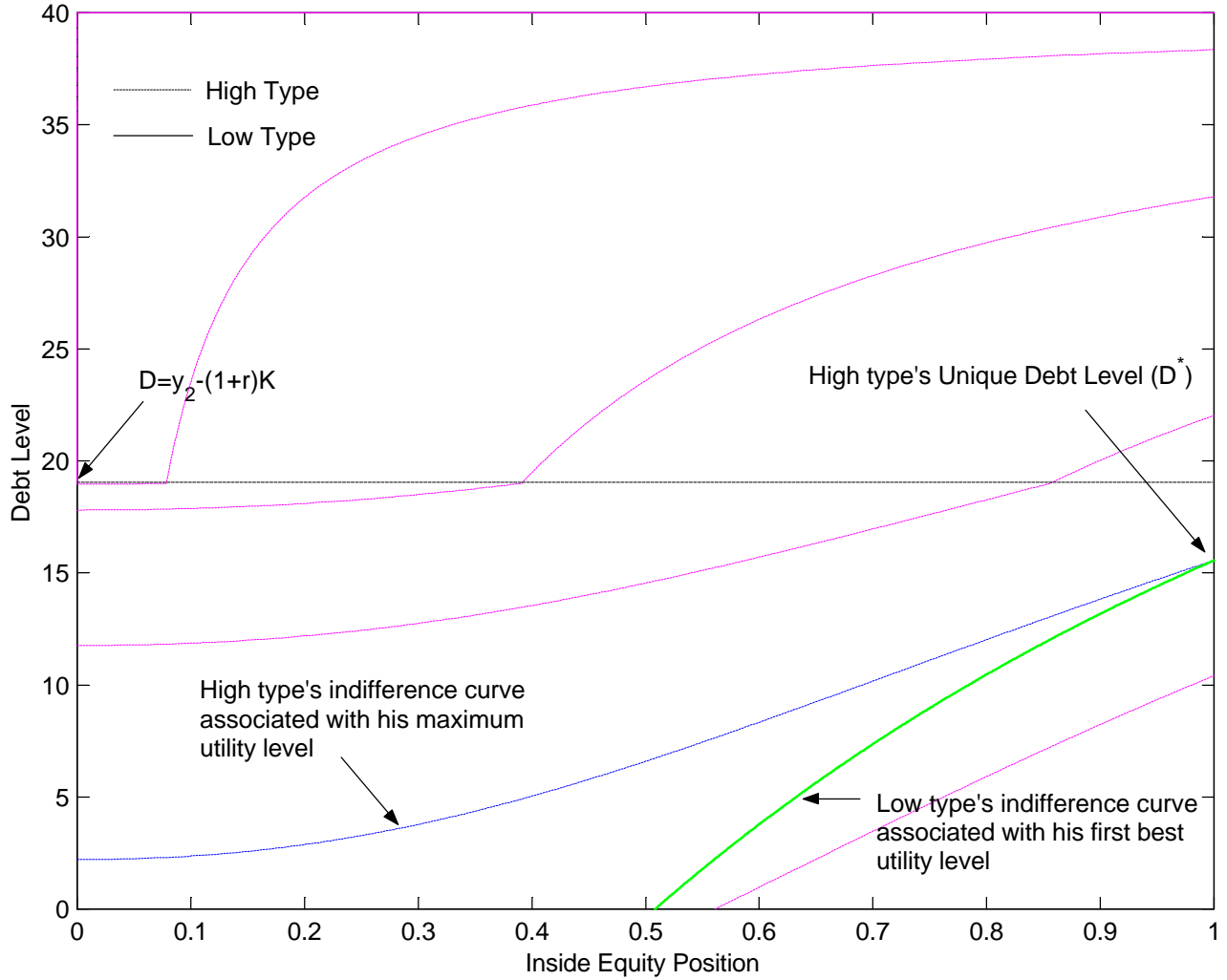


Figure 3. Equilibrium with Unique Separating Signal for a High Type under Tax System



## Appendix

### Proof of Proposition 1

Equation (2) is identical to the incentive compatibility constraint (1) except  $X$  is substituted for  $\alpha^H(y_2 - D^H)$ . It is easily seen that the right-hand side of Equation (2) is decreasing in  $X$  and, hence,  $X$  is uniquely determined in Equation (2). The objective function can be also written as a function of  $X$  then a high type's maximization degenerates to a choice over a singleton set  $\{X\}$ . [Q.E.D.]

### Proof of Proposition 2

The marginal rate of substitution for  $D$  of  $\alpha$  ( $\equiv MRS$ ) is given by

$$\frac{dD}{d\alpha} = -\frac{P_1^i u'(\Gamma_1) \frac{\partial \Gamma_1}{\partial \alpha} + P_2^i u'(\Gamma_2) \frac{\partial \Gamma_2}{\partial \alpha}}{P_1^i u'(\Gamma_1) \frac{\partial \Gamma_1}{\partial D} + P_2^i u'(\Gamma_2) \frac{\partial \Gamma_2}{\partial D}}$$

where  $i$  indicates type,  $H$  or  $L$ , and  $\Gamma_j$  is the wealth at the end of the second period in which outcome  $y_j$  is realized. Thus,  $\Gamma_1 = (1+r)B^H$  and  $\Gamma_2 = \Gamma_1 + \alpha(y_2 - D)$ , and we obtain the following relationships:

$$\begin{aligned} \frac{\partial \Gamma_1}{\partial \alpha} &= -P_2^H(y_2 - D) \\ \frac{\partial \Gamma_2}{\partial \alpha} &= P_1^H(y_2 - D) \\ \frac{\partial \Gamma_1}{\partial D} &= \alpha P_2^H \\ \frac{\partial \Gamma_2}{\partial D} &= -\alpha P_1^H. \end{aligned}$$

It is easily seen that  $MRS$  is positive and decreases as both  $\alpha$  and  $D$  increase. To prove identical slopes, we compute  $\frac{\partial MRS}{\partial P_1^i}$  since there are only two possible outcomes and  $P_1^i$  captures the type difference.

$$\frac{\partial MRS}{\partial P_1^i} = -\frac{1}{\Lambda^2} u'(\Gamma_1) u'(\Gamma_2) \left( \frac{\partial \Gamma_1}{\partial \alpha} \frac{\partial \Gamma_2}{\partial D} - \frac{\partial \Gamma_1}{\partial D} \frac{\partial \Gamma_2}{\partial \alpha} \right)$$

where  $\Lambda$  denotes the denominator of  $MRS$ . After plugging in partial derivatives, we have

$$\frac{\partial MRS}{\partial P_1^i} = 0. \text{ [Q.E.D.]}$$

### Proof of Proposition 3

The marginal rate of substitution for  $D$  of  $\alpha$  ( $\equiv MRS_\tau$ ) is given by

$$\frac{dD}{d\alpha} = - \frac{P_1^i u'(\Gamma_1^\tau) \frac{\partial \Gamma_1^\tau}{\partial \alpha} + P_2^i u'(\Gamma_2^\tau) \frac{\partial \Gamma_2^\tau}{\partial \alpha}}{P_1^i u'(\Gamma_1^\tau) \frac{\partial \Gamma_1^\tau}{\partial D} + P_2^i u'(\Gamma_2^\tau) \frac{\partial \Gamma_2^\tau}{\partial D}}$$

where

$$\begin{aligned} \frac{\partial \Gamma_1^\tau}{\partial \alpha} &= -P_2^H ((1-\tau)(y_2 - D) + \tau(1+r)K) \\ \frac{\partial \Gamma_2^\tau}{\partial \alpha} &= P_1^H ((1-\tau)(y_2 - D) + \tau(1+r)K) \\ \frac{\partial \Gamma_1^\tau}{\partial D} &= \tau P_2^H + \alpha(1-\tau)P_2^H \\ \frac{\partial \Gamma_2^\tau}{\partial D} &= \tau P_2^H - \alpha(1-\tau)P_1^H. \end{aligned}$$

First, the numerator of  $MRS_\tau$  is negative regardless of firm type since

$$\begin{aligned} &P_1^L u'(\Gamma_1^\tau) \frac{\partial \Gamma_1^\tau}{\partial \alpha} + P_2^L u'(\Gamma_2^\tau) \frac{\partial \Gamma_2^\tau}{\partial \alpha} \\ &< P_1^H u'(\Gamma_1^\tau) \frac{\partial \Gamma_1^\tau}{\partial \alpha} + P_2^H u'(\Gamma_2^\tau) \frac{\partial \Gamma_2^\tau}{\partial \alpha} \\ &< 0. \end{aligned}$$

Similarly, a positive sign is obtained for the denominator of  $MRS_\tau$  for both types since

$$\begin{aligned} &P_1^L u'(\Gamma_1^\tau) \frac{\partial \Gamma_1^\tau}{\partial D} + P_2^L u'(\Gamma_2^\tau) \frac{\partial \Gamma_2^\tau}{\partial D} \\ &> P_1^H u'(\Gamma_1^\tau) \frac{\partial \Gamma_1^\tau}{\partial D} + P_2^H u'(\Gamma_2^\tau) \frac{\partial \Gamma_2^\tau}{\partial D} \\ &> 0. \end{aligned}$$

Therefore,  $MRS_\tau$  is positive for both types.

The single-crossing property is proven by showing that  $MRS_\tau$  increases in  $P_1^i$ .

$$\frac{\partial MRS_\tau}{\partial P_1^i} = -\frac{1}{\Lambda_\tau^2} u'(\Gamma_1^\tau) u'(\Gamma_2^\tau) \left( \frac{\partial \Gamma_1^\tau}{\partial \alpha} \frac{\partial \Gamma_2^\tau}{\partial D} - \frac{\partial \Gamma_1^\tau}{\partial D} \frac{\partial \Gamma_2^\tau}{\partial \alpha} \right)$$

where  $\Lambda_\tau$  denotes the denominator of  $MRS_\tau$ . Plugging in partial derivatives, we obtain a negative sign of the numerator of  $\frac{\partial MRS_\tau}{\partial P_1^i}$ . Therefore, we obtain  $\frac{\partial MRS_\tau}{\partial P_1^i} > 0$ . [Q.E.D.]

### Proof of Proposition 4

A low-type firm's optimal choice is the same as its first best choice under symmetric information. In equilibrium, it spreads all risk either by holding no inside equity or by issuing the maximum debt. It also completely shields its output from tax obligations by issuing debt more than  $y_2 - (1 + r)K$ . Therefore, the conditions (i) and (ii) are obtained. From the viewpoint of a high type, there are two different types of equilibria, depending upon whether it pays corporate income taxes or not. Equation (4) is obtained from Equation (3) with corresponding  $\Gamma_1^\tau$  and  $\Gamma_2^\tau$  plugged in after replacing  $D^H$  with  $D^*$  and fixing  $\alpha$  at 1. Since the indifference curves of both types are increasing in  $\alpha^H$  and  $D^H$ ,  $D^*$  satisfying Equation (4) locates the upper-right end of the incentive compatibility constraint when high types are paying positive taxes. If  $D^*$  is less than or equal to  $y_2 - (1 + r)K$ , the constraint is a single piece indifference curve and its upper-right end is the unique separating signal for a high type, due to the single-crossing property. If  $D^*$  is greater than  $y_2 - (1 + r)K$ , the constraint is made of two pieces. A high type does not choose any point on the lower piece due to monotonicity and the single-crossing property. However, all points on the upper piece give the same utility since the indifference curves of both types are identical when there are no taxes. Therefore, any combination of  $D^H$  and  $\alpha^H$  obtained from Equation (2), which is identical to Equation (3) with the corresponding  $\Gamma_1^\tau$  and  $\Gamma_2^\tau$ , is equally optimal for a high type. [Q.E.D.]

### Proof of Proposition 5

(i) Taking  $D^*$  as a function of  $P_2^H$ , the total derivative of  $l^H$  is obtained as

$$\frac{1}{1+r} \frac{dl^H}{dP_2^H} = \frac{1}{\Delta^2} \left( \frac{dD^*}{dP_2^H} P_2^H ((1-\tau)y_2 + \tau(1+r)K) - D^*(1-\tau)(y_2 - D^*) - D^*\tau(1+r)K \right),$$

where  $\Delta$  is the numerator of  $V_E$  and

$$\frac{dD^*}{dP_2^H} = - \frac{(D^* - y_1)(P_1^L u'(\Gamma_1^*) + P_2^L u'(\Gamma_2^*))}{P_2^H (P_1^L u'(\Gamma_1^*) + P_2^L u'(\Gamma_2^*)) - (1-\tau)P_2^L u'(\Gamma_2^*)}.$$

The numerator of  $\frac{dD^*}{dP_2^H}$  is positive since  $D^* > y_1$  and the denominator is also positive since it is identical to  $\frac{\partial EU}{\partial D^*}$  in Equation (7); therefore,  $\frac{dD^*}{dP_2^H}$  is negative. The numerator of  $\frac{dl^H}{dP_2^H}$  is then negative, and result (i) follows. [Q.E.D]

(ii) In a similar way, we have

$$\frac{1}{1+r} \frac{dl^H}{d\tau} = \frac{1}{\Delta^2} \left( \frac{dD^*}{d\tau} P_2^H ((1-\tau)y_2 + \tau(1+r)K + P_2^H D^*(y_2 - D^*) - (1+r)K) \right),$$

where

$$\frac{dD^*}{d\tau} = \frac{P_2^L u'(\Gamma_2^*)(y_2 - D^* - (1+r)K)}{P_2^H (P_1^L u'(\Gamma_1^*) + P_2^L u'(\Gamma_2^*)) - (1-\tau)P_2^L u'(\Gamma_2^*)}.$$

It is easily seen that both numerator and denominator of  $\frac{dD^*}{d\tau}$  are positive, and hence result (ii) follows. [Q.E.D]

## Proof of Proposition 6

(i) Taking  $D^{**}$  as a function of  $P_2^H$  in Equation (5), we obtain the total derivative as follows:

$$\frac{dD^{**}}{dP_2^H} = - \frac{(P_1^L u'(\gamma_1) + P_2^L u'(\gamma_2))(D^{**} - y_1)}{(P_1^L u'(\gamma_1) + P_2^L u'(\gamma_2))P_2^H - P_2^L u'(\gamma_2)},$$

where  $\gamma_1 = P_1^H y_1 + P_2^H D^{**} + (1+r)(w_0 - K)$  and  $\gamma_2 = \gamma_1 + y_2 - D^{**}$ . One can easily see that the denominator is positive since  $u'$  is decreasing,  $\gamma_1 < \gamma_2$ , and  $P_2^L < P_2^H$ . The numerator is also positive if  $D^{**} > y_1$ , which is satisfied when  $y_2 - (1+r)K > y_1$  since  $D^{**} > y_2 - (1+r)K$ . [Q.E.D.]

(ii) Taking  $\alpha^{**}$  as a function of  $P_2^H$  in Equation (6), we obtain the total derivative as follows:

$$\frac{d\alpha^{**}}{dP_2^H} = \frac{(P_1^L u'(\lambda_1) + P_2^L u'(\lambda_2))(y_2 - y_1 - \alpha^{**}(1+r)K)}{(P_1^L P_2^H u'(\lambda_1) - P_1^H P_2^L u'(\lambda_2))(1+r)K},$$

where  $\lambda_1 = P_1^H y_1 + P_2^H (y_2 - (1+r)K) + (1+r)(w_0 - K) + (1 - \alpha^{**})P_2^H (1+r)K$  and  $\lambda_2 = \lambda_1 + \alpha^{**}(1+r)K$ . Since  $P_1^L > P_1^H$ ,  $P_2^H > P_2^L$ ,  $\lambda_2 > \lambda_1$  and  $u'$  is decreasing,

the denominator is positive. One can easily see that the numerator is also positive if  $y_2 > y_1 + (1+r)K$ . [Q.E.D.]

### Proof of Proposition 7

(i) Proof shown as part of the proof to Proposition 5.

(ii) A proof by contradiction is as follows: suppose not, then there exists a  $\tilde{\tau} (< 1)$  such that  $D^*(\tilde{\tau}) = y_2 - (1+r)K$  due to the continuity of utility functions. Let  $D_1$  and  $D_2$ , respectively, denote  $D^*(1)$  and  $D^*(\tilde{\tau})$ . Then  $D_1 > D_2$ , and we have the following relationship from Equation (4):

$$\begin{aligned}
& P_1^L u(P_1^H y_1 + P_2^H D_1 + (1+r)(w_0 - K)) \\
& + P_2^L u(P_1^H y_1 + P_2^H D_1 + (1+r)(w_0 - K) + (1+r)K) \\
& = P_1^L u(P_1^H y_1 + P_2^H D_2 + (1+r)(w_0 - K)) \\
& + P_2^L u(P_1^H y_1 + P_2^H D_2 + (1+r)(w_0 - K) + (1-\tilde{\tau})(y_2 - D_2) + \tilde{\tau}(1+r)K).
\end{aligned}$$

Plugging  $D^*(\tilde{\tau}) = y_2 - (1+r)K$  in the above, we find that the first and second terms in the left-hand side are respectively larger than those in the right-hand side. Therefore, the above equality cannot hold and there is no tax increase that can push  $D^*$  above  $y_2 - (1+r)K$ . [Q.E.D.]

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