MORAL HAZARD AND OPTIMAL
CIGARETTE TAXATION

by

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ABSTRACT: This paper views cigarette taxation as a correction for health insurance distortions (an efficiency tax). The theoretical framework utilizes an individual expected utility maximizing consumer - optimal social planner model. From the model an optimal tax formula is derived. There are two main results. First, when indemnification is prohibitive, a subsidy to medical care (reimbursement insurance) may be optimal. Second, when reimbursement is optimal, the optimal cigarette tax (subsidy) depends on the complementarity (substitutability) between medical care and cigarettes as well as moral hazard.

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1. INTRODUCTION

The purpose of this paper is to analyze the potential use of cigarette taxes to partially correct distortions in the health care sector.

The health care markets in the United States are thought by many to be inefficient. In seeking to provide risk-sharing and in-kind transfers to the poor and elderly, our public health insurance system, through the Medicare and Medicaid programs, has incurred large and ever increasing expenditures (Health Care Financing Administration 1987; Ginsburg 1988). These public programs may have overinsured the groups they are assisting (Feldstein 1973; Pauly 1980). Because of the tax-subsidy afforded employer provided insurance, private health insurance may also be overprovided (Feldstein and Friedman 1977; Vogel 1980; Greenspan and Vogel 1980; Pauly 1986). Even if the tax-subsidy distortion is insubstantial (Chernick, Holmer, and Weinberg 1987), and the direct programs (Medicare and Medicaid) use optimal limits (e.g., coinsurance and deductibles) and price controls (e.g., diagnostic related groups), both public and private health insurance are still subject to moral hazard and subsidy distortions (Pauly 1968, 1974).

The net effect of these government interventions and informational imperfections is overutilization. In other words, the consumer price of medical care is too low (at least on the margin). However, the appropriate policy prescription is not a simple increase in consumer prices. Rather, what we seek is a second-best solution -- a balance between risk-sharing and marginal incentives (Zeckhauser 1970). Cigarette taxes may be part of a superior solution.

The potential for using cigarette taxes to redress distortions in the health care sector depends upon the interaction between cigarette demand and
health care utilization. This interaction may come directly through the price of medical care or indirectly through the price of health insurance.

If health insurance premiums are not perfectly rated, then there is a potential for moral hazard. That moral hazard may take the form of too much smoking (too little prevention). The extent to which smoking alters the probability of illness and thus the demand for medical care, is an indicator of the potential effectiveness of cigarette taxes to correct for moral hazard.

In addition, health insurance lowers the consumer price of medical care on the margin. This will increase the quantity demanded of medical services (overutilization). The lower consumer price will also increase (decrease) the demand for complements (substitutes) to (for) medical care. If cigarettes and medical services are complements (substitutes) then a tax (subsidy) on (to) cigarettes can partially correct the overutilization of medical services.

Thus, cigarettes are a potential target for a corrective tax for two reasons. First, health insurance premiums do not perfectly reflect the risks of smoking. Second, the consumer price of medical care is too low and the consumption of medical care and cigarettes may be too high.

This paper develops a theoretical model which explicitly incorporates these two effects. From this model an optimal tax formula is derived.

This paper borrows heavily from Arnott and Stiglitz (1986). In their paper, commodity taxation is viewed as a corrective mechanism for the moral hazard distortion associated with public insurance (Arnott and Stiglitz 1986).

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1 There are many other arguments for cigarette taxation (e.g., second-hand smoke, fiscal finance, misinformation, irrationality, etc.) (Leu and Schaub 1984; Littleschild 1986). Amongst them is an argument which focuses on the alleged pecuniary externalities associated with smoking, the so-called "do smokers pay their way?" issue (Atkinson and Townsend 1977; Cullis 1978; Heins 1978; Kristlein and Grove 1978; Wikler 1978; Wilkinson et al. 1978; Sommers 1980; Benham 1981; Harris 1982; Cady 1983; Rosen 1983; Leu and Schaub 1983, 1984; Stoddart, et al. 1986; Manning, et al. 1989). This paper is distinct from this large body of literature in that it is concerned with allocative efficiency rather than purely pecuniary effects.
Although this paper is less general than Arnott and Stiglitz (1986) in the number of commodities and states of nature modeled, it actually extends their theoretical work by including an additional distortion. Arnott and Stiglitz (1986) deal with indemnity insurance which is lump-sum by design. Whereas, this model deals with reimbursement insurance which is marginal by design. Indemnity insurance only results in a moral hazard problem (Pauly 1974). On the other hand, reimbursement insurance is both subject to moral hazard and results in a subsidy distortion (Pauly 1968; Zeckhauser 1970). This formulation is more appropriate because medical insurance is typically structured as reimbursement.

There are two main theoretical results. First, when indemnification is not possible, a subsidy to medical care (reimbursement insurance) may be optimal. This result is important because it both describes and explains the most common form of health insurance. Second, when reimbursement is optimal, the optimal cigarette tax (subsidy) depends both on moral hazard (Arnott and Stiglitz 1986) and on the complementarity (substitutability) (Balcer 1980) between medical care and cigarettes. Because health insurance pay-outs are structured as subsidies to medical care consumption, there is this additional distortion which is not explicitly recognized by Arnott and Stiglitz (1986). This paper incorporates both the moral hazard effect and the subsidy effect in the optimal tax formula derivation.

The theoretical framework is that of expected utility theory (von Neumann and Morgenstern, 1944) with health insurance generating moral hazard and subsidy distortions. Cigarette taxes are presented as an abatement to these distortions, and thus, as a partial substitute for perfectly rated insurance premiums. The final model presented shows that when states are
unobservable, the optimal social insurance implies a subsidy to medical care (reimbursement insurance). The optimal cigarette tax generated from this model is shown to depend in part on the pure cross-elasticity of demand between cigarettes and medical care, and is qualitatively indeterminant. Even though cigarette consumption increases the probability of illness, the optimal insurance tax-subsidy scheme may involve a subsidy to cigarettes. The theoretical possibility of such a seemingly counter-intuitive result, implies that proponents of increased cigarette taxation, who contend such taxes will correct health insurance distortions, must support their contention with empirical evidence.²

The models presented allow for two types of health insurance distortions. The first type of distortion is ex ante and arises from the structural assumption that cigarette consumption increases the probability of illness. This type of distortion is a classic moral hazard problem (Pauly 1974; Shavell 1979) and can be viewed as a lack of prevention.

The second type of distortion is created by subsidizing medical care and is ex post in nature. Given that an accident or illness has occurred, reimbursement insurance lowers the price of medical care and potentially leads to over use and excess claims (Pauly 1968).³

²Proponents of the cigarette tax (Wilkinson, et al. 1978; Reins 1978; Kratstein and Groves 1976; Harris 1982; Cady 1983; Rosam 1963) frequently point to an alleged pecuniary externality, imposed on non-smokers by smokers through the health insurance system, as a partial justification for such a tax. Less editorially inclined authors (Atkinson and Townsend 1977; Leu and Schaub 1984; Schelling 1988) have also recognized the potential presence of these pecuniary effects. In addition, empirical evidence has been presented both supporting (Cooper and Rice 1976; Luce and Schweitzer 1976; Manning, et al. 1980) and rejecting (Leu and Schaub 1983, 1984; Wright 1986; Stoddart, et al. 1986) the hypothesis that smokers impose a pecuniary externality on non-smokers through the health insurance system. This literature does not recognize the efficiency effects which may be embodied in the pecuniary relationships it describes.

³Using an example from auto insurance, the first type of distortion, moral hazard, is analogous to driving recklessly in the knowledge that your insurance coverage will provide a payment in the event that you damage your car. The subsidy distortion is analogous to seeking excessive and elaborate repairs given an accident has occurred, because reimbursement has lowered the consumer price of such repairs. If one prefers, the moral hazard problem can be thought of as a lack of self-protection and the subsidy distortion as a lack of self-insurance (Ehrlich and Becker 1972).
Indemnity insurance precludes the subsidy distortion, but not the moral hazard problem. Reimbursement insurance is likely to be plagued by both problems. The models presented here will treat each distortion in isolation and derive an optimal tax for each. The final model will allow for both types of problems simultaneously, and imply an optimal corrective tax which redresses both. Both types of distortions can be viewed as moral hazard problems if we broaden the definition of moral hazard to include all effects which cause expected losses to increase with insurance coverage.

Taking the cue from some recent theoretical literature (Greenwald and Stiglitz 1986; Arnott and Stiglitz 1986) this paper analyzes the efficiency effects associated with medical care, medical insurance and cigarette demand interactions. The next section describes the specific models used for this analysis.

2. AN OVERVIEW OF THE THEORETICAL MODELS

Four models are presented. In all versions consumers are assumed to be identical and producer prices are assumed constant and equal to marginal cost. Thus, the models are purged of all production and distributional effects. The representative consumer is assumed to be fully informed and rational. Additionally there are no physical externalities. Therefore, these models focus exclusively on the pure efficiency effects associated with the interaction between health insurance and smoking.

Model I is a certainty model with a health care subsidy creating a distortion. The optimal corrective tax (subsidy) is shown to depend on the degree of Hicksian complementarity (substitutability) (Slutsky 1915; Hicks and Allen 1934; Schultz 1938), between cigarettes and medical care. This model is
used to introduce notation and show the role of complementarity in the optimal tax formula. The complementarity issue reappears in Model IV.

Model II is an expected state-dependent utility model with a potential for moral hazard in the form of excess smoking. Health is treated as an unobservable which is embedded in the utility function. The implicit assumption is that given illness has occurred medical care cannot completely restore health. Utility is dependent upon health and thus the state (Cook and Graham 1977; Shavell 1978; Dionne 1982; Hey and Patel 1983).

A fully informed private insurer provides actuarially fair indemnity insurance. Indemnification precludes any distortion of medical care consumption. The results of this model include full insurance and no distortion of cigarette demand. A "perfect" premium fully abates any potential moral hazard from smoking.

This model can be viewed as the "first-best" case for comparative purposes. This is evident by inspecting the optimal tax (subsidy) formulae, which imply non-intervention.

Model III is identical to Model II except that a social insurer substitutes taxes and subsidies for the private insurers premiums and insurance payments. Throughout these models information and transaction costs are zero. The social and private insurer render identical results, with the social insurer’s cigarette tax substituting for the private insurer’s ideal premium. It is conjectured that in a world with positive transaction and information costs, the low cost insurer should provide.

Model IV is a model of social insurance where indemnification is not possible (infinite information cost of state observation). The interpretation being that since states are unobservable no state-contingent cash payments are
possible. However, the social insurer can subsidize medical care thereby providing reimbursement insurance. The model shows that when it is optimal to provide such insurance, it is also optimal to alter the consumer price of cigarettes. The optimal cigarette tax (subsidy) depends in part on the complementarity of cigarettes and medical care as in Model I and on the moral hazard as in Model III.

3. MODEL I: A CERTAINTY MODEL

This is a certainty model of cigarette and medical care demand. An exogenous, (and in this model unexplained) subsidy to medical care creates a distortion in consumption patterns. The optimal cigarette tax (subsidy) which corrects for this distortion is shown to depend upon the degree of Hicksian complementarity (substitutability) between cigarettes and medical care demand.

This result is both intuitively and analytically similar to results in the public finance literature which indicate that optimal tax formulae are dependent upon demand price sensitivities. The result that differential commodity taxation is Pareto superior to uniform commodity taxation (Ramsey 1927) is the precursor to this body of literature. A particularly apropos comparison can be made with the result that differential commodity taxation is Pareto superior to either income or value-added taxation when at least one commodity remains untaxable (e.g., leisure). Here the optimal tax formulae imply relatively greater taxation of goods which are more complementary to the untaxed commodity and relatively less taxation, perhaps subsidization, of goods which are substitutes for the untaxed good (Corlett and Hague 1953-54; Diamond and Mirrlees 1971; Atkinson and Stiglitz 1972, 1980).

Another closely related area of research, where the complementarity
issue is important, is that of indirect Pigovian taxation. In this literature, either it is assumed that direct taxation of the externality generating good or activity is not possible, or it is shown that under some conditions it is optimal to augment the direct taxation. The optimal tax formulae imply taxation of complements to and subsidization of substitutes for the externality generating good or activity (Green and Sheshinski 1976; Sandmo 1976; Balcer 1980; Wijkander 1985).

3.1. NOTATION

\begin{align*}
  c & : \text{quantity of cigarettes} \\
  q & : \text{producer price of cigarettes} \\
  t & : \text{per unit tax/subsidy on cigarettes} \\
  Q & : \text{consumer price of cigarettes; } Q = q + t \\
  m & : \text{quantity of medical care} \\
  r & : \text{producer price of medical care} \\
  s & : \text{per unit tax/subsidy on medical care} \\
  R & : \text{consumer price of medical care; } R = r + s \\
  H & : \text{health (unpriced, untaxed)} \\
  z & : \text{numeraire (untaxed)} \\
  y & : \text{income before lump-sum transfer} \\
  x & : \text{lump-sum transfer} \\
  Y & : \text{income after lump-sum transfer; } Y = y + x
\end{align*}

3.2. STRUCTURAL EQUATIONS

\begin{align*}
(1) \quad U & = U(c, H, z); \ U_c > 0; \ U_H > 0; \ U_z > 0 & \text{Direct Utility Function}
\end{align*}
Utility is a strictly increasing function of cigarettes, health, and the numeraire.

\[(2) \ H = H(c, m); \ H_c < 0; \ H_m > 0 \quad \text{Health Production}\]

Health is depleted by smoking and augmented by consuming medical services. Although this is a single period model, it is very much in the spirit of Grossman's (1972a, 1972b) stock of health model.

From equations (1) and (2) it can be seen that cigarettes will be demanded for the direct utility they provide, whereas, the demand from medical care is derived from the demand for health.

\[(3) \ Y = Qc + Rm + z \quad \text{Budget Constraint}\]

where:

\[(3.1) \ Q = q + t\]
\[(3.2) \ R = r + s\]
\[(3.3) \ Y = y + x\]

The consumer is assumed to spend all of his/her income. The taxes and subsidies are imbedded in the problem and treated as exogenous to the consumer.

3.3. CONSUMER'S PROBLEM

\[(4) \ \max_{c, m} U = U(c, H(c, m), Y - Qc - Rm)\]
Substituting the budget constraint directly into the utility function and maximizing with respect to cigarette and medical care consumption renders the following first order conditions for an interior solution.

\[(4.1) \quad U_c + U_h H_c - U_s Q = 0\]

F.O.C. for an Interior Solution

\[(4.2) \quad U_h H_m - U_s R = 0\]

Equation (4.1) states that optimal cigarette consumption implies the equality of the marginal benefit of smoking, \(U_c\), and the sum of the health cost, \(-U_h H_c\), and financial cost, \(U_s Q\). Since medical care provides no direct utility the optimal condition has one less term. The optimizing consumer should equate the marginal health benefits, \(U_h H_m\), to the marginal financial cost, \(U_s R\).

Implicitly solving equations (4.1) and (4.2) for \(c\) and \(m\) renders the following Marshallian demand functions.

\[(5.1) \quad c = c(Q,R,1,Y)\]

Marshallian Demands

\[(5.2) \quad m = m(Q,R,1,Y)\]

3.4. SOCIAL PLANNER’S PROBLEM

Assuming that: (i) individuals are identical in both their preferences and the constraints they face, and, (ii) producer prices are constant and equal to marginal cost; the social planner's objective is to maximize indirect utility. Additionally, it is required that the transfer scheme is a break-
even proposition.

(6) \[ \text{Max } U(Q,R,Y) = U(c,H(c,m), Y - Qc - Rm) \] \text{ Indirect Utility Function} \\
\[ t,s,x \]

(7) \[ \text{s.t. } x = tc + sm \] \text{Social Planner’s Budget Constraint}

Where, c and m are Marshallian demand functions.

Forming the Lagrangian renders:

(8) \[ L = U(c, H(c,m), Y - Qc - Rm) + \lambda(x - tc - sm) \]

Recall that: 
\[ Q = q + t \]
\[ R = r + s \]
\[ Y = y + x \]

By virtue of the envelope theorem, the first order conditions can be written as:

(8.1) \[ L_t = -U_z c + \lambda \left[ -c - \frac{\partial c}{\partial Q} - \frac{s m}{\partial Q} \right] = 0 \]

(8.2) \[ L_m = -U_z m + \lambda \left[ -m - \frac{\partial c}{\partial R} - \frac{s m}{\partial R} \right] = 0 \]

(8.4) \[ L_y = -U_z y + \lambda \left[ 1 - \frac{\partial c}{\partial Y} - \frac{s m}{\partial Y} \right] = 0 \]

Multiplying equation (8.3) by c and adding the product to equation (8.1) yields equation (9.1). Using the same procedure, with appropriate changes, yields equation (9.2).

\[ L_x c + L_t = 0 \Rightarrow \]

(9.1) \[ t \left[ \frac{\partial c}{\partial Q} + \frac{\partial c}{\partial Y} \right] + s \left[ \frac{\partial m}{\partial Q} + \frac{\partial m}{\partial Y} \right] = 0 \]
\[ L_q m + L_s = 0 \Rightarrow \]

\[
(9.2) \quad t \left[ \frac{\partial c}{\partial R} + \frac{\partial c}{\partial Y} \right] + s \left[ \frac{\partial m}{\partial R} + \frac{\partial m}{\partial Y} \right] = 0
\]

Writing equations (9.1) and (9.2) in matrix form yields:

(10) \quad A \cdot b = d

(10.1)

\[
\begin{bmatrix}
U & U \\
\left[ \frac{\partial c}{\partial Q} \right] & \left[ \frac{\partial m}{\partial Q} \right] \\
U & U \\
\left[ \frac{\partial c}{\partial R} \right] & \left[ \frac{\partial m}{\partial R} \right]
\end{bmatrix}
\begin{bmatrix}
t \\
s
\end{bmatrix}
= \begin{bmatrix}
t \\
s
\end{bmatrix}
\]

\[ \Rightarrow t = 0, \ s = 0 \]

Where \( A \) is the matrix of substitution terms. In other words, the matrix of income compensated or utility held constant price derivatives.

3.5. THE OPTIMAL TAX FORMULA

It is obvious from inspection of (10.1) that the optimal solution is non-interference with the market. However, if we assume that medical care is exogenously subsidized, then the optimal cigarette tax, in general, is not zero.

This immediately raises the question of why would you subsidize medical
care in the first place. This paper will culminate with a model (Model IV) that shows the conditions under which subsidization of medical care is optimal. Suffice it to say for now that if a subsidy is enacted it will create a distortion which implies the optimal tax formula of equation (11).

Assuming \( s = -s < 0 \Rightarrow \)

\[
(11) \quad t = -s \left( \frac{\partial m}{\partial Q} \right)^u \frac{\partial c}{\partial Q}^u
\]

The denominator of equation (11) contains the pure own-price effect which is negative. Strictly speaking, this term is non-positive. But strict negativity is required in order to render deterministic results. Given this additional assumption, the optimal cigarette tax can summarized in the following 3 cases.

3 Cases:

Case 1: if \( \left( \frac{\partial m}{\partial Q} \right)^u < 0 \) (Hicksian complements) then \( t > 0 \) (tax)

Case 2: if \( \left( \frac{\partial m}{\partial Q} \right)^u = 0 \) (Hicksian independents) then \( t = 0 \) (neither tax nor subsidize)

Case 3: if \( \left( \frac{\partial m}{\partial Q} \right)^u > 0 \) (Hicksian substitutes) then \( t < 0 \) (subsidize)

Subsidizing medical care has the primary effect of inducing over-consumption of medical care vis a vis the level implied by producer prices. If cigarettes and medical care are complements, the secondary distortion,
which accompanies this primary distortion, is over-consumption of cigarettes. Therefore, complementarity implies a tax on cigarettes (Case 1).

Such a tax, will also have primary and secondary effects. The primary effect of the cigarette tax is to abate cigarette smoking. More completely, the primary (secondary) effect of a cigarette tax, is to diminish the secondary (primary) distortion of a medical care subsidy.

Since primary effects are usually thought to outweigh secondary effects, the optimal cigarette tax implies an over-correction in the cigarette market. That is, the optimal cigarette tax should reduce smoking below the "first-best" level. This will still imply over-consumption of medical care. In other words, the optimal cigarette tax will be a "second-best" solution, with the marginal loss of reducing cigarette consumption equal to the marginal gain of reducing medical care consumption. Of course the "first-best" solution is to leave both medical and cigarettes unsubsidized and untaxed.

Inspecting the denominator of the optimal cigarette tax formula (equation (11)) reveals that the formula has a Ramsey-like property (Ramsey 1927); the less sensitive cigarette demand is to its own-price, the larger the tax should be. This is what the preceding paragraph intuitively implied.

If cigarette and medical care demand are independent, then a subsidy to medical care creates no secondary distortion in the cigarette market. Likewise, by Slutsky symmetry, the cross-effect cannot serve as a conduit, through which cigarette taxes can correct for the primary distortion in the medical care market. Therefore, the optimal cigarette tax is equal to zero (Case 2).

Case 3 is simply a reversal in the directions implied by Case 1.

It should be pointed out that these results have nothing to do with
cigarettes and medical care per se. Rather, they apply to any two goods when the first is subsidized and the second is related to the first as delineated by Cases 1-3.

The assumption that both goods were arguments in the health production function (equation (2)) was incidental to the model. Such a structural assumption is completely superfluous to the qualitative predictions of demand behavior. Although, it seems intuitive that cigarettes and medical care are complementary goods, no such relationship is implied by utility maximization.

Case 3 is not a theoretical anomaly like that of a Giffen good. Rather, substitution is the usual relationship between goods. In a two-good model they must be substitutes. To see this recall that Hicksian demands are homogeneous of degree zero in prices. Since the own-price effect is always negative, the cross-price effect must be positive. In the more-than-two-good case, this same homogeneity condition limits the degree of complementarity in a system of demands. Additionally, the special case of additively-separable preferences exclusively renders substitution.

The importance of these results will become apparent in the analysis of Model IV.

4. MODEL II: PRIVATE INSURANCE

Model II is an expected utility model with state-dependent utility. Unlike Model I, health is not explicitly modelled, it is imbedded in the utility function. There are two states; state 1 is the healthy state, and state 2 is the ill state.

Although this is a single-period model, there is some intuitive notion of time. That is, cigarette consumption is chosen ex ante and therefore is
the same in both states. On the other hand, medical care is only consumed if the illness/accident occurs and is therefore an ex post choice. The choice of the level of consumption of the numeraire good is also ex post, but unlike medical care, it is consumed in both states.

The presence of a financial loss, although allowed for, is not necessary to motivate a demand for insurance. Differences in the marginal utility of money across states is sufficient for insurance demand to arise. Since utility is state-dependent, these differences can occur irrespective of changes in the level of wealth.

The potential for moral hazard is built into the model by allowing the probability of illness to depend on the level of cigarette consumption. An actuarially fair insurer who can observe the level of cigarette consumption charges a premium which completely abates the moral hazard. The results are full insurance, no moral hazard, and optimal consumption of both cigarettes and medical care. This model of private insurance generates the "first-best" solution. Therefore, the optimal social intervention is no intervention.

4.1. NOTATION

\( U(c,z_1) \): utility in state 1 (healthy)
\( V(c,m,z_2) \): utility in state 2 (ill)

\( 1 - \rho(c) \): probability of state 1
\( \rho(c) \): probability of state 2; \( \rho' > 0 \)

\( P \): Net insurance payout in state 1 (insurance premium)
\( I \): Gross insurance payment in state 2 (level of insurance coverage)
\( I - P \): Net insurance payment in state 2 (Net insurance)
4.2. STRUCTURAL EQUATIONS

(12) $EU = (1 - \rho(c))U(c, z_1) + \rho(c)V(c, m, z_2)$ Direct Expected Utility

(13) $Y_1 - P = Qc + z_1$ Budget Constraint for State 1

(14) $Y_2 - P + I = Qc + Rm + z_2$ Budget Constraint for State 2

(15) $(1 - \rho(c)) P + \rho(c)P = \rho(c)I$ Actuarially Fair Insurance

The private insurer is expected to break even. Therefore, expected premiums must equal expected insurance payments, equation (15). Combining terms renders equation (15').

(15') $P = \rho(c)I$

The actuarially fair insurance premium is linear in the level of coverage and increasing in cigarette consumption.

4.3. CONSUMER'S PROBLEM

The consumer seeks to maximize expected utility by choosing the level of cigarettes, medical care, and indemnity insurance.

(16) $\max_{c, m, I} EU = (1 - \rho(c)) U(c, Y_1 - \rho I - Qc)$

$+ \rho(c) V(c, m, Y_2 + (1 - \rho)I - Qc - Rm)$

The first order conditions for an interior solution imply equations (16.1), (16.2) and (16.3).
\[ \frac{\partial EU}{\partial c} = 0 \]

\[(16.1) \quad (1 - \rho)U_c + \rho V_c = [(1 - \rho)U_z + \rho V_z] [Q + \rho' I] + \rho' [U - V] \]

The pecuniary externality is fully internalized.

The consumer's optimal choice of cigarettes involves setting the expected marginal utility of smoking (the left-hand-side of equation (16.1)) equal to the marginal cost of cigarettes in expected utility term (the right-hand-side of equation (16.1)). The marginal cost has three terms in total; two associated with the financial cost, and one representing the health cost.

The financial cost is composed of the purchase price of cigarettes, Q, and the marginal increase is the insurance premium, \( \rho' I \). Since the insurer observes the number of cigarettes consumed, any potential pecuniary externality imposed by the smoker on the risk pool can be perfectly internalized. In this sense the premium is Pigovian (Pigou 1932) in nature.

The health cost is represented by the term \( \rho' [U - V] \). In the case of state-independent utility, full insurance would imply both equality of marginal utilities and total utilities across states, and this term would be zero. However in this model health cannot be fully restored. By assumption, the total utility in the healthy state will always exceed that of the ill state, \( U > V \). Even a fully insured smoker will face a cost in terms of a marginal probability of illness. If this term is large enough, even consumers who would ordinarily enjoy smoking will find a corner solution optimal. As long as there is an interior solution, this term alone will not fully abate moral hazard (Dionne 1982). It will be left for the insurance premium to do so.
\( \frac{\partial EU}{\partial m} = 0 \Rightarrow 

(16.2) \quad \rho[V_m - V_R] = 0 \Rightarrow V_m = V_R \quad \text{Indemnity Insurance is Non-distorting.}

Equation (16.2) simply reconfirms the intuition that indemnity insurance, because of its lump sum nature, will not distort the consumption of medical care.

\( \frac{\partial EU}{\partial I} = 0 \Rightarrow 

(16.3) \quad \rho(1-\rho)U_z - \rho(1-\rho)V_z = U_z - V_z \quad \text{Full Insurance Condition}

The optimal amount of insurance implies the equality of the marginal utility of money across states. It should be noted that in the case of state-dependent utility the consumer may not demand insurance against illness. This would be the case if the marginal utilities were equated without insurance.

If the marginal utility of money is greater when the consumer is healthy than when the consumer is ill, then the consumer will demand insurance against the chance of being healthy. An extreme example would be if illness actually meant death or simply brain death. Without a bequest motive, the rational consumer would demand an insurance contract that transferred income from the illness/death state to the health/alive state. The optimal insurance contract would imply that, \( I < 0 \). It should also be noted that the insurer now faces moral hazard in the other direction, the consumer is too cautious and doesn't smoke enough. The optimal insurance contract offers discounts to smokers based on how much they smoke. This seemingly counter intuitive result makes
sense when you realize the insurance company makes out better if you die.

Since consumers tend to demand health insurance, it is probably appropriate to interpret the insurance payment, I, as positive. This then is the standard case and insurers will charge higher premiums to smokers based on the quantity smoked. The additional premium for each cigarette equals, \( p' I \). Thus, private actuarial insurance provides a financial incentive to smoke less (self-protection) and may result in less smoking than would occur if the consumer was uninsured (Ehrlich and Becker 1972, p. 642).

This is why it is typically thought that fully informed insurance can curtail or eliminate moral hazard problems (Ehrlich and Becker 1972; Pauly 1974). But even if the consumer increased cigarette consumption after being insured this would not indicate that moral hazard was a problem in the sense that it resulted in sub-optimal behavior. Rather, it would indicate that the uninsured consumer over-protected. That is, smoked too little. This last statement should not be interpreted as saying insurers should offer discounts to smokers for smoking more, as was the case when the insurance was paid in the healthy state. As long as it is optimal to provide insurance against the illness, it is constrained-efficient to charge higher premiums for smoking more.

Continuing with the analysis, the next section will show that the perfectly informed social insurer cannot improve upon the efficiency implied by the perfectly informed private insurer. It is in this sense that the preceding problem can be regarded as the "first-best" case.

Equations (16.1) - (16.3) can be implicitly solved for the following Marshallian demand functions.
(17.1) \[ c = c(Q, R, l, Y_1, Y_2) \] Cigarettes

(17.2) \[ m = m(Q, R, l, Y_1, Y_2) \] Medical Care

(17.3) \[ I = I(Q, R, l, Y_1, Y_2) \] Indemnity Insurance

4.4. SOCIAL PLANNER'S PROBLEM

Assuming identical individuals, the social planner's problem is to choose the optimal tax/subsidy scheme to maximize indirect expected utility.

(18) \[
\text{MAX} \quad EU = (1 - \rho(c))U(c, Y_1 - \rho(c)I - Qc) \\
\quad + \rho(c)V(c, m, Y_2 + (1-\rho(c))I - Qc - Rm)
\]
Indirect Expected Utility

(18.1) \[ Q = q + t \]
(18.2) \[ R = r + s \]
(18.3) \[ Y_1 = y_1 + x_1 \]
(18.4) \[ Y_2 = y_2 + x_2 \]

(19) \[
s.t. \quad (1 - \rho)x_1 + \rho x_2 - (1 - \rho)tc + \rho tc + \rho sm
\]
Social Planner's Budget Constraint

Where, \( c, m, \) and \( I \) represent Marshallian demand functions.

In addition to choosing the optimal commodity tax/subsidy the social planner also chooses the lump-sum transfer. The lump sum transfer into state 1, \( x_1 \), can also be thought of as an insurance premium like \( P \), and the difference between the lump-sum transfers, \( x_2 - x_1 \) can be thought of as a gross insurance payment like, \( I \).
In this model, the social planner is constrained such that expected lump-sum transfer equal expected commodity transfers.

\[(19') \quad (1 - \rho)x_1 + \rho x_2 = tc + \rho sm\]

Expected Lump Sum Subsidy (tax) = Expected Commodity Tax (subsidy) Receipts (payments)

Forming the Lagrangian for the above problem renders:

\[(20) \quad \text{MAX } L = \text{EU}(Q,R,Y_1,Y_2) + \lambda[(1-\rho)x_1 + \rho x_2 - tc - \rho sm]\]

The first order conditions for an interior solution are represented by equations (20.1) through (20.4)

\[(20.1) \quad L_t = -[(1-\rho)U_z + \rho V_z]c + \lambda[-c - \frac{\partial c}{\partial Q} - \rho s \frac{\partial m}{\partial Q} + \rho'[x_2-x_1-sm] \frac{\partial c}{\partial Q} = 0]

\[(20.2) \quad L_s = -\rho V_z m + \lambda[-\rho m - \frac{\partial c}{\partial R} - \rho s \frac{\partial m}{\partial R} + \rho'[x_2-x_1-sm] \frac{\partial c}{\partial R} = 0]

\[(20.3) \quad L_{x_1} = (1 - \rho)U_z

\quad + \lambda[(1-\rho) - \frac{\partial c}{\partial Y_1} - \rho s \frac{\partial m}{\partial Y_1} + \rho'[x_2-x_1-sm] \frac{\partial c}{\partial Y_1} = 0]

\[(20.4) \quad L_{x_2} = -\rho V_z + \lambda[\rho - \frac{\partial c}{\partial Y_2} - \rho s \frac{\partial m}{\partial Y_2} + \rho'[x_2-x_1-sm] \frac{\partial c}{\partial Y_2} = 0]

if \(s = t - x_1 - x_2 = 0\) \Rightarrow (20.1) through (20.4) become:

\[(21.1) \quad -[(1-\rho)U_z + \rho V_z]c - c\lambda = 0\]
(21.2) \[-\rho V_x m - \rho \lambda m = 0\]

(21.3) \[(1-\rho)U_z + (1-\rho)\lambda = 0\]

(21.4) \[\rho V_x + \rho \lambda = 0\]

(21.1) through (21.4) \[U_z = V_z\]

Recall that $U_z - V_z$ from (16.3). Therefore (21.1) through (21.4) hold and $s = t = x_1 - x_2 = 0$ is a solution. Thus, the social insurer cannot improve upon private insurance when the private insurer is perfectly informed. However, as will be shown in the next section, the social insurer can do equally as well.

5. MODEL III: SOCIAL INSURANCE

This is a model of the optimal socialized health insurance plan.\(^4\) The model implies full indemnification, no medical care subsidy, and an optimal cigarette tax. The model shows that the cigarette tax can substitute for the differential smoking premium of Model II.

This model is a special case of the general model presented by Arnott and Stiglitz (1986). Therefore much of the structure, methodology and intuition presented here are owed to them. However, except for a few special cases (Arnott and Stiglitz 1986, pp. 10-13), they do not provide an analytical interpretation of their results. But they do suggest that their work is applicable to cigarette and alcohol taxes, and subsidies to preventive health care (Arnott and Stiglitz 1986, p. 15). This model makes the application to

\(^4\)It should be pointed out that Models III and IV utilize a Stackelberg equilibrium (Varian 1984, pp. 101-102). The consumer (follower) takes the prices and insurance levels (actions) of the social planner (leader) as given and maximizes expected utility accordingly. The results of the first step optimization problem are embodied in the consumer's demand (reaction) functions. In the second step the social planner (leader) takes the consumer's (follower's) demands as given and selects the overall optimal. In this sense both models are variants of the Stackelberg model.
cigarette taxes.

Model IV, which follows, extends Arnott and Stiglitz's (1986) results in ways they have not suggested or envisioned. It provides a rational for both cigarette taxes and reimbursement insurance, and combines the results of the preceding models presented here. But first the results of Model III must be presented before making the promised extension.

5.1. STRUCTURAL EQUATIONS

(22) \[ EU = (1 - \rho(c))U(c, z_1) + \rho(c)V(c, m, z_2) \] Direct Expected Utility Function

(23) \[ Y_1 = Qc + z_1 \] State 1 Budget Constraint

(24) \[ Y_2 = Qc + Rm + z_2 \] State 2 Budget Constraint

(25.1) \[ Y_1 = y_1 + x_1 \]

(25.2) \[ Y_2 = y_2 + x_2 \]

(25.3) \[ Q = q + t \]

(25.4) \[ R = r + s \]

There is no private insurance. The consumer perceives the government lump sum taxes and subsidies as well as the commodity taxes and subsidies as exogenous. This exogeneity of insurance will generate a need for a corrective tax.

5.2. CONSUMER'S PROBLEM

(26) \[ \text{MAX } EU = (1 - \rho(c))U(c, Y_1 - Qc) + \rho(c)V(c, m, Y_2 - Qc - Rm) \]

As before, the consumer is represented as an expected utility maximizer.

There is no market insurance, so the consumer only chooses \( c \) and \( m \). Equations
(26.1) and (26.2) represent the first order condition for an interior solution.

\[ \frac{\partial EU}{\partial c} = 0 \Rightarrow \]
\[ (1-\rho)U_c + \rho V_c = [(1-\rho)U_z + \rho V_z]Q + \rho'[U-V] \]

\[ \frac{\partial EU}{\partial m} = 0 \Rightarrow \]
\[ \rho[V_m - V_zR] = 0 \]

Implicitly solving equations (26.1) and (26.2) for \( c \) and \( m \) renders the following Marshallian demands.

\[ (27.1) \quad c = c(Q,R,1,Y_1,Y_2) \]

\[ (27.2) \quad m = m(Q,R,1,Y_1,Y_2) \]

5.3. SOCIAL PLANNER'S PROBLEM

\[ (28) \quad \text{MAX} \quad EU = (1-\rho(c))U(c,Y_1 - Qc) + \rho(c)V(c,m,Y_2 - Qc - Rm) \]
\[ t,s,x_1,x_2 \]

Where \( c \) and \( m \) are Marshallian demands.

\[ (29) \quad \text{s.t.} \quad (1-\rho)x_1 + \rho x_2 = (1-\rho)tc + \rho tc + \rho sm \]

As before the Social Planner maximizes indirect expected utility subject to expected lump sum transfers equaling expected commodity transfers.

Forming the Lagrangian and recalling that:

\[ Q = q + t \]
\[ R = r + s \]
\[ Y_1 = y_1 + x_1 \]
\[ Y_2 = y_2 + x_2 \]
renders the following:

\[(30)\quad L = EU(Q,R,Y_1,Y_2) - \lambda[(1-\rho)x_1 + \rho x_2 - tc - psm]\]

\[L_t = 0 \Rightarrow \quad (30.1)\quad -[(1-\rho)U_z + \rho V_z]c + \lambda[-c - t\frac{\partial c}{\partial Q} - \rho s\frac{\partial m}{\partial Q} + \rho'[x_2-x_1-sm]\frac{\partial c}{\partial Q}] = 0\]

\[L_a = 0 \Rightarrow \quad (30.2)\quad -\rho V_z - \lambda[-\rho m - t\frac{\partial c}{\partial R} - \rho s\frac{\partial m}{\partial R} + \rho'[x_2-x_1-sm]\frac{\partial c}{\partial R}] = 0\]

\[L_{x1} = 0 \Rightarrow \quad (30.3)\quad (1-\rho)U_z - \lambda[(1-\rho) - t\frac{\partial c}{\partial Y_1} - \rho s\frac{\partial m}{\partial Y_1} + \rho'[x_2-x_1-sm]\frac{\partial c}{\partial Y_1}] = 0\]

\[L_{x2} = 0 \Rightarrow \quad (30.4)\quad \rho V_z - \lambda[\rho - t\frac{\partial c}{\partial Y_2} - \rho s\frac{\partial m}{\partial Y_2} + \rho'[x_2-x_1-sm]\frac{\partial c}{\partial Y_2}] = 0\]

Equations (30.1) through (30.4) represent the first order conditions for an interior solution. Algebraically manipulating them as indicated renders equations (31) and (32).

\[L_{x1c} + L_{x2c} + L_t = 0 \Rightarrow \quad (31)\quad - t \left[\frac{\partial c}{\partial Q}\right]_{EU} - s\rho \left[\frac{\partial m}{\partial Q}\right]_{EU} + \rho'[x_2-x_1-sm] \left[\frac{\partial c}{\partial Q}\right]_{EU} = 0\]

\[L_{x2m} + L_a = 0 \Rightarrow \quad (32)\quad - t \left[\frac{\partial c}{\partial R}\right]_{EU} - s\rho \left[\frac{\partial m}{\partial R}\right]_{EU} + \rho'[x_2-x_1-sm] \left[\frac{\partial c}{\partial R}\right]_{EU} = 0\]
5.4. OPTIMAL TAX FORMULAE

Isolating the terms associated with \( t \) and \( s \) and writing in matrix form renders equations (33) and (33.1)

(33) \( A \cdot b = d \cdot e \)

(33.1)

\[
\begin{bmatrix}
\left( \frac{\partial c}{\partial Q} \right)_{EU} & \left( \frac{\partial m}{\partial Q} \right)_{EU} + \left( \frac{\partial m}{\partial Q} \right)_{EU} \\
\left( \frac{\partial c}{\partial R} \right)_{EU} & \left( \frac{\partial m}{\partial R} \right)_{EU} + \left( \frac{\partial m}{\partial R} \right)_{EU}
\end{bmatrix}
\begin{bmatrix}
t \\
s
\end{bmatrix}
= 
\begin{bmatrix}
\left( \frac{\partial c}{\partial Q} \right)_{EU} \\
\left( \frac{\partial c}{\partial R} \right)_{EU}
\end{bmatrix} \cdot \rho'[x_2-x_1]
\]

Solving equation (33.1) for \( t \) and \( s \) renders the following:

(34) \( t = \rho'[x_2-x_1] \) Optimal Cigarette Tax

(35) \( s = 0 \) Zero Medical Care Subsidy

Combining equations (30.3) and (30.4) with equations (34) and (35) renders equation (36), the full insurance condition.

\[ L_{x1} = 0; \ L_{x2} = 0; \ s = 0; \ t = \rho'[x_2-x_1] \]

\[ \Rightarrow \]

(36) \( V_x = U_x \) the full insurance condition.

The social planner chooses \( x_1 \) and \( x_2 \) to achieve full insurance.
Ordinarily, you would expect $x_2 > x_1$. That is, optimality implies a lump sum transfer from the healthy state to the ill state. If this occurs, then the optimal cigarette tax is positive.

$$\rho' > 0$$

$$x_2 > x_1 \Rightarrow$$

$$t = \rho'[x_2 - x_1] > 0$$

Optimality implies no subsidy to medical care, $s = 0$. This simply reconfirms the notion that indemnification is more efficient than reimbursement.

Interpreting $x_1$ as the premium and $[x_2 - x_1]$ as the gross insurance payment, then the optimal social insurance plan renders the identical solution as the preceding private insurance plan. The only difference is the private fully informed insurer internalizes the marginal moral hazard cost, $\rho'I$, with the premiums. The optimal social plan, does so with a cigarette tax. In fact, both plans, Model II and Model III, assume zero transaction cost, omniscient insurers, and actuarially fair premiums. Therefore, both models render the same results and private and social insurance are perfect substitutes for one another. Any combinations of Models II and III will be efficient. The social insurance will simply crowd-out the private insurance one for one. The consumer will be left in an identical welfare position.

One might also infer from this result, that if private insurance was provided with less than full information, cigarette taxation might improve upon the efficiency of the market.

The next model shows that reimbursement may be optimal.
6. MODEL IV: REIMBURSEMENT INSURANCE AND COMPLEMENTARITY

This model is identical to Model III except that the social planner cannot make state-contingent lump sum transfers. The consumer's income in each state equals initial income in that state plus the lump-sum transfer which is restricted to be identical across states; \( Y_1 = y_1 + x \) and \( Y_2 = y_2 + x \). The interpretation being that since individual states cannot be discerned, indemnification is not possible. However, an alternative method of insuring is possible. That is, subsidizing the commodity (medical care) for which demand is specific to the illness state. When subsidizing medical care (reimbursement insurance) is efficient, an optimal cigarette tax is implied. This optimal cigarette tax is shown to depend in part on the complementarity of cigarettes and medical care.

Although analytically similar to the results presented by Arnott and Stiglitz (1986), this model renders substantially different interpretations. In their model (Arnott and Stiglitz 1986), subsidies are seen as corrections for insurance distortions. Here, the subsidy is the insurance! Anytime the government provides a subsidy to a commodity for which the demand is stochastic, it is providing insurance.

When insurance takes on this form (reimbursement) it provides the benefit inherent in all insurance, risk-spreading, and also generates a cost by deviating price from marginal cost (Pauly 1968, Zeckhauser 1970). Thus, reimbursement insurance is a "second-best" solution. "First-best" solutions are either of the Model II or Model III types, and involve full indemnification. Here less than full insurance is optimal.

In this model, as in that of Arnott and Stiglitz (1986), the cigarette
tax is a correction for moral hazard. But it is also a potential correction for the subsidy distortion. Thus, it offers partial improvement of the "second-best" solution.

This model improves upon existing literature in two important respects. First it is both consistent with and explanatory of reimbursement insurance, the most common type of health insurance. Second, rather than relying on prima facie evidence, it analyzes cigarette taxation in a formal framework, thereby, challenging the notion that cigarette taxation is necessarily optimal.

6.1. STRUCTURAL EQUATIONS

The presentation of this model begins with the Marshallian demand functions implied by the consumer's problem in Model III.

\[(37) \quad c = c(Q,R,Y_1,Y_2) \quad \text{Marshallian Demands} \]

\[(38) \quad m = m(Q,R,Y_1,Y_2) \]

As in the previous social insurance model, the consumer views lump sum payments and commodity taxes/subsidies as exogenously determined. The only difference in the two models is the number of policy instruments available to the social planner. Therefore, consumer behavior as embodied in equations (37) and (38) is identical to that implied by Model III.

6.2. SOCIAL PLANNER'S PROBLEM

By restraining the lump sum transfer to be identical across states, this
model eliminates indemnification as a method of insuring. This structural assumption can be seen in equations (39.3), (39.4), and (40).

**Indirect Expected Utility Function**

\[
\text{MAX}_{x,t,s} \quad \text{EU} = (1-\rho(c))U(c,Y_1 - Qc) + \rho(c)V(c,m,Y_2 - Qc - Rm)
\]

(39.1) \( Q = q + t \)

(39.2) \( R = r + s \)

(39.3) \( Y_1 = y_1 + x \)

(39.4) \( Y_2 = y_2 + x \)

(40) \( s.t. \quad x = tc + \rho sm \)

**Expected Lump-Sum Transfer - Expected Commodity Transfers.**

Thus, by restricting the model in this way, the social planner can only provide insurance against illness by subsidizing medical care. This model, however, is not so overly restrictive as to force a cigarette tax. Distributional neutrality can be maintained by using the lump sum transfer as a premium.

Forming the Lagrangian and writing the first order conditions renders equations (41) and (41.1) through (41.3), respectively.

(41) \( L = EU(Q,R,Y_1,Y_2) - \lambda[x-tc-\rho sm] \)

\[ L_x = 0 \Rightarrow \]

(41.1) \( \frac{\partial (1-\rho)U_z + \rho V_z}{\partial Y_z} - \lambda \left[ 1 - t \left( \frac{\partial c}{\partial Y_1} + \frac{\partial c}{\partial Y_2} \right) + \rho s \left( \frac{\partial m}{\partial Y_1} + \frac{\partial m}{\partial Y_2} \right) \right. \]

\[ - \left. \rho' sm \left( \frac{\partial c}{\partial Y_1} + \frac{\partial c}{\partial Y_2} \right) \right] = 0 \]
\[ L_t = 0 \Rightarrow \]

\[ (41.2) \quad -[(1-\rho)U_x + \rho V_x]c - \lambda[-c - \frac{\partial c}{\partial Q} - \rho' s m \frac{\partial c}{\partial Q}] = 0 \]

\[ L_s = 0 \Rightarrow \]

\[ (41.3) \quad -\rho V_x m - \lambda[-\rho m - \frac{\partial c}{\partial R} - \rho s \frac{\partial m}{\partial R} - \rho' s m \frac{\partial c}{\partial R}] = 0 \]

Multiplying equations (41.1) by c and adding the product to equation (41.2) renders equation (42).

\[ L_c c + L_t = 0 \Rightarrow \]

\[ (42) \quad t\left(\frac{\partial c}{\partial Q}\right)^{EU} + \rho s\left(\frac{\partial m}{\partial Q}\right)^{EU} + \rho' s m \left(\frac{\partial c}{\partial Q}\right)^{EU} = 0 \]

Where the partial derivatives are expected utility held constant price effects.

Following the same procedures, with appropriate changes, renders equation (43).

\[ L_c m + L_s = 0 \Rightarrow \]

\[ (43) \quad t\left[ \frac{\partial c}{\partial R}\right]^{EU} + \frac{\partial c}{\partial Y_1} m + \rho s\left[ \frac{\partial m}{\partial R}\right]^{EU} + \frac{\partial m}{\partial Y_1} m + \rho' s m \left[ \frac{\partial c}{\partial R}\right]^{EU} + \frac{\partial c}{\partial Y_1} m = \]

\[ \frac{(1 - \rho)m(\lambda - U_x)}{\lambda} \]

Where \( \lambda \) is the expected marginal utility of the numeraire.
Isolating the terms associated with $t$ and $s$ and writing equations (42) and (43) in matrix form renders equations (44) and (44.1)

\begin{equation}
\mathbf{A} \cdot \mathbf{b} = \mathbf{d}
\end{equation}

\begin{equation}
(44.1)
\end{equation}

\[
\begin{bmatrix}
\frac{\partial c}{\partial Q}^{EU} & \rho \left( \frac{\partial m}{\partial Q} \right)^{EU} + \rho' m \left( \frac{\partial c}{\partial Q} \right)^{EU} \\
\frac{\partial c}{\partial R}^{EU} + \frac{\partial c}{\partial Y_1} m & \rho \left( \frac{\partial m}{\partial R} \right)^{EU} + \frac{\partial m}{\partial Y_1} m + \rho' m \left( \frac{\partial c}{\partial R} \right)^{EU} + \frac{\partial c}{\partial Y_1} m
\end{bmatrix}
\begin{bmatrix}
t \\
s
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0 \\
(1-\rho)m(\lambda-U_p) \\
\end{bmatrix}
\]

\[
\frac{\lambda}{\lambda}
\]
From a detailed inspection of the comparative static results of the consumer's problem, the following relationship holds.

\[(45) \quad \left( \frac{\partial c}{\partial Q} \right)_{EU} \frac{\partial m}{\partial Y_1} - \left( \frac{\partial m}{\partial Q} \right)_{EU} \frac{\partial c}{\partial Y_1} = 0\]

Using equation (45), the determinant of A can be written as:

\[(46) \quad |A| = \rho \left[ \left( \frac{\partial c}{\partial Q} \right)_{EU}^2 - \left( \frac{\partial m}{\partial Q} \right)_{EU} \left( \frac{\partial c}{\partial R} \right)_{EU} \left( \frac{\partial m}{\partial Q} \right)_{EU} \right] > 0\]

The determinant of matrix A differs from the determinant of the Slutsky matrix (the matrix of pure-substitution effects) only by the multiplicative term \(\rho\). Therefore, the determinant of A is positive.

6.3. OPTIMAL TAX FORMULAE

\[(47) \quad t = -\frac{1}{|A|} \left( \frac{(1-\rho)m(\lambda-U_z)}{\lambda} \right) \left[ \rho \left( \frac{\partial m}{\partial Q} \right)_{EU} + \rho \left( \frac{\partial c}{\partial Q} \right)_{EU} \right]\]

\[(48) \quad s = -\frac{1}{|A|} \left( \frac{(1-\rho)m(\lambda-U_z)}{\lambda} \right) \left( \frac{\partial c}{\partial Q} \right)_{EU}\]

Note that: \(\lambda > U_z \Leftrightarrow V_z > U_z\)
\(\lambda - U_z \Leftrightarrow V_z - U_z\)
\(\lambda < U_z \Leftrightarrow V_z < U_z\)

7. INTERPRETATIONS

The results of this model can be summarized in 3 main cases.

7.1. THE NO INSURANCE CASE
Case 1: \[ V_z - U_z \Rightarrow s = 0 \]
\[ t = 0 \]

If the marginal utility of money is equal across states, then there is no fundamental demand for insurance. In this model insurance can only come in the form of a per-unit subsidy to medical care. Thus, the optimal subsidy is equal to zero. Since there is no insurance, no moral hazard or subsidy distortion is generated and the optimal tax on cigarettes is also equal to zero.

7.2. THE STANDARD CASE

Case 2: \[ V_z > U_z \Rightarrow s < 0 \] Subsidize Medical Care

If the marginal utility of money is greater in the ill state than in the healthy state, the consumer will benefit from insurance against medical expenses. This is evident by inspecting the optimal tax formula which, in this case, unambiguously implies a subsidy to medical care. Since the insurance in this sub-model has been formulated as a subsidy to medical care, it creates two types of distortions rather than simply the one type implied by indemnification. These two distortions may run in opposite directions and imply additional subcases for the optimal cigarette policy.

Case 2.1: \[ \left( \frac{\partial m}{\partial Q} \right)_{EU} < 0 \] (Hicksian Complements) \[ t > 0 \] Tax Cigarettes

---

5 The structural equations (39.1) and (39.2) are set up such that positive (negative) values of s and t indicate a tax (subsidy).
Case 2.2: \((\partial m^{EU} / \partial Q)^{EU} - 0\) (Hicksian Independents) \(\Rightarrow t > 0\) Tax Cigarettes

Case 2.3: \((\partial m / \partial Q)^{EU} > 0\) (Hicksian Substitutes)

Case 2.3.1: \(\left| \rho' \left( \partial c / \partial Q \right)^{EU} \right| > \rho \left( \partial m / \partial Q \right)^{EU} \Rightarrow t > 0\) Tax Cigarettes

Case 2.3.2: \(\left| \rho' \left( \partial c / \partial Q \right)^{EU} \right| = \rho \left( \partial m / \partial Q \right)^{EU} \Rightarrow t = 0\) Neither Tax Nor Subsidize

Case 2.3.3: \(\left| \rho' \left( \partial c / \partial Q \right)^{EU} \right| < \rho \left( \partial m / \partial Q \right)^{EU} \Rightarrow t < 0\) Subsidize Cigarettes

The first type of distortion is the moral hazard problem as in Models II and III. The second type is the subsidy distortion as in Model I. The optimal cigarette tax formula will reflect the potential to abate both of these distortions. This is revealed by inspecting equation (47).

The term \(\rho' \left( \partial c / \partial Q \right)^{EU}\), is associated with the moral hazard and, as in Model III, implies taxation. By using equation (48) to substitute in the medical care subsidy, s, a more revealing presentation can be made.

The actual portion of the tax, which is for redressing moral hazard, equals, \(-\rho'sm\). This is strikingly similar to the terms, \(\rho'I\), and \(\rho'[X_2 - X_1]\) in Models II and III respectively. In fact the interpretation is identical. The terms \(I, [X_2 - X_1]\), and \(-sm\) are the insurance payouts for Models II, III, and IV, respectively. Thus, the terms \(\rho'I, \rho'[X_2 - X_1]\), and \(-\rho'sm\) are the corrective premiums/taxes which reflect the fact that smoking increases expected insurance claims. One major difference in this model, is that insurance claims depend directly on the demand for medical care, m. This
endogeneity of the loss would not occur with indemnification.

Another closely related phenomenon is the subsidy distortion. A correction for the subsidy distortion is evident in the optimal cigarette tax formula by inspecting the term associated with the pure cross-substitution effect.

Substituting the optimal subsidy as specified by equation (48) into equation (47) renders:

\[
(49) \quad t = -s \left[ \frac{\partial u}{\partial q} + \rho' m \right]
\]

As already mentioned the second term is the portion of corrective tax due to moral hazard, and is nearly identical to the like terms in Models II and III. The first term is the portion of the corrective tax due to the subsidy distortions. Except for the factor \( \rho \), this term is identical to the optimal tax formula implied by Model I.

The presence of this term makes the optimal cigarette tax policy ambiguous and leads to additional subcases. Given that it is optimal to subsidize medical care (\( V_z > U_z = s < 0 \)) (Case 2), then if cigarettes and medical care are complements or independent, a tax on cigarettes is optimal (Cases 2.1 and 2.2). However, if cigarettes and medical care are substitutes and the spillover from price distortion is greater than the moral hazard problem, a cigarette subsidy is optimal (Case 2.3.3).
7.3. THE STANDARD CASE REVERSED

Case 3: \( V_x < U_x \Rightarrow s > 0 \) Tax Medical Care

If the marginal utility of money is greater in the healthy state than in the ill state, then the consumer will demand insurance against the chance of being healthy. This implies payments in the health state and thus a medical tax.

All the subsequent results are reversals of the subcases of Case 2.

Case 3.1: \( \left( \frac{\partial m}{\partial Q} \right)^{EU} < 0 \) (Hicksian Complements) \( \Rightarrow t < 0 \) Subsidize Cigarettes

Case 3.2: \( \left( \frac{\partial m}{\partial Q} \right)^{EU} = 0 \) (Hicksian Independents) \( \Rightarrow t < 0 \) Subsidize Cigarettes

Case 3.3: \( \left( \frac{\partial m}{\partial Q} \right)^{EU} > 0 \) (Hicksian Substitutes)

Case 3.3.1: \( |\rho' m\left( \frac{\partial c}{\partial Q} \right)^{EU}| > \rho\left( \frac{\partial m}{\partial Q} \right)^{EU} \Rightarrow t < 0 \) Subsidize Cigarettes

Case 3.3.2: \( |\rho' m\left( \frac{\partial c}{\partial Q} \right)^{EU}| = \rho\left( \frac{\partial m}{\partial Q} \right)^{EU} \Rightarrow t = 0 \) Neither Tax Nor Subsidize

Case 3.3.3: \( |\rho' m\left( \frac{\partial c}{\partial Q} \right)^{EU}| < \rho\left( \frac{\partial m}{\partial Q} \right)^{EU} \Rightarrow t > 0 \) Tax Cigarettes

8. SUMMARY

Model I derived the optimal tax formula when a subsidy distortion was present. It was shown that the optimal tax depended upon the degree of complementarity between cigarettes and medical care. The main purposes of this model were the introduction of notation and the raising of the
complementarity issue.

Models II and III were perfect information models of private and public insurance respectively. The important result from these models is that since smoking represents a potential moral hazard problem there is a need for differential premiums or taxes.

Model IV integrated the results of the previous three models. Unlike Model I where the subsidy distortion was unexplained, Model IV showed that a subsidy to medical care is the optimal insurance scheme when states are unobserved. Since insurance in Model IV was a subsidy, it created a moral hazard problem much like indemnification in Models II and III and a subsidy distortion as in Model I. Thus, the optimal cigarette tax was shown to depend both on moral hazard and complementarity.

9. CONCLUSION

The final theoretical model (Model IV) looks at reimbursement-type health insurance. A type which generates moral hazard and subsidy distortions. Cigarette taxes are presented as an abatement to these distortions, and thus, as a partial substitute for perfectly rated insurance premiums. The model shows that when states are unobservable, the optimal social insurance implies a subsidy to medical care (reimbursement insurance). The optimal cigarette tax generated from this model is shown to depend in part on the pure cross-elasticity of demand between cigarettes and medical care, and is qualitatively indeterminant. Even though cigarette consumption increases the probability of illness, the optimal insurance tax-subsidy scheme may involve a subsidy to cigarettes. The theoretical possibility of such a seemingly counter-intuitive result, implies that proponents of increased
cigarette taxation, who contend such taxes will correct health insurance distortions, must support their contention with empirical evidence.

Although analytically similar to the results presented by Arnott and Stiglitz (1986), this model renders substantially different interpretations. In their model (Arnott and Stiglitz 1986), subsidies are seen as corrections for insurance distortions. Here, the subsidy is the insurance! Anytime the government provides a subsidy to a commodity for which the demand is stochastic, it is providing insurance.

When insurance takes on this form (reimbursement) it provides the benefit inherent in all insurance, risk-spreading, and also generates a cost by deviating price from marginal cost (Pauly 1968, Zeckhauser 1970). Thus, reimbursement insurance is a "second-best" solution. "First-best" solutions are either of the Model II or Model III types, and involve full indemnification. Here less than full insurance is optimal.

In this model, as in that of Arnott and Stiglitz (1986), the cigarette tax is a correction for moral hazard. But it is also a potential correction for the subsidy distortion. Thus, it offers partial improvement of the "second-best" solution. Moreover, the model has integrated the indirect tax literature with the imperfect information (insurance) literature.

This model improves upon existing literature in two important respects. First it is both consistent with and explanatory of reimbursement insurance, the most common type of health insurance. Second, rather than relying on prima facie evidence, it analyzes cigarette taxation in a formal framework, thereby, challenging the notion that cigarette taxation is necessarily optimal.
REFERENCES


