ON VERTICAL INTEGRATION, 
DEMAND COMPLEMENTARITY, 
AND SOCIAL GAIN FROM MERGER

by

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On Vertical Integration, Demand Complementarity, and Social Gain from Merger

A commonly accepted notion in Industrial Organization is that vertical integration of a bilateral monopoly ("successive monopoly") enhances economic efficiency. More specifically, when an intermediate good is used in fixed proportion for manufacturing a final product, the price of the final good is lower when the firms are vertically integrated than when they behave independently. Oddly, proofs of this claim invariably suppose that the non-merged firms exhibit Stackelberg, leader-follower behavior. The main purpose of this discussion is to show that merger achieves welfare gains when the non-merged firms operate at a non-cooperative equilibrium. One justification for dwelling on this point is the wide acceptance of the Nash equilibrium concept in economics. Perhaps its omission as a solution to bilateral monopoly can be attributed to the game-theoretic formulation that is customarily employed for this situation, in which case Nash equilibrium fails to exist (see Footnote 3). A second motivation underlying Proposition 1 is that the arbitrary assignment of leader-follower roles can be avoided. As F. M. Scherer notes,

The theory of bilateral monopoly is indeterminate with a vengeance.

[If the buyer ... faces either competitive sellers or a monopolistic supplier accepting the buyer's quoted price....]

If on the other hand the seller is a monopolist and sets a price to which the buyer responds....

[What happens under bilateral monopoly depends in part
upon whether the buyer or the seller exercises price
leadership.... [T]he price is theoretically indeterminate....
[1980, pp. 299-300]

Related discussion presents sufficient conditions for efficiency gains via
merger of two monopolists supplying complementary goods. The novel feature is
that consumption need not exhibit fixed proportions, but rather, that each
good's demand is linear in the two prices and that certain restrictions hold
for demand parameters. Since Proposition 2 shows that price collaboration
might raise social welfare under conditions not previously noted, empirical
studies based on this notion might discover that collusive pricing is indeed
desirable more often than had been expected.

I. VERTICAL INTEGRATION

In the bilateral monopoly analyzed below, an upstream firm sells its
product, $x_1$, to a downstream firm for use in subsequent production. The
marginal cost of producing $x_1$ is a constant, $c_1$. It is imagined that the
action over which the upstream firm exercises discretion is markup over cost,
$m_1 \equiv p_1 - c_1$, where $x_1$ is sold to the downstream firm at price $p_1$. It is
assumed that one unit of $x_1$ is needed to produce each unit of the final good,
x_2. The downstream firm faces a constant marginal cost, $c_2$, of transforming
$x_1$ into $x_2$. The downstream firm also chooses its markup over cost, $m_2$, and so
consumers pay $p_2 = c_1 + c_2 + m_1 + m_2$ for the final good. In this formulation a
change in either firm's choice variable ($m_1$ or $m_2$) alters price $p_2$ and,
consequently, the quantities demanded of $x_1$ and $x_2$.

With regard to technical assumptions, the demand function for final
product is downward sloping, $x_2'(\cdot) < 0$. In addition, the "profit" expression
\( m \cdot x_2(c_1+c_2+m) \) is strictly concave in \( m \). That is, \( d^2\pi/dm^2 = mx_2''(c_1+c_2+m) + 2x_2'(c_1+c_2+m) < 0 \).

**A. NASH EQUILIBRIUM**

Firms that behave non-cooperatively are concerned about their separate profit functions,

\[
\begin{align*}
\pi_1 &= m_1 \cdot x_2(p_2) \\
\pi_2 &= m_2 \cdot x_2(p_2),
\end{align*}
\]

where \( p_2 = c_1+c_2+m_1+m_2 \). At a Nash equilibrium, to be denoted by \( m_1^* \) and \( m_2^* \), each firm embraces the hypothesis that the other's markup is fixed. Each then selects the markup that maximizes its individual profit. The pair \( m_1^* \) and \( m_2^* \) satisfies the following first-order conditions (FOCs),

\[
\begin{align*}
d\pi_1/dm_1 &= m_1 x_2'(p_2) + x_2(p_2) = 0, \\
d\pi_2/dm_2 &= m_2 x_2'(p_2) + x_2(p_2) = 0,
\end{align*}
\]

where \( p_2 = c_1+c_2+m_1+m_2 \). The prior assumption of concavity assures that second-order conditions (SOCs) hold for a maximum. It follows from the particular form of (3) and (4) that \( m_1^* = m_2^* \). Introducing now the notation \( m^* = m_1^* + m_2^* \), \( m^* \) is found by solving the equation

\[
[1/2]m^* x_2''(c_1+c_2+m) + x_2(c_1+c_2+m) = 0.
\]

Given a demand curve \( x_2(\cdot) \), equation (5) defines \( m^* \) from which \( m_1^* \) and \( m_2^* \) can be obtained.

**B. COLLUSION**

If the two firms integrate vertically, a single profit expression becomes relevant,

\[
\pi(m) = m \cdot x_2(c_1+c_2+m).
\]

The optimal markup \( m^* \) satisfies the usual FOC \( d\pi/dm = 0 \), or

\[
m \cdot x_2'(c_1+c_2+m) + x_2(c_1+c_2+m) = 0.
\]
The solution to (7) will be unique because of the concavity assumption made earlier.

C. COMPARISON

A concise comparison between Nash and collusive prices exploits the similarity between equations (5) and (7). Taking such a comparison as the objective, an additional equation is now introduced that involves the variable m and a new parameter t,

\[(t+1)/2 \] \[m \cdot x_2'(c_1+c_2+m) \cdot x_3'(c_1+c_2+m) = 0.\]

It should be obvious that (8) is identical to (5) when t = 0 and that (8) is identical to (7) when t = 1. The comparison between m* and m* can therefore be accomplished by examining the implicit function m(t) implied by (8), with special attention given to m(0) and m(1). Differentiation of (8) yields the following expression for m'(t),

\[m'(t) = \frac{-m x_2'(p_2)}{(t+1)[m x_2''(p_2) + x_2'(p_2)] + 2 x_2'(p_2)}\]

\[p_2 \equiv c_1+c_2+m.\] A comparison of m* and m* is now possible.

PROPOSITION 1. In a bilateral monopoly involving fixed-proportions production, vertical integration is preferable to non-cooperative behavior (Nash equilibrium in markups) because the price of the final product is lower after merger than under non-cooperative behavior.

Proof. Since m* = m(0) and m* = m(1), it is useful to examine the algebraic sign of m'(t) over the range 0 ≤ t ≤ 1. The numerator in (9) is positive. The denominator in (9) can be re-arranged as follows,

\[(t+1)[m x_2''(p_2) + x_2'(p_2)] + 2 x_2'(p_2)\]

\[= (t+1)[m x_2''(p_2) + x_2'(p_2)] + 0 + 2 x_2'(p_2)\]

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\[ = (t+1)\left[mx_2''(p_2)+x_2'(p_2)\right] + (t+1)x_2'(p_2)-(t+1)x_2'(p_2) + 2x_2'(p_2) \]

\[ = (t+1)[mx_2''(p_2)+2x_2'(p_2)] + (1-t)x_2'(p_2). \]

Because of concavity of profit (i.e., \(mx_2''(c_1+c_2+m) + 2x_2'(c_1+c_2+m) < 0\)) the RHS of (10) is negative. Since this makes the denominator of (9) negative, \(m'(t) \neq 0\) for \(0 \leq t \leq 1\). It follows that \(m' = m(0) > m(1) = m^*\).

QED

II. Complementarity in Demand

I now wish to direct the discussion toward commodities that exhibit complementarity in demand. This topic is related to vertically linked firms since consumers require both goods in order to obtain utility. It is shown below that collaboration in the pricing of such items results in lower prices than non-cooperative behavior, if demand is linear and certain parameter restrictions are met. Since the goods need not be utilized in fixed proportion, the discussion expands the set of cases in which collusive behavior improves economic efficiency.

By way of introduction, it is imagined that market demand for the goods \(y_1\) and \(y_2\) is linear in prices \(q_1\) and \(q_2\),

\[ y_1 = \alpha_1 - \alpha_{11}q_1 - \alpha_{12}q_2 \]
\[ y_2 = \alpha_2 - \alpha_{21}q_1 - \alpha_{22}q_2. \]

Since the discussion is examining the notion of complementarity, the restrictions \(\alpha_{12} > 0\) and \(\alpha_{21} > 0\) are imposed. Marginal costs are again assumed to be constant. It is now possible, by proper choice of units of measurement, to write market demand more conveniently as a function of markups, \(m_1\) and \(m_2\). The simplified demand system appears as

\[ x_1 = a_1 - m_1 - a_{12}m_2 \]
\[(14) \quad x_2 = a_2 - a_{21} m_1 - m_2.
\]

In order that FOCs encountered below will characterize a profit maximum rather than a minimum, it is assumed that cross effects satisfy \(a_{12} + a_{21} < 2\). Each firm's profit can now be expressed as the product of a markup and a quantity demanded. Also, it will facilitate the eventual comparison between non-cooperative and collusive pricing to express firms' objective functions in a more generalized form that involves an added parameter \(t\), as follows:

\[\begin{align*}
(15) \quad & m_1 = m_1 x_1 + t m_2 x_2 \\
(16) \quad & m_2 = t m_1 x_1 + m_2 x_2,
\end{align*}\]

where \(0 \leq t \leq 1\). The parameter \(t\) can be interpreted as a measure of either seller's concern for the other seller's profit. In the algebra below, seller \(j\) chooses \(m_j\) that maximizes the quantity \(\pi_j\). Consequently, the assignment \(t = 0\) leads to the Nash equilibrium of the non-cooperative game in markups, while setting \(t = 1\) leads to the joint profit maximum.

Given \(t\), consider \(m_1\) and \(m_2\) that simultaneously satisfy the FOCs

\[\begin{align*}
(17) \quad & \delta \pi_1/\delta m_1 = 0 \\
(18) \quad & \delta \pi_2/\delta m_2 = 0.
\end{align*}\]

Replacing \(x_1\) and \(x_2\) in (15) and (16) by their expressions from (13) and (14), equations (17) and (18) take on the following form after some rearrangement,

\[\begin{align*}
2m_1 + [a_{12} + ta_{21}] m_2 &= a_1 \\
[a_{12} t + a_{21}] m_1 + 2m_2 &= a_2.
\end{align*}\]

Since this is only a two-variable system, it is worthwhile to solve explicitly for \(m_1\) and \(m_2\) in terms of parameters \(a_j, a_{k1}\), and \(t\). Some algebra gives

\[\begin{align*}
(19) \quad & m_1(t) = \frac{1}{D} [2a_j - a_{12} a_2 - a_{21} a_2 t] \\
(20) \quad & m_2(t) = \frac{1}{D} [-a_{21} a_1 - a_{12} a_1 t + 2a_2],
\end{align*}\]

where \(D \equiv 4 - a_{12} a_2 - [a_{12}]^2 t - [a_{21}]^2 t + a_{12} a_{21} t^2\).
The question to be addressed is whether markups rise or fall as the two firms proceed from non-cooperative behavior toward collusion. The answer can be found by differentiating (19) and (20) with respect to \( t \), the index of cooperation. The expression for \( m_1'(t) \) is displayed here,

\[
D^2 \cdot m_1'(t) = -4a_{12}a_2 + 2a_1[a_{12}]^2 + 2a_1[a_{21}]^2 - [a_{12}]^3a_2 - 4a_1a_{12}a_{21}t \\
+ 2[a_{12}]^2a_2a_{21}t + a_{12}[a_{21}]^2a_2t^2,
\]

where \( D^2 \equiv D \cdot D \). By symmetry \( m_2'(t) \) is found by interchanging subscripts. A comparison between non-cooperative behavior and collaboration is now possible.

**PROPOSITION 2.** If cross effects \( a_{12} \) are sufficiently small, then \( m_1'(t) \leq 0 \) and \( m_2'(t) \leq 0 \). That is, an increase in cooperation reduces markups under certain conditions of demand.

**Proof.** In the terms on the RHS of (21), the parameters \( a_{12} \) and \( a_{21} \) are raised to first, second, and third powers. What is noteworthy is that the lone first-order term is negative, \( -4a_{12}a_2 < 0 \). Consequently, for sufficiently small values of \( a_{12} \) and \( a_{21} \), it follows from (21) that \( D^2 \cdot m_1'(t) < 0 \). Since \( D^2 \) (\( D^2 \equiv D \cdot D \)) approaches the value 4 for small values of \( a_{12} \), it follows that \( m_1'(t) \leq 0 \) for small \( a_{12} \). By symmetry, \( m_2'(t) \leq 0 \).

\( \text{QED} \)

In an empirical application of Proposition 2, it should be remembered that parameters \( a_{12} \) measure price sensitivities after normalization of demand, as illustrated in (13) and (14). As an alternative to Proposition 2, the search for sufficient conditions could have stopped with the simple requirements \( D^2 \cdot m_1'(t) \leq 0 \) and \( D^2 \cdot m_2'(t) \leq 0 \). In either case, the principle is that cross effects in demand should be mild.
III. REMARKS

The notion that bilateral monopoly is more efficient under collaboration than at a non-cooperative equilibrium (Proposition 1) appears to be ignored in Industrial Organization and in antitrust cases. Whenever such a comparison has been made, economists seem to presume that independent behavior follows the Stackelberg model. Proposition 1 strengthens the argument supporting vertical integration because the Nash equilibrium is a more believable characterization of non-cooperative behavior. Regarding a recently debated case of vertically related production, Paul MacAvoy and Kenneth Robinson (1983) describe the dissolution of the Bell System into upstream and downstream divisions. Western Electric now manufactures equipment used by AT&T to provide long-distance telephone service. Because both firms will probably continue to be dominant firms in their respective industries, the model of bilateral monopoly is relevant here. One might argue using Proposition 1 that future equilibrium prices for long-distance service will exceed those that would have existed under vertical integration. Of course, a thorough analysis would also address additional issues, such as the regulation of a natural monopoly.

Proposition 2 presents a sufficient condition, weaker than fixed-proportions consumption, that implies the existence of welfare gains achievable through collaboration in pricing. Empirical analysis might find that some mergers in the transportation industry, say, ought to be encouraged on this basis. Richard Levin and Daniel Weinberg (1979) study end-to-end railroad mergers (i.e., Firm 1 serves points A and B, Firm 2 serves B and C, and the merged firm serves A, B, and C). The authors emphasize that cost
reductions and improved service justify these mergers. Dennis Carlton, William Landes, and Richard Posner (1980) present an econometric analysis of the social benefits of certain airline mergers. The authors find that greater convenience and reliability result from integration of flight segments under a single carrier. Franklin Fisher (1987) describes the transfer of trans-Pacific airline routes from Pan American to United Airlines and argues that expansion of United Airlines' route system will benefit customers through the improved service anticipated from a single carrier. It is puzzling that these studies ignore the idea that merged carriers might have a strategic motivation for charging lower-than-Nash prices, namely, the sort of cross effects examined in Proposition 2.
REFERENCES


1. Industrial Organization often explores the efficiency implications of mergers or agreements over prices. Generally speaking, horizontal mergers raise price and vertical integration tends to reduce price. The latter form of organization can achieve lower production costs, reduce uncertainty about demand, rectify market failure (William Comanor and H. E. Frech III, 1985), or overcome distributors' incentives to set high prices (Frank Easterbrook, 1981). Merger of two monopolists producing complements consumed in fixed proportions also leads to lower prices (Augustin Cournot, 1838).

2. F. M. Scherer (1980, pp. 299-302) and Roger Blair and David Kaserman (1985, pp. 299-301, 317-318) demonstrate how vertically related firms might exhibit Stackelberg behavior, not Nash. M. L. Greenhut and H. Ohta (1976) study an upstream supplier and downstream buyer in which the latter is alternately cast as a monopolist, a group of competitive sellers, or several oligopolists in the output market. The upstream firm is a Stackelberg leader, the downstream buyer is a Stackelberg follower. Frederick Zeuthen (1930, pp. 89-91) presents a brief, graphical analysis of bilateral monopoly at a Nash equilibrium. Demand is linear and the comparison between non-cooperative and collusive outcomes is limited.

Monopolists who supply complementary goods consumed in fixed proportions ("completing monopolists") are formally similar to vertically related firms. But as Scherer (1980) and Blair and Kaserman (1985) indicate, the simple Nash solution that characterizes such completing monopolists' non-cooperative behavior has not been adapted to vertical markets per se, despite the theoretical similarities between the two situations. As noted by Fritz Machlup and Martha Taber (1960), additional studies of completing monopolists
acknowledge the indeterminacy of a seeming bargaining problem, and this assertion might be made about a vertical market also. Completing monopolies are also explored in Cournot (1838, pp. 99–103) and Zeuthen (1930, pp. 63–89) through graphs.

3. In modelling bilateral monopoly one might utilize prices instead of markups as the strategic variables. However, no Nash equilibrium would exist in this case. Profits appear as

\[ \pi_1 = (p_1 - c_1) \cdot x_2(p_2) \]
\[ \pi_2 = (p_2 - c_2 - p_1) \cdot x_2(p_2). \]

The upstream firm’s perception that \( p_2 \) is fixed (an assumption that is central to the Nash methodology) disconnects the quantity \( x_1 \), \( x_1 = x_2 \), from its price \( p_1 \), and hence the upstream firm never chooses a finite value for \( p_1 \). One might argue that such a model is unrealistic because the upstream firm would not expect the downstream firm to leave \( p_2 \) fixed as \( p_1 \) is varied. By designating markups as strategic variables the notion that players react to each other becomes implicit, and yet the appealing Nash property is retained.

Although the final price \( p_2 \) is indeed affected by the upstream firm’s action \( m_1 \), when markups are the strategic variables, the text’s Nash equilibrium still differs from the usual Stackelberg solution (in which the upstream firm is leader and downstream firm is follower). The reason is that \( m_2^* \) is a constant, not a function \( m_2^*(m_1) \). A numerical example emphasizes this distinction between Nash and Stackelberg. Suppose demand is given by \( x_2(p_2) = M - p_2 \). Based on (5) Nash markups are \( m_1^* = m_2^* = [M - c_1 - c_2]/3 \). The equilibrium price of \( x_2 \) is therefore

\[ p_2^* = [2M + c_1 + c_2]/3. \]
For the Stackelberg solution straightforward algebra reveals that $m_1^* = (M-c_1-c_2)/2$ and $m_2^* = (M-c_1-c_2)/4$. Hence,

$$p_2^* = [3M+c_1+c_2]/4.$$ 

The two solution concepts arrive at different outcomes.

4. The model in Section 1 of bilateral monopoly can be re-interpreted as a model of complementary goods, consumed in one-to-one proportion, being supplied by two monopolists. It is imagined that the (formerly) upstream and (formerly) downstream firms sell their output directly to customers who obtain utility by consuming the products jointly. The (formerly) upstream firm sells good $x_1$ at price $c_1+m_1$; the (formerly) downstream firm sells a good $y$ at price $c_2+m_2$. Demand therefore satisfies $x_1 = y = x_2(c_1+c_2+m_1+m_2)$, where $x_2(\cdot)$ indicates a mathematical function only — there no longer exits any good $x_2$.

Under independent behavior each firm's profit is computed as follows,

$$\pi_1 = m_1 \cdot x_2(c_1+c_2+m_1+m_2)$$

$$\pi_2 = m_2 \cdot x_2(c_1+c_2+m_1+m_2).$$

A comparison of these profit formulas to (1) and (2) reveals that the Nash equilibrium for the provision of complementary goods $x_1$ and $y$ is identical to the text's Nash equilibrium for vertically related firms. That is, the firms choose the same markup in either interpretation. If the suppliers of the complementary goods merge, the single profit expression would be written as

$$\pi = [m_1+m_2] \cdot x_2(c_1+c_2+m_1+m_2).$$

Since this function is identical to (6), prices and quantities after merger would be equal to those under vertical integration.

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5. Consider the original demand system in (11) and (12),

\[ y_1 = \alpha_1 - \alpha_{11}q_1 - \alpha_{12}q_2 \]
\[ y_2 = \alpha_2 - \alpha_{21}q_1 - \alpha_{22}q_2 , \]

where goods are measured in their "old" units. A "new" unit for each good is introduced such that one new unit comprises \( w_j \) old units. Quantities can now be denoted by numbers \( x_j \), where \( x_j \equiv y_j/w_j \),

\[ x_1 = y_1/w_1 = \alpha_1/w_1 - [\alpha_{11}/w_1]q_1 - [\alpha_{12}/w_1]q_2 \]
\[ x_2 = y_2/w_2 = \alpha_2/w_2 - [\alpha_{21}/w_2]q_1 - [\alpha_{22}/w_2]q_2 . \]

New dollar prices \( p_j \) are given by \( p_j = w_jq_j \). It is possible to write demand using \( x_j \) and \( p_j \) only,

\[ x_1 = \alpha_1/w_1 - [\alpha_{11}/w_1^2] p_1 - [\alpha_{12}/w_1 w_2] p_2 \]
\[ x_2 = \alpha_2/w_2 - [\alpha_{21}/w_1 w_2] p_1 - [\alpha_{22}/w_2^2] p_2 . \]

The change of units simplifies the demand system when \( w_1^2 = \alpha_{11} \) and \( w_2^2 = \alpha_{22} \) because, then, the own-price sensitivities equal unity. That is,

\[ x_1 = b_1 - p_1 - b_{12}p_2 \]
\[ x_2 = b_2 - b_{21}p_1 - p_2 \]

(where \( b_1 \equiv \alpha_1/w_1 = \alpha_1/\sqrt{\alpha_{11}} \), etc.). Finally as in (13) and (14), demand can be expressed as a function of markups \( m_j \equiv p_j - c_j \) instead of prices \( p_j \),

\[ x_1 = a_1 - m_1 - a_{12}m_2 \]
\[ x_2 = a_2 - a_{21}m_1 - m_2 \]

(where \( a_1 \equiv b_1-c_1-b_{12}c_2 \), \( a_{12} \equiv b_{12} \), etc.).