

University of Hawaii at Manoa  
Department of Economics

**Mathematics for Economics**  
**ECON 627, Fall 2009**  
**MW 10:30-11:45AM SAUNDERS 541**

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**Course Content and Objectives**

- The goal of this course is to prepare you for, or remind you of, the mathematical underpinnings of economic theory courses—Econ 606, 607, 608 and 609. To that end, we will seek to translate undergraduate economic theory topics into the language of mathematics, taking considerable care to develop the necessary mathematical framework and adding a few bells and whistles in the process. In addition, several increasingly important topics in economic theory are inherently mathematical—notably dynamic optimization models that form the basis of macroeconomic growth theory, natural resources exploitation models and other “dynamic” topics. We will cover both the mathematics and economics of these subjects, though only briefly.
- Ideally, we would cover each topic in this course just in time for the theory courses to use that topic. Inevitably, we will miss the timing on at least some topics. If the theory courses arrive at some topic that requires mathematics we have not yet covered in this course, you will nonetheless be expected to read mathematical appendices or other textbooks for that course to obtain at least an intuitive feel for the mathematics. Of course, you are also welcome to request adjustments in our schedule, and I will try to accommodate.

**Prerequisites**

MATH 203, MATH 215, MATH 241, MATH 251A or equivalent. Familiarity with the topics covered in the summer math cram course (offered by the Department of Economics).

**Grading**

|                                       |                |
|---------------------------------------|----------------|
| Problem Sets                          | 30%            |
| Midterms (2)                          | 40% (20% each) |
| Final Exam (Dec 14 Monday 9:45-11:45) | 30%            |

There will be periodic problem sets. The problem sets will be mostly analytical but may also include some numerical problems. The latter type may involve the use of *Excel* or *Matlab* on the computer. I encourage you to work together on problem sets but each of you will hand in your own assignment. **Though you are encouraged to work in groups, you must acknowledge your collaborators in your solutions to the problem sets.**

### **Textbooks**

There are two books that we will reference fairly extensively in the course:

**(SB)** Simon, C. P. and L. Blume (1994) *Mathematics for Economists*, 1st Edition, Norton, New York.

**(Chiang)** Chiang, A. C. (1992) *Elements of Dynamic Optimization*, Waveland Press, Illinois.

### **Other references**

#### Calculus and its applications to economics

\*(**Sundaram**) Sundaram, R. (1996) *A First Course in Optimization Theory*, Cambridge University Press, Cambridge, UK.

\*(Sundaram's book is also highly recommended.)

Chiang, A.C. and K. Wainwright (2005) *Fundamental Methods of Mathematical Economics*, 4th Edition, McGraw-Hill, New York.

Silberberg, E. and W. Suen (2000) *The Structure of Economics: A Mathematical Analysis*, 3rd Edition, McGraw-Hill/Irwin, New York.

Takayama, A. (1993) *Analytical Methods in Economics*, University of Michigan Press, Ann Arbor.

#### Dynamic programming

**(SLP)** Stokey, N. L. and R. E. Lucas (with E. C. Prescott) (1989) *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge, MA.

Ljungqvist, L. and T. J. Sargent (2004) *Recursive Macroeconomic Theory*, 2nd edition, MIT Press, Cambridge, MA.

#### Optimal control theory

Seierstad, A., Sydsæter, K. (1987) *Optimal Control Theory with Economic Applications*, North-Holland, Amsterdam.

#### Analysis

Bartle, R. G. (1997) *The Elements of Real Analysis*, 2nd Edition, John Wiley & Sons, New York.

Rudin, W. (1976) *Principles of Mathematical Analysis*, 3rd Edition, McGraw-Hill, New York.

Ok, E. A. (2007) *Real Analysis with Economic Applications*, Princeton University Press, Princeton, NJ.

#### Numerical methods

Miranda, M. and P. Fackler (2002) *Applied Computational Economics and Finance*, MIT Press, Cambridge, MA.

Judd, K. (1998) *Numerical Methods in Economics*, MIT Press, Cambridge, MA.

## Topics covered

### 1. Preliminaries

#### Elements of set theory, logic, and proof

\*SB Appendix A1  
Sundaram Appendices A and B

#### Properties of Euclidean space and metric space

Sequence, convergence and limits, Cauchy sequence, vector space, norm, metric space, complete metric space, Banach space

\*SB Ch 10 and 12.1, 12.2  
\*SLP Ch 3.1  
Sundaram, Ch 1.1 and 1.2  
Sundaram, Appendix C

#### Topology of Euclidean spaces

Open, closed, bounded, compact sets  
\*SB Ch 12.3-12.6, Ch 29.

### 2. Multivariate calculus

#### Functions: basic concepts

Basic terminology of functions, continuity, monotonicity, partial and total derivatives, chain rule, higher order derivatives

\*SB Ch 13, 14.1-14.9  
Sundaram, Ch 1.4

#### Functions: applications

Intermediate and mean value theorems, Taylor's Theorem, inverse and implicit function theorem

\*SB Ch 15, 30  
Sundaram, Ch 1.5 and 1.6

#### Existence of solutions to optimization

Weierstrass Theorem  
\*SB Ch 30.1  
Sundaram, Ch 3

## MIDTERM I

### 3. Static optimization

#### Unconstrained optimization

Quadratic forms, first order necessary conditions, second order sufficient conditions

\*SB 16.1, 16.2, 17  
Sundaram, Ch 4

### Constrained optimization

Lagrangian method, first order necessary conditions, second order sufficient conditions, equality and inequality constraints, Kuhn-Tucker Theorem, interpretation of Lagrangian multipliers, envelope theorems, concave and quasiconcave functions, concave programming

\*SB Ch 18, 19.1-19.5, 21.1-21.5  
Sundaram, Ch 5, 6, 7

### Economic applications of static optimization

Utility maximization and expenditure minimization, Roy's identity, Shephard's lemma, Slutsky matrix

\*SB Ch 22

## **MIDTERM II**

### **4. Dynamic optimization**

#### Difference and differential equations

\*Difference equation: SB 23.1, 23.2  
\*Differential equation: SB 24.1-24.4, 25.1-25.5

#### Dynamic optimization: Calculus of variations, optimal control theory

The Calculus of Variations: Chiang Ch 2, 3, 6

The Hamiltonian Function: Chiang Ch. 7

More on Optimal Control: Chiang Ch 8

Infinite-Horizon Problems: Chiang Ch 5, 9

\*Dorfman, R. (1969) An Economic Interpretation of Optimal Control Theory, *American Economic Review*, 59(5) 817-831.

Plourde, C. G. (1970) A Simple Model of Replenishable Natural Resource Exploitation, *American Economic Review*, 60(3) 518-522.

Appendices A.1 and A.3 of Barro and Sala-i-Martin (2003) *Economic Growth*, 3rd Edition, MIT Press, Cambridge, MA.

### **5. Other possible topics**

Maximum Theorem, Fixed Point Theorem, ...

### **Disability Access**

If you feel you need reasonable accommodations because of the impact of a disability, please: (1) contact the KOKUA Program (V/T) at 956-7511 or 956-7612 in room 013 of the QLCSS (Queen Lili'uokalani DCenter for Student Services); (2) speak with me privately to discuss your specific needs. I will be happy to work with you and the KOKUA Program to meet access needs related to a documented disability.