UNIVERSITY of HAWAI'I MĀNOA

University of Hawai`i at Mānoa Department of Economics Working Paper Series

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Honolulu, HI 96822
Phone: (808) 956 -8496
www.economics.hawaii.edu

Working Paper No. 16-23R
(Revised Version of WP No. 16-4)

Competitive Search with Ex-post Opportunism

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November 2016

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November 28, 2016

Abstract

We consider a frictional market where an element of the terms of trade (price or quantity) is posted ex-ante (before the matching process) while the other is determined ex-post. By doing so, sellers can then exploit their local monopoly power by adjusting prices or quantities once the local demand is realized. We find that when sellers can adjust quantities ex-post, there exists a unique symmetric equilibrium where an increase in the buyer-seller ratio leads to higher quantities and prices. When buyers instead can choose quantity ex-post, higher buyer-seller ratio leads to higher price but lower quantity. These equilibrium allocations are generically not constrained efficient, in terms of both intensive and extensive margins. When sellers post ex-ante quantities and adjust prices ex-post, a symmetric equilibrium exists where buyers obtain no surplus. The equilibrium allocation is also constrained inefficient. If buyers choose prices ex-post, there is no equilibrium when entry is costly. This paper highlights how sellers' ability to commit ex-ante to certain elements of the terms of trade is crucial in generating constrained efficient allocations.

JEL Classification: D40, L10
Keywords: Competitive Search, Price Posting, Quantity Posting

*We would like to thank the editor as well as the referees for their helpful comments and suggestions, and seminar participants at the University of Hawaii Manoa.

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1 Introduction

Search and matching models have been developed and extensively applied in the context of labor economics. In these settings the matching technology tries to capture the inherent market frictions (see Pissarides (2000), Rogerson et al. (2005)). Search is typically assumed to be two-sided so that firms search for workers and unemployed search for jobs.[1] This kind of search framework has also been applied to goods/services markets as well as monetary economics.[2] Regardless of the market at hand, the typical assumption in these environments is that the terms of trade are determined ex-post via a bargaining procedure (generalized Nash, Kalai, or Rubinstein). Recently, however, a large and growing literature, both in labor and monetary economics, has explored the allocative properties of allowing one side of the market to ex-ante post and commit to the terms of trade. This can be achieved by setting a wage (price) or more general trading mechanisms. The competitive search framework captures such departures.[3] *The key assumptions in such environments are that: (i) sellers are able to fully commit to all posted terms of trade, (ii) buyers are able to observe all terms of trade posted by all sellers, (iii) none of the agents posting the terms of trade is capable of serving the entire market, and (iv) typically goods are indivisible.*[4]

In this paper we explore the role of ex-ante commitment to some elements of the terms of trade in determining the resulting equilibrium prices and quantities. So far in the literature, the only possibility for sellers to exploit ex-post opportunities (after the match has taken place) has been to post auctions, thus allowing for multilateral meetings. For instance, Peters and Severinov (1997), Julien et al. (2000), and Albrecht et al. (2014) explore the consequences of posting only reserve prices. Within this spirit, Kim and Kircher (2015) analyze the implications of first-price and second-price auctions, and show that the choice of the trading mechanism is crucial for the existence of equilibrium. Here we explore an alternative procedure allowing sellers or buyers to exploit ex-post opportunities. To do so, we consider a competitive search model where sellers are able to produce any continuous quantity at a convex cost while serving only one buyer at a

[1] This is essentially the Diamond-Mortensen-Pissarides framework.

[2] See Nosal and Rocheteau (2011) and Lagos et al. (2016) for an extensive survey.

[3] See McAfee (1993), Shimer (1996), Moen (1997), Acemoglu and Shimer (1999), Peters (2000), Julien et al. (2000), Burdett et al. (2001), Mortensen and Wright (2002), and Rocheteau and Wright (2005), to name a few.

[4] There exists a literature relaxing some of the key assumptions. Doyle and Wong (2013) consider the case of imperfect commitment, thus departing from assumption (i) above. In Geromichalos (2012) a seller could choose to serve the whole market (if all the buyers show up at her store), and hence, in a special case of that paper assumption (ii) might be violated. Gomis-Porqueras et al. (2017) relax assumption (ii) and allow for costly probabilistic signals. Also, there are many exceptions to an often extreme assumption that only one buyer gets served, including Lester (2010) and Geromichalos (2012, 2014). Faig and Jerez (2005) consider divisible goods.

time. In contrast to the previous literature, sellers have the ability to commit ex-ante only part of the terms of trade (prices or quantities), while choosing the remaining one ex-post or letting buyers choose. This is what we refer to as ex-post opportunism: sellers' or buyers' ability to take advantage of opportunities by choosing one component of the terms of trade after the meeting process has been revealed. In particular, we consider sellers posting and committing to a per unit price ex-ante, but choosing the quantity/quality ex-post, after the matching process has taken place. Then, we change the choice of quantity ex-post to buyers. Many markets have this ex-ante limited commitment feature. This is especially relevant if one considers the quality margin interpretation of the model. Such type of arrangements are found in labor markets, where wage rates are posted but hours worked are left to be determined ex-post, as in part-time jobs. This situation can also be observed in other markets. For instance, apartments or houses sold before construction or new cars priced and ordered/sold before they are manufactured. Examples of buyers choosing quantity to consume ex-post after observing posted prices are pervasive.[5]

Under these terms of trade with ex-post opportunism, we show that marginal cost pricing is the unique symmetric equilibrium as in the standard perfectly competitive market when sellers choose quantity ex-post. We show that this equilibrium can lead to under, over, or efficient production. These different possibilities critically depend on the aggregate buyer-seller ratio. Typically, entry is inefficient for small or large entry cost. For small cost, there is excessive entry and under-production relative to the efficient quantity (vice versa for large entry cost). This is in sharp contrast to the standard competitive search equilibrium where sellers ex-ante post and commit to both price and quantity. In such an environment, the equilibrium is always constrained efficient both at the extensive and intensive margins.[6]

We also find that posting prices ex-ante by sellers with quantities determined ex-post by buyers always yields marginal utility pricing.[7] *The equilibrium is generically not constrained efficient both at the intensive and the extensive margins. Interestingly, the direction of quantity inefficiency is the inverse of when sellers choose quantity ex-post. For low cost of entry, we find excessive entry, but over-production relative to the efficient quantity (vice versa for large entry cost).*

Finally, we consider the possibility that sellers post and commit to quantities ex-ante, but

[5] This type of limited ex-ante commitment can apply to any market where goods or services are priced ex-ante but produced upon a match.

[6] This result holds without fiat money. Rocheteau and Wright (2005) show that under-production can occur with money being essential for trade and high inflation.

[7] The environment with buyers choosing quantity ex-post is related to that analyzed in Peters (1984).

determine the per unit price ex-post, unilaterally by sellers or buyers, respectively. Under this trading protocol, we show that competitive search equilibrium exists. In the case of sellers setting prices ex-post, they have the incentive to extract all of the buyers' surplus by choosing a unit price equivalent to Diamond (1971), that is, the equilibrium price is above marginal cost. In addition, we find that the efficient quantity is always traded, but entry is excessive relative to what the social planer would choose. When buyers set prices ex-post, they extract all the surplus from sellers. The equilibrium price equals average cost and sellers choose not to participate in the DM when entry is costly. All of our results hold for a wide range of utility and cost functions.

The paper is organized as follows. Section 2 reviews the competitive search literature with divisible goods. In Section 3 we provide the environment, while in Section 3.1 we consider the planner's allocation. In Section 3.2 we characterize the symmetric equilibria where sellers commit to both per unit prices and quantity. In Section 3.3, we consider the case where sellers ex-ante commit to per unit prices and quantity is chosen ex-post by sellers, and Section 3.4 studies equilibria where buyers choose quantity ex-post. In Section 3.5 we analyze the equilibrium where sellers commit to quantity, but the per unit price is chosen ex-post by sellers, while in Section 3.6, the ex-post price is chosen by buyers instead. A conclusion follows and all proofs are in Appendix.

2 Relevant Literature

This paper relates to two different strands of literature. One that explores the equilibrium consequences for quantities and prices of having various trading mechanisms. The other studies how the equilibrium changes once the assumption of full commitment by sellers is relaxed.

The paper most closely to ours is that of Peters (1984), who considers a large directed search market where sellers produce a continuous quantity, q, while facing a convex cost and an exogenous capacity, K. Sellers post unit prices, and upon a match, buyers choose the quantity to demand, which is the minimum of q and K. In equilibrium, sellers post a price equal to average cost. Here we differ in that the quantity choice is determined ex-post, rather than on demand as in Peters (1984). As a result, the equilibrium explored in this paper is consistent with marginal cost pricing, while Peters (1984) finds average cost pricing. In addition, we find over, under, or efficient production is possible, depending on the buyer-seller ratio, while in Peters (1984), there is always over production. Faig and Jerez (2005) also have a related environment. They consider a competitive search economy with sellers competing in offering non-linear price schedule (e.g.

$z(q)$). Buyers have private information and, upon meeting a seller, choose the quantity they want and pay $z(q)$. This is related to our case with buyers choosing quantity ex-post. They show that when sellers offer two-tier prices, even with buyers' private information about their value, the competitive search equilibrium is constrained efficient, otherwise it is not.

More recently Geromichalos (2012, 2014), and Godenhielm and Kultti (2015) also allow sellers to produce multiple indivisible units at a convex cost. These authors also extend the choice of capacity prior to the matching process. Within the labor market, Lester (2010) and Hawkins (2013) allow firms to post multiple indivisible vacancies at a convex cost. However, all of these papers assume that all terms of trade are posted and committed ex-ante, before the matching takes place. In contrast to these papers, here we consider some ex-post opportunism. This is in the spirit of Godenhielm and Kultti (2014) who also assume continuous quantity, but allow sellers to choose capacity q simultaneously, and then prices. Production occurs before the matching process and the authors consider two cases where capacity choices are observed by buyers or not when selecting sellers. The equilibrium price is similar to Burdett et al. (2001), and hence, not tied to marginal or average cost. In contrast here we also consider production on demand.

The other related literature is the one that relaxes the assumption of full commitment. They do so by allowing some form of renegotiation on the posted terms of trade. For instance, Doyle and Wong (2013) study a directed search labor market where firms post wages but allow for ex-post wage renegotiation. The authors impose "downward commitment" so that firms can only commit to paying no less than their advertised wage. In their environment, only a wage is posted and there is no divisible hours of work, so there is no intensive margin. Along the same lines, Albrecht, et al. (2016) consider the housing market and assume that sellers of an indivisible house cannot commit to their advertised price. After the matching process, home owners can accept prices above or below their advertised price. Buyers learn their idiosyncratic valuation for the house, after an inspection, and each buyer submits a bid. They do so not knowing how many other bidders have visited the seller of the house. Buyers can accept the asking price, submit a counteroffer, or walk away. Unlike Doyle and Wong (2013) they do not impose "downward commitment". However, as in Doyle and Wong (2013), their good is indivisible and the only terms of trade is the house price.[8] *In contrast to these later papers, here we focus on equilibria where some elements of the terms of trade (price or quantity) are not committed ex-ante before the match takes place.*

[8] Other models along the lines of allowing asking prices to be renegotiable are considered in Camera and Selcuk (2009) and Lester, Visschers and Wolthoff (2016).

3 The Model

We use the competitive search framework based on Montgomery (1991), Shimer (1996), and Moen (1997) with a continuum of uncoordinated buyers and potential sellers, *of measures b and $\bar{s}$, respectively. The measure of active sellers s is endogenously determined by free entry. Sellers need to cover a fixed cost k to participate in the market. The aggregate active market tightness is given by $\Theta = b/s$.* Buyers have preference $u(q)$ over goods produced by sellers who incur a cost $c(q)$, where $u(\cdot)$ and $c(\cdot)$ satisfy usual properties of strict concavity and convexity. *In addition, $u(0) = c(0) = 0$ and $u(\hat{q}) = c(\hat{q})$ for some $\hat{q} > 0$.*[9] Any positive measure of sellers posting the same terms of trade, $\omega = (p, q) \in \mathcal{P} \times Q \subseteq \mathbb{R}^2_+$, form a submarket; where p denotes the per unit price, q is the quantity or quality of the good, and θ is the corresponding buyer-seller ratio in that submarket. Buyers have access to potentially a large number of these submarkets and choose which market to enter. All actions are observable to everyone. Within each submarket, buyers and sellers meet according to a matching technology that is homogeneous of degree one, where sellers' (buyers') meeting rate is $\alpha(\theta)$ $(\alpha(\theta)/\theta)$, with $\alpha'(\cdot) > 0$, $\alpha''(\cdot) < 0$, $\alpha(0) = 0$, $\lim_{\theta\to\infty} \alpha(\theta) = 1$, $\lim_{\theta\to 0} \alpha'(\theta) = 1$, and $\lim_{\theta\to\infty} \alpha'(\theta) = 0$. We also define the elasticity of sellers' matching rate $\varepsilon(\theta) = \alpha'(\theta)\theta/\alpha(\theta)$ with $\varepsilon'(\theta) < 0$. This property holds for many meeting technologies.[10] Finally, we define sellers' surplus as $S(p, q) = pq - c(q)$ and buyers' surplus as $B(p, q) = u(q) - pq$.

Given this structure, we construct a competitive search equilibrium by looking for an optimal deviation of a submarket where all sellers post terms of trade ω and the rest of sellers in other submarkets post ω^c. In equilibrium, buyers and sellers are indifferent across submarkets, and we have that $\theta = \Theta$ and $\omega = \omega^c$. This allows us to focus on one submarket.

We first consider the efficient allocation chosen by a planner in terms of extensive (entry) and intensive margins (q). We then compare the decentralized allocation against the planner' solution. As a benchmark, we first analyze a situation where sellers post ex-ante per unit prices and quantities. We then explore a situation where sellers do not have as much ex-ante commitment. In particular, we consider an environment where sellers post ex-ante per unit prices and the quantity is determined ex-post; i.e., after matches have taken place. Finally, we characterize the equilibrium

[9] In Proposition 1 and 3 we show sufficient conditions related to u''' and c''' for our results to hold.

[10] These properties are standard in the static version of the matching frameworks of Diamond-Mortensen-Pissarides. The matching technology can take any form, urn-ball, Cobb-Douglas, Telephone Line, etc. Given the submarket construct, one can choose any technology, either bilateral or multilateral meeting. What is important is that matching/trading is always pairwise. See Lester et al. (2015) and Cai et al. (2016) for more details on the relation between trading mechanisms and meeting technologies.

where sellers post ex-ante quantities and the per unit price is set ex-post.

3.1 Planner's Problem

A benevolent social planner maximizes the net expected total surplus by choosing a quantity to be traded q and a measure of active sellers s. The planner however is constrained by the matching technology that agents face. In particular, if the meeting technology in any submarkets allows multilateral meetings, after sellers and buyers meet, the planner randomly selects a buyer to be allocated with a seller, and if pairwise meeting technology, trading occurs with just one buyer. Because of this, and as is standard in search and matching models, we refer to the planner's solution as the "constrained efficient allocation".

The planner solves the following problem

$$\max_{q,s} s\alpha(\Theta)\left[u(q) - c(q)\right] - sk,$$

which we can rewrite as follows

$$\max_{q,\Theta} \frac{\alpha(\Theta)}{\Theta}\left[u(q) - c(q)\right] - \frac{k}{\Theta}.$$

Given the properties of the matching technology, utility and cost functions, we find the following necessary and sufficient conditions for optimal allocation

$$\begin{aligned} u'(q^e) &= c'(q^e) \text{ (intensive margin)}, \\ \alpha(\Theta^e)\left[1 - \varepsilon(\Theta^e)\right]\left[u(q^e) - c(q^e)\right] &= k \text{ (extensive margin)}, \end{aligned} \tag{1}$$

where $\varepsilon(\Theta) = \alpha'(\Theta)\Theta/\alpha(\Theta)$ is the elasticity of the matching rate for sellers, and (q^e, Θ^e) is the constrained efficient allocation in terms of the quantity traded and the market tightness, respectively.

3.2 Ex-ante Price and Quantity Posting By Sellers

In this section, the posted terms of trade are akin to the competitive search section of Rocheteau and Wright (2005) in which all terms of trade are posted ex-ante. *Given that most competitive search models, with a small number of exceptions as in Faig and Jerez (2005), the goods are indivisible. We need a benchmark model with divisible goods, perfect information, and all terms of trade posted ex-ante, the reason why we refer to Rocheteau and Wright (2005).* When the meeting technology in any submarkets allows multilateral meetings, upon meeting buyers, sellers randomly select a buyer to trade with, and if pairwise meeting technology, trading occurs with that buyer. Then, sellers produce the quantity posted.

A positive measure of sellers choose the same terms of trade $\omega = (p, q)$ to form a submarket, while other submarkets post the same $\omega^c = (p^c, q^c)$. Sellers solve the following problem[11]

$$\max_{(p,q)} \alpha(\theta) S(p, q)$$

$$\text{s.t. } \frac{\alpha(\theta)}{\theta} B(p, q) \geq \bar{U} \Leftrightarrow \theta > 0$$
$$\frac{\alpha(\theta)}{\theta} B(p, q) < \bar{U} \Leftrightarrow \theta = 0,$$

where $\bar{U} = \max_{(p^c,q^c)\in\mathbb{R}^2_+} U(p^c, q^c, \Theta) > 0$ is the buyers' maximum expected market utility from participating in any other submarkets.[12] In any optimal deviation, the participation constraint for buyers is always binding as in standard competitive search models. *From the constraint, we can obtain $\theta(\omega; \bar{U})$, which represents buyers' beliefs about the market tightness to prevail in the deviating submarket posting ω. These beliefs are pinned down by the market utility $\bar{U}$. Technically, one can substitute $\theta(\omega; \bar{U})$ in the sellers' objective function and proceed with the maximization. Alternatively, a much easier procedure is to allow the choice of θ directly in the maximization. It is well known that these two procedures give the same allocation. Through the second procedure, we can reduce the maximization problem to*

$$\max_{(p,q),\theta} \alpha(\theta) S(p, q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(p, q) = \bar{U}.$$

Solving the constraint for pq and substituting into $S(p, q)$, it is easy to show that optimality implies an efficient equilibrium quantity $q^* = q^e$ and an implied per unit price of

$$p^*(\theta) = \frac{[1 - \varepsilon(\theta)] u(q^*) + \varepsilon(\theta) c(q^*)}{q^*},$$

where $\varepsilon(\theta) = \theta\alpha'(\theta)/\alpha(\theta)$ is again the elasticity of the seller's matching rate.

In a symmetric equilibrium, we have that $\omega^* = \omega^c$ and $\theta = \Theta$, which implies the following equilibrium price

$$p^*(\Theta) = [1 - \varepsilon(\Theta)] \frac{u(q^*)}{q^*} + \varepsilon(\Theta) \frac{c(q^*)}{q^*}, \tag{2}$$

[11] In Rocheteau and Wright (2005), the terms of trade are posted by market makers. Here we assume sellers post the terms of trade and buyers form beliefs about the associated queue length θ.

[12] This is commonly known as the *market utility*, first used by Montgomery (1991) and McAfee (1993), subsequently by Shimer (1996), Moen (1997), Acemoglu and Shimer (1999), and is now standard in competitive search theory.

which is a convex combination of average utility and cost evaluated at q^*. Notice that if we rewrite the above pricing equation as

$$\varepsilon\left(\Theta\right)=\frac{u\left(q^*\right)-p^*q^*}{u\left(q^*\right)-c(q^*)}, \tag{3}$$

we recover the standard Hosios sharing rule. This always holds endogenously in any standard competitive search model when all terms of trade are committed ex-ante. The buyer's and the seller's expected payoffs are $B(p^*,q^*)\alpha(\Theta)/\Theta$ and $S(p^*,q^*)\alpha(\Theta)$, where their surpluses are given by $S(p^*,q^*)=p^*q^*-c(q^*)$ and $B(p^*,q^*)=u(q^*)-p^*q^*$, respectively.

The competitive search equilibrium is always surplus maximizing, but, depending on Θ, it could be that $p^*(\Theta)\lesseqqgtr u'(q^*)=c'(q^*)$. Notice that the equilibrium quantity does not depend on Θ. A sudden inflow of buyers leading to a larger Θ would result in a higher price given that $\varepsilon'\left(\Theta\right)<0$ and no change in quantity traded. *Under free entry, the equilibrium market tightness* Θ^* *is given by*

$$\alpha(\Theta^*)[p^*q^*-c(q^*)]=k.$$

Using (2), this becomes

$$\alpha(\Theta^*)\left[1-\varepsilon(\Theta^*)\right]\left[u(q^*)-c(q^*)\right]=k.$$

As we can see, under ex-ante commitment to all terms of trade, the resulting equilibrium allocation is constrained efficient, i.e., $(q^*,\Theta^*)=(q^e,\Theta^e)$.

3.3 Ex-ante Price Posting and Ex-post Quantity by Sellers

Prices are posted ex-ante and sellers optimally choose quantities ex-post after the matches have taken place. From observed prices, buyers decide which submarket to enter. To construct the equilibrium, we need to not only account for the optimal deviation in price, but also the deviating sellers' optimal ex-post reaction to their quantity choice given the ex-ante price.

Since the trading mechanism now has two stages, we solve for equilibrium backwards. We first solve for sellers' optimal choice of q given p, and then solve for the competitive search equilibrium choice of price p. *We first do so without entry, and subsequently allow free entry.*

Consider the *ex-post* problem where deviating sellers take posted per unit price p as given. *Upon meeting a buyer, they solve*

$$\max_q S(p,q) \text{ s.t. } B(p,q)\geq 0.$$

For interior solutions, the optimal quantity $\tilde{q}$ satisfies

$$p = c'(\tilde{q}) \text{ and } u(\tilde{q}) > p\tilde{q},$$

when the constraint is not binding, while *when it is binding*, the optimal quantity is given by

$$p = \frac{u(\tilde{q})}{\tilde{q}} \text{ and } c'(\tilde{q}) < p.$$

This yields a one-to-one relationship between prices and quantities $\tilde{q}(p)$.

Lemma 1 *In any competitive search equilibrium, given the posted price p, the optimal ex-post choice of quantity by sellers is given by $p = c'(\tilde{q})$.*

Taking as given the ex-post optimal choice $\tilde{q}$, we solve for the competitive search equilibrium price. To be consistent with the previous notation, let $B(p, \tilde{q}(p)) = u(\tilde{q}(p)) - p\tilde{q}(p) \equiv \tilde{B}(p)$ and $S((p, \tilde{q}(p)) = p\tilde{q}(p) - c(\tilde{q}(p)) \equiv \tilde{S}(p)$.

While other submarkets post the same p^c and Θ, the positive measure of deviating sellers solve

$$\max_{p,\theta} \alpha(\theta) \tilde{S}(p) \text{ s.t. } \frac{\alpha(\theta)}{\theta} \tilde{B}(p) \geq \bar{U}.$$

It is easy to show that the optimal solution satisfies

$$\frac{1 - \varepsilon(\Theta)}{\varepsilon(\Theta)} = -\frac{\tilde{B}'(p)}{\tilde{S}'(p)} \frac{\tilde{S}(p)}{\tilde{B}(p)}, \tag{4}$$

where $\varepsilon(\Theta)$ is as previously defined.

In a symmetric equilibrium, $p = p^c$ and $\theta = \Theta$, which implies $p(\Theta)$. From the previous problem, it is a bit more involved to show existence and uniqueness. Fortunately, we can change the problem by substituting for $p = c'(\tilde{q})$ instead and maximize *as if* sellers were choosing $\tilde{q}$ ex-ante.[13] To simplify the notation, let $\tilde{q} = q$ from now on. The problem for sellers then become

$$\max_{q,\theta} \alpha(\theta) S(q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(q) \geq \bar{U}, \tag{5}$$

where $B(q) = B(c'(q), q)$ and $S(q) = S(c'(q), q)$. The optimal solution $q(\Theta)$ satisfies

$$\frac{1 - \varepsilon(\Theta)}{\varepsilon(\Theta)} = -\frac{B'(q)}{S'(q)} \frac{S(q)}{B(q)}. \tag{6}$$

[13] Given the nature of the trading mechanism, sellers rationally anticipate that buyers participate in the submarket, knowing that sellers will choose q ex-post to maximize profit given the posted price that has attracted buyers in the first place.

Interestingly, the previous condition could be rewritten as

$$\varepsilon(\Theta) = \frac{\eta_s(q)}{\eta_s(q) + \eta_b(q)} \equiv \chi(q), \tag{7}$$

where $\eta_s(q) = qS'(q)/S(q)$ and $\eta_b(q) = -qB'(q)/B(q)$ *are the elasticities of surplus with respect to output for sellers and buyers, respectively.* This expression is a Hosios-like sharing rule expressed in surplus elasticities. This is the case as the quantity q is determined ex-post. It is important to highlight how sellers' optimal ex-post choice of q relates to the relative surpluses of buyers and sellers as well as the seller's contribution to the matching rate.

Proposition 1 For all $\Theta \in (0, \infty)$, if $qc'''/c'' < \eta_s(q) - 1$*, where* $\eta_s(q) = S'(q)q/S(q) > 1$*,* there exists a unique symmetric equilibrium with all sellers choosing $p(\Theta)$ ex-ante and $q(\Theta)$ ex-post, satisfying $p(\Theta) = c'(q(\Theta))$*, where* $q(\Theta) \in (\underline{q}, \bar{q})$*,* $B'(\underline{q}) = 0$ *and* $B(\bar{q}) = 0$*.* In addition, both $(p(\Theta), q(\Theta))$ are strictly increasing in Θ.

Posting prices ex-ante while quantities are determined ex-post by sellers always yields marginal cost pricing under the sufficient condition on the cost function, $qc'''/c'' < \eta_s(q) - 1$*. This condition simply reflects that the elasticity of the sellers' surplus with respect to q needs to be large enough relative to 1. Note that S is strictly convex, which implies that* $\eta_s(q) > 1$*. However, the surplus needs to be convex enough relative to the convexity of the cost function. These conditions are consistent with a wide range of cost functions, including* $c''' \leq 0$ *and* $c''' > 0$ *but not too big (not too convex). To understand why for any* $\Theta \in (0, \infty)$*,* $q(\Theta) \in (\underline{q}, \bar{q})$*, recall that by assumption* $\varepsilon'(\Theta) < 0$*. As we show in the proof in Appendix, for the right-hand side of (7),* $\chi(\underline{q}) = 1$ *and* $\chi(\bar{q}) = 0$*. Under the sufficient condition,* $\chi(q)$ *is monotone decreasing in q over the interval. By assumption as well, the matching function is constant returns to scale, which implies* $\varepsilon(\Theta) \in (0, 1)$ *for all* $\Theta \in (0, \infty)$*. The results of Proposition 1 are featuring in Figure 1.*

Comparing the allocation obtained under ex-ante price relative to the planner's problem, we note that for a particular value of Θ, say Θ^c, such that $q(\Theta^c) = q^e$ and

$$p(\Theta^c)q^e = c'(q^e)q^e = [1 - \varepsilon(\Theta^c)]\, u(q^e) + \varepsilon(\Theta^c)\, c(q^e), \tag{8}$$

the full commitment (planner's problem) and ex-ante pricing outcomes are equivalent. But this holds only for a very specific value of Θ^c. It is then a knife-edge condition.

Under free entry, the allocation is constrained efficient at the extensive margin if and only if (8) holds and

$$\alpha(\Theta^c)[c'(q^e)q^e - c(q^e)] = k^c,$$

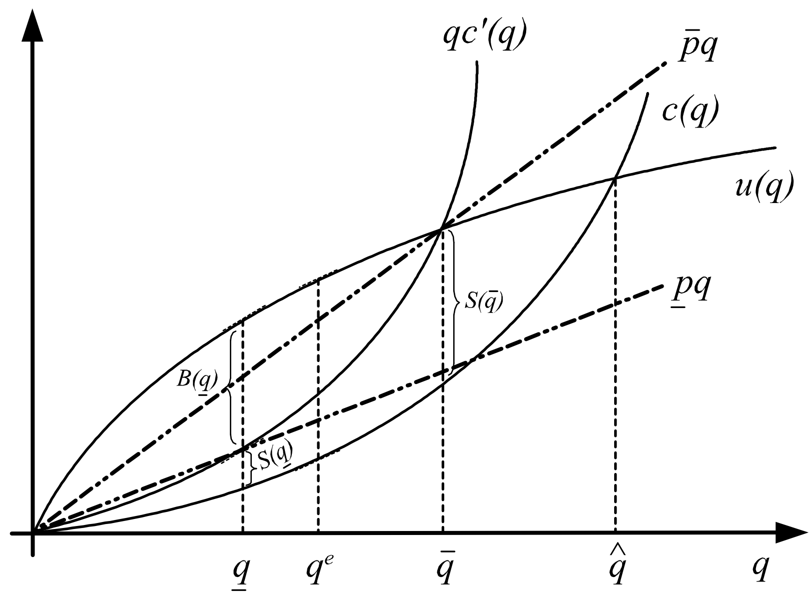

Figure 1: Case of $p = c'(q)$.

otherwise there is inefficient entry. Another way to view this result, is that given the unique efficient quantity q^e, there is a unique entry cost k^c that can generate Θ^c. These results are in sharp contrast to the standard competitive search equilibrium, where sellers post unit prices and quantities ex-ante as in Subsection 3.2 and constrained efficient quantity q^e and entry Θ^e are always achieved. *With ex-post quantity trading, efficiency is achieved only if Θ happens to lead to $u'(q(\Theta)) = p(\Theta) = c'(q(\Theta))$. The ability to commit ex-ante to all terms of trade is critical to obtain the constrained efficient outcome.*

Proposition 2 There exists a unique free entry equilibrium given an entry cost $k > 0$ for sellers. The equilibrium is generically not constrained efficient, at the intensive, q, or the extensive margin of entry, Θ.

The generic free entry condition is given by

$$\alpha(\Theta)[c'(q(\Theta))q(\Theta) - c(q(\Theta))] = k.$$

It is easy to show that the left-hand side is strictly increasing in $\Theta \in (0, \infty)$ for $q(\Theta) \in (\underline{q}, \bar{q})$. It is clear then that only for a very particular value of k^c, we have Θ^c such that $q(\Theta^c) = q^e$. Thus entry is efficient. Entry can be insufficient when $k > k^c \Rightarrow \Theta > \Theta^c$ and $q(\Theta) > q^e$, or excessive for costs such that $k < k^c \Rightarrow \Theta < \Theta^c$ and $q(\Theta) < q^e$. For low cost of entry, the equilibrium entails buyers' surplus being large (sellers' small), while for very large entry cost, buyers get their surplus *almost fully extracted by sellers. A low cost facilitates entry and tilts the bargaining power afforded by the market (via Θ) towards the buyers, and vice versa for a large cost.*

Exploring this "partial commitment" is interesting because it has many applications such as new real estate construction and labor market. The labor market suits this setup particularly well. Assume that a measure v of vacancies are to be matched with a measure u of unemployed, with $\Theta = v/u$ (thus firms are buyers and workers are sellers). The surplus from a match is $f(h)$ where h is the hours worked by workers upon a match. Let $c(h)$ be the cost for workers implementing h, and wh be the wage revenue paid from the firm to the worker. Consider setting up the competitive search problem as workers (sellers) competing by posting (w, θ) to attract firms, and then choose h ex-post.[14] This environment fits perfectly the above setup where $q \equiv h$, $u(q) \equiv f(h)$, $c(h) \equiv c(q)$, and $wh \equiv pq$. All results follow. Workers would set wages ex-ante and choose hours ex-post such that $w = c'(h)$. Only a particular value of Θ would result in $f'(h(\Theta)) = w(\Theta) = c'(h(\Theta))$, and so workers would be paid at their marginal product. Otherwise, for other values of Θ, workers could be paid above or below their marginal product. A sudden increase in Θ would lead to increase in both wage and hours worked. Compared to workers posting (w, h, θ), hours in equilibrium would always be determined by efficient hours $f'(h^e) = c'(h^e)$, independent of Θ, with only wage increasing in Θ.

3.4 Ex-ante Price Posting by Sellers and Ex-post Quantity by Buyers

In this section we consider the natural possibility that buyers instead of sellers choose quantity ex-post, while sellers still post prices ex-ante. This environment has even more applications than the one highlighted in the previous subsection.[15] *Given p, ex-post buyers solve*

$$\max_q u(q) - pq \text{ s.t. } pq - c(q) \geq 0.$$

For interior solutions, the optimal quantity $\tilde{q}$ satisfies

$$u'(\tilde{q}) = p \text{ and } p\tilde{q} > c(\tilde{q}),$$

[14] Note that for our results to apply, we need to have workers posting as in Julien, et al. (2000), and choosing their hours ex-post. Otherwise, one can have firms posting as in Burdett et al. (2001), and choosing hours ex-post.

[15] In terms of applications for this demand determined ex-post quantity, the labor market example with hours of work in the previous section also fits this environment. In this context, one can envision workers (sellers) posting wages ex-ante and firms (buyers) choosing hours h ex-post or firms (sellers) posting wages ex-ante and workers (buyers) choosing hours ex-post. Essentially, any environments in which prices are posted and quantity consumed are chosen by buyers and produced on demand is a good fit (e.g. recent trends of markets where prices are known by consumers but quantity or quality are made on order, Dell Inc. in retail and many wholesale markets).

when the constraint is not binding. When it is binding, the optimal quantity is given by

$$p = \frac{c(\tilde{q})}{\tilde{q}} \text{ and } u'(\tilde{q}) < p.$$

This yields a one-to-one relationship $\tilde{q}(p)$ with $\tilde{q}'(p) < 0$.

Lemma 2 In any competitive search equilibrium, given the posted price p, the optimal ex-post choice of quantity by buyers is given by $p = u'(\tilde{q})$.

This result is analogous to Lemma 1, the case of sellers choosing q ex-post, with the difference of marginal utility pricing instead of marginal cost pricing and $\tilde{q}'(p) < 0$ rather than being positive.

Proposition 3 For all $\Theta \in (0, \infty)$, if $qu'''/u'' < \eta_b(q) - 1$, where $\eta_b(q) = B'(q)q/B(q) > 1$, there exists a unique symmetric equilibrium with all sellers choosing $p(\Theta)$ ex-ante and buyers choosing $q(\Theta)$ ex-post, satisfying $p(\Theta) = u'(q(\Theta))$, where $q(\Theta) \in (\underline{q}, \bar{q})$, $S'(\underline{q}) = 0$ and $S(\bar{q}) = 0$. In addition, $p(\Theta)$ is strictly increasing in Θ while $q(\Theta)$ is strictly decreasing in Θ.

Posting prices ex-ante with quantities determined ex-post by buyers always yields marginal utility pricing as long as the utility function is such that $qu'''/u'' < \eta_b(q) - 1$. The elasticity of the buyers' surplus with respect to q needs to be large enough relative to 1. To understand the sufficient condition for uniqueness of equilibrium, note that B is strictly convex implying $\eta_b(q) > 1$, but the surplus needs to be convex enough relative to the concavity of the utility function.[16] *The above condition encompasses a wide range of utility functions, including $u''' \geq 0$ and $u''' < 0$ not too negative (not too concave). To understand why for any $\Theta \in (0, \infty)$, $q(\Theta) \in (\underline{q}, \bar{q})$, recall that by assumption $\varepsilon'(\Theta) < 0$. As we show in the proof in Appendix, for the right-hand side of (7), $\chi(\underline{q}) = 0$, $\chi(\bar{q}) = 1$, and under the sufficient condition, $\chi(q)$ is monotone increasing in q over the interval. By assumption again, the matching function is constant returns to scale which implies $\varepsilon(\Theta) \in (0, 1)$ for all $\Theta \in (0, \infty)$. The results of Proposition 3 are featuring in Figure 2.*

Comparing this equilibrium allocation to that of the planner's problem, we note that if for a particular value of Θ, say Θ^c, such that $q(\Theta^c) = q^e$ and

$$p(\Theta^c)q^e = u'(q^e)q^e = [1 - \varepsilon(\Theta^c)]\, u(q^e) + \varepsilon(\Theta^c)\, c(q^e), \tag{9}$$

the full (planner's problem) and partial commitment outcomes are equivalent. But this holds only for a very specific value of Θ^c.

[16] Note that here $\eta_b(q) > 0$ because when $p = u'(q)$, $B'(q) > 0$, unlike the case of $p = c'(q)$ and $B'(q) < 0$, so there we have defined $\eta_b(q) = -B'(q)q/B(q) > 0$.

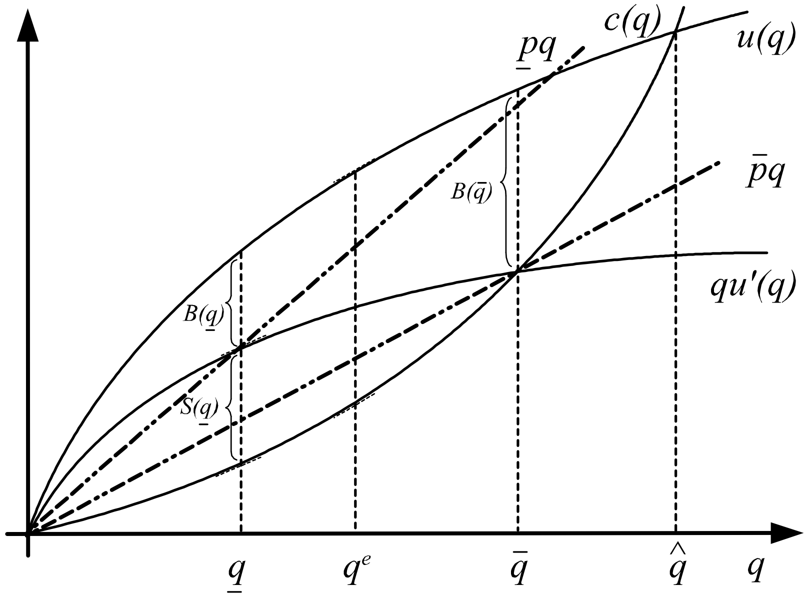

Figure 2: Case of $p = u'(q)$.

Under free entry, the allocation is constrained efficient at the extensive margin if and only if (9) holds and

$$\alpha(\Theta^c)[u'(q^e)q^e - c(q^e)] = k^c,$$

otherwise there is inefficient entry. Again here, given the unique efficient quantity q^e, there is a unique entry cost k^c that can generate Θ^c. These results echo the ones with marginal cost pricing that we found in the previous section.

Proposition 4 Given an entry cost $k > 0$, there exists a unique equilibrium that is generically not constrained efficient, at the intensive, q, or the extensive margin, Θ.

The generic free entry condition for this problem is given by

$$\alpha(\Theta)[u'(q(\Theta))q(\Theta) - c(q(\Theta))] = k.$$

It is easy to show that the left-hand side is strictly increasing in $\Theta \in (0, \infty)$ for $q(\Theta) \in (\underline{q}, \bar{q})$. Then only for a very particular value of k^c, leading to Θ^c such that $q(\Theta^c) = q^e$, entry is efficient. Otherwise, entry is insufficient, for $k > k^c \Rightarrow \Theta > \Theta^c$ and $q(\Theta) < q^e$, or excessive for $k < k^c \Rightarrow \Theta < \Theta^c$ and $q(\Theta) > q^e$. For low cost of entry, the equilibrium entails sellers' surplus being small (buyers large), while for very large entry cost, sellers get large surplus (buyers small), and this echoes the result of entry under marginal cost pricing in the previous section. However, there is *one important difference. Because in equilibrium we have $q'(p) < 0$, with marginal utility pricing, the direction of inefficiency is the reverse when comparing the two cases. Therefore, it matters if quantity is chosen ex-post by sellers (supply side) or by buyers (demand side).*

Since the environment we have considered is related to the one analyzed in Peters (1984), we offer a more detailed comparison. For a special case of his model, Peters (1984) considers an environment with price competition among firms when there are capacity constraints and buyers have limited ability to visit firms. Each firm selects, at the beginning of the game, a price which it will charge consumers and a rationing rule, which specifies how the firm will allocate output if its capacity constraint is binding. Knowing price and this rationing rule, consumers choose a shopping strategy. It is assumed that buyers are only able to visit a single store.

Peters (1984) considers a market with finite agents and his Proposition 3 establishes that as the number of sellers goes to infinity, keeping the number of buyers finite, the gain for sellers to deviate from average cost pricing is less than ϵ. Therefore, $p = c(q)/q$ is an approximate equilibrium. This is quite an intuitive result. Sending the number of sellers to infinity while keeping buyers finite drives the market tightness to zero. This situation emulates a perfectly competitive market. Then, it should be no surprise that equilibrium pricing gives zero profit to sellers.

In addition, Peters (1984) assumes an exogenous capacity K such that $c'(q) = \infty$, for all $q \geq K$. The ex-post demand by a buyer is $q = \min\{d(p), K\}$. This assumption of capacity was made in the spirit of the Edgeworth's model. Peters (1984) acknowledges that if the number of buyers and sellers grow large together, this may change his result.[17] Let us consider a similar capacity constraint K in our model. The optimal choice for buyers depends on capacity as $q = \min\{u'^{-1}(p), K\}$. Note that in the above analysis leading to Proposition 3, we assume that the capacity constraint $q \leq K$ is not binding, and buyers choose ex-post quantity such that $p = u'(q)$. In contrast to Peters (1984), we argue that this ex-post marginal utility pricing is indeed an equilibrium if the capacity constraint is not severe. Otherwise, the average cost pricing of Peters (1984) holds. If $\min\{u'^{-1}(p), K\} = K$, implying that $u'(q) > p$, the optimal buyer's ex-post choice is to consume at $q = K$, and hence $p = c(K)/K$. For severe capacity constraints, the result of Peters (1984) can hold in large market as both buyers and sellers are taken to infinity.

[17] If all firms are pricing at the average cost of producing their capacity, any single firm might be able to increase its profits by raising price. However, if price is raised to $p > c(q)/q$ so that $U(p, K) < \bar{U}$, the probability with which any buyer would visit the firm falls to zero, as would the firm's profits.

3.5 Ex-ante Quantity Posting and Ex-post Pricing by Sellers

In this section, we assume that sellers can post a quantity ex-ante and then choose the per unit price ex-post, after buyers choose which seller to visit. Working backwards, sellers take as given posted quantity ex-ante and solve

$$\max_{p} S(p,q) \text{ s.t. } B(p,q) \geq 0.$$

Sellers are able to extract all the surplus by pricing $p^* = u(q)/q = g(q)$. Note that the seller's pricing decision does not directly depend on θ. Differentiating this ex-post pricing decision, it is easy to show that

$$q\frac{dp^*}{dq} = u'(q) - p^*. \tag{10}$$

In a similar way as in the previous sections, we can transform the competitive search program using $p = g(q)$, with ex-ante sellers posting $\omega = (q,\theta) \in \mathbb{R}^2_+$, to solve

$$\max_{q,\theta} \alpha(\theta) S(g(q), q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(g(q), q) = 0. \tag{11}$$

That is, the market utility $\bar{U} = 0$, with solution

$$p^* + qdp^*/dq - c'(q) = 0.$$

Together with (10) implies an efficient q^e as $u'(q^e) = c'(q^e)$.

When sellers post quantities ex-ante and choose prices ex-post, they choose to post the efficient quantity to attract buyers, and then extract all the surplus from trade by adjusting prices ex-post. This is akin to the Diamond (1971) equilibrium. Buyers fully anticipate that the seller's best ex-post choice is to fully extract all of their surplus, which implies that $\bar{U} = 0$, *and thus buyers are indifferent between participating or not. This equilibrium would occur for even a non-zero value Z of outside option for buyers.*[18] *The equilibrium would entail $\bar{U} = Z$.* If sellers were given the choice of what to post ex-ante, no sellers would want to deviate from posting only q ex-ante.

The equilibrium entry is given by

$$\alpha(\Theta^*)[u(q^e) - c(q^e)] = k.$$

Compared to the planner' solution (1), $\Theta^* < \Theta^e$ and there is excessive entry of sellers. The result follows directly from sellers extracting all the surplus.

[18] The problem becomes $\max_p S(p,q)$ s.t. $B(p,q) \geq z$, with pricing $p^* = [u(q)-z]/q = g(q)$. From this, $q\partial p^*/\partial q = u'(q) - p^*$. Sellers take the ex-post pricing rule as given and solve $\max_{q,\theta} \alpha(\theta) S(g(q), q)$ s.t. $[\alpha(\theta)/\theta]B(g(q), q) = Z$, with solution $S'(g(q), q)g'(q) = p^* + q\partial p^*/\partial q - c'(q) = 0$, and the efficient q results.

Proposition 5 When sellers post quantities ex-ante and prices ex-post, a symmetric equilibrium exists where buyers get zero surplus and the constrained efficient allocation is implemented for the intensive margin (q), but entry is excessive.

Although this case is interesting, examples of such mechanisms are harder to find. Potentially, items advertised with undisclosed prices and where buyers are asked to request a quote seem to fit this model.

3.6 Ex-ante Quantity Posting by Sellers and Ex-post Pricing by Buyers

Finally, just for completeness, we consider an environment where sellers post quantity ex-ante, but let buyers choose what price to pay ex-post. Working backwards, buyers take as given posted quantity ex-ante and solve

$$\max_p B(p,q) \text{ s.t. } S(p,q) \geq 0.$$

Buyers are able to extract all the surplus by pricing $p^* = c(q)/q = h(q)$. Note that the buyers' pricing decision does not directly depend on θ. Differentiating this ex-post pricing decision, it is easy to show that

$$q\frac{dp^*}{dq} = c'(q) - p^*. \tag{12}$$

Similarly, we can transform the competitive search program using $p = h(q)$, with ex-ante sellers posting $\omega = (q,\theta) \in \mathbb{R}^2_+$, to solve

$$\max_{q,\theta} \alpha(\theta) S(h(q), q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(h(q), q) = \bar{U}.$$

The solution is

$$u'(q) - p^* - qdp^*/dq = 0.$$

Together with (12) implies an efficient q^e as $u'(q^e) = c'(q^e)$.

When sellers post quantities ex-ante and buyers choose prices ex-post, sellers choose to post the efficient quantity to attract buyers, knowing buyers will then extract all the surplus from trade by adjusting prices ex-post. If there is an entry cost $k > 0$, no sellers would be active in the market since $S(h(q), q) = 0 < k$. There is no equilibrium with an active market. Because k is a sunk cost, sellers are being held up entirely if buyers can choose the price ex-post. This may explain why such a mechanism is seldom in markets.

4 Conclusion

We consider a frictional market where buyers are uncoordinated and sellers cannot commit to both per unit price and quantity of a divisible good ex-ante. As in Kim and Kircher (2015), the choice of the trading mechanism is crucial in determining whether the equilibrium is constrained efficient or not. In particular, we find that when sellers post ex-ante prices, there exists a unique symmetric equilibrium with marginal cost pricing if sellers choose quantity ex-post, and with marginal utility pricing if buyers choose quantity ex-post instead. The equilibrium is generically not constrained efficient at the intensive or the extensive margin, and the extent of inefficiency in quantity are inversely related whether sellers or buyers make the ex-post choice. When sellers post ex-ante quantities and choose a price ex-post, in the unique symmetric equilibrium, buyers get their surplus extracted as in Diamond (1971) equilibrium. However, the efficient quantity is always produced but there is excessive entry. If sellers choose quantities ex-ante but buyers are able to choose a price ex-post, with positive entry cost, this generates a severe holdup problem and there is no equilibrium with active trade. All of our results hold for a wide range of utility and cost functions.

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Appendix

Proof of Lemma 1

Define $\bar{q}$ such that $u(\bar{q}) = \bar{q}c'(\bar{q})$ (i.e. $B(\bar{q}) = 0$) and $\bar{p} = c'(\bar{q})$. We summarize the seller's optimal ex-post choice as follows:

$$p = \begin{cases} c'(\tilde{q}) & \text{for } p \in (0, \bar{p}], \\ u(\tilde{q})/\tilde{q} & \text{for } p \in (\bar{p}, \infty). \end{cases}$$

It is important to highlight that both of these solutions imply a monotone relationship between quantity and price. Notice the following

$$\begin{aligned} p &\leq \bar{p} \Rightarrow \tilde{q}'(p) = 1/c''(\tilde{q}) > 0 \text{ (interior)}, \\ p &> \bar{p} \Rightarrow \tilde{q}'(p) = \frac{\tilde{q}^2}{\tilde{q}u'(\tilde{q}) - u(\tilde{q})} < 0 \text{ (binding)}. \end{aligned}$$

In a competitive search equilibrium with $\Theta \in (0, \infty)$, the positive measure of deviating sellers can choose a price that is either in $(0, \bar{p}]$ for an interior solution or in $(\bar{p}, \infty)$ under binding constraint. Define the first possible deviation as $p_1 = c'(\tilde{q}_1)$ and the second possible deviation as $p_2 = u(\tilde{q}_2)/\tilde{q}_2$. It is easy to show that the seller's expected payoff is

$$\pi_1 \equiv \alpha(\Theta)\left[\tilde{q}_1 c'(\tilde{q}_1) - c(\tilde{q}_1)\right] \text{ and } \pi_2 \equiv \alpha(\Theta)\left[\tilde{q}_2 u(\tilde{q}_2)/\tilde{q}_2 - c(\tilde{q}_2)\right],$$

while for buyers we have

$$U_1 \equiv \frac{\alpha(\Theta)}{\Theta}\left[u(\tilde{q}_1) - \tilde{q}_1 c'(\tilde{q}_1)\right] > 0 = \frac{\alpha(\Theta)}{\Theta}\left[u(\tilde{q}_2) - \tilde{q}_2 u(\tilde{q}_2)/\tilde{q}_2\right] \equiv U_2.$$

It is clear that $\tilde{B}(p_2) = 0$ holds when the buyers' surplus is fully extracted, and no buyers would participate in a deviating submarket that offers p_2 and $\tilde{q}_2$. Buyers fully anticipate that the best ex-post choice of sellers is to fully extract all of their surplus. It must be that any price as part of a competitive search equilibrium is $p \in (0, \bar{p}]$. The optimal ex-post choice is then $p_1 = c'(\tilde{q}_1) \in (0, \bar{p}]$. In other words, $\tilde{q}_1(p_1)$ is the equilibrium anticipated seller's response by buyers. ■

Proof of Proposition 1

To show existence, first, rewrite (7) as

$$\varepsilon(\Theta) = \frac{B(q)S'(q)}{B(q)S'(q) - B'(q)S(q)} \equiv \chi(q). \tag{13}$$

This condition characterizes a relationship $q(\Theta)$. According to Lemma 1, $p = c'(q)$. Since $B(q) = u(q) - c'(q)q$, we find $B(0) = 0 = S(0)$. Then, $\exists!\ \bar{q} > 0$ such that $B(\bar{q}) = 0$. In addition, $\exists!$

$\underline{q} \in (0, \bar{q})$ such that $B'(\underline{q}) = 0$. For $q \in (\underline{q}, \bar{q})$,

$$B'(q) = u'(q) - c'(q) - c''(q)q < 0,$$
$$B''(q) = u''(q) - 2c''(q) - c'''(q)q < 0 \text{ if } c'''(q) \geq 0 \text{ or } c'''(q) < 0 \text{ not too large.}$$

Thus, $B(q)$ is downward concave under these sufficient conditions. Similarly, we find that for $S(q) = c'(q)q - c(q)$,

$$S'(q) = c''(q)q > 0,$$
$$S''(q) = c''(q) + c'''(q)q > 0 \text{ if } c'''(q) \geq 0 \text{ or } c'''(q) < 0 \text{ not too large.}$$

Hence, $S(q)$ is upward convex under these sufficient conditions on the cost function. We also have that $\chi(\underline{q}) = 1$, and $\chi(\bar{q}) = 0$, with $\chi(q) > 0$, $\forall q \in [\underline{q}, \bar{q})$. Since we focus on matching technologies with constant returns to scale, it follows that $\varepsilon(\Theta) \in (0,1)$, $\forall \Theta \in (0, \infty)$. Therefore, given $\varepsilon(\Theta)$, $\exists\ q(\Theta) \in (\underline{q}, \bar{q})$ such that (13) holds and a symmetric equilibrium exists.

To prove uniqueness, we show sufficient conditions for $\chi'(q) < 0$, $\forall q \in (\underline{q}, \bar{q})$. Using (13), we find (omitting q as an argument)

$$\begin{aligned} \chi'(q) &= \frac{(B'S' + BS'')(BS' - B'S) - BS'(B'S' + BS'' - B''S - B'S')}{(BS' - B'S)^2} \\ &= \frac{-SS'(B'^2 - BB'') + BB'(S'^2 - SS'')}{(BS' - B'S)^2}. \end{aligned} \tag{14}$$

We have $B'(q) < 0$, $\forall q \in (\underline{q}, \bar{q})$, $S'(q) > 0$, $B''(q) < 0$ and $S''(q) > 0$, if $c'''(q) \geq 0$ or $c'''(q) < 0$ not too large by assumption. For $\chi(q)$ to be monotonically decreasing in q over $(\underline{q}, \bar{q})$, we need $S'^2 - SS'' > 0$. This condition holds if

$$\frac{c'''q}{c''} < \eta_s(q) - 1,$$

where $\eta_s(q) = S'(q)q/S(q) > 1$, due to the convexity of $S(q)$. Hence, we also need $c'''(q) \geq 0$, not too large. Therefore, as long as $c'''(q) < 0$ not too negative to preserve the properties of $B(q)$ and $S(q)$, and $c'''(q) \geq 0$ not too positive, we have $\chi'(q) < 0$, $\forall q \in (\underline{q}, \bar{q})$, and there exists a unique equilibrium $q \in (\underline{q}, \bar{q})$, $\forall \Theta \in (0, \infty)$.

Notice that we cannot have an equilibrium with $q \in (0, \underline{q})$. Since $B'(\underline{q}) = u'(\underline{q}) - c'(\underline{q}) - c''(\underline{q})\underline{q} = 0$, we have $u'(\underline{q}) > c'(\underline{q})$, $\underline{q} < q^e$, where $u'(q^e) = c'(q^e)$, and $B'(q) > 0$ over $(0, \underline{q})$. In any competitive search equilibrium, if $q \in (0, \underline{q})$, a positive measure of sellers can deviate and increase q, and increase buyers' surplus $B(q)$ while sellers' surplus $S(q)$ also increases.

On the other hand, for any competitive search equilibrium with $q \in (\bar{q}, \hat{q}]$, where $u(\hat{q}) = c(\hat{q})$, $B(q) < 0$ over this interval and $S(q) = 0$, since the implied queue length $\Theta = 0$. A positive measure of sellers can deviate and post $q_1 = \bar{q} - \epsilon$, attracting a positive measure of buyers by offering them $B(q_1) > 0$, and get positive surplus $S(q_1) > 0$. Hence, we cannot have $q \in (\bar{q}, \hat{q}]$ in any competitive search equilibrium.

We are left to show that for any $\Theta \in (0, \infty)$, neither $\underline{q}$ nor $\bar{q}$ is an equilibrium. Consider the Lagrangian of the competitive search program:

$$\mathcal{L} = \max_{q,\theta} \alpha(\theta)S(q) + \lambda[\frac{\alpha(\theta)}{\theta}B(q) - \bar{U}],$$

with necessary (and sufficient) conditions

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial q} &= \alpha(\theta)S'(q) + \lambda\frac{\alpha(\theta)}{\theta}B'(q) = 0, \\ \frac{\partial \mathcal{L}}{\partial \theta} &= \alpha'(\theta)S(q) + \lambda\left[\frac{\alpha'(\theta)\theta - \alpha(\theta)}{\theta}\right]B(q) = 0. \end{aligned}$$

These conditions lead to (13) and are valid for any $\theta = \Theta \in (0, \infty)$. Now transform the second condition as

$$\frac{\lambda}{\theta} = \frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{S(q)}{B(q)},$$

with $\varepsilon(\theta) = \frac{\alpha'(\theta)\theta}{\alpha(\theta)}$. Substituting into the first condition gives

$$\frac{\partial \mathcal{L}}{\partial q} = \alpha(\theta)S'(q) + \frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{S(q)}{B(q)}\alpha(\theta)B'(q).$$

Since $B'(\underline{q}) = 0$,

$$\frac{\partial \mathcal{L}}{\partial q}|_{q=\underline{q}} = \alpha(\theta)S'(\underline{q}) > 0,$$

and $B(\bar{q}) = 0$,

$$\frac{\partial \mathcal{L}}{\partial q}|_{q=\bar{q}} = -\infty < 0.$$

Therefore, neither $\underline{q}$ nor $\bar{q}$ is an equilibrium for $\theta = \Theta \in (0, \infty)$.

Finally, by assumption we have $\varepsilon'(\Theta) < 0$, and note as in the text that this holds for a large set of matching technologies. From (13), we have

$$\varepsilon'(\Theta)\, d\Theta = \chi'(q)\, dq \Rightarrow \frac{dq}{d\Theta} > 0 \Rightarrow \frac{dp}{d\Theta} = c''(q)\frac{dq}{d\Theta} > 0.$$

Therefore, any increase in Θ leads to higher q and p over $(\underline{q}, \bar{q})$. ■

Proof of Proposition 2

The proof follows from the text.

Proof of Lemma 2

Define $\bar{q}$ such that $c(\bar{q}) = \bar{q}u'(\bar{q})$ (i.e. $S(\bar{q}) = 0$) and $\bar{p} = u'(\bar{q})$. We summarize the buyers's optimal ex-post choice as follows:

$$p = \begin{cases} c(\tilde{q})/\tilde{q} & \text{for } p \in (0, \bar{p}), \\ u'(\tilde{q}) & \text{for } p \in [\bar{p}, \infty). \end{cases}$$

It is important to highlight that both of these solutions imply a monotone relationship between quantity and price. Notice the following

$$\begin{aligned} p &< \bar{p} \Rightarrow \tilde{q}'(p) = \frac{\tilde{q}^2}{\tilde{q}c'(\tilde{q}) - c(\tilde{q})} > 0 \text{ (binding)}, \\ p &> \bar{p} \Rightarrow \tilde{q}'(p) = 1/u''(\tilde{q}) < 0 \text{ (interior)}. \end{aligned}$$

In a competitive search equilibrium with $\Theta \in (0, \infty)$, the positive measure of deviating sellers can choose a price that is either in $[\bar{p}, \infty)$ for an interior solution or in $(0, \bar{p})$ under binding constraint. Define the first possible deviation as $p_1 = u'(\tilde{q}_1)$ and the second possible deviation as $p_2 = c(\tilde{q}_2)/\tilde{q}_2$. It is easy to show that the buyer's expected payoff is

$$U_1 \equiv \frac{\alpha(\Theta)}{\Theta}[u(\tilde{q}_1) - \tilde{q}_1 u'(\tilde{q}_1)] \text{ and } \frac{\alpha(\Theta)}{\Theta}[u(\tilde{q}_2) - \tilde{q}_2 c(\tilde{q}_2)/\tilde{q}_2] \equiv U_2,$$

while for sellers we have

$$\pi_1 \equiv \alpha(\Theta)[\tilde{q}_1 u'(\tilde{q}_1) - c(\tilde{q}_1)] > 0 = \pi_2 \equiv \alpha(\Theta)[\tilde{q}_2 c(\tilde{q}_2)/\tilde{q}_2 - c(\tilde{q}_2)].$$

It is clear that $\pi_2 = 0$ holds when the sellers' surplus is fully extracted, and no sellers would deviate and offer p_2 and $\tilde{q}_2$ in a submarket. Sellers fully anticipate that the best ex-post choice of buyers is to fully extract all of their surplus. It must be that any price as part of a competitive search equilibrium is $p \in [\bar{p}, \infty)$. The optimal ex-post choice is then $p_1 = c'(\tilde{q}_1) \in [\bar{p}, \infty)$. In other words, $\tilde{q}_1(p_1)$ is the equilibrium anticipated buyers's response by sellers. ■

Proof of Proposition 3

The proof is very similar to the one of Proposition 1. First, to show existence, we use again

$$\varepsilon(\Theta) = \frac{B(q)S'(q)}{B(q)S'(q) - B'(q)S(q)} \equiv \chi(q), \tag{15}$$

where under $p = u'(q)$, $S(q) = u'(q)q - c(q)$ and $B(q) = u(q) - u'(q)q$. $\exists!\ \underline{q} > 0$ such that $S'(\underline{q}) = 0$, which implies $u'(\underline{q}) - c'(\underline{q}) = -u''(\underline{q})\underline{q} > 0 \Rightarrow \underline{q} < q^e$ (the efficient quantity). Also $\exists!\ \bar{q} > 0$ such that $S(\bar{q}) = 0$. For $q \in (\underline{q}, \bar{q})$, we find (omitting q as an argument)

$$S' = u''q + u' - c' < 0,$$
$$S'' = 2u'' - c'' + u'''q < 0 \text{ if } u''' \leq 0 \text{ or } u''' > 0 \text{ not too large.}$$

Thus, $S(q)$ is downward concave under these sufficient conditions on the utility function. For buyers,

$$B' = -u''q > 0$$
$$B'' = -u'' - u'''q > 0 \text{ if } u''' \leq 0 \text{ or } u''' > 0 \text{ not too large.}$$

Hence, $B(q)$ is upward convex under the sufficient conditions on the utility function. We also have that $\chi(\underline{q}) = 0$ and $\chi(\bar{q}) = 1$, with $\chi(q) > 0$, $\forall q \in (\underline{q}, \bar{q})$. Again, since $\varepsilon(\Theta) \in (0, 1)$, $\forall \Theta \in (0, \infty)$, there exists a $q \in (\underline{q}, \bar{q})$ satisfying (15).

To prove uniqueness, we need $\chi'(q) > 0$, $\forall q \in (\underline{q}, \bar{q})$. Using (15) and the properties of $B(q)$ and $S(q)$, we can derive the following sufficient condition

$$\frac{u'''q}{u''} < \eta_b(q) - 1 \Rightarrow \left(B'^2 - B''B\right) > 0,$$

where $\eta_b(q) = B'(q)q/B(q) > 1$, due to the convexity of $B(q)$. Note that if u''' is negative while preserving the properties of $B(q)$ and $S(q)$ as above, or $u''' \geq 0$ but not too positive, this condition holds and we have $\chi'(q) > 0$, $\forall q \in (\underline{q}, \bar{q})$. Then, there exists a unique equilibrium $q \in (\underline{q}, \bar{q})$, $\forall \Theta \in (0, \infty)$.

Notice that we cannot have an equilibrium with $q \in (0, \underline{q})$. Since $S'(\underline{q}) = u'(\underline{q}) - c'(\underline{q}) + u''(\underline{q})\underline{q} = 0$, we have $u'(\underline{q}) > c'(\underline{q})$ and $\underline{q} < q^e$ where $u'(q^e) = c'(q^e)$, and $S'(q) > 0$ over this interval. In any competitive search equilibrium, if $q \in (0, \underline{q})$, a positive measure of buyers can increase q, and increase sellers' surplus $S(q)$ while buyers' surplus $B(q)$ also increases.

On the other hand, for any competitive search equilibrium with $q \in (\bar{q}, \hat{q}]$, where $u(\hat{q}) = c(\hat{q})$, $S(q) < 0$ over this interval and $B(q) = 0$, since sellers will not produce for negative surplus. A positive measure of sellers can deviate and post $q_1 = \bar{q} - \epsilon$, attracting a positive measure of buyers by offering them $B(q_1) > 0$, and get positive surplus $S(q_1) > 0$. Hence, we cannot have $q \in (\bar{q}, \hat{q}]$ in any competitive search equilibrium.

We are left to show that for any $\Theta \in (0, \infty)$, neither $\underline{q}$ nor $\bar{q}$ is an equilibrium. Consider the Lagrangian of the competitive search program:

$$\mathcal{L} = \max_{q,\theta} \alpha(\theta)S(q) + \lambda[\frac{\alpha(\theta)}{\theta}B(q) - \bar{U}],$$

with necessary (and sufficient) conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q} &= \alpha(\theta)S'(q) + \lambda\frac{\alpha(\theta)}{\theta}B'(q) = 0, \\
\frac{\partial \mathcal{L}}{\partial \theta} &= \alpha'(\theta)S(q) + \lambda\left[\frac{\alpha'(\theta)\theta - \alpha(\theta)}{\theta}\right]B(q) = 0.
\end{aligned}$$

These conditions lead to (15) and are valid for any $\theta = \Theta \in (0, \infty)$. As in the proof of Proposition 1, we can derive

$$\frac{\partial \mathcal{L}}{\partial q} = \alpha(\theta)S'(q) + \frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{S(q)}{B(q)}\alpha(\theta)B'(q).$$

Since $S'(\underline{q}) = 0$,

$$\frac{\partial \mathcal{L}}{\partial q}|_{q=\underline{q}} = \frac{\varepsilon(\theta)}{1-\varepsilon(\theta)}\frac{S(\underline{q})}{B(\underline{q})}\alpha(\theta)B'(\underline{q}) > 0,$$

and $S(\bar{q}) = 0$,

$$\frac{\partial \mathcal{L}}{\partial q}|_{q=\bar{q}} = \alpha(\theta)S'(\bar{q}) < 0.$$

Therefore, neither $\underline{q}$ nor $\bar{q}$ is an equilibrium when $\theta = \Theta \in (0, \infty)$.

Finally, by assumption we have $\varepsilon'(\Theta) < 0$. From (15)

$$\varepsilon'(\Theta)\, d\Theta = \chi'(q)\, dq \Rightarrow \frac{dq}{d\Theta} < 0 \Rightarrow \frac{dp}{d\Theta} = u''(q)\frac{dq}{d\Theta} > 0.$$

Therefore, any increase in Θ leads to lower q and higher p over $(\underline{q}, \bar{q})$. ■

Proof of Proposition 4

The proof follows from the text.

Proof of Proposition 5

The proof follows from the text.