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Money and Credit: Theory and Applications

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Abstract

We develop a theory of money and credit as competing payment instruments, then put it to work in applications. Buyers can use cash or credit, with the former (latter) subject to the inflation tax (transaction costs). Frictions making the choice of payment method interesting also imply equilibrium price dispersion, and together these deliver closed-form solutions for money demand. The model can simultaneously account for the price-change facts, share of credit in micro data, and money-interest correlations in macro data. We also analyze the effects of inflation on welfare, price dispersion, markups and participation, and describe nonstationary equilibria as self-fulfilling prophecies.

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1 Introduction

This paper develops a model of money and credit as competing payment instruments and puts it to work in applications. There is no doubt that this is a classic issue: as Lionel Robbins put it in his Introduction to von Mises (1953), "Of all branches of economic science, that part which relates to money and credit has probably the longest history and the most extensive literature." To bring this up to date, we work within a framework, sometimes called the New Monetarist approach, that endeavors to take the exchange and payment processes seriously (Section 2 reviews the literature). This involves describing environments where agents trade with each other, not only with their budget lines, in the presence of frictions, including spatial or temporal separation, limited commitment and imperfect information. To get both cash and credit in the model, we adopt the venerable idea that the former is subject to the inflation tax while the latter involves transaction costs.[1] We consider both fixed and variable transaction costs, which turn out to work rather differently – indeed, an unanticipated finding is that a variable cost of credit outperforms a fixed cost in terms of theory and matching data.

An important ingredient in our theory is what Burdett and Judd (1983) call "noisy" search, which means sellers post prices, and each buyer sees a random number of them. This leads to a distribution of prices $F(p)$ such that any p in the nondegenerate support yields the same profit; intuitively, lower-price sellers earn less per unit but make it up on the volume. We integrate this into the setting used by Lagos and Wright (2005), and many others, with alternating centralized and decentralized markets, which is natural because at the core of this environment is an asynchronization of expenditures and receipts crucial for any analysis of money or credit. In the centralized market agents consume, work, adjust cash

[1] One needs some such device to get both money and credit into general equilibrium in a nontrivial way. In a broad class of models, without transaction costs, Gu et al. (2016) prove the following: if credit conditions are loose money cannot be valued; if credit conditions are tight money can be valued, but then credit is not essential and changes in credit conditions are neutral. Transaction costs sometimes (not always) get around this result.

balances, settle debts and pay transaction costs. In the decentralized market they trade different commodities, as in Burdett-Judd, but with payment frictions: since buyers have no goods or services to offer by way of quid pro quo, they must use cash or credit. Given transaction costs, and consistent with conventional wisdom, they tend to use credit for large and cash for small expenditures.

This implies a simple demand for money and avoids an indeterminacy that plagues similar models, as discussed below. Moreover, it allows us to revisit several issues in a new light, including the effects of inflation on markups, price dispersion and welfare. It also allows us to expand on a conceptual point about nominal stickiness. On that, our sellers post prices in dollars, sensibly, since that is how buyers at least sometimes pay. As the money supply M increases, $F(p)$ shifts so that the real distribution stays the same, but as long as the supports overlap some firms can keep the same p. Thus prices look sticky, even though everyone is allowed to adjust whenever they like at no cost. For a seller that sticks to a nominal p when M rises, his real price falls, but the probability of a sale increases so that, in equilibrium, changing p is simply not profitable. While Head et al. (2012) and others mentioned below make a similar point, our setup avoids a serious technical problem in that approach. Also, while their model can match some features of price-change behavior quantitatively, we go beyond that by matching these features plus observations on payment methods in micro data and money demand (the relationship between real balances and interest rates) in macro data.[2]

In another application, we find small effects of inflation on welfare – with a variable transaction cost, e.g., eliminating inflation of $\pi = 10\%$ is worth only 0.23% of consumption. This is because in our baseline setting π does not affect the the size or number of trades, and welfare effects come from impinging on the cash-credit

[2] Head et al. (2012) have no credit, and hence cannot match the micro data, and although it is not reported, they do not match the macro money demand data at all well either. Earlier papers making similar theoretical points about sticky prices, like Caplin and Spulber (1987) or Eden (1994), do not take their models to the data. So, although we are not the first to conceptualize sticky prices in this way, the intended contribution in this application is quantitative.

margin. Yet even in an extension with endogenous participation, where π does affect output, its impact on welfare is smaller than in similar models. One reason is that we use posting instead of bargaining; another is that our agents can substitute out of cash and into credit. We also show the impact of π on markups and price dispersion is consistent with evidence. In other generalizations, we consider different specifications for the process by which buyers sample prices, and go beyond steady state by analyzing dynamic equilibria where inflation and deflation arise as self-fulfilling prophecies. The framework is tractable enough to get closed-form solutions for money demand reminiscent of Baumol-Tobin, with similar interpretations, but a general equilibrium approach arguably has advantages – e.g., how does one show inflation can be a self-fulfilling prophecy in partial equilibrium?

Quantitatively, a fixed-cost specification can match the standard money demand observations but not these plus the money and credit shares in the payment data. A proportional-cost specification can match both. Either specification is consistent the salient price-change facts, including long durations, large average changes, many small changes, many negative changes, a decreasing hazard, and adjustment behavior that depends on inflation. Although we match the price-change facts reasonably well, the fit is not perfect, mainly due to the discipline imposed by other observations; without this discipline – e.g., if we give up on money demand – we can match the price-change data virtually perfectly, but that seems too easy. We think that any theory attempting to match the price-change facts should also confront the other facts, since they all pertain to monetary phenomena, and all have implications for monetary policy. We strive to match all of these observations simultaneously.

The paper is organized as follows. Section 2 reviews the literature. Section 3 describes the model. Section 4 analyzes stationary equilibrium. Section 5 discusses calibration. Section 6-9 consider applications to price-change behavior, the cost of inflation, endogenous participation, and dynamics. Section 10 concludes.

2 Literature

There is much related work in several areas.[3] We do not spend much time on the New Monetarist approach, in general, since that is surveyed in Lagos et al. (2016), but do want to mention previous efforts to integrate Burdett and Judd (1983) into these kinds of models. Head et al. (2012) and Wang (2014) embed it in Lagos and Wright (2005), while Head and Kumar (2005) and Head et al. (2010) embed it in Shi (1997). There is, however, a technical problem with using any formulation with indivisible goods and price posting, as in Burdett-Judd, in a monetary economy: it leads to an indeterminacy (i.e., a continuum) of stationary equilibria.[4] The above-mentioned papers get around this by assuming divisible goods, but then another problem comes up – what should firms post? The papers assume linear menus, where sellers set p and let buyers choose any q as long as they pay pq, but this does not generally maximize profit. That seems like a serious problem in research that purports to be about microfoundations.

In an environment with costly credit, this indeterminacy is eliminated while maintaining indivisible goods, and thereby avoiding ad hoc assumptions like linear pricing. Intuitively, holding more cash reduces the amount of costly credit that buyers expect to use, which delivers a well-behaved money demand function and a unique monetary equilibrium. Note that we do not take a stand on whether divisible or indivisible goods are more realistic, but clearly indivisibility is an assumption about the physical environment, and as such is preferable to a restriction on pricing strategies. Also note that the indeterminacy in question concerns stationary equilibria, not dynamic equilibria, which are indeterminate in most models of money and credit, including this one. This is discussed in Section 9. There we also make

[3] Although it is important to view the contribution in the context of the literature, one can skip this for now, and jump to the theory in Section 3, without loss of continuity.

[4] This comes up in a series of papers spawned by Green and Zhou (1998). See Jean et al. (2010) for citations and further discussion, but here is a simple version of the problem: If all sellers post p then buyers' best response is to bring $m = p$ dollars to the market as long as p is not too high. If all buyers bring m then sellers' best response is post $p = m$ as long as m is not too low. Hence, any $p = m$ in some range constitutes a monetary equilibria.

contact with other theories of credit, including Kiyotaki and Moore (1997), Gu et al. (2013) and references therein.

Despite technical differences, we share with Head et al. (2012) the goal of analyzing pricing without imposing menu costs (e.g., Mankiw 1985), letting sellers only change at exogenous points in time (e.g., Taylor 1980; Calvo 1983), or assuming inattention (e.g., Woodford 2002; Sims 2003). While those approaches are interesting, we want to see how far we can go without them.[5] Caplin and Spulber (1987) and Eden (1994) take a similar approach to sticky prices, but do not confront the data, and do not adopt microfoundations, the way we do. We take this approach because work with similar microfoundations has proved fruitful in other contexts. Compared to papers that adopt similar theory and confront the data, as mentioned, we go further by trying to simultaneously account for price-change behavior, payment methods in micro data, and money demand in macro data.

There is much purely empirical work on price adjustment. Campbell and Eden (2014) find in grocery-store data an average duration between price changes of about 10 weeks, but we do not want to focus exclusively on groceries. Bils and Klenow (2004) find in BLS data at least half of prices last less than 4.3 months, or 5.5 months if one excludes sales. Klenow and Kryvtsov (2008) report durations from 6.8 to 10.4 months, while Nakamura and Steinsson (2008) report 8 to 11 months, excluding substitutions and sales. These papers also find large fractions of small and negative price changes, plus some evidence of a decreasing hazard. Eichenbaum et al. (2011) report a duration of 11 months for reference prices (those most often quoted in a quarter). Cecchetti (1986) finds durations for magazine prices from 1.8 months to 14 years, while Carlton (1986) finds durations for wholesale prices from 5.9 to 19.2 months. Other empirical work is surveyed by Klenow and Malin (2010). We provide a summary of the findings in the Appendix C.

[5] Burdett and Menzio (2016) combine search as in our model with menu costs, making the analysis much more difficult, even without money. Other nonmonetary search models with menu costs include Benabou (1988,1992a) and Diamond (1993).

One issue emphasized in the pricing literature is that average price changes are fairly big, suggesting high menu costs, if one wants to take a menu cost approach. However, there are also many small changes, suggesting low menu costs. Midrigan (2011) explains this with firms selling multiple goods, where paying a cost to change one price lets them change the rest for free. See also Vavra (2014). This is nice, but we are still interested in alternatives. Our model accounts for realistic durations, large average changes, many small and negative changes, and repricing behavior that depends on inflation. It can also yield a decreasing hazard, which is problematic for some other approaches (Nakamura and Steinsson 2008), and generate price dispersion at low or zero inflation, consistent with evidence but not some other models (Campbell and Eden 2014). These findings suggest that search-based theories should be part of the conversation on price stickiness.[6]

On the cost of inflation, see Lucas (2000) or Cooley (1995) for discussions based on money-in-the-utility-function or cash-in-advance models, where eliminating an annual inflation of $\pi = 0.10$ is worth around 0.5% of consumption. We cannot review all this work in detail, but mention Dotsey and Ireland (1996) and Aiyagari et al. (1998) as particularly related in their concern for transaction costs. Lagos et al. (2016) discuss search-and-bargaining models that get a costs closer to 5.0%. Our benchmark findings are much smaller, for reasons explained below. On inflation and price dispersion, empirical findings are mixed: Parsley (1996) and Debelle and Lamont (1997) find a positive relation; Reinsdorf (1994) finds a negative relation; Caglayana et al. (2008) find a U-shaped relation. On markups and inflation, a standard reference is Benabou (1992b), who reports a small but significant negative relationship. Benabou (1992a) and Head and Kumar (2005) explain this

[6] In discussions with people in the area, we found there is more or less agreement that these are the stylized facts: (1) Prices change slowly, but exact durations vary across studies. (2) The frequency and size of changes vary across goods. (3) Two sellers changing at the same time usually do not pick the same $\hat{p}$. (4) Many changes are negative. (5) Hazards decline slightly with duration. (6) There are many small (beow 5%) and many big (above 20%) changes. (7) The frequency and size of changes, and fraction of negative changes, vary with inflation. (8) There is price dispersion even at low inflation. Our model is consistent with all these.

by inflation increasing dispersion and thus search effort. Here, inflation decreases markups by directly affecting the cash-credit choice.

Concerning money demand, we deliver exact solutions similar to well-known results by Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966). The economic intuition is similar, too, involving a comparison between the opportunity cost of holding cash and the cost of tapping financial services. But those papers involve partial-equilibrium analyses, or, more accurately, decision-theoretic analyses of how to manage one's money given that only money can be used for payments. While such models are still being used to good effect (e.g., Alvarez and Lippi 2009,2014), we like our structure because it is easy to integrate with standard macro, and because it allows us to investigate general equilibrium issues, like the emergence of inflation or deflation as a self-fulfilling prophecy. Incorporating price posting and costly credit is part of ongoing efforts to better understand this workhorse framework in modern monetary economics.

On modeling money and credit, one approach follows Lucas and Stokey (1987) by simply assuming some goods require cash and others allow credit. Papers that let individuals choose subject to a cost of credit include Prescott (1987), Freeman and Huffman (1991), Chatterjee and Corbae (1992), Lacker and Schreft (1996) and Freeman and Kydland (2000). See Nosal and Rocheteau (2011) for a general discussion; see Gomis-Porqueras and Sanches (2013), Li and Li (2013), and Lotz and Zhang (2015) for more recent work closer in spirit to our approach. Various interpretations for these transaction costs are possible, including resources used up in record keeping, screening, enforcement, etc. Another is that the cost of credit is a tax that can be avoided by using cash (e.g., Gomis-Porqueras et al. 2014). Yet another is that credit requires resources for monitoring.[7]

[7] Monitoring is a critical element in many theoretical analysis of money and credit. As Wallace (2013) says: "If we want both monetary trade and credit in the same model, we need something between perfect monitoring and no monitoring. As in other areas of economics... extreme versions are both easy to describe and easy to analyze. The challenge is to specify and analyze intermediate situations." Here monitoring is available, but costly, which is not expecially 'deep' but is useful in applications. Araujo and Hu (2014) try to go 'deeper' using a mechanism design approach.

Finally, we mention heterogeneity. As is well known in other applications of Burdett-Judd, including the labor literature following Burdett and Mortensen (1998), if firms are homogeneous then theory does not pin down which one charges which p, only the distribution $F(p)$. With heterogeneity, however, lower-cost firms prefer lower p since they prefer higher volume. Still, for any subset of sellers with the same marginal cost, theory does not pin down which one posts which p. This is especially relevant for retail, where marginal cost is the wholesale price. Even if a few retailers get better deals, like quantity discounts, many others face the same wholesale price, making them homogeneous for our purposes. This bears on our discussion of sticky prices; it is unimportant for the other applications.

3 Environment

Each period in discrete time has two subperiods: first there is a decentralized market, called BJ for Burdett-Judd; then there is a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms (retailers) with measure 1, and a set of households with measure $\bar{b}$. Households consume a divisible good x_t and supply labor ℓ_t in AD, while in BJ they consume an indivisible good y_t produced by firms at unit cost $\gamma \geq 0$. Agents in the BJ market can use credit iff they access a technology to authenticate identity and record transactions at a cost. By incurring the cost, they can get BJ goods in exchange for commitments to deliver d_t dollars in the next AD; otherwise they need cash at the point of sale. We consider both a fixed cost δ and a proportional cost τ. Thus, the transaction cost is $C(d_t) = \delta \mathbf{1}(d_t) + \tau d_t$, where $\mathbf{1}(d_t)$ is an indicator function that is 1 iff $d_t > 0$. The transaction cost is paid by buyers, but for many purposes it is equivalent to let sellers pay, as in elementary tax-incidence theory.[8]

Household utility within a period is $U(x_t) + \mu \mathbf{1}(y_t) - \ell_t$, where $U'(x_t) > 0 > U''(x_t)$, $\mu > \gamma + \delta$ and $\mathbf{1}(y_t)$ is an indicator function. Let $\beta = 1/(1+r)$, with

[8] There is a caveat: when it is paid by sellers (e.g., as with credit cards), in principle they may want to post different prices for cash and credit. Understanding this is left for future research.

$r > 0$, be a discount factor between AD today and BJ tomorrow; any discounting between BJ and AD can be subsumed in the notation. We impose $\pi > \beta - 1$, where in stationary equilibrium π is the inflation rate, and the nominal interest rate is given by the Fisher equation $1 + i = (1 + \pi)(1 + r)$. Note that $\pi > \beta - 1$ implies $i > 0$, and the Friedman rule is the limiting case $i \to 0$. As usual, $1 + i$ is the amount of money agents require in the next AD market to give up a dollar in the current AD market, and whether or not such trades occur in equilibrium they can be priced. Let x_t be the AD numeraire, and assume it is produced one-for-one with ℓ_t, so the real wage is 1. The AD price of money in numeraire is ϕ_t, so $1/\phi_t$ is the nominal price level.

All agents enter the BJ market for free for now (later we introduce a cost). Each firm in BJ maximizes profit by posting a price, taking as given the CDF of other firms' prices, $F_t(p)$, with support $\mathcal{F}_t$. Every period each household in BJ randomly samples n firms – i.e., sees n independent draws from $F_t(p)$ – with probability α_n. For many purposes it suffices to have α_1, $\alpha_2 > 0$ and $\alpha_n = 0$ $\forall n \geq 3$, but this is generalized in Section 4.3. The money supply per buyer evolves according to $M_{t+1} = (1 + \pi) M_t$, with changes implemented in AD via lump-sum taxes if $\pi > 0$ or transfers if $\pi < 0$, although most results are the same if instead government uses seigniorage to buy AD goods.

3.1 Firm Problem

Using the specification with α_1, $\alpha_2 > 0$ and $\alpha_n = 0$ $\forall n \geq 3$, for now, we write expected real profit for a firm posting p at date t as

$$\Pi_t(p) = b_t \left[\alpha_1 + 2\alpha_2 \hat{F}_t(p_t)\right](p\phi_t - \gamma), \tag{1}$$

where $\hat{F}_t(p) \equiv 1 - F_t(p)$. Thus, net revenue per unit is $p\phi_t - \gamma$, and the number of units is determined as follows: The probability a household contacts this firm and no other is α_1. Then the firm makes a sale for sure. The probability a household contacts this firm plus another is $2\alpha_2$, as it can happen in two ways, this one first

and the other one second, or vice versa. Then the firm makes a sale iff it beats the other firm's price, which happens with probability $\hat{F}_t(p)$. This is all multiplied by tightness b_t to convert buyer probabilities into seller probabilities.

Profit maximization means every $p \in \mathcal{F}_t$ yields the same profit. As is standard in these models, $F_t(p)$ is continuous and $\mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$ is an interval.[9] Taking as given for now $\bar{p}_t$, and anticipating that $\bar{p}_t$ is not too high, so buyers do not reject it, $\forall p \in \mathcal{F}_t$ profit from p must equal profit from $\bar{p}_t$, which is

$$\Pi_t(\bar{p}_t) = b_t \alpha_1 (\bar{p}_t \phi_t - \gamma), \tag{2}$$

because the highest price firm never beats the competition. Equating (1) to (2) and rearranging immediately yields the equilibrium price distribution:

Lemma 1 $\forall p \in \mathcal{F}_t = [\underline{p}_t, \bar{p}_t]$

$$F_t(p) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\phi_t \bar{p}_t - \phi_t p}{\phi_t p - \gamma}. \tag{3}$$

It is easy to check $F_t'(p) > 0$ and $F_t''(p) < 0$. Also, using $F(\underline{p}_t) = 0$ we get

$$\underline{p}_t = \frac{\alpha_1 \phi_t \bar{p}_t + 2\alpha_2 \gamma}{\phi_t (\alpha_1 + 2\alpha_2)}. \tag{4}$$

To translate from dollars to numeraire, let $q_t = \phi_t p_t$ and write the real price distribution as

$$G_t(q) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\bar{q}_t - q}{q - \gamma}. \tag{5}$$

We denote its support by $\mathcal{G}_t = [\underline{q}_t, \bar{q}_t]$, and let $\hat{G}_t(q_t) \equiv 1 - G_t(q_t)$.

3.2 Household Problem

Consider a stationary equilibrium, where real variables are constant and nominal variables grow at rate π. Framing the household problem in real terms, the state

[9] There cannot be a mass of firms with the same p because any one of them would have a profitable deviation to $p - \varepsilon$, since they lose only ε per unit and make discretely more sales by undercutting others at p. Also, if there were a gap between p_1 and $p_2 > p_1$, a firm posting p_1 can deviate to $p_1 + \varepsilon$ and earn more per unit without losing sales.

variable in AD is net worth, $A = \phi m - d - C(d) + I$, where ϕm and d are real money balances and real debt carried over from the previous BJ market, $C(d)$ is the transaction cost of using credit, and I is any other income. Generally, I includes transfers net of taxes, plus profit, since as in standard general equilibrium theory the firms are are owned by the households (this plays little role, except for making our welfare criterion unambiguous). All debt is settled in AD, so that households start BJ with a clean slate; they could roll over d from one AD market to the next at interest rate r, but since preferences are linear in ℓ, there is no point. Hence the state variable in BJ is simply real balances, z.

The AD and BJ value functions are $W(A)$ and $V(z)$. These satisfy

$$W(A) = \max_{x,\ell,z} \{U(x) - \ell + \beta V(z)\} \text{ st } x = A + \ell - (1+\pi) z, \tag{6}$$

where the cost of real balances z next period is $(1+\pi) z$ in terms of numeraire this period. Eliminating ℓ and letting x^* solve $U'(x^*) = 1$, after rearranging, we get

$$W(A) = A + U(x^*) - x^* + \beta \max_z O_i(z) \tag{7}$$

where the objective function for the choice of z is $O_i(z) \equiv V(z) - (1+i) z$, with i given by the Fisher equation. As in is standard in Lagos-Wright models, we have (Appendix A contains all nonobvious proofs):

Lemma 2 *$W'(A) = 1$ and the choice of z does not depend on A.*

The BJ value function satisfies

$$V(z) = W(z+I) + (\alpha_1 + \alpha_2) \left[\mu - \mathbb{E}_H q - \delta \hat{H}(z) - \tau \mathbb{E}_H \max(0, q-z)\right]. \tag{8}$$

In (8), $\hat{H}(q) \equiv 1 - H(q)$, and $H(q)$ is the CDF of transaction prices,

$$H(q) = \frac{\alpha_1 G(q) + \alpha_2 \left[1 - \hat{G}(q)^2\right]}{\alpha_1 + \alpha_2}. \tag{9}$$

Notice $H(q)$ differs from the CDF of posted prices $G(q)$, because a buyer seeing multiple draws of q obviously picks the lowest. Also, notice the costs δ and $\tau(q-z)$ are paid iff $q > z$. Therefore, in terms of simple economics, the benefit of higher z is that it reduces the expected cost of having to tap credit.

4 Equilibrium

The above discussion characterizes behavior given $\bar{q}$, which will be determined presently. First we have these definitions:

Definition 1 *A stationary equilibrium is a list* $\langle G(q), z\rangle$ *such that: given* $G(q)$, z *solves the household's problem; and given* z, $G(q)$ *solves the firm's problem with* $\bar{q}$ *determined as in Lemma 3 below.*

Definition 2 *A nonmonetary equilibrium, or NME, has* $z = 0$, *so all BJ trades use credit. A mixed monetary equilibrium, or MME, has* $0 < z < \bar{q}$, *so BJ trades use cash for* $q \leq z$ *and credit for* $q > z$. *A pure monetary equilibrium, or PME, has* $z \geq \bar{q}$, *so all BJ trades use cash.*

Other variables, like x and ℓ, can be computed, but are not needed in what follows. Also, notice in NME prices must be described in numeraire q, while in MME or PME they can equivalently be described in numeraire or dollars.

The next step is to describe $\bar{q}$. The following results are proved in the Appendix except where obvious:

Lemma 3 *In NME,* $z = 0$ *and* $\bar{q} = (\mu - \delta)/(1 + \tau)$. *In MME,* $z \in (0, \mu - \delta)$ *and* $\bar{q} = (\mu - \delta + \tau z)/(1 + \tau)$. *In PME,* $\bar{q} = z \geq \mu - \delta$.

Lemma 4 *In MME,* $O_i(z)$ *is continuous. It is smooth and strictly concave* $\forall z \in (\underline{q}, \bar{q})$, *and linear* $\forall z \notin (\underline{q}, \bar{q})$.

4.1 Fixed Cost

Consider first $\tau = 0 < \delta$. Given $\delta < \mu - \gamma$, there is a nonmonetary equilibrium where all BJ transactions use credit. We are more interested in monetary equilibrium. As shown in Figure 1, as a special case of Lemma 4, the objective function $O_i(z)$ is linear $\forall z \notin (\underline{q}, \bar{q})$ with slope $O_i'(z) = -i < 0$, since real balances only

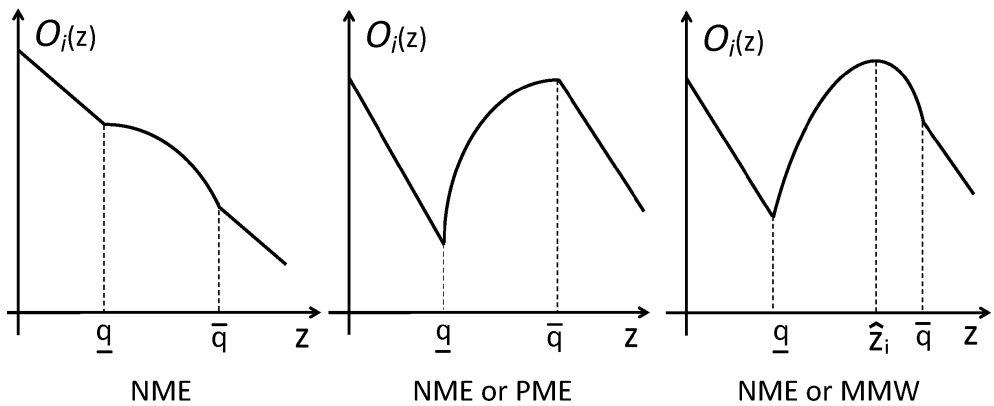


Figure 1: Possible Equilibria with Fixed Cost

reduce the expected cost of credit when $z \in \mathcal{G}_t$. It is also easy to check $O_i''(z) < 0$ $\forall z \in (\underline{q}, \bar{q})$.

These results imply $\exists!\, z_i = \arg\max_{z \in [\underline{q}, \bar{q}]} O_i(z)$. If $z_i \in (\underline{q}, \bar{q})$, as required for MME, it satisfies the FOC

$$(\alpha_1 + \alpha_2)\, \delta H'(z_i) = i. \tag{10}$$

To check $z_i \in (\underline{q}, \bar{q})$, let $\hat{z}_i$ be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. If $O_i^+(\underline{q}) \leq 0$ then $\hat{z}_i = 0$, as in the left panel of Figure 1. If $O_i^+(\underline{q}) > 0$ then we need to check $O_i^-(\bar{q})$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i \in (\underline{q}, \bar{q})$, as in the right panel. As a result (see the Appendix for the proof), we have the following:

Proposition 1 *In the fixed-cost model:*

(i) $\exists!$ *NME;*

(ii) $\exists!$ *MME iff* $\delta < \bar{\delta}$ *and* $i \in (\underline{i}, \bar{\imath})$*;*

(iii) $\exists$ *PME iff either* $\bar{\delta} < \delta < \mu - \gamma$ *and* $i < \hat{\imath}$*, or* $\delta < \bar{\delta}$ *and* $i < \underline{i}$*;*

where the thresholds satisfy $\bar{\imath} \in (\underline{i}, \infty)$*,*

$$\underline{i} = \frac{\delta \alpha_1^2}{2\alpha_2 (\mu - \delta - \gamma)} \text{ and } \bar{\delta} = \mu - \frac{\gamma (2\alpha_2^2 + 2\alpha_1\alpha_2)}{2\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2}.$$

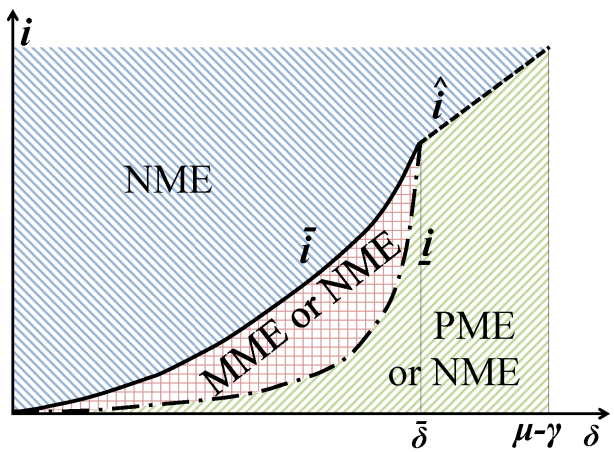


Figure 2: Equilibria in Parameter Space with Fixed Cost

This is illustrated in Figure 2. Notice that money (credit) may be used iff the nominal rate i (transaction cost δ) is not too high. Also notice that there is a continuum of PME when they exist (for reasons discussed in fn. 4). One of the main benefits of costly credit is that we get uniqueness of the MME, which is our main object of interest. When MME exists, it is easy to insert $G(q)$ into (10) and rearrange to get the explicit solution for money demand – i.e., for real balances as a function of the nominal rate i,

$$\hat{z}_i = \gamma + \left[\alpha_1^2 \delta \left(\mu - \delta - \gamma\right)^2 / 2\alpha_2\right]^{1/3} i^{-1/3}. \tag{11}$$

The closed-form in (11) is reminiscent of the famous square-root rule in Baumol (1952) and Tobin (1956), or cube-root rule in Miller and Orr (1966) and Whalen (1966). In those models, the usual story has an agent sequentially incurring expenses requiring currency, with a fixed cost of rebalancing. The decision rule compares i, the opportunity cost of cash, with the benefit of reducing the number of financial transactions interpretable as trips to the bank. Our buyers make at most one transaction in before rebalancing z, but its size is random. Still, they compare i with the benefit of reducing the use of financial services, again interpretable as trips to the bank, although one might say they now go there to get a loan and not to make a withdrawal. While we do not model banking explicitly –

see Berentsen et al. (2007) or Chiu and Meh (2011) on banks in a similar model – it is still useful heuristically, as in standard textbook discussions of Baumol-Tobin.

4.2 Variable Cost

Now consider $\tau > 0 = \delta$. This is in many respects easier, and avoids a technical issue with fixed costs that we waited until now to raise: In economies with non-convexities, like fixed costs, it can be desirable to let agents trade using lotteries.[10] One might try to argue that lotteries are infeasible, or unrealistic, but that seems awkward. Still, we do not analyze lotteries because the setup with a variable cost actually works better, and it has no role for lotteries. The main reason for covering fixed costs at all is that they are used in some of the models discussed in Section 2, models that are also subject to issue that they ignore a potential role for lotteries.

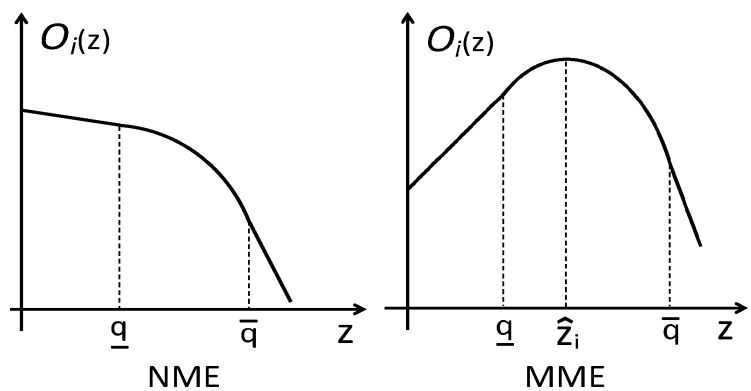


Figure 3: Possible Equilibria with Variable Cost

The price distribution emerging from the firm's problem is similar to the fixed-cost model, and in particular,

$$\bar{q} = \frac{\mu + z\tau}{1+\tau} \text{ and } \underline{q} = \frac{\alpha_1 (\mu + z\tau) + 2\alpha_2 \gamma (1+\tau)}{(\alpha_1 + 2\alpha_2)(1+\tau)}.$$

[10] See Berentsen et al. (2002) for an analysis in related monetary models. The idea would be for a seller to post: "you get my good for sure if you pay p; if you pay $\tilde{p} < p$ then you get my good with probability $P = P(\tilde{p})$." In Section 4.1, when a buyer with $m = p - \varepsilon$ meets a seller posting p, he pays $p - \varepsilon$ in cash, ε in credit and δ in fixed costs; if ε is small, both parties would prefer to trade using cash only, to avoid δ, and have the good delivered with probability $P < 1$.

It is easy to check that $O_i(z)$ is now differentiable everywhere, including $q = \bar{q}$ and $q = \underline{q}$. As Figure 3 shows, this means there are only two possible types of equilibria: if $i > (\alpha_1 + \alpha_2)\tau$ there is a unique NME; if $i < (\alpha_1 + \alpha_2)\tau$ there is a unique NME and a unique MME. Thus, PME never exist, because buyers are always willing to use credit with some probability. It turns out that this is helpful quantitatively – for reasonable parameters it is hard to get a MME in the fixed-cost model, because when δ is moderately high the economy abandons credit and switches to PME, something that never happens in the variable-cost model.

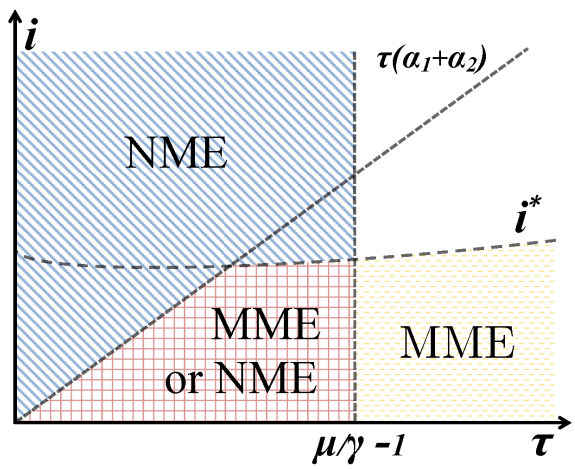


Figure 4: Equilibria in Parameter Space with Variable Cost

Proposition 2 *In the variable-cost model*

(i) $\exists!$ *NME iff* $\tau \leq \mu/\gamma - 1$*;*

(ii) $\exists!$ *MME iff* $i < \min\{\tau(\alpha_1 + \alpha_2), i^*\}$*;*

(iii) $\nexists$ *PME;*

where $i^* = i^*(\tau)$ *is the nominal rate that drives buyers' payoff to* 0*.*

As Figure 4 illustrates, MME exists for any value of $\tau > 0$ as long as i is not too big. Also, from (10) we again get a closed-form money demand function,

$$\hat{z}_i = \gamma + \frac{(\mu - \gamma)\left[\tau + (1+\tau)\sqrt{1 + 4\alpha_2 i/\alpha_1^2\tau}\right]}{1 + 2\tau + 4\alpha_2(1+\tau)^2 i/\alpha_1^2\tau}. \tag{12}$$

While this may be slightly more complicated than the fixed-cost version, it is still very tractable, and has a similar heuristic interpretation in terms of trips to the bank. As shown below, both specifications do well at fitting the money demand data, although the variable-cost model matches some other facts better.

4.3 Generalized Sampling Distributions

Here we consider alternative specifications for the probability that a household randomly samples n prices. The results are summarized and the arguments only sketched, but details are provided in Appendix A.[11]

Let N be the maximum number of prices a household can sample, which could be $N = \infty$; the baseline model has $N = 2$. Now for a firm posting any p profit is

$$\Pi_t(p) = b_t(p\phi_t - \gamma) \sum_{n=1}^{N} \alpha_n n \hat{F}_t(p_t)^{n-1}, \tag{13}$$

while for one posting $\bar{p}_t$ profit is given by (2). Using $F(\underline{p}_t) = 0$, we get

$$\underline{p}_t = \frac{\gamma}{\phi_t} + \frac{\alpha_1(\phi_t \bar{p}_t - \gamma)}{\phi_t \sum_{n=1}^{N} \alpha_n n}.$$

By virtue of the equal profit condition, $\forall p \in [\underline{p}_t, \bar{p}_t]$,

$$(p\phi_t - \gamma) \sum_{n=1}^{N} \alpha_n n [1 - F_t(p_t)]^{n-1} = \alpha_1(\bar{p}_t \phi_t - \gamma). \tag{14}$$

From this follows $G_t(q)$ and $H_t(q)$ (see Appendix A). Households are the same as before. In the fixed- and variable-cost models, the FOC's required for MME are respectively

$$\sum_{n=1}^{N} \alpha_n \delta H'(z_i) = i \text{ and } \sum_{n=1}^{N} \alpha_n \tau [1 - H(z_i)] = i. \tag{15}$$

In general, given p, (14) is a polynomial of order $N - 1$. When $N = 2$, it is linear, which is why the baseline model is so easy. However, we also get closed-form solutions with some parametric specifications for α_n. First, related to Mortensen

[11] This extension shows the general tractability and flexibility of the approach, but one can skip to the applications below without loss of continuity.

(2005), consider a Poisson distribution for n, $\alpha_n = e^{-\eta}\eta^n/n!$, where $\eta = \mathbb{E}n$. Then (13) reduces to

$$\Pi_t(p) = b_t(p\phi_t - \gamma)\,\eta e^{-\eta F_t(p)}.$$

From this and the fact $e^x = \sum_{n=0}^{\infty} x^n/n$, we get

$$F_t(p) = 1 - \frac{1}{\eta}\left[\log(\phi_t \bar{p}_t - \gamma) - \log(\phi_t p - \gamma)\right].$$

Proposition 3 *Assume a Poisson distribution for n. Then in the fixed-cost model:*
(i) $\exists!$ NME;
(ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{\imath})$;
(iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{\imath}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$;
where the thresholds satisfy $\bar{\imath} \in (\underline{i}, \infty)$,

$$\underline{i} = \frac{e^{-\eta}\delta}{\mu - \delta - \gamma} \text{ and } \bar{\delta} = \mu - \frac{(1 - e^{-\eta})\gamma}{1 - 2e^{-\eta}}.$$

Proposition 4 *Assume a Poisson distribution for n, and in the variable-cost model:*
(i) $\exists!$ NME iff $\tau \leq \mu/\gamma - 1$;
(ii) $\exists!$ MME iff $i < \min\{\tau(1 - e^{-\eta}), i^*\}$;
(iii) $\nexists$ PME;
where $i^* = i^*(\tau)$ is the nominal rate that drives buyers' payoff to 0.

The qualitative results are similar to the baseline model, and, in particular, the variable-cost specification again rules our PME. In MME, with a Poisson distribution for n, the fixed- and variable-cost models respectively again deliver nice money demand functions:

$$\hat{z}_i = \gamma + \left[e^{-\eta}\delta(\mu - \delta - \gamma)\right]^{\frac{1}{2}} i^{-\frac{1}{2}} \text{ and } \hat{z}_i = \gamma + \frac{(\mu - \gamma)\tau e^{-\eta}}{(1 + \tau)i + \tau e^{-\eta}}. \tag{16}$$

These are similar to (11) and (12), and are actually somewhat simpler, especially with variable cost. In both models, when buyers can observe more prices, money demand becomes more elastic wrt i.

As another example, related to Burdett et al. (2016), consider a Logarithmic distribution for n, $\alpha_n = -\omega^n / n \log(1-\omega)$, where $\omega \in (0,1)$ is a shape parameter. It is then easy to derive

$$F_t(p) = 1 - \frac{\phi_t(\bar{p}_t - p)}{\omega(\phi_t \bar{p}_t - \gamma)}.$$

Notice that $F_t(p)$ is linear – i.e., p is uniformly distributed – somewhat related to Caplin and Spulber (1987), although here it is a result and not an assumption.

Proposition 5 *Assume a Logarithmic distribution for n. Then in the fixed-cost model:*

(i) $\exists!$ NME;

(ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\underline{i}, \bar{\imath})$;

(iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{\imath}$, or $\delta < \bar{\delta}$ and $i < \underline{i}$;

where the thresholds satisfy $\bar{\imath} \in (\underline{i}, \infty)$,

$$\underline{i} = -\frac{\delta}{\mu - \delta - \gamma \log(1-\omega)} \text{ and } \bar{\delta} = \mu - \frac{\gamma \log(1-\omega)}{1 + \log(1-\omega)}.$$

Proposition 6 *Assume a Logarithmic distribution for n, and in the variable-cost model:*

(i) $\exists!$ NME iff $\tau \leq \mu/\gamma - 1$;

(ii) $\exists!$ MME iff $i < \min\{\tau, i^*\}$;

(iii) $\nexists$ PME;

where $i^* = i^*(\tau)$ is the nominal rate that drives buyers' payoff to 0.

The qualitative results are again similar to the baseline model, with the variable-cost specification ruling our PME. In MME, with a Logarithmic distribution for n, the fixed- and variable-cost models respectively again deliver very nice money demand functions:

$$\hat{z}_i = \gamma - \frac{\delta}{i \log(1-\omega)} \text{ and } \hat{z}_i = \gamma + \frac{(\mu - \gamma)(1-\omega)^{i/\tau}}{1 + \tau - \tau(1-\omega)^{i/\tau}}. \tag{17}$$

The bottom line is that all these specifications provide results that can easily be taken to the money demand data. Remarkably, they all imply closed-form solutions and tight characterizations of the equilibrium set. Plus, they all have intuitive economic interpretations about substituting between the use of money and credit as payment instruments in the face of inflation and transaction costs. While future research may find there are advantages and disadvantages to any of these parametric specifications, for our purposes they are similar, and in the applications to follow we focus on the baseline formulation with $N = 2$.

4.4 Repricing Behavior

While this is not the only paper to make the point, and this is not the only point of the paper, let us sketch the search-based explanation of sticky prices. In the models presented above, the nominal price distribution $F_t(p)$ is uniquely determined, but an individual firm's price is not. Consider Figure 5, drawn for the calibrated parameters in Section 5.2. With $\pi > 0$, the density F'_{t+1} lies to the right of F'_t. Firms with $p < \underline{p}_{t+1}$ at t (Region A) must reprice at $t+1$, because while p maximized profit at t, it no longer does so at $t+1$. But as long as the supports $\mathcal{F}_t$ and at $\mathcal{F}_{t+1}$ overlap, there are firms with $p > \underline{p}_{t+1}$ at t (Region B) that can keep the same p at $t+1$ without reducing their profit. They are allowed to change, at no cost, but they have no incentive to do so.

Given this, consider a particular repricing strategy. If $p_t \notin \mathcal{F}_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where $\hat{p}$ is a new price; and if $p_t \in \mathcal{F}_{t+1}$ then:

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob } \sigma \\ \hat{p} & \text{with prob } 1-\sigma \end{cases} \tag{18}$$

Notice σ defines a *payoff-irrelevant tie-breaking rule*. Different from Calvo pricing, where firms can be desperate to change p but are simply not allowed, this rule only applies to firms that are indifferent to repricing. Now consider symmetric equilibrium, which means that all seller that change p pick a new $\hat{p}$ from the same

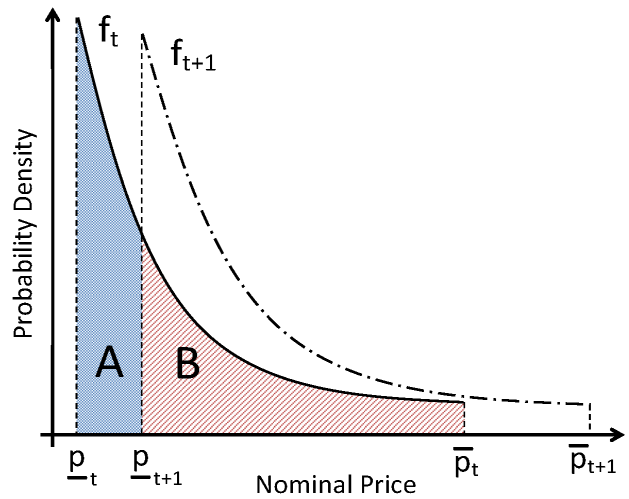


Figure 5: Nominal Price Densities and Inflation

repricing distribution $R_{t+1}(\hat{p})$. As in Head et al. (2012), given σ, there is a unique $R_{t+1}(\hat{p})$ consistent with equilibrium:

$$R_{t+1}(p) = \begin{cases} \frac{F_t\left(\frac{p}{1+\pi}\right)-\sigma[F_t(p)-F_t(\underline{p}_{t+1})]}{1-\sigma+\sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\underline{p}_{t+1}, \bar{p}_t) \\ \frac{F_t\left(\frac{p}{1+\pi}\right)-\sigma[1-F_t(\underline{p}_{t+1})]}{1-\sigma+\sigma F_t(\underline{p}_{t+1})} & \text{if } p \in [\bar{p}_t, \bar{p}_{t+1}] \end{cases} \tag{19}$$

One can now compute statistics from the model and compare them to any facts about pricing deemed interesting in the literature. Here the focus is on the distribution of price changes, $(p_{t+1} - p_t)/p_t$, because that is the topic of discussion in the research to which we are trying to contribute. Notice that different values of σ imply different repricing behavior, but this observation does not mean that anything goes. Once one pins down σ, say, based on data, there is a unique symmetric repricing distribution and it provides precise predictions for observables. Based on this reasoning, while we are well aware that the theory does not impose tight restrictions on an *individual* seller's strategies, we think it is nonetheless interesting to ask how well it can account for *average* repricing behavior – at the very least, to the extent that the model is consistent with these observations, it provides a voice of caution about using the data to make inferences about Mankiw-style menu costs or Calvo-style arrival rates, since here we abstract from both.

5 Quantitative Results

In terms of broad outline, we want to discipline the quantitative work by matching the shares of money and credit in the micro payment data, plus statistics derived from a standard empirical macro notion of money demand. As in Lucas (2000), e.g., this notion is $L_i = \hat{z}_i/Y$, where $Y = x^* + (\alpha_1 + \alpha_2)\,\mathbb{E}_H q$ is output aggregated over AD and BJ. If $U(x) = \log(x)$ then $x^* = 1$, obviously a normalization.[12] Explicit formulae for L_i and its elasticity η_i are given in Appendix B, and we target these in the data. Other key statistics are the average BJ markup $\mathbb{E}_G q/\gamma$ computed using posted prices, and the aggregate markup across both AD and BJ. These are natural targets because BJ equilibrium can deliver anything from monopoly to marginal-cost pricing as α_1/α_2 varies, so the BJ markup contains information about this ratio, while the aggregate markup contains information about the relative importance of AD and BJ. We also use the average duration between changes as the natural target to pin down σ in the tie-breaking rule as discussed above.

5.1 Data

We focus on 1988-2004, because the price-change observations are from that period, although in principle information from other periods can also be used to calibrate parameters. For money, the best available data is the M1J series in Lucas and Nicolini (2012) that adjusts M1 for money-market deposit accounts, similar to the way M1S adjusts for sweeps as discussed in Cynamon et al. (2006). Lucas-Nicolini have an annual series from 1915-2008 and a quarterly series from 1984-2013, and make the case that there is a stable relationship between these and (3-month T-Bill) nominal interest rates. We use their quarterly series, because the years correspond better to the price-change sample. In these data the average annualized nominal

[12] Generally, we can write utility as $\log(x) + \mu\mathbf{1}(y) - \psi\ell$, with μ capturing the importance of BJ vs AD goods, and ψ the importance of leisure. As in standard business cycle research, ψ can be set to match average hours, but as the key results do not depend on hours we ignore this.

rate is $\mathbb{E}i = 0.041$, which implies $L_{\mathbb{E}i} = 0.277$ and $\eta_{\mathbb{E}i} = -0.116$.[13]

Markup information comes from the U.S. Census Bureau Annual Retail Trade Report 1992-2008. At the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target for the gross margin is 1.3, in the middle of these numbers. This implies a markup of 1.39, as discussed in Bethune et al. (2014). While this number is above what macro people often use, it is consistent with the micro data analyzed by Stroebel and Vavra (2015). Moreover, the exact value does not matter a lot over a reasonable range, similar to the findings in Aruoba et al. (2011). We choose the target for the aggregate markup to be 1.1, based on Basu and Fernald (1997). Since the BJ markup is 1.39 and the AD markup is 1.0, this implies the BJ market accounts for about 25% of total output here.

On the shares of money and credit there are various sources. In terms of concept, we interpret monetary transactions broadly to include cash, check and debit card purchases. Here is the rationale: (1) Checks and debit cards use demand deposits that, like currency, are quite liquid and pay basically no interest. (2) As a matter of theory, as discussed in various papers on modern monetary economics, for some issues it does not matter whether your liquid assets are in you pocket or a your banker's vault (3) The interesting feature of credit is that it allows buyers to pay for BJ goods by working in the *next* AD market, while cash, check and debit purchases all require working in the *previous* AD market, which matters here especially because BJ transactions are random. (4) Using this notion of money in the micro data is consistent with the use of M1J in the macro data. Based on all this, monetary exchange includes cash, check and debit but not credit cards.

Earlier calibrations of monetary models proceeding in the same spirit (see Coo-

[13] In the longer annual sample, $\mathbb{E}i = 0.038$, $L_{\mathbb{E}i} = 0.279$ and $\eta_{\mathbb{E}i} = -0.149$. Using these instead does not affect the results much. We also tried truncating the data in 2004, to better match the pricing sample and eliminate the recent crisis, which also did not affect the results much.

ley 1995) target 16% for credit purchases, but more information is now available. In detailed grocery-store data from 2001, Klee (2008) finds credit cards account for 12% of purchases, although we do not want to focus on just groceries. In 2012 Boston Fed data, as discussed by Bennett et al. (2014) and Schuh and Stavins (2014), credit cards account for 22% of purchases in their survey and 17% in their diary sample. In Bank of Canada data, as discussed by Arango and Welte (2012), the number is 19%. While not literally identical, the Boston Fed and Bank of Canada data are close, and suggest a target of 20%. Note that this number is actually fairly stable over the relevant period, where the bigger changes have been into debit cards and out of checks, and to some extent out of currency (Jiang and Shao 2014*a*,*b*).[14]

For price-change data we mainly use Klenow and Kryvtsov (2008), and benchmark their average duration of 8.6, but alternatives are also considered since there are differences across and within studies depending on details. Their average absolute price change is 11.3%, well above average inflation, because there are many negative changes. Since the Klenow-Kryvtsov data are monthly, the model period is a month, and model-generated money demand is aggregated to quarterly to line up with Lucas-Nicolini. A month also seems natural since it corresponds to credit card billing period. However, this does not matter much: as usual, a convenient feature of search models is that they can be fit to different frequencies simply by scaling parameters like arrival, discount and interest rates.

5.2 Basic Findings

Generally, while we cannot hit all the targets exactly, we get very close except where indicated. The results are in Table 1. Consider first the fixed-cost model, which hits all targets except the fraction of credit transactions, because our parameter

[14] These numbers are shares of credit transactions by volume. In Canadian data the fraction by value is double, 40%, since as theory predicts credit is used for larger purchase. However, in Boston Fed data, the fractions by value and volume are about the same. There seems to be no consensus why American and Canadian data differ on value, but in any case, we use volume.

search is constrained to stay within the region where MME exists. Trying to get 20% BJ credit transactions forces δ into a region where MME does not exist for some values of i in the sample. Hence, for this model we use the smallest δ consistent with MME at the maximum observed $i = 0.103$, which yields 11.9% credit transactions. This δ is about 4.7% of the BJ utility parameter μ, which comes primarily from matching average real balances. The value of γ, about one-third of μ, comes primarily from the BJ markup. The probability of sampling one price (two prices) in BJ is $\alpha_1 = 0.013$ ($\alpha_2 = 0.081$).

	BJ utility μ	BJ cost γ	credit cost δ/τ	$pr(n=1)$ α_1	$pr(n=2)$ α_2	tie-break σ
Fix	8.62	2.91	0.404	0.013	0.081	0.90
Var	5.93	3.14	0.202	0.034	0.048	0.90

Table 1: Baseline Calibration

For the variable-cost model, in contrast, we approximate all targets very well, including 20% for BJ credit. Note the trade surplus, $\mu - \gamma$, is lower than the fixed-cost case, so BJ goods are now less important relative to AD goods, but γ/μ is higher. With $\mathbb{E}_H q$ around 4.21, the average transaction cost $\tau \mathbb{E}_H \max(0, q - z)$ is about 0.017. Scaled by BJ utility, $\tau \mathbb{E}_H \max(0, q - z)/\mu = 0.0029$, which is less than the average credit cards fees of around 1.5-2% (without counting the small fixed costs per transaction). Also notice α_1 (α_2) is higher (lower) than those in the fixed-cost model. A constant across specifications is the tie-breaking parameter $\sigma = 0.90$, implying that indifferent sellers change prices only 10% of the time.

Figure 6 shows money demand, with the solid curve from the fixed-cost model and the dashed curve from the variable-cost model. The fit is good in both cases, although the curves are somewhat different at low values of i. While this difference can be important for other issues, it does not matter a lot for our applications. In general, we conclude that the variable-cost specification demonstrates can match both money demand and micro payment data well, but the fixed-cost model has trouble with the latter given our calibration methods.

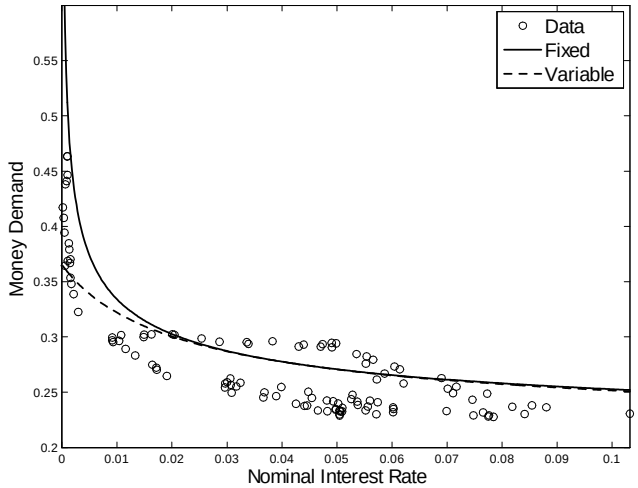


Figure 6: Money Demand for Different Specifications

6 Application A: Sticky Prices

It was discussed in Section 4.4, and in several earlier papers, how models can in principle generate the appearance of sticky prices without exogenous restrictions or costs on sellers' behavior. How well can they do quantitatively? Figure 7 shows the Klenow-Kryvtsov data plus model predictions of the price-change distribution. Both the fixed- and variable-cost versions capture the overall shape of the empirical histogram, although the fit is not perfect. We now argue, however, that the theory is broadly consistent with several facts considered important in the literature.

The average absolute change is 11.3% in the data, 20.3% in the model with a fixed cost of credit, and 12.3% with a variable cost. So at least with a variable cost we get very close to the data. The fraction of small changes (below 5% in absolute value) is 44% in the data, 28% with a fixed cost, and 31% with a variable cost. So on this we are off but not dramatically so.[15] The fraction of big changes (above 20% in absolute value) is 16% in the data, 34% with a fixed cost, and 21% with a variable cost, while the fraction of negative changes is 37% in the

[15] Eichenbaum et al. (2015) find a fraction of small prices changes lower than other studies, and suggest this is because one needs correct for measurement error. While their point is valid, for this exercise we take the Klenow-Kryvtsov numbers at face value.

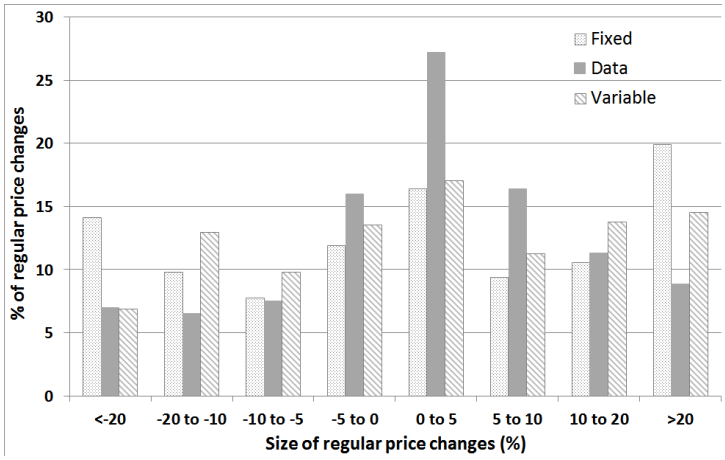
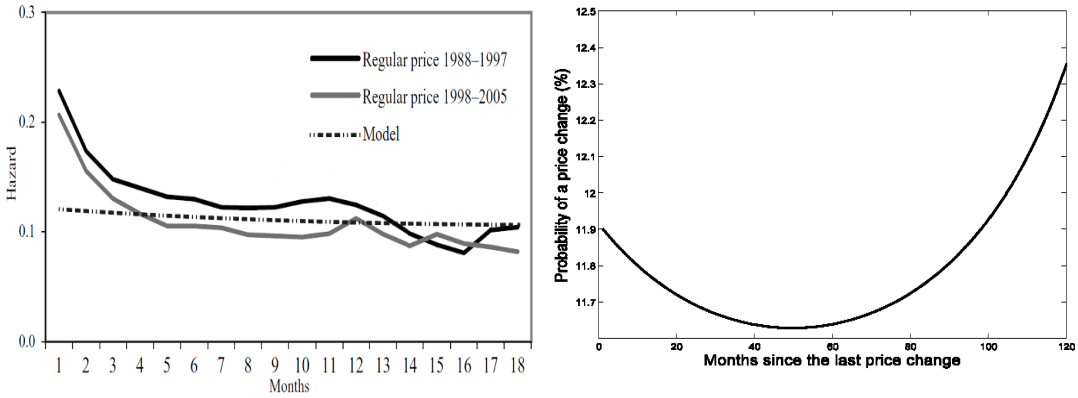


Figure 7: Distribution of Price Changes

data, and 43% in both models. So again we are not far off. Given the literature says it is not trivially easy to generate large average, many small, many big and many negative adjustments, we think the theory performs reasonably well, but not perfectly. Also, to be clear, we did not calibrate parameters to match these price-change statistics, but to match macro money demand and micro payment method data, plus markups, although we did set σ in the tie-breaking rule to match average price duration (robustness on this dimension is discussed below).



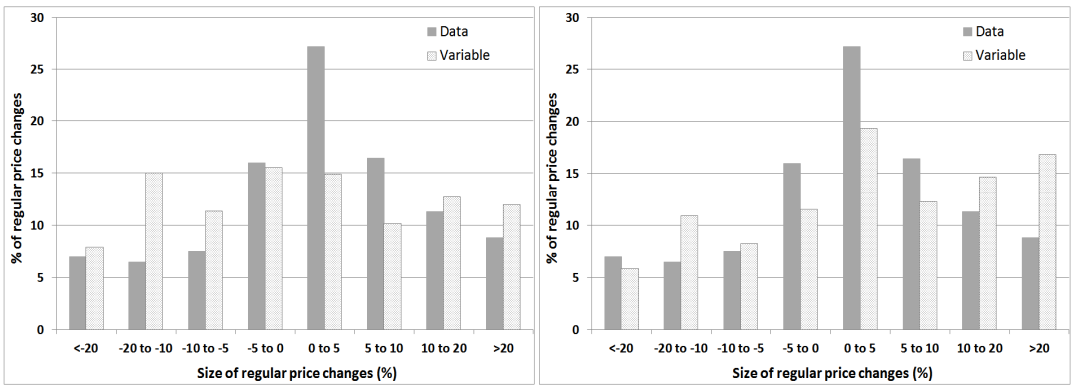
a. Price Change Hazard over 18 Months

b. Price Change Hazard over 10 Years

Figure 8: Price Change Hazards

Another observation to consider is the hazard rate, the probability of changing p as a function of the time since the last change. The left panel Figure 8 plots the

data from Nakamura and Steinsson (2008) and the prediction from the variable-cost model. We do not generate enough action at low durations, but at least the hazard slopes downward, something they say is hard to get in theory. Now one should not expect to explain every nuance, and there is undoubtedly a lot missing in the model related to the hazard, including experimentation or learning (e.g., see Bachmann and Moscarini 2014). Still, our hazard decreases for a while, before turning up at around 4 years, as shown in the right panel. It is U-shaped over a longer horizon because continuing inflation means any p eventually falls out of the equilibrium support.[16]

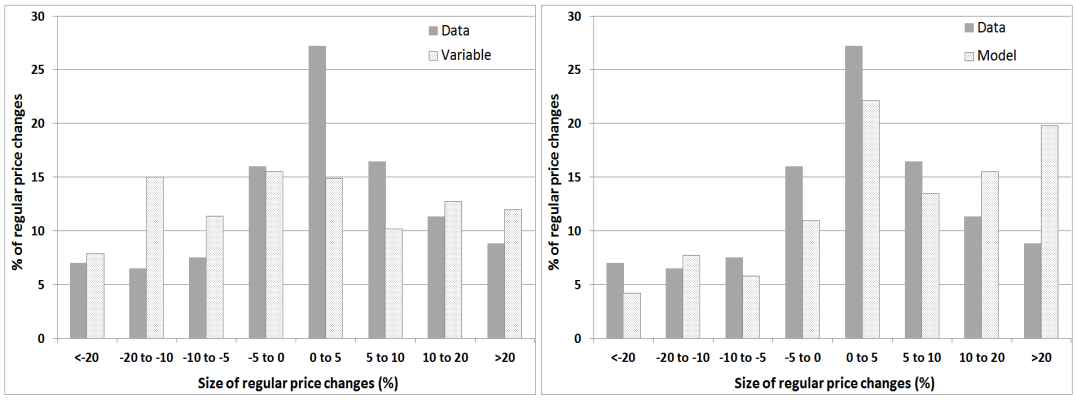


a. Average Duration 1 Month, $\sigma = 1.3\times e^{-10}$ b. Average Duration 16 Months, σ=0.95

Figure 9: The Effect of Varying Duration

Figures 9 and 10 show the impact of counterfactually changing duration and inflation in the variable-cost model. The left panel of Figure 9 is for $\sigma \simeq 0$ and an expected duration of 1 month; the right is for $\sigma = 0.95$ and an expected duration of 16 months. Given there considerable variability in estimates of average price durations (see Appendix C), it is worth considering robustness with respect to σ. Evidently the right panel fits better than the benchmark duration of 8.6 months. However, with too much stickiness the fit gets bad: at $\sigma = 0.9999$, e.g., the fraction of negative changes drops to 1.5%. The left panel of Figure 10 sets π to 0, and the

[16] Yet even at 10 years, our hazard is only up to 12.35%. Therefore some sellers can stick to prices for a very long time, as long as Cecchetti's (1986) magazines mentioned in Section 2. Also, we again emphasize that we do not calibrate to match these observations.



a. 0% Annual Inflation b. 20% Annual Inflation

Figure 10: The Effect of Varying Inflation

right to 20%. Notice the fraction of negative adjustments decreases, while both the frequency and size increase, with π. This is not surprising, but still important, because it is consistent with the evidence reported in Klenow and Kryvtsov (2008), evidence that obviously cannot be explained with simple Calvo pricing.

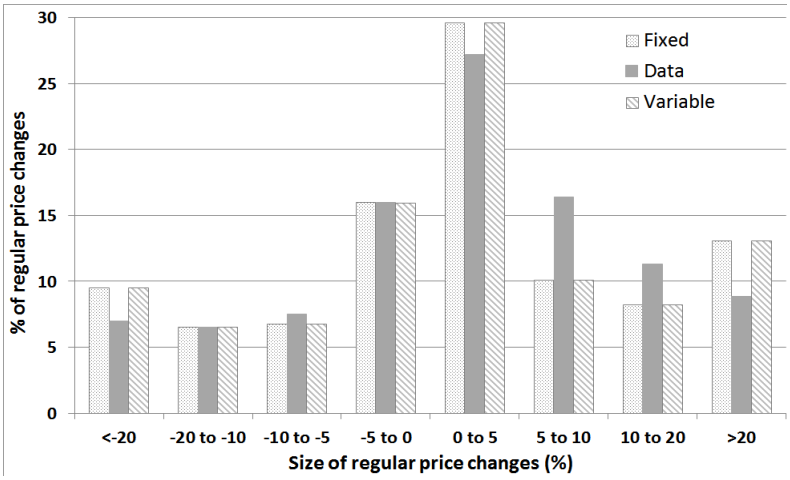


Figure 11: Distribution of Changes Ignoring Money Demand

To summarize the findings, while the fit is far from perfect, overall it seems hard to argue there is anything especially puzzling in this price-change data; it is similar to what rudimentary search theory predicts. Moreover, this is true even with the discipline imposed by macro and micro observations on money and credit. If we ignore those observations we can do better. How much better? Figure 11 shows

a calibration that gives up on matching money demand. The histograms fit very well. We conclude that it is easy for search models to capture the appearance of sluggish nominal prices if we do not impose the discipline of other data, and even with that discipline they capture at least broadly many of the facts. This is not to suggest that there are no other explanations for these observations; it only to suggest that it is worth considering search-based models of money and pricing as another candidate explanation.

7 Application B: Welfare

A genuinely classic economic question asks, what is the welfare cost of inflation? As is standard, we compute the percent change in consumption that is equivalent to changing π from a given level to some alternative, which we take to be 0. Given π, welfare is measured by

$$Y - (\alpha_1 + \alpha_2) \left\{ \delta \left[1 - H_\pi (z_\pi) \right] + \tau \int_{z_\pi}^{\bar{q}} (q - z_\pi) \, dH_\pi \right\}, \tag{20}$$

where $Y = U(x^*) - x^* + (\alpha_1 + \alpha_2)(\mu - \gamma)$ adds the AD and BJ surpluses, while the remaining terms subtract the resource costs of credit.

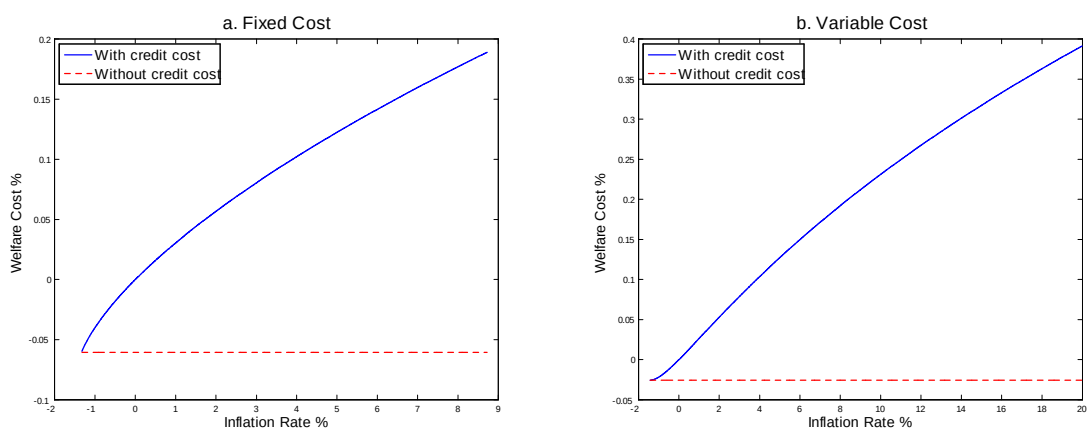


Figure 12: The Welfare Effects of Inflation

Figure 12 shows the welfare cost monotonically increases with inflation in both models. The range of the horizontal axis is the range over which MME exists, going

down to the Friedman rule $\pi = \beta - 1$, and up to about 9% with a fixed cost (left panel) or 20% with a variable cost (right panel).[17] The welfare effects are small – e.g., with a variable cost of credit, eliminating 10% annual inflation is worth only 0.23% of consumption. This is smaller than estimates in Lucas (2000), e.g., and much smaller than Lagos and Wright (2005). Intuitively, changes in π here affect neither the intensive margin of trade, since the good is indivisible, nor the extensive margin, since the population of participants is fixed. Hence, the welfare cost is mainly due to inflation increasing the usage and hence the cost of credit, as shown in the right panel of Figure 13. In the variable-cost model, inflation also affects $G(q)$, and in particular the upper bound $\bar{q}$, as buyers economize on real balances, but this additional impact it is not large.

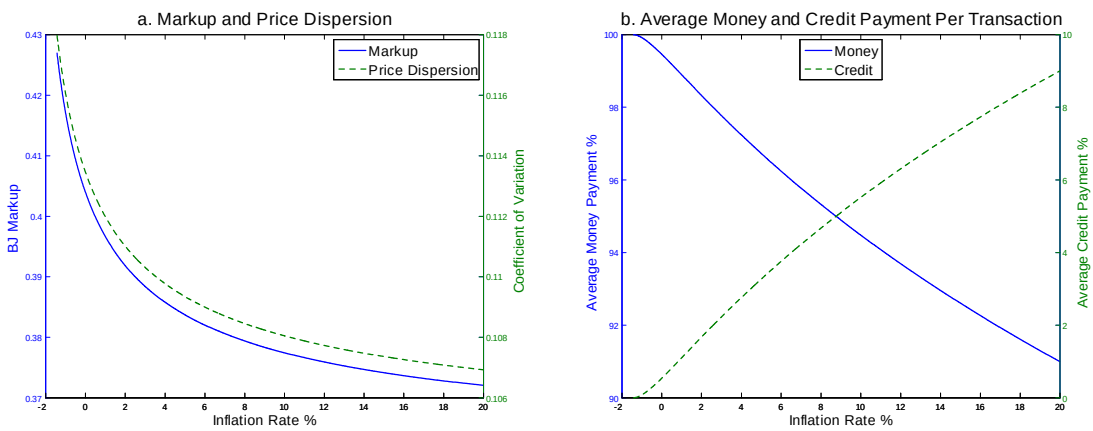


Figure 13: The Other Effects of Inflation

We revisit welfare in Section 8. First, let us briefly consider the relationship between inflation, markups and price dispersion. With a fixed cost of credit π does not affect $G(q)$, markups or dispersion. This is another reason to prefer a variable cost, where one can show implies $G(q)$ decreases with π in the sense of first-order stochastic dominance. Consequently, the average markup and dispersion

[17] These thresholds are low, but this not surprising in a representative-agent context. Suppose we introduce heterogeneity across buyers, with some having zero or only very costly access to credit – e.g., the unbanked, who have to deal with loans sharks, pawnshops or payday advances. They would presumably continue to use cash up to higher thresholds. While this is worth pursuing, heterogeneity is an extension that we reserve for future work.

(coefficient of variation) both decrease with π, as shown in Figure 13. In fact, both $\bar{q}$ and $\underline{q}$ fall with π, but $\bar{q}$ falls faster. Benabou (1992) finds a small but significant negative relationship between markups and inflation, consistent with the model. On inflation and dispersion, Parsley (1996) and Debelle and Lamont (1997) find the relationship is positive, Reinsdorf (1994) finds it is negative, and Caglayana et al. (2008) find it is U-shaped. Hence the facts are not unequivocally established, but the theory at least can match the findings in Reinsdorf (1994).

8 Application C: Participation

We now let buyers choose whether to participate in the BJ market, at cost $k > 0$, to make output depend directly on inflation.[18] Let $W^1(A)$ and $W^0(A)$ be the AD value functions for households that enter and do not enter the next BJ market, respectively, so that $W(A) = \max\{W^1(A), W^0(A)\}$. In equilibrium where some but not all households enter, $W(A) = W^1(A) = W^0(A)$. This simplifies to $\beta\Psi = k$, where Ψ is the expected surplus from participation,

$$\Psi = (\alpha_1 + \alpha_2)\left[\mu - \mathbb{E}_H q - \tau \mathbb{E}_H \max(0, q - z)\right] - i\hat{z}_i. \tag{21}$$

Buyers' arrival rates now depend on the buyer-seller ratio, or market tightness, $\alpha_n = \alpha_n(b_t)$. With entry, b adjusts to satisfy (21). An increase in π reduces b, and hence output, although a one-time unanticipated increase in M does not, as ϕ falls proportionately to leave ϕM and $G(q)$ the same (classical neutrality).

We need to parameterize the α's. Suppose that buyers attempt to solicit price quotes, and succeed with probability $s = s(b)$, with $s(0) = 1$, $s(\bar{b}) = 0$, $s'(b) < 0$, and $s''(b) > 0$. While this much is is standard, to generate price dispersion as in the baseline setup, any buyer who succeeds sees 1 price with probability $1 - \xi$ and

[18] Other work with endogenous entry by buyers includes Liu et al. (2011), while models with entry by sellers include Rocheteau and Wright (2005); of course the prototypical search model with entry is Pissarides (2000).

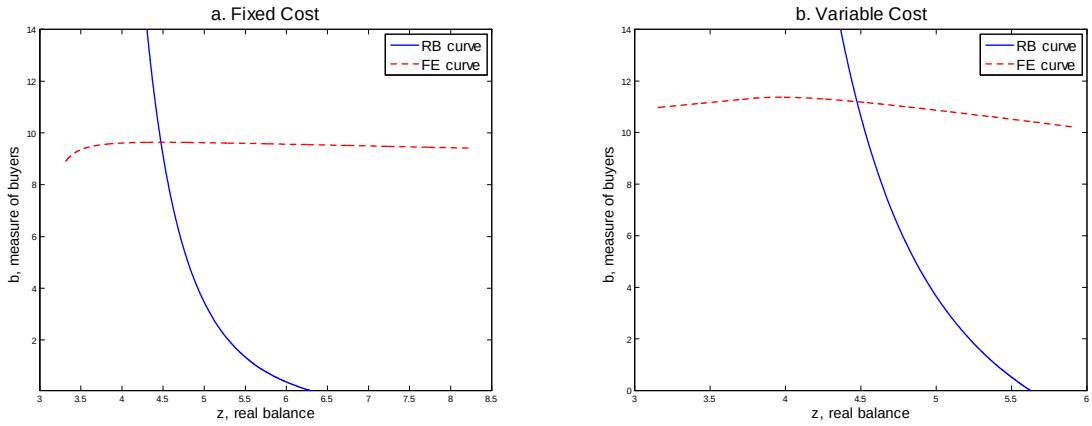


Figure 14: Real Balances and Free Entry

sees 2 with probability ξ.[19] Then $\alpha_1(b) = (1-\xi)s(b)$ and $\alpha_2(b) = \xi s(b)$. As a special case of the money demand functions derived above, $\hat{z}_i$ now depends on b, as shown in Figure 14 by the RB (real balance) curve. Similarly, $\beta\Psi = k$ is shown as the FE (free entry) curve, and the intersection of these curves constitutes a MME. As Figure 14 shows, RB is decreasing and convex while FE is concave, implying a unique MME, from which $F(p)$, $G(q)$ and the rest of the endogenous variables are constructed as usual. It is easy to check that higher inflation shifts both curves toward the origin, reducing buyer entry and hence BJ output.

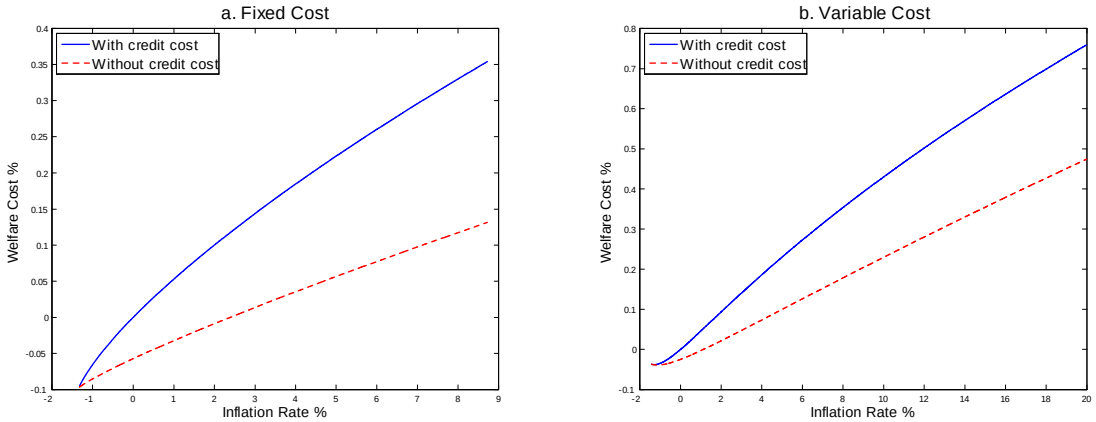


Figure 15: The Welfare Effects of Inflation with Entry

[19] A common heuristic for Burdett-Judd is that with probability s a buyer finds a catalogue containing a random numer of items he likes. Our only extension is to make s depend on b.

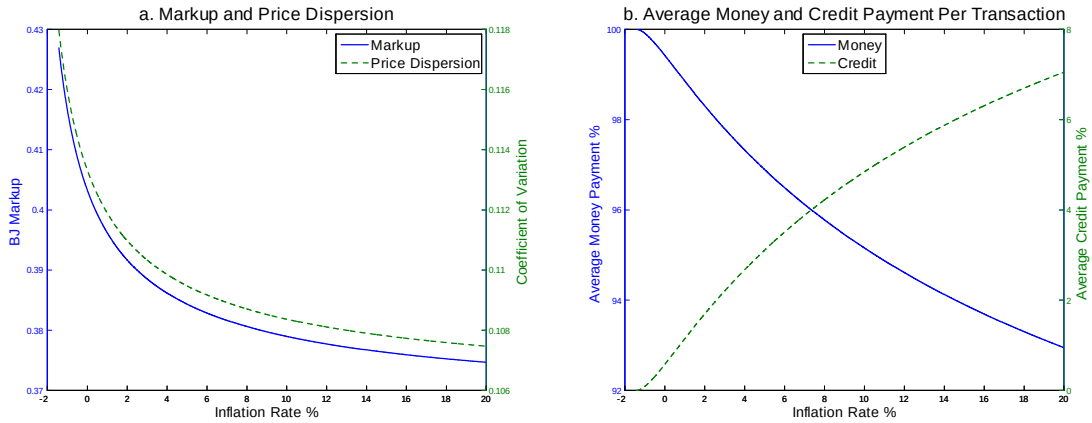


Figure 16: Other Effects of Inflation with Entry

While out theory is consistent with the appearance of sticky prices, the implications are different from models with constraints on changing prices. In those models, a one-time unanticipated jump in M has real effects. This is because at least some firms do not adjust p, even though they would like to, absent the assumed constraints, and hence the nominal distribution $F(p)$ does not change enough to keep the same real distribution $G(q)$. Hence, prices turn in favor of buyers, making b and output increase. In contrast, in our model economy, a surprise jump in M affects neither $G(q)$ nor b. A policy advisor seeing only a fraction of sellers adjusting p each period in our economy may conclude that a jump in M would have real effects; that would be wrong. Although not surprising, it is worth emphasizing that for policy prescriptions it is not actually enough to say prices are sticky in the data, it is important to know why.

As Figure 15 shows, compared to the benchmark model, the welfare cost of inflation approximately doubles, because it not only increases resources used to support more credit, an increase in π also decreases participation in the BJ market. Figure 16 demonstrates how π affects markups, price dispersion, and payment methods in the variable-cost model. Compared to Figure 13, notice that endogenizing participation does not change the impact of inflation on the markup or price dispersion a lot. In particular, now fewer buyers enter the BJ market at higher π,

but since that leads to higher arrival rates for those that do enter, their reduction in real balances is attenuated. The other conclusion is that the high welfare costs found in some search-and-bargaining models depend heavily on bargaining, since we get much smaller costs using price posting in otherwise similar environments.

9 Application D: Dynamics

General equilibrium monetary models can have nonstationary equilibria. Recently it has been argued this is because money conveys liquidity, but that can be true of many assets and has little to do with the fiat nature of currency (e.g. Rocheteau and Wright 2013). To consider these ideas in our model, let us now assume m earns a flow return ρ. If $\rho > 0$ then, as in standard finance, asset m can be interpreted as a share in a technology or 'tree' bearing a dividend in terms of 'fruit' in AD numeraire. If $\rho < 0$ it can be interpreted as a storage cost, as in models of commodity money. And $\rho = 0$ is the baseline case of fiat currency. For this exercise we keep the asset supply M fixed.[20] We also revert to $k = 0$, so $b = \bar{b}$ is exogenous, and focus on the variable-cost model.

In our preferred variable cost model, the household's problem is now

$$W(A) = A + U(x^*) - x^* + \beta \max_z O_i(z)$$

where $A = \rho m + \phi m - d - C(d) + I$ includes dividend income ρm, and $O_r(z) = V(z) - (1+r)z$, with r replacing i because $\pi = 0$. Textbook methods yield the Euler equation

$$\phi_t = \frac{\phi_{t+1} + \rho}{1+r}\left[1 + (\alpha_1 + \alpha_2)\tau \hat{H}(\hat{z})\right], \tag{22}$$

where we do not impose stationarity of the asset price ϕ_t. Suppose $\alpha_1 = \alpha_2 = 0$ (i.e., shut down the BJ market). Then (22) is a standard asset-pricing equation, there is a unique equilibrium, and $\phi_t = \rho/r \ \forall t$.[21] More generally, (22) is augmented

[20] It is not hard to let M change over time, but that is less interesting for real assets than for money, where one can think of π as a policy choice.

[21] Any other solution to the difference equation is explosive, and hence violates transversality, as discussed in detail in Rocheteau and Wright (2013) for a class of models including this one.

on the RHS by the liquidity value of the asset, the expected reduction in the cost of using credit $(\alpha_1 + \alpha_2)\,\tau \hat{H}(\hat{z})$. This dramatically changes the set of equilibria.

Inserting $H(\hat{z})$, which is the same as the baseline case, after routine algebra we get

$$\phi_t = \frac{\phi_{t+1} + \rho}{1 + r} \left\{ 1 + \frac{\tau \alpha_1^2}{4\alpha_2} \frac{\left[\mu - \left(\rho + \phi_{t+1}\right)\right] \left[\mu + \left(\rho + \phi_{t+1}\right)(1 + 2\tau) - 2\gamma(1 + \tau)\right]}{(1 + \tau)^2 \left(\rho + \phi_{t+1} - \gamma\right)^2} \right\}$$

This dynamic system gives the asset price today in terms of its price tomorrow, say $\phi_t = \Phi\left(\phi_{t+1}\right)$. The left panel of Figure 17 shows $\phi_t = \Phi\left(\phi_{t+1}\right)$, as well as the inverse $\phi_{t+1} = \Phi^{-1}(\phi_t)$, for the calibrated parameters, including $\rho = 0$, except now $\pi = 0$ and hence $i = r$. There is a unique steady state MME at around $\phi = 4.4$. As is typical with fiat currency, the monetary steady state is unstable, implying there is also an equilibrium where $\phi \to 0$. This is inflation as a self-fulfilling prophecy, even when the money supply and other fundamentals are all constant.

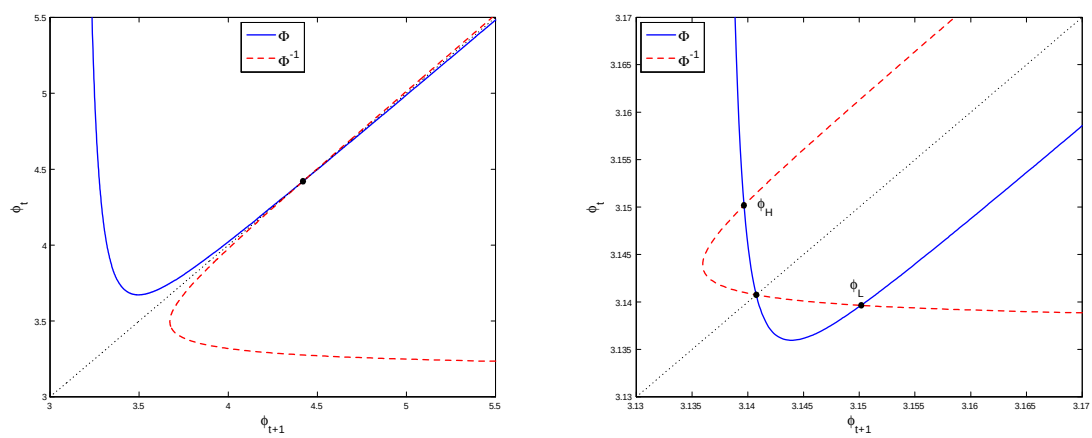


Figure 17: Phaseplane for Dynamic Equilibria

The right panel of Figure 17 makes only one change in parameter values: α_1 is reduced from 0.034 to 0.0001. There is still a unique steady state MME, with ϕ reduced to around 3.14. However, standard methods in dynamic theory (e.g., see Azariadis 1993) now imply the following: because $\Phi' < -1$ at the steady state, Φ and Φ^{-1} cross off the 45° line, and hence there exists a cycle of period 2. In this equilibrium cycle ϕ oscillates between ϕ_L and ϕ_H as a self-fulfilling prophecy, and

these are not purely nominal fluctuations – they matter for payoffs. Also, while it is not atypical for monetary models to have cyclic equilibria, this has a somewhat novel feature: as ϕ fluctuates, so does the entire distribution $F(p)$. Dynamics in a distribution are easy to handle because one number $\bar{p}$ is sufficient to pin down $F(p)$ by (3). A second novelty here is the precise nature of liquidity: the there nothing in the environment that says buyers cannot use credit all the time; they simply prefer to use cash, at least some of the time, to reduce transaction costs.

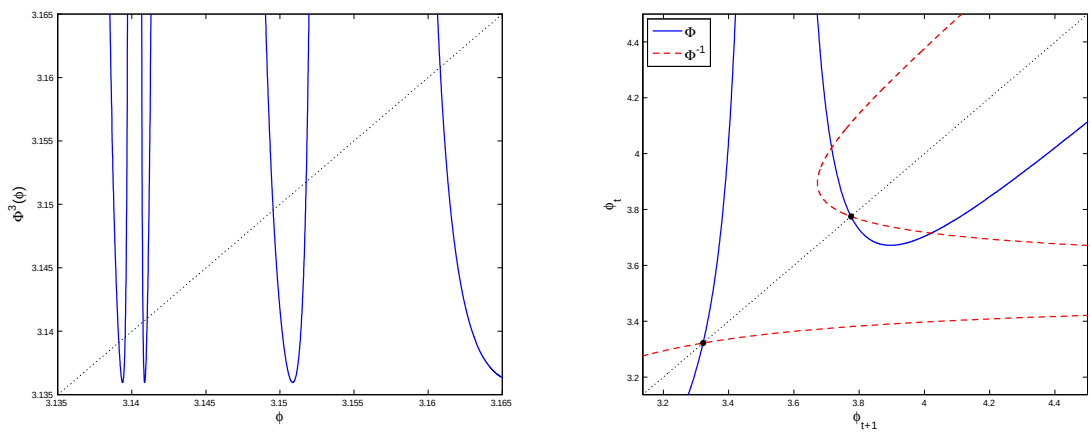


Figure 18: Examples with a 3-Cycle and with Two Steady States

For the same parameters that generate the 2-cycle, the left panel of Figure 18 shows the third iterate $\Phi^3(\phi)$. In addition to the steady state, $\Phi^3(\phi)$ has 6 intersections with the 45° line. This means there exist a pair of 3-cycles. Standard results (again see Azariadis 1993) imply that the existence of a 3-cycle implies the existence of N-cycles for all N, by the Sarkovskii theorem, as well as chaotic dynamics, by the Li-Yorke theorem. Therefore we can generate a large set of perfect-foresight dynamics, if not for parameters at their calibrated values, for parameters fairly close to their calibrated values, and hence for fairly realistic values. There are also sunspot equilibria for these parameters, with stochastic fluctuations in ϕ, $F(p)$ and other endogenous variables, illustrating how costly credit can generate excess volatility as a self-fulfilling prophecy.[22]

[22] An eay proof that sunspot equilibria exist, going back to Azariadis and Guesnerie (1986), is

When $\rho = 0$, one might think it is natural to get this kind of dynamic multiplicity, because there are two steady state equilibria, $\phi > 0$ and $\phi = 0$. However, we can eliminate the nonmonetary equilibrium $\phi = 0$ by setting $\rho > 0$, and as long as ρ is not too big, by continuity the qualitative results are similar. Heuristically , the cyclic, chaotic and stochastic equilibria discussed above should *not* be interpreted as approximating fluctuations across two steady states, but as fluctuations around one steady state. Thus, $\rho > 0$ is another example where multiple steady states are not necessary for complicated dynamics, including sunspot equilibria. At the same time, setting $\rho < 0$ leads to two steady states, say ϕ_1 and ϕ_2, with $\phi_2 > \phi_1 > 0$ as shown in the right panel of Figure 18, drawn using the calibrated parameters and $\rho = -0.4$. In this configuration, the lower steady state ϕ_1 is stable, which means we can construct by different methods sunspot equilibria fluctuating around it.[23]

Summarizing, models with costly credit admit cyclic, chaotic and stochastic dynamics, with price distributions, allocations, and the use of money and credit varying over time simply due to 'animal spirits.' This has little to do with money, per se, as these results hold for $\rho \neq 0$, too. It has to do with a trade-off between paying transaction costs on credit and accepting low returns on liquid assets. The return on fiat money is low, for obvious reasons, at least away from the Friedman rule. It may be less obvious for real assets but the point is similar: if an asset is useful in exchange its price is above the fundamental ρ/r, as seen in (22), and high prices mean low returns. This is standard, but it is novel to analyze it when credit is always available at a cost. Moreover, our usual interpretation has assets facilitating trades as media of exchange – buyers hand them over by way of quid

to suppose the outcome depends on an extrinsic two-state Markov process, $s \in \{s_1, s_2\}$, where $\varepsilon_s = prob(s_{t+1} \neq s | s_t = s)$. If $\varepsilon_1 = \varepsilon_2 = 1$ this reduces to a 2-cycle, the existence of which we proved above by example. By continuity there are equilibria for ε_s close to 1, although in examples ε_s does not have to be close to 1.

[23] A method Azariadis (1981) uses in OLG models is this: We seek $(\phi_1, \phi_2, \varepsilon_1, \varepsilon_2)$ such that $\phi_1 = \varepsilon_1 \Phi(\phi_2) + (1 - \varepsilon_1)\Phi(\phi_1)$ and $\phi_2 = \varepsilon_2 \Phi(\phi_1) + (1 - \varepsilon_2)\Phi(\phi_2)$, where $\varepsilon_s \in (0, 1)$ and wlog $\phi_2 > \phi_1$. These equations are easy to solve for ε_1 and ε_2. Whenever $\Phi'(\phi_s) > 1$ at a steady state ϕ_s, for any ϕ_1 in some range to the left of ϕ_s and any ϕ_2 in some range to the right of ϕ_s, one can check $\varepsilon_1, \varepsilon_2 \in (0, 1)$.

pro quo – but the narrative can be changed. In fact, the equations and conclusions are identical under the alternative interpretation that assets are used as collateral, and the results can be recast in terms of secured vs. costly unsecured credit, rather money vs. credit, to make contact with the large literature following Kiyotaki and Moore (1997).[24]

10 Conclusion

This paper has explored models of alternative payment methods, with money and costly credit as the leading example. For this we combined price posting as in Burdett-Judd with the monetary model in Lagos-Wright. We are not the first to combine these ingredients, but a novelty is the introduction of costly credit. This was useful, technically, since it resolves an indeterminacy problem in other models with money and posting, and implies a unique stationary equilibrium where agents use both money and credit. For both fixed and variable transaction costs, and for different assumptions about the way households sample prices, we derived exact money demand functions that resemble Baumol-Tobin, but we think with better microfoundations. These functions match the macro data well, but only the variable-cost model can do that plus match credit shares in micro data.

In one application, we showed how the theory can account for the price-change data. It accounts for this very well if we do not impose the discipline of matching other observations, and fairly well if we do impose it. By accounting for the price-change data, we only mean there are equilibrium outcomes that are roughly consistent with the evidence. To be clear, the theory does not pin down which seller posts which price in the cross section, and hence does not pin down price-change behavior in the time series. However, once one sets the parameter σ in a pay-

[24] In the interest of space, for details, we refer readers to the extended discussion in the survey by Lagos et al. (2016). However, the basic idea is easy: purchases in frictional markets for goods, inputs, or anything else can have constraints that are relaxed by asset holdings, and this leads to very similar outcomes whether the assets are used to finalize spot trades, or forfeited in case of an off-the-equilibrium-path default.

off irrelevant tie-breaking rule, there is a unique symmetric, stationary, monetary equilibrium with very precise predictions about price-change behavior. What we did is to calibre σ to the average duration of a price, and then compared these predictions to the facts.[25]

Another application revisited the welfare cost of inflation, from which we learned the following: while search-based models with bargaining generate large welfare costs, this is not the case in otherwise similar models with price posting. We found this in a baseline model where inflation impinged mainly on the costly use of credit, and to some extent the choice of real balances, and in an extension where it also impinged on participation, where the cost was double, but still fairly low. The extension is also interesting in its own right, highlighting as it does an entry channel through which monetary policy affects frictional markets. A related application considered the relationship between inflation, markups and price dispersion, where the model was shown to be consistent with some findings in the empirical literature. A final application discussed endogenous dynamics. While the mathematics in that discussion are well known, there are some novel economic ideas – e.g., fluctuations in a price distribution, not just a price level, and liquidity considerations emerging from assets reducing the resource cost of unsecured credit. Many other extensions are possible, such as incorporating heterogeneity, or combining search and menu-cost monetary models; these must be left for future work.

[25] As fn. 6 says, the stylized facts are: (1) Empirical price durations are long, but vary across studies. (2) The frequency and size of price changes vary across goods. (3) Two sellers changing at the same time do not generally pick the same new price. (4) Many changes are negative. (5) Hazards decline with duration. (6) There are many small but also many big changes. (7) The frequency and size of price changes, as well as the fraction of negative changes, vary with inflation. (8) There is price dispersion even at low inflation. Our model is consistent with all these, although we did not play up (2); it seems clear, however, that different values for the preference and cost parameters μ and γ, or arrival rates rates α_n, as can be reasonably expected across goods, will affect the predictions for price-change behavior.

Appendix A: Proofs of Non-obvious Results

Derivation of (8): The BJ value function can be written as

$$V(z) = W(z+I) + \alpha_1 \int_{\underline{q}}^{z} (\mu - q)\, dG_1(q) + \alpha_1 \int_{z}^{\bar{q}} [\mu - q - \delta - \tau(q-z)]\, dG_1(q)$$
$$+ \alpha_2 \int_{\underline{q}}^{z} (\mu - q)\, dG_2(q) + \alpha_2 \int_{z}^{\bar{q}} [\mu - q - \delta - \tau(q-z)]\, dG_2(q),$$

where $G_n(q) = 1 - \hat{G}(q)^n$ is the CDF of the lowest of n draws from $G(q)$. The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with $q \leq z$, so only cash is used, times the expected surplus, which is simple because $W'(A) = 1$. The third is the probability of meeting a seller with $q > z$, so credit must be used, which adds fixed cost δ and variable cost $\tau(q-z)$. The last two terms are similar except the buyer meets two sellers. The rest is algebra. ■

Proof of Lemma 3: (i) In NME, buyers' BJ surplus is $\Sigma = \mu - q - \delta - \tau q$. Note $\Sigma = 0$ at $q = (\mu - \delta)/(1+\tau)$, so no buyer pays more than this. If $\bar{q} < (\mu - \delta)/(1+\tau)$ then the highest price seller has profitable deviation toward $(\mu - \delta)/(1+\tau)$, which increases profit per unit without affecting sales. Hence $\bar{q} = (\mu - \delta)/(1+\tau)$. (ii) In MME, for $q > z$, as it is near $\bar{q}$, $\Sigma = \mu - q - \delta - \tau(q-z)$. Note $\Sigma = 0$ at $q = (\mu - \delta + \tau z)/(1+\tau)$, and repeat the argument for NME to show $\bar{q} = (\mu - \delta + \tau z)/(1+\tau)$. The definition of MME has $z < \bar{q} = (\mu - \delta + \tau z)/(1+\tau)$, which reduces to $z < \mu - \delta$. (iii) In PME, given buyers bring z to BJ, they would pay z. Hence $\bar{q} \geq z$, as $\bar{q} < z$ implies the highest price seller has profitable deviation. We also have to be sure there is no profitable deviation to $q > z$, which requires buyers using some credit. The highest such q a buyer would pay solves $\Sigma = \mu - q - \delta - \tau(q-z) = 0$, or $q = (\mu - \delta + \tau z)/(1+\tau)$. There is no profitable deviation iff $(\mu - \delta + \tau z)/(1+\tau) \leq z$, which reduces to $z \geq \mu - \delta$. ■

Proof of Proposition 1: (i) With fiat currency $\phi = 0$ is always self-fulfilling, so we simply set $G(q)$ according to (5), corresponding to equilibrium in the original BJ model.

(ii) From Figure 1, MME exists iff three conditions hold: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) is equivalent to $(\alpha_1 + \alpha_2)\,\delta H^-(\bar{q}) < i$, which holds iff $i > \underline{i}$. Then (b) is equivalent to $(\alpha_1 + \alpha_2)\,\delta H^+(\underline{q}) > i$, which holds

iff $i < \tilde{\imath}$ where $\tilde{\imath} = \delta(\alpha_1 + 2\alpha_2)^3 / 2\alpha_1\alpha_2(\mu - \delta - \gamma) > \underline{i}$. Also, (c) is equivalent to $(\alpha_1 + \alpha_2)\delta H(z_i) - iz_i > (\alpha_1 + \alpha_2)\delta H(0)$, which holds iff $\Delta(i) > 0$ where

$$\Delta(i) = -i\gamma + \frac{\delta(\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^{\frac{2}{3}}\delta^{\frac{1}{3}}\alpha_1^{\frac{2}{3}}\alpha_2^{-\frac{1}{3}}(\mu - \delta - \gamma)^{\frac{2}{3}}(2^{-\frac{1}{3}} + 2^{-\frac{4}{3}}).$$

Notice $\Delta(0) > 0 > \Delta(\tilde{\imath})$ and $\Delta'(i) < 0$. Hence $\exists!\ \bar{\imath}$ such that $\Delta(\bar{\imath}) = 0$, and $\Delta(i) > 0$ iff $i < \bar{\imath}$. It remains to verify that $\bar{\imath} > \underline{i}$, so that (a) and (c) are not mutually exclusive. It can be checked that this is true iff $\delta < \bar{\delta}$. Hence a MME exists under the stated conditions. It is unique because $\bar{q} = \mu - \delta$, which pins down $G(q)$, and then $\hat{z}_i = \arg\max_{z \in (\underline{q}, \bar{q})} O_i(z)$.

(iii) From Figure 1, PME exists iff three conditions hold: (a) $O_i^-(\bar{q}) > 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(\bar{q}) > O_i(0)$. Now (a) holds iff $i < \underline{i}$ and (b) holds iff $i < \tilde{\imath}$. Condition (c) holds iff $i < \hat{\imath}$. For $\delta > \bar{\delta}$, it can be checked that $\hat{\imath} < \underline{i}$ and $\underline{i} < \tilde{\imath}$, so the binding condition is $i < \hat{\imath}$. For $\delta < \bar{\delta}$, it is easily checked that $\hat{\imath} > \underline{i}$, and $\underline{i} < \tilde{\imath}$, so the binding condition is $i < \underline{i}$. ■

Proof of Proposition 2: (i) With fiat currency $\phi = 0$ is always self-fulfilling, so there exists a NME as long as the buyer's payoff from using credit only in the BJ market is nonnegative, that is $(\alpha_1 + \alpha_2)[\mu - (1 + \tau)\mathbb{E}_H q] \geq 0$. Substituting $\mathbb{E}_H q$ into the equation, after some routine algebra, we can show that a NME exists iff $\tau \leq \mu/\gamma - 1$.

(ii) From Figure 3, MME exists iff three conditions hold: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $\Psi_M > 0$ where

$$\Psi_M = (\alpha_1 + \alpha_2)[\mu - \mathbb{E}_H q - \tau\mathbb{E}_H \max(0, q - z_i)] - iz_i$$

represents a buyer's payoff from trading in the BJ market. Now (a) holds automatically since $O_i^-(\bar{q}) = -i$. Then (b) is equivalent to $(\alpha_1 + \alpha_2)\tau\hat{H}^+(\underline{q}) > i$, which holds iff $i < (\alpha_1 + \alpha_2)\tau$. Also, (c) is equivalent to

$$\begin{aligned}\Psi_M &= \alpha_2(\mu - \gamma) + \frac{\alpha_1\tau(\mu - z_i)}{1 + \tau} - \frac{\alpha_1^2\tau(\mu - z_i)^2}{4\alpha_2(1 + \tau)^2(z_i - \gamma)} - iz_i \\ &= \Psi(z_i) - iz_i > 0.\end{aligned}$$

Notice that Ψ_M is strictly concave in i on $[0, \infty)$. Since Ψ_M is continuous in i, $\lim_{i \to 0} \Psi_M > 0$, and $\lim_{i \to \infty} \Psi_M < 0$, one can use contradiction to easily show that there exists one and only one solution to $\Psi_M = 0$, which is defined as i^*. Then,

$\Psi_M > 0$ holds for all $i < i^*$. Hence, a MME exists under the stated conditions. It is unique because $O_i''(z_i) = V''(z_i) < 0$, and then $\hat{z}_i = \arg\max_{z\in(\underline{q},\bar{q})} O_i(z)$.

(iii) From Figure 3, it is clear that there is no PME in the variable-cost model.■

Proof of Proposition 3: Substituting α_n into (13) we have

$$\Pi_t(p) = b_t(p\phi_t - \gamma)\eta e^{-\eta}\sum_{n=1}^{\infty}\frac{\left[\eta\hat{F}_t(p_t)\right]^{n-1}}{(n-1)!} = b_t(p\phi_t - \gamma)\eta e^{-\eta F_t(p)},$$

since $e^x = \sum_{i=0}^{\infty} x^i/i!$. As a special case, $\Pi_t(\bar{p}_t) = b_t(\bar{p}_t\phi_t - \gamma)\eta e^{-\eta}$. Equal profit implies the nominal distribution is

$$F_t(p) = 1 - \frac{1}{\eta}\left[\log(\phi_t\bar{p}_t - \gamma) - \log(\phi_t p - \gamma)\right].$$

The real distribution is

$$G_t(q) = 1 - \frac{1}{\eta}\left[\log(\bar{q}_t - \gamma) - \log(q - \gamma)\right],$$

with $\bar{q}$ as in the baseline model and $\underline{q}_t = e^{-\eta}\bar{q}_t + (1 - e^{-\eta})\gamma$. After some algebra, the distribution of real transaction prices is

$$H_t(q) = \frac{\sum_{n=1}^{\infty}\alpha_n\left[1 - \left[1 - G_t(q)\right]^n\right]}{\sum_{n=1}^{\infty}\alpha_n} = \frac{1 - e^{-\eta}(\bar{q}_t - \gamma)/(q - \gamma)}{1 - e^{-\eta}}.$$

In the fixed-cost model, notice (i) holds as in Proposition 1. To prove (ii), we follow Proposition 1 and check three conditions for MME: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) is equivalent to $\sum_{n=1}^{\infty}\alpha_n\delta H^-(\bar{q}) < i$, which holds iff $i > \underline{i} = e^{-\eta}\delta/(\mu - \delta - \gamma)$. Then (b) is equivalent to $\sum_{n=1}^{\infty}\alpha_n\delta H^+(\underline{q}) > i$, which holds iff $i < \tilde{\imath} = e^{\eta}\delta/(\mu - \delta - \gamma) > \underline{i}$. And (c) is equivalent to $\sum_{n=1}^{\infty}\alpha_n\delta H(z_i) - iz_i > \sum_{n=1}^{\infty}\alpha_n\delta H(0)$, which holds iff $\Delta(i) > 0$ where

$$\Delta(i) = \delta - 2\left[e^{-\eta}\delta(\mu - \delta - \gamma)i\right]^{\frac{1}{2}} - i\gamma.$$

Given $\Delta(0) > 0 > \Delta(\tilde{\imath})$ and $\Delta'(i) < 0$, $\exists!\ \bar{\imath}$ such that $\Delta(\bar{\imath}) = 0$, and $\Delta(i) > 0$ iff $i < \bar{\imath}$. It remains to verify $\bar{\imath} > \underline{i}$, so that (a) and (c) are not exclusive. It can be checked that this is true iff $\delta < \bar{\delta}$, where $\bar{\delta} = \mu - (1 - e^{-\eta})\gamma/(1 - 2e^{-\eta})$. Hence, MME exists under the stated conditions. The proof of uniqueness and of (iii) follow directly Proposition 1, except now $\hat{\imath} = \delta(1 - e^{-\eta})/(\mu - \delta)$.■

Proof of Proposition 4:

To show (i) we follow the proof of Proposition 2 and check the household's payoff from using credit in BJ is nonnegative,

$$\Phi_N = \sum_{n=1}^{\infty} \alpha_n \left[\mu - (1+\tau)\,\mathbb{E}_H q\right] \geq 0. \tag{23}$$

After substituting $\mathbb{E}_H q$, we get $\Phi_N = (1 - e^{-\eta} - \eta^{-\eta})[\mu - \gamma(1+\tau)]$. Thus NME exists iff $\tau \leq \mu/\gamma - 1$. To prove (ii), again we check: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > 0$, where

$$O_i(z_i) = \sum_{n=1}^{\infty} \alpha_n \left[\mu - \mathbb{E}_H q - \tau \int_{\hat{z}_i}^{\bar{q}} (q - \hat{z}_i)\, dH\right] - i\hat{z}_i \tag{24}$$

is a buyer's payoff from BJ. Now (a) always holds, and it is easy to show (b) holds iff $i < (1 - e^{-\eta})\tau$. For (c), substitute α_n and H and simplify (24) to get

$$\begin{aligned} O_i\left(z_i\right) \;=\; & \left(1 - e^{-\eta}\right)\mu - \left(1 - e^{-\eta} - \eta e^{-\eta}\right)\gamma - \eta e^{-\eta}\frac{\mu + z_i\tau}{1+\tau} - \frac{\tau e^{-\eta}\gamma\left(\mu - z_i\right)}{\left(1+\tau\right)\left(z_i - \gamma\right)} \\ & - \frac{\tau e^{-\eta}\left[\mu - \gamma + \tau\left(z_i - \gamma\right)\right]}{1+\tau}\log\frac{\mu - \gamma + \tau\left(z_i - \gamma\right)}{\left(1+\tau\right)\left(z_i - \gamma\right)}. \end{aligned}$$

After some algebra, we can show that $O_i''(z_i) < 0$. Since z_i is strictly decreasing in i, $O_i''(z_i)$ is strictly convex in i on $[0, \infty)$. Moreover, it is easy to show that $\lim_{i\to 0} O_i(z_i) > 0$ and $\lim_{i\to\infty} O_i(z_i) < 0$. Since $O_i(z_i)$ is a continuous function, there exists a unique solution to $O_i(z_i) = 0$ on $[0, \infty)$. Let i^* be the i that solves $O_i(z_i) = 0$, and $O_i(z_i) > 0$ holds for all $i < i^*$. Hence, there exists a unique MME iff $i < \min\left\{\tau\left(1 - e^{-\eta}\right), i^*\right\}$. Finally, by the argument in the proof of Proposition 2, (iii) is true. ■

Proof of Proposition 5: Substituting α_n into (13) we have

$$\Pi_t(p) = b_t\left(p\phi_t - \gamma\right)\sum_{n=1}^{\infty}\left[-\frac{\omega^n}{\log\left(1-\omega\right)}\right]\left[\hat{F}_t\left(p_t\right)\right]^{n-1},$$

and $\Pi_t(\bar{p}_t) = -b_t\left(\bar{p}_t\phi_t - \gamma\right)\omega/\log\left(1-\omega\right)$. Now equal profit implies

$$F_t\left(p\right) = 1 - \frac{\phi_t\left(\bar{p}_t - p\right)}{\omega\left(\phi_t\bar{p}_t - \gamma\right)} \text{ and } G_t\left(q\right) = 1 - \frac{\bar{q}_t - q}{\omega\left(\bar{q}_t - \gamma\right)},$$

with $\bar{q}$ as in the baseline models and $\underline{q}_t = (1-\omega)\bar{q}_t + \omega\gamma$. Also,

$$H_t\left(q\right) = 1 - \frac{\log\left[1 - \omega\left[1 - G_t(q)\right]\right]}{\log\left(1-\omega\right)} = 1 - \frac{\log\left(q - \gamma\right) - \log\left(\bar{q}_t - \gamma\right)}{\log\left(1-\omega\right)},$$

where we used $\sum_{n=1}^{\infty} x^n/n = -\log(1-x)$.

In the fixed-cost model, notice (i) holds as in Proposition 1. To show (ii), we again check: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > O_i(0)$. Now (a) holds iff $i > \underline{i} = -\delta/[(\mu-\delta-\gamma)\log(1-\omega)]$, and (b) holds iff $i < \tilde{\imath} = -\delta/[(1-\omega)(\mu-\delta-\gamma)\log(1-\omega)] > \underline{i}$. Further, (c) holds iff $\Delta(i) > 0$, where

$$\Delta(i) = \delta - \frac{\delta\left[\log(\hat{z}_i - \gamma) - \log(\mu-\delta-\gamma)\right]}{\log(1-\omega)} - i\gamma + \frac{\delta}{\log(1-\omega)}.$$

It is easy to check $\Delta'(i) < 0$, $\lim_{i\to 0}\Delta(i) > 0$, and $\exists!\ \bar{\imath}$ such that $\Delta(\bar{\imath}) = 0$. Hence $\Delta(i) > 0$ iff $i < \bar{\imath}$. For (a) and (c) to not be mutually exclusive, we check $\bar{\imath} > \underline{i}$. This holds iff $\delta < \bar{\delta}$, where

$$\bar{\delta} = \mu - \frac{\gamma\log(1-\omega)}{1+\log(1-\omega)}.$$

Thus MME exists. The proof of uniqueness follows Proposition 1, and so does the proof of (iii), except now $\hat{\imath} = \delta/(\mu-\delta)$.■

Proof of Proposition 6:

To show (i), we check (23). Using $\mathbb{E}_H q = \gamma - \omega(\bar{q}-\gamma)/\log(1-\omega)$ and (23) we get

$$\Phi_N = \left[1 - \frac{\omega}{\log(1-\omega)}\right][\mu - \gamma(1+\tau)] \geq 0.$$

Thus NME exists iff $\tau \leq \mu/\gamma - 1$. To prove (ii), we check: (a) $O_i^-(\bar{q}) < 0$; (b) $O_i^+(\underline{q}) > 0$; and (c) $O_i(z_i) > 0$. Now (a) always holds and (b) holds iff $i < \tau$. As for (c), rewrite $O_i(z_i)$ as

$$O_i(z_i) = \mu - \gamma + \frac{(\omega+\tau)\mu - (1+\tau)\omega\gamma + (\omega-1)\tau\hat{z}_i}{(1+\tau)\log(1-\omega)} + \frac{\gamma\tau}{\log(1-\omega)}\log\left[\frac{\mu-\gamma+\tau(\hat{z}_i-\gamma)}{(1+\tau)(\hat{z}_i-\gamma)}\right].$$

Since $\log(1-\omega) < 0$, $O_i'(z_i) > 0$. Given $\partial\hat{z}_i/\partial i < 0$, $\partial O_i(z_i)/\partial i < 0$. Define i^* as the solution to $O_i(z_i) = 0$, and $O_i(z_i) > 0$ holds for all $i < i^*$. Hence, there exists a MME iff $i < \min\{\tau, i^*\}$, and it is unique since $O_i''(z_i) < 0$. Finally, (iii) is true following the proof of Proposition 2. ■

Appendix B: Formulae for Calibration

Consider first the variable-cost model. Inserting $\bar{q}$ and $\underline{q}$, we get

$$\begin{aligned} G(q) &= 1 - \frac{\alpha_1}{2\alpha_2}\frac{\mu - q + \tau(\hat{z}_i - q)}{(1+\tau)(q-\gamma)}, \\ H(q) &= 1 - \frac{\alpha_1^2[\mu - q + \tau(\hat{z}_i - q)][\mu + \tau\hat{z}_i + (q-2\gamma)(1+\tau)]}{4\alpha_2(\alpha_1+\alpha_2)(1+\tau)^2(q-\gamma)^2}. \end{aligned}$$

The fraction of monetary transactions and the markup are therefore

$$\begin{aligned} H(\hat{z}_i) &= 1 - \frac{\alpha_1^2(\mu - \hat{z}_i)[\mu + \tau\hat{z}_i + (\hat{z}_i - 2\gamma)(1+\tau)]}{4\alpha_2(\alpha_1+\alpha_2)(1+\tau)^2(\hat{z}_i-\gamma)^2}, \\ \frac{\mathbb{E}_G q}{\gamma} &= 1 + \frac{\alpha_1(\mu + \tau\hat{z}_i - \gamma + \tau\gamma)\log(1+2\alpha_2/\alpha_1)}{2\alpha_2\gamma(1+\tau)}, \end{aligned}$$

where $\hat{z}_i$ is given in the text. From this we get

$$\begin{aligned} L_i &= \frac{(1+\tau)\hat{z}_i}{\alpha_1(\mu + \hat{z}_i\tau) + (1+\tau)(1+\alpha_2\gamma)}, \\ \eta_i &= \frac{\alpha_1\mu + (1+\tau)(1+\alpha_2\gamma)}{\alpha_1(\mu+\hat{z}_i\tau)+(1+\tau)(1+\alpha_2\gamma)}\frac{\partial\hat{z}_i/\partial i}{\hat{z}_i/i}. \end{aligned}$$

Consider next the fixed-cost model. Inserting $\bar{q}$ and $\underline{q}$, we get

$$\begin{aligned} G(q) &= 1 - \frac{\alpha_1}{2\alpha_2}\frac{\mu - \delta - q}{q-\gamma}, \\ H(q) &= 1 - \frac{\alpha_1^2(\mu-\delta-q)(\mu-\delta+q-2\gamma)}{4\alpha_2(\alpha_1+\alpha_2)(q-\gamma)^2}. \end{aligned}$$

The the fraction of monetary transactions and the markup are

$$\begin{aligned} H(\hat{z}_i) &= \frac{[2\alpha_1\alpha_2(\mu-\delta-\gamma)/\delta]^{2/3} i^{2/3} - \alpha_1^2}{4\alpha_2(\alpha_1+\alpha_2)}, \\ \frac{\mathbb{E}_G q}{\gamma} &= 1 + \frac{\alpha_1(\mu-\delta-\gamma)\log(1+2\alpha_2/\alpha_1)}{2\alpha_2\gamma}. \end{aligned}$$

From this we get

$$\begin{aligned} L_i &= \frac{\gamma + \left[\alpha_1^2\delta(\mu-\delta-\gamma)^2/2\alpha_2\right]^{1/3} i^{-1/3}}{1+\alpha_1(\mu-\delta)+\alpha_2\gamma} \\ \eta_i &= \frac{-1}{3 + 3\gamma\left[\alpha_1^2\delta(\mu-\delta-\gamma)^2/2\alpha_2\right]^{-1/3} i^{1/3}}. \end{aligned}$$

Appendix C: Summary of Empirical Findings on Price Changes Statistics

Studies	Data source	Sample period	Empirical evidences [1]							
			Monthly change frequency (%)		Duration (month)		Absolute size of p changes (%)		Fraction of p changes below 5%	Hazard rates
			Mean	Median	Mean	Median	Mean	Median		
Using BLS CPI Research Database										
Bils and Klenow (2004)		1995M1-1997M12	26.1	20.9	...	4.3 (5.5)				
Klenow and Kryvtsov (2008) [2]		1988M2-2005M1	36.2 (29.9)	27.3 (13.9)	6.8 (8.6)	3.7 (7.2)	14.0 (11.3)	11.5 (9.7)	39.8 (44.3)	Declining pooled hazard; flat w/o survivor bias
Nakamura and steinsson (2008)		1988M1-2005M12	23.9-27.7 (19.7-23.1)	19.4-21.7 (10.6-13.0)	7.5-9.0 (9.0-10.4)	4.1-4.6 (7.2-8.9)		10.7 (8.5)		Declining hazard
Klenow and Malin (2010)		1988M1-2009M10	...	...	6.2 (8.0)	3.4 (6.9)	...	...	...	...
Eichenbaum et. al. (2014) [3]		1988M1-2011M7	22.0 (13.5)	...	4.5 (7.4)	...	...	...	40.6 (46)	...
Kehoe and Midrigan (2014) [4]		1988M1-2005M12	22.0 (6.9)	...	...	...	...	...	...	...
Using Scanner data set [5]										
Burstein and Hellwig (2007)	Dominick's	1989-1997	41 (26)	...	2.5 (4)	...	13 (11)	...	...	...
Nakamura (2008)	AC Nielsen ScanTrak	2004	43.9 (19.0)	42.7 (17.5)	...	...	...	...	...	...
Midrigan (2011) [6]	Dominick's	1989-1997	34 (2.9)	...	...	...	20 (11)	...	25	...
Eichenbaum et. al. (2011) [6]	A Large U.S. Retailer	2004-2006	43	...	0.6	...	...	...	...	...
Campbell and Eden (2014)	AC Nielsen ERIM	1985-1988	29	...	2.6	...	...	...	...	Declining hazard
Eichenbaum et. al. (2014) [7]	A Large U.S. Retailer	2004	...	...	...	...	...	30	5.2	...

(1) For studies that report statistics of both regular (i.e., excl. sales) and posted prices, the figures in parenthese correspond to those of regular prices.
(2) The paper also reported other estimates of durations, with the highest at 13.4 (mean) and 10.6 (median) duration when restricting the sample to 1998-2004 only.
(3) The paper also claimed that many small price changes are due to measurement errors and quality adjustment. Once removing these, the fraction drops to 32.2% for regular price changes and 24.4% for posted prices.
(4) This paper used the same data as Nakamura and Steinsson (2008), but different algorithm to calcualte price changes.
(5) Most scanner data are available on weekly basis, but here only monthly frequencies of price changes are reported, except Midrigan (2011) and Eichenbaum et. al. (2011) which reports a weekly frequency.
(6) Numbers of price change frequency are weekly frequencies.
(7) The paper also reported statistics calculated using the unit value index (UVI)-based approach; the median change in UVI-based prices is 10 percent and 31.5 percent of the changes are smaller than 5 percent in absolute terms.

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