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Costly Credit and Sticky Prices

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Abstract

We construct a model where money and credit are alternative payment instruments, use it to analyze sluggish nominal prices, and confront the data. Equilibria entail price dispersion, where sellers set nominal terms that they may keep fixed when aggregate conditions change. Buyers use cash and credit, with the former (latter) subject to inflation (transaction costs). We provide strong analytic results and exact solutions for money demand. Calibrated versions match price-change data well, with realistic durations, large average changes, many small and negative changes, a decreasing hazard, and behavior that changes with inflation, while staying consistent with macro and micro data on money and credit. Policy implications are discussed.

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1 Introduction

This paper has two related goals: (i) construct a framework where money and credit serve as alternative payment instruments; (ii) pursue within this setting a theory of endogenously sluggish prices that we can take to the data. It builds on the analysis of price dispersion in frictional goods markets by Burdett and Judd (1983), integrated into the general equilibrium monetary model of Lagos and Wright (2005). This environment generates monetary equilibria with a distribution of prices, and that means sellers can set prices in nominal terms that they may keep fixed when aggregate conditions change. Consumers use both cash and credit because the former is subject to the inflation tax while the latter involves a fixed or variable transaction cost, making the choice of payment method nontrivial and, we think, realistic. In particular, buyers tend to use cash (credit) more for small (large) purchases. Also, introducing costly credit allows us to avoid an indeterminacy of equilibrium that plagues some similar models, as we now explain.

To begin, consider Diamond (1971), where each seller (firm) of an indivisible good posts a price $p$, then buyers (households) sample sellers one at a time until finding one below their reservation price $p^*$. Clearly, for any seller, the profit maximizing strategy is $p = p^*$. Hence, there is a single price in the market. The Burdett-Judd model makes one change in Diamond’s specification: buyers sometimes sample multiple prices simultaneously, and when they do they obviously choose the lowest. This implies there cannot be a single $p$, nor even a set of sellers with positive measure charging the same $p$, since that would leave open a profitable deviation to $p - \varepsilon$ (see fn. 9 below for more discussion). In fact, one can compute explicitly the Burdett-Judd distribution $F(p)$, where any $p$ in the support $\mathcal{F}$ yields the same profit: lower-price sellers earn less per unit, but make it up on the volume, by making sales with higher probability.

If one embeds Burdett-Judd in a monetary economy, it makes sense for firms to post prices in dollars. Then, if the money supply $M$ increases, $F(p)$ shifts so
that the real distribution stays the same. Some firms can keep \( p \) fixed when \( M \) and the average price increase, however, and that means sticky prices according to the usage adopted here, even though sellers are allowed to adjust whenever they like at no cost. For those that stick to their nominal prices, real prices fall, but profits do not since sales increase. This much is similar to Head et al. (2012), but that paper has a technical problem. As discussed below, the combination of indivisible goods and price posting in monetary economies entails an indeterminacy – i.e., a continuum – of stationary equilibria. Hence, the model uses divisible goods, but then another problem pops up: what should firms post? That paper simply assumes linear menus: a seller sets \( p \) and lets buyers choose any \( q \) as long as they pay \( pq \). But there is no reason to think linear menus maximize profit.

Incorporating costly credit eliminates the indeterminacy that obtains with indivisible goods, so we can avoid ad hoc restrictions like linear pricing. Intuitively, holding more cash reduces the probability of needing to use credit, which delivers a well-behaved money demand function and unique monetary equilibrium. Hence we can revert to the original Diamond-Burdett-Judd specification, with indivisible goods, and get by with fewer “delicate” assumptions.\(^1\) Also, to make a point, changing \( M \) here is, by design, neutral. The point is not that neutrality holds in reality; it is that nominal stickiness does not logically imply nonneutrality. While others make similar points, it may be worth re-emphasizing, but there are many other reasons to construct a framework with both money and credit: (i) there is a long tradition of trying to build models along these lines (see fn. 7); (ii) they are relevant for applied work because they allow substitution between cash and credit as policy changes (Section 8); and (iii) they allow us to confront the data in novel ways (Section 7). The price dispersion implied by Burdett-Judd provides a natural context in which to study money and credit.

\(^1\)We do not take a stand on whether divisible or indivisible goods are more “realistic,” as that depends on the context, but we would certainly argue that indivisibility is an assumption on the physical environment and hence less “delicate” than a restriction on pricing strategies.
In terms of confronting data, Head et al. (2012) calibrate their model to match some features of price-change behavior, and show it also accounts for other facts. We perform a similar but more disciplined exercise, by calibrating to some features of pricing behavior, plus observations on the fractions of cash and credit usage, and money demand (the relation between real balances and interest rates). One specification involves a fixed cost of credit. It performs well in terms of money demand and can match the key facts in the pricing data, including long durations, large average price changes, many small or negative changes, a decreasing hazard, and repricing behavior that varies with inflation. It cannot match these plus the shares of cash and credit in micro data, however. Hence, we also consider a proportional cost of credit. That version can simultaneously match the pricing, money demand and payment data.\(^2\)

Section 2 reviews the literature. Section 3 describes our environment. Sections 4 and 5 consider fixed and variable costs. Section 6 discusses stickiness. Sections 7 and 8 discuss quantitative results and policy implications. Section 9 concludes.

2 Literature

Many sticky-price papers follow Taylor (1980) and Calvo (1983) by letting sellers adjust \(p\) only at certain times, or Rotemberg (1982) and Mankiw (1985) by letting them adjust only at a cost. Our sellers can change any time for free, but may choose not to. A few papers push imperfect-information or rational-inattention theories; see Mackowiak and Wiederhold (2009) for references. We are not against these devices, and indeed our formalization can be interpreted as saying sellers may be rationally inattentive to aggregate conditions. However, the focus is on search, and we exclude menu costs, because when Burdett and Menzio (2014) combine them the analysis is much harder, and the results are similar when menu costs are

\(^2\)To be clear, Head et al. (2012) match the price-change distribution and average duration, but cannot match money demand data at all well, and cannot match the shares of money and credit at all. Trying to be consistent with those data constitutes additional discipline.
small. Also, they find the majority of price dispersion in the data (about 70%) is due to search, and we want to see how far we get without menu costs.\(^3\)

The literature on Burdett-Judd models is large, including the labor-market applications following Burdett and Mortensen (1998). In monetary economics, prior to Liu (2010), Wang (2011) and Head et al. (2012) putting Burdett-Judd in Lagos and Wright (2005), Head and Kumar (2005) and Head et al. (2010) put it in the related model of Shi (1997). Other theories of price dispersion include Albrecht and Axel (1984) and Diamond (1987), where buyers differ not in terms of what they observe but their intrinsic (e.g., preference) type. A monetary version in Curtis and Wright (2004) delivers a two-point \( p \) distribution, for any number of types, which is less useful for our purposes. Also, as in Shi (1995) or Trejos and Wright (1995), and diametrically from us, that paper assumes goods are divisible while money is not, which limits applicability for policy and empirical issues.

Caplin and Spulber (1987) and Eden (1994) are papers that are in some respects similar in spirit, yet also very different. First, they do not try to match the data as we do, which is big component of this paper. Second, we use Burdett-Judd to get endogenous price dispersion, while Caplin and Spulber start by assuming a uniform price distribution (i.e., they do not derive this as an equilibrium outcome). Third, we build on the foundation for monetary economics in Lagos-Wright, which itself builds on Kiyotaki and Wright (1989,1993), Kochelakota (1998), Wallace (2001) etc. We adopt this approach because we believe it is best to analyze monetary phenomena in environments that are explicit about the frictions that money-like institutions are meant to ameliorate.\(^4\)

\(^3\)Nonmonetary search models with menu costs, where prices are sticky in unit of account, as in Benabou (1988,1992) or Diamond (1993), are special cases of Burdett and Menzio (2014).

\(^4\)See Wallace (2010), Williamson and Wright (2010), Nosal and Rocheteau (2011) or Lagos et al. (2014) for surveys of the approach, sometimes called New Monetarist Economics, that strives to provide relatively clean descriptions of how agents trade and specifications for specialization, commitment and information that give money and related institutions essential roles. Many of our points can also be made in, e.g., cash-in-advance or overlapping-generations models, but we find it natural to use monetary theory based on search, because search is already a key component of the environment – it is what drives price dispersion and stickiness.
There is much empirical work on price adjustment. Recently, Campbell and Eden (2014) find in grocery-store data an average duration between price changes of around 10 weeks, but we do not want to focus exclusively on groceries. Going back to Bils and Klenow (2004), in BLS data at least half of prices last less than 4.3 months, or 5.5 months if one excludes sales. Klenow and Kryvtsov (2008) report durations from 6.8 to 10.4 months, while Nakamura and Steinsson (2008) report 8 to 11 months, excluding substitutions and sales. These papers also find large fractions of small and negative price changes, plus some evidence of a decreasing hazard. Eichenbaum et al. (2011) report a duration of 6 months for “reference” prices, which are those most often quoted in a quarter (presumably to avoid recording Saturday Night Specials as two changes in a week). Cecchetti (1986) finds durations for magazine prices from 1.8 months to 14 years, while Carlton (1986) finds durations for wholesale prices from 5.9 months for household appliances to 19.2 for chemicals, and also finds many small changes, with about 2/3 below 2%. More empirical work is surveyed by Klenow and Malin (2010).

One empirical issue emphasized in the literature is that average price changes are fairly big, suggesting high menu costs, but there are also many small changes, suggesting low menu costs. Midrigan (2011) accounts for this by having firms sell multiple products, and paying a cost to change one price lets them change the rest for free. That is nice, but it does not mean we should not consider alternatives. Our model accounts for realistic durations, large average changes, many small changes, and many negative changes. It also delivers a decreasing hazard, and pricing behavior that depends on inflation, both of which are obvious problems for some alternative models. It also naturally generates price dispersion at low or zero

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5In conversations with people in the area, there is more or less agreement that these are the stylized facts: (1) Prices change slowly, but exact durations vary across studies. (2) The frequency and size of changes vary across goods. (3) Two sellers changing at the same time usually do not pick the same \( p \). (4) Many changes are negative. (5) Hazards decline slightly with duration. (6) There are many small (below 5%) and many big (above 20%) changes. (7) The frequency and size of changes, and fraction of negative changes, vary with inflation. (8) There is price dispersion even at low inflation. Our models are consistent with all eight facts.
inflation, consistent with the evidence (Campbell and Eden 2014). Given this, we think our approach should be part of the conversation on sticky prices and their implications.

The paper may also be considered a contribution to pure monetary economics. We deliver exact money demand functions expressing real balances in terms of interest rates, similar to the well-known results of Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966). The economic intuition is also similar, involving a comparison between the opportunity cost of holding cash and the cost of using financial services. However, those earlier papers involve partial-equilibrium analysis, or, more accurately, decision-theoretic analyses of how to manage one’s inventory of cash given that it – as opposed to barter, credit or something else – must be used for transactions. While such inventory-theoretic models are still being applied to good effect (e.g., Alvarez and Lippi 2009,2014), and we recognize that this is partly a matter of taste, we like the Lagos-Wright structure because it is tractable, it is easy to integrate with mainstream macroeconomics, and it has proved useful in variety of other applications.6

The above-mentioned multiplicity of monetary equilibria in models with indivisible goods and posting occurs in a series of papers spawned by Green and Zhou (1998). Jean et al. (2010) provide citations and further discussion, but here is the intuition: If all sellers post \( p \) then buyers’ best response is to bring \( m = p \) dollars to the market as long as \( p \) is not too big. If all buyers bring \( m \) then sellers’ best response is post \( p = m \) as long as \( m \) is not too small. Hence, any \( p = m \) in some range is an equilibrium, and the selection matters for payoffs. Note that this involves a continuum of stationary equilibria, very different from the multiplicity of dynamic equilibria in most monetary models. Head et al. (2012) avoid this be-

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6 Incorporating price posting with indivisible goods, as in Burdett-Judd, is also a further step in understanding this monetary framework, which has become a workhorse in the literature. Previous analyses use Nash, Kalai or strategic bargaining, posting with random or directed search, competitive price taking, auctions and pure mechanism design. See the surveys referenced in fn. 4 for extended discussions, and for other applications in finance, labor, growth etc.
cause, even if all sellers post \( p \), buyers need not bring \( m = p \) when \( q \) is divisible, but as we said that leads to other problems. In our alternative approach, even if all sellers post \( p \), buyers need not bring \( m = p \) when they have access to credit.

Before proceeding, we mention heterogeneity. As is well understood in other applications (e.g., Mortensen and Pissarides 1999), if firms are homogeneous, as they are here, theory does not pin down which one charges which \( p \), but only the distribution \( F(p) \). With heterogeneity, however, low cost firms prefer low \( p \) since they are more interested in high volume. Still, for any subset of firms with the same marginal cost, it does not matter which one posts which \( p \). Hence, heterogeneity eliminates neither dispersion nor stickiness within sets of similar sellers. This is especially relevant for retail, where the marginal cost is the wholesale price. Even if a few retailers get better deals – e.g., Kmart has a quantity discount – many others face the same wholesale terms. Irrespective of fixed costs, wages etc., these sellers are homogeneous for our purposes.\(^7\)

3 Environment

Each period in discrete time has two subperiods: first there convenes a decentralized market, called BJ for Burdett-Judd; then there convenes a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms interpreted as retailers with measure \( 1 \), and a set of households with measure \( \bar{b} \). Households consume a divisible good \( x_t \) and supply labor \( \ell_t \) in AD, while in BJ they consume an indivisible good \( y_t \) produced by firms at unit cost \( \gamma \geq 0 \). As agents are anonymous in the BJ market, they cannot use credit, unless they access a technology to authenticate

\(^7\)As a final note on literature, we alluded above to the venerability of work on money and costly credit. This includes Prescott (1987), Freeman and Huffman (1991), Chatterjee and Corbae (1992), Lacker and Schrefl (1996), Freeman and Kydland (2000), Dong (2011), Nosal and Rocheteau (2011) and Lucas and Nicolini (2013). See also Gomis-Porqueras and Sanches (2013), Li and Li (2013), Lotz and Zhang (2013), Aranjo and Hu (2014), Gu et al. (2014) and Bethune et al. (2015). None of these consider our applications. It is also worth highlighting that while costly credit resolves technical problems in previous posting models, price dispersion makes it natural for money and credit to coexist. Thus, costly credit and sticky prices are intimately related.
identity and record transactions at a cost. By incurring the cost, households can get BJ goods in exchange for commitments to deliver \( d_t \) dollars in the next AD; otherwise cash is required at point of sale.\(^8\) We consider both a fixed cost of credit \( \delta \) and a proportional cost \( \tau \). To nest these, let the transaction cost in general be \( C(d_t) = \delta \mathbf{1}(d_t) + \tau d_t \), where \( \mathbf{1}(d_t) \) is an indicator function that is 1 iff \( d_t > 0 \).

Household utility within a period is \( U(x_t) + \mu \mathbf{1}(y_t) - \ell_t \), where \( U'(x_t) > 0 > U''(x_t) \), \( \mu > \gamma + \delta \) is a parameter and \( \mathbf{1}(y_t) \) is again an indicator function. Let \( \beta = 1/(1+r) \), with \( r > 0 \), be a discount factor between AD today and BJ tomorrow; any discounting between BJ and AD can be subsumed in the notation. We impose \( \pi > \beta - 1 \), where in stationary equilibrium \( \pi \) is the inflation rate, and the nominal interest rate is given by the Fisher equation \( 1 + i = (1 + \pi)(1 + r) \). Notice \( i > 0 \), and the Friedman rule is the limiting case \( i \to 0 \). As usual, \( 1 + i \) is the amount of money agents require in the next AD market to give up a dollar in the current AD market, whether or not such trades occur in equilibrium. Let \( x_t \) be AD numeraire, and assume it is produced one-for-one with \( \ell_t \), so the wage is 1, but the interesting results do not depend on this. The AD price of money in numeraire is \( \phi_t \).

Firms enter the BJ market for free, but households must pay cost \( k \), which in general determines participation \( b_t \leq \bar{b} \). However, in the baseline model \( k = 0 \), so \( b_t = \bar{b} \). Firms use BJ profits to buy AD goods, over which they have linear utility, but noting of interest changes if instead they disburse profits to households as dividends. Each firm posts a price taking as given household behavior and the CDF of other firms’ prices, \( F_t(p) \), with support \( \mathcal{F}_t \). Every household in BJ randomly samples \( n \) firms – i.e., sees \( n \) independent draws from \( F_t(p) \) – with probability \( \alpha_n = \alpha_n(b_t) \), generally depending on the buyer-seller ratio, or market tightness, \( b_t \). For our purposes it suffices to have \( \alpha_1, \alpha_2 > 0 \) and \( \alpha_n = 0 \ \forall n \geq 3 \), but this

\(^8\)While the cost is paid by households, it is not hard to show the allocation is identical if it is instead paid by firms, as in elementary tax-incidence theory. Also, it does not matter if debt due in the frictionless AD market is denominated in dollars or numeraire.
can be generalized easily enough (see, e.g., Burdett et al. 2014). Finally, in terms of policy, let the money supply per capita evolve according to \( M_{t+1} = (1 + \pi) M_t \), with changes occurring in AD via lump-sum transfers, although the main results are the same if instead government uses new money to buy AD goods.

### 3.1 The Firm Problem

Expected real profit for a firm posting \( p \) at date \( t \) is

\[
\Pi_t(p) = b_t \left[ \alpha_1 (b_t) + 2\alpha_2 (b_t) \hat{F}_t(p) \right] (p_\phi - \gamma),
\]

where \( \hat{F}_t(p) \equiv 1 - F_t(p) \). Thus, net revenue per unit is \( p_\phi - \gamma \), and the number of units is determined as follows: The probability a household contacts this firm and no other is \( \alpha_1 (b_t) \). Then the firm makes a sale for sure. The probability a household contacts this firm plus another is \( 2\alpha_2 (b_t) \), as it can happen in two ways, this one first and the other second, or vice versa. Then the firm makes a sale iff it beats the other’s price, which occurs with probability \( \hat{F}_t(p) \). This is all multiplied by tightness \( b_t \) to convert buyer probabilities into seller probabilities.

Profit maximization means every \( p \in F_t \) yields the same profit and no \( p \notin F_t \) yields higher profit. As is standard in this kind of model, \( F_t(p) \) is continuous and \( F_t = [\underline{p}_t; \bar{p}_t] \) is an interval.\(^9\) Taking as given for now \( \bar{p}_t \), and that \( \bar{p}_t \) is not so high that buyers reject it, \( \forall p \in F_t \) profit must equal the profit from \( \bar{p}_t \), which is

\[
\Pi_t(\bar{p}_t) = b_t \alpha_1 (b_t) (\bar{p}_t \phi - \gamma).
\]

As in Burdett-Judd, equating (1) to (2) and solving for \( F_t(p) \) immediately yields:

\[
\text{Lemma 1 } \forall p \in F_t = [\underline{p}_t; \bar{p}_t] \quad F_t(p) = 1 - \frac{\alpha_1 (b_t) \phi \bar{p}_t - \phi p}{2\alpha_2 (b_t) \phi p - \gamma}.
\]

\(^9\)There cannot be a mass of firms with the same \( p \) because any one of them would have a profitable deviation to \( p - \varepsilon \), since they lose only \( \varepsilon \) per unit and make discretely more sales by undercutting others at \( p \). Similarly, if there were a gap between \( p_1 \) and \( p_2 > p_1 \), a firm posting \( p_1 \) can deviate to \( p_1 + \varepsilon \) and earn more per unit without losing sales. These and some other results below are standard in BJ models, although later the analysis is slightly complicated by the possibility of paying with money, and the transaction costs of paying without money.
Now it is easy to see $F'(p) > 0$ and $F''(p) < 0$ for $p_1 < p < \bar{p}_t$. Also, since the lower limit satisfies $F(p_t) = 0$, (3) implies

$$p_t = \frac{\alpha_1 (b_t) \phi_t \bar{p}_t + 2 \alpha_2 (b_t) \gamma}{\phi_t [\alpha_1 (b_t) + 2 \alpha_2 (b_t)]}.$$  

Also, to translate from dollars to numeraire, let $q_t = \phi_t p_t$ and write the distribution of real BJ prices as

$$G_t(q) = F_t(\phi_t p_t) = 1 - \frac{\alpha_1 (b_t) \bar{q}_t - q}{2 \alpha_2 (b_t) q - \gamma}.$$  

The support is $G_t = [q_t, \bar{q}_t]$, and we define $\hat{G}_t(q_t) = 1 - G_t(q_t)$.

### 3.2 The Household Problem

We focus on stationary equilibrium, where real variables are constant and nominal variables grow at rate $\pi$. This makes it convenient to frame the household problem in real terms. The state variable in AD is net worth, $A = \phi m - \phi d - C(d) + T$, where $\phi m$ and $\phi d$ are real money balances and debt carried over from the previous round of BJ trade, $C(d)$ is the transaction cost of having used credit, and $T$ is a lump sum transfer by which new money is injected. Since preferences are linear in $\ell$, we assume with no loss in generality that all obligations are settled in AD and households start BJ debt free. The state variable in BJ is real balances carried into that market, $z$. The AD and BJ value functions are $W(A)$ and $V(z)$.

Let $W^1(A)$ and $W^0(A)$ be the AD value functions for households that enter and skip the next BJ market, resp. Then $W(A) = \max \{W^1(A), W^0(A)\} = W^1(A)$ as long as some buyers enter. If $k$ is the BJ entry cost, then

$$W(A) = \max_{x,\ell,z} \{\mu(x) - \ell + \beta V(z)\} \text{ s.t. } x = A - k + \ell - (1 + \pi) z.$$  

Eliminating $\ell$ and letting $x^*$ solve $U'(x^*) = 1$, we get

$$W(A) = A + U(x^*) - x^* - k + \beta \max_z O_i(z),$$  

where the objective function for the choice of $z$ is $O_i(z) \equiv V(z) - (1 + i) z$, with $i$ again given by the Fisher equation. As in Lagos-Wright, we immediately have:
Lemma 2 \( W' (A) = 1 \) and the choice of \( z \) does not depend on \( A \).

The BJ value function satisfies (see the Appendix for more details):

\[
V (z) = W (z + T) + [\alpha_1 (b) + \alpha_2 (b)] \left[ \mu - E H q - \hat{H} (z) \delta - \tau (E H q - z) \right] \tag{7}
\]
where \( \hat{H} (q) \equiv 1 - H (q) \), and \( H (q) \) is the CDF of transaction prices,

\[
H (q) = \frac{\alpha_1 (b) G (q) + \alpha_2 (b) \left[ 1 - \hat{G} (q) \right]^2}{\alpha_1 (b) + \alpha_2 (b)}. \tag{8}
\]
Notice \( H (q) \) differs from the CDF of posted prices \( G (q) \), because buyers seeing multiple \( q \) pick the lowest. Naturally, from (7), the benefit of higher \( z \) is that it reduces the expected cost of having to use credit.

For the BJ entry decision, it is easy to see \( \Phi \equiv (1 + r) [W^1 (A) - W^0 (A)] \) is independent of \( A \) and satisfies

\[
\Phi = [\alpha_1 (b) + \alpha_2 (b)] \left[ \mu - E H q - \hat{H} (z) \delta - \tau (E H q - z) \right] - \kappa - iz, \tag{9}
\]
where \( \kappa = k / \beta \). The first term is the expected benefit of participating in BJ, as in (7), while \( \kappa + iz \) is the cost. Then

\[
b = \bar{b} \Rightarrow \Phi \geq 0; \ b = 0 \Rightarrow \Phi \leq 0; \ b \in (0, \bar{b}) \Rightarrow \Phi = 0. \tag{10}
\]

3.3 Equilibrium

The above discussion characterizes the behavior of all agents given \( \bar{q} \), which will be determined presently.

Definition 1 A stationary equilibrium is a list \((G (q), b, z)\) such that: given \( G (q) \), \((b, z)\) solves the household’s problem; and given \((b, z)\), \( G (q) \) solves the firms’ problem with \( \bar{q} \) determined as in Lemma 3 below.

Definition 2 (i) A nonmonetary equilibrium, or NME, has \( z = 0 \), so all BJ trades use credit. (ii) A mixed monetary equilibrium, or MME, has \( 0 < z < \bar{q} \), so BJ trades use cash for \( q \leq z \) and credit for \( q > z \). (iii) A pure monetary equilibrium, or PME, has \( z \geq \bar{q} \), so all BJ trades use cash.
It is implicit in these definitions that payoffs are positive, so the BJ market does not shut down; this is always checked in the analysis below. Also, in NME prices must be described in numeraire \( q \), while in MME or PME they can equivalently be described in numeraire or dollars, where \( F_t(p) = G(\phi_t, p) \) and \( \phi_t = z_t/M_t \). Other variables that can be computed include AD consumption \( x \) and labor supply \( \ell \), but we do not need these for what follows.

We now describe \( \bar{q} \), depending on the type of equilibrium. The following is proved in the Appendix:

**Lemma 3**

(i) NME implies \( z = 0 < \bar{q} = (\mu - \delta) / (1 + \tau) \).

(ii) MME implies \( z < \mu - \delta \) and \( \bar{q} = (\mu - \delta + \tau z) / (1 + \tau) \).

(iii) PME implies \( \bar{q} = z \geq \mu - \delta \).

Further, by virtue of (7) and (5), the following is immediate:

**Lemma 4** In MME, \( O_i(z) \) is continuous; it is smooth and strictly concave \( \forall z \in (q, \bar{q}) \); and it is linear \( \forall z \notin (q, \bar{q}) \).

Additional results hold in two special cases, \( \tau = 0 \) and \( \delta = 0 \), to be considered in order. As a benchmark, let \( k = 0 \), so that \( b = \bar{b} \) is fixed and the argument can be omitted from \( \alpha_n(b) \); we return to \( k > 0 \) in Section 8.

### 4 Fixed-Cost Model

Consider first \( \tau = 0 < \delta \). Given \( \delta < \mu - \gamma \), when \( k = 0 \) there is a nonmonetary equilibrium with \( b = \bar{b} \), where all households participate, and all transactions use credit – basically, the original Burdett-Judd equilibrium. We are more interested in monetary equilibrium. For this, consider the objective function \( O_i(z) \). As shown in Figure 1, as a special case of Lemma 4, \( O_i(z) \) is linear \( \forall z \notin (q, \bar{q}) \) with slope \( O'_i(z) = -i < 0 \). Intuitively, a marginal unit of real balances will only affect the expected cost of having to use credit when \( z \in \mathcal{G}_t \), and cash is a poor savings vehicle when \( i > 0 \). It is also easy to check \( O''_i(z) < 0 \ \forall z \in (q, \bar{q}) \).
These results imply a unique $z_i = \arg \max_{z \in [q, q]} O_i(z)$. If $z_i \in (q, \bar{q})$, as required for MME, $z_i$ satisfies the FOC

$$(\alpha_1 + \alpha_2) \delta H'(z_i) = i.$$  

To check $z_i \in (q, \bar{q})$, let $\hat{z}_i$ be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. If $O_i^+(q) \leq 0$ then $\hat{z}_i = 0$, as in the left panel of Figure 1. If $O_i^+(q) > 0$ then we need to check $O_i^-(q)$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = z_i$, as in the right panel. As a result of this analysis, with details in the Appendix, we have the following:

**Proposition 1**  *In the fixed-cost model:*

(i) $\exists!$ a unique NME;

(ii) $\exists!$ MME iff $\delta < \bar{\delta}$ and $i \in (\hat{i}, \bar{i})$;

(iii) $\exists$ PME iff either $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{i}$, or $\delta < \bar{\delta}$ and $i < \bar{i}$;

where the thresholds satisfy $\bar{i} \in (\hat{i}, \infty)$,

$$\hat{i} = \frac{\delta \alpha_i^2}{2\alpha_2 (\mu - \delta - \gamma)} \quad \text{and} \quad \bar{\delta} = \mu - \frac{\gamma (2\alpha_2^2 + 2\alpha_1 \alpha_2)}{2\alpha_2^3 + 2\alpha_1 \alpha_2 - \alpha_1^3}.$$

Proposition 1 is illustrated in Figure 2, where one sees that money can be valued only if $i$ is not too high. Note that while there is a continuum of PME
when they exist, Proposition 1 still provides sharp existence conditions. In any case, we are mainly interested in MME, which exists for intermediate values of \((\delta, i)\). Note however that the region for MME appears relatively small, as is made precise in calibration below. In terms of analytics, the simplicity of MME arises because buyers have a unique best response \(\hat{z}_i\) to \(G(q)\), and sellers’ best response conditions hold by construction at \(G(q)\) and \(\bar{q} = \mu - \delta\), independent of \(\hat{z}_i\).

In MME we can insert \(G(q)\) into the FOC (11) and rearrange to get the explicit solution for money demand:

\[
\hat{z}_i = \gamma + \left[\alpha_2^2 \delta (\mu - \delta - \gamma)^2 / 2\alpha_2\right]^{1/3} i^{-1/3}
\]

This expresses real balances in terms of the cube-root of \(1/i\), reminiscent of Baumol (1952), Tobin (1956), Whalen (1966) or Miller and Orr (1966).\(^{10}\) The results derive from similar economic forces. The usual story behind Baumol-Tobin has an agent sequentially incurring expenses assumed to need currency, with a fixed cost of rebalancing \(z\). The decision rule compares \(i\), the opportunity cost of cash, with the benefit of reducing the number of financial transactions interpretable as trips to the bank. Our buyers make at most one transaction in BJ before rebalancing \(z\),

\(^{10}\) Actually, the first two get a square-root rule, while the latter two also get a cube-root. We get something more like a square-root in Section 5.
but its size is random, and spending beyond \( z \) (credit) is costly. Still, they compare \( i \), again the cost of cash, with the benefit of reducing the use of financial services, again interpretable as trips to the bank, although one might say they now go there for a loan and not to make a withdrawal.\(^{11}\)

## 5 Variable Cost

Now consider \( \tau > 0 = \delta \). One but not the only interpretation of \( \tau \) is a proportional tax that can be avoided by using cash. Regardless of interpretation, variable costs are in some respects easier, make many (not all) results more transparent, and avoid a technical issue that we waited until now to raise. The issue is that in economies with nonconvexities, like fixed costs, it can be desirable to use lotteries (see Berentsen et al. 2002 on lotteries in closely related monetary models). Thus, a seller can post: "you get my good for sure if you pay \( p \); if you pay \( \tilde{p} < p \) then you get my good with probability \( P = P(\tilde{p}) \)." In Section 4, when a buyer with \( m = p - \varepsilon \) meets a seller posting \( p \), he pays \( p - \varepsilon \) in cash, \( \varepsilon \) in credit and \( \delta \) in fixed costs; if \( \varepsilon \) is small, both parties would prefer to trade using cash only, to avoid \( \delta \), and have the good delivered with probability \( P < 1 \).

Now, one might try to argue that lotteries are infeasible, unrealistic or otherwise unwarranted – but that would be awkward, because ruling out randomized exchange may be considered uncomfortably close to ruling out nonlinear menus, something we criticized above. We cover the fixed-cost case since it has a tradition in the literature (recall fn. 7), but do not want to tackle lotteries in this exercise. To the extent that this is problematic, we now turn to the variable-cost case, which has no such problem. It is also useful to compare the two environments in terms of their implied money demand functions and their quantitative performance.

\(^{11}\)See Berentsen et al. (2007) or Chiu and Meh (2011) on explicitly adding banking to Lagos-Wright in a way consistent with our application (e.g., Chiu and Meh incorporate a fixed cost of accessing banks). We could do this, too, but prefer to remain agnostic as to the exact nature of credit; trips to the bank are only being used heuristically, just like in Baumol-Tobin.
The price distribution emerging from the firms’ problem is the same as above. In particular, once one calculates

$$q = \frac{\mu + z\tau}{1 + \tau} \quad \text{and} \quad \bar{q} = \frac{\alpha_1 (\mu + z\tau) + 2\alpha_2 \gamma (1 + \tau)}{(\alpha_1 + 2\alpha_2) (1 + \tau)},$$

it is easy to check that $O_i(z)$ is now, conveniently, differentiable everywhere, including $q = \bar{q}$ and $q = \bar{q}$. As Figure 3 shows, this means there are only two possible types of equilibria, NME and MME. If $i > (\alpha_1 + \alpha_2)\tau$ there is a unique candidate NME, which is an actual NME iff payoffs are positive which holds iff $\tau \leq \mu/\gamma - 1$. If $i < (\alpha_1 + \alpha_2)\tau$ there is a unique candidate MME, which is an actual MME iff payoffs are positive which holds iff

$$\Phi = (\alpha_1 + \alpha_2) [\mu + \tau \hat{z}_i - (1 + \tau) \mathbb{E}_H q] - i \hat{z}_i$$

$$= \alpha_2 [\mu + \tau \hat{z}_i - \gamma (1 + \tau)] - i \hat{z}_i \geq 0.$$

It can be shown that $i \hat{z}_i$ increases and $\Phi$ decreases with $i$. Letting $i^* = i^* (\tau)$ be the $i$ that solves $\Phi = 0$, this proves:

**Proposition 2** In the variable-cost model

(i) $\exists!$ a unique NME iff $\tau \leq \mu/\gamma - 1$;

(ii) $\exists!$ a unique MME iff $i < \min \{\tau(\alpha_1 + \alpha_2), i^*\}$;

(iii) $\nexists$ a PME.
In MME, the FOC yields a decreasing demand function comparable to (12),

\[ \hat{z}_i = \gamma + \frac{(\mu - \gamma) \left[ \tau + (1 + \tau) \sqrt{1 + 4\alpha_2 \sqrt{1 + \tau}} \right]}{1 + 2\tau + 4\alpha_2 (1 + \tau)^2 i / \alpha_1^2 \tau}. \]  

Figure 4 illustrates Proposition 2. Given \( \tau > 0 \), MME exists as long as \( i \) is not too big. Note that credit is used only if \( q > \hat{z}_i \), and the maximum debt \( \bar{q} - \hat{z}_i \) increases with \( i \). One can also show \( \partial \hat{z}_i / \partial \tau > 0 \) and \( \partial (\bar{q} - \hat{z}_i) / \partial \tau < 0 \). As \( \tau \) gets big \( \bar{q} \to \hat{z}_i \), so buyers eventually stop using credit. All these results imply the variable-cost model is also tractable, and delivers an exact money demand function natural properties. It also avoids the issue of lotteries, and as suggested by Figures 2 and 4, it allows MME for a larger set of parameters which, as shown below, leads to better quantitative performance.

## 6 Sticky Prices

With either a fixed or variable cost, the nominal price distribution \( F_t(p) \) is uniquely determined, but individual-firm price dynamics are not. Consider Figure 5, drawn for the calibrated parameters in Section 7. With \( \pi > 0 \), the density \( F'_{t+1} \) is a right shift of \( F'_t \). Firms with \( p < p_{t+1}^* \) at \( t \) (Region A) must reprice at \( t + 1 \), because, while \( p \) maximized profit at \( t \), it no longer does so at \( t + 1 \). But as long as the
supports $\mathcal{F}_t$ and at $\mathcal{F}_{t+1}$ overlap, there are firms with $p > p_{t+1}$ at $t$ (Region B) that can keep the same $p$ at $t+1$ without reducing profit.

Figure 5: The Nominal Price Density Shifting with Inflation

Since equilibrium puts weak restrictions on an individual firm’s pricing strategy, we add a payoff-irrelevant tie-breaking rule: if $p_t \notin \mathcal{F}_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where $\hat{p}$ is a new price; and if $p_t \in \mathcal{F}_{t+1}$ then:

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob } \sigma \\ \hat{p} & \text{with prob } 1 - \sigma \end{cases}$$

(14)

Hence, firms that are indifferent stick with probability $\sigma$ to their current $p$.\footnote{In case it is not obvious, this is very different from Calvo pricing, where firms can be desperate to change $p$ but are simply not allowed. In our environment, any firm that wants to can and does adjust, but there are some that are indifferent and they are happy to randomize.}

Then we focus on symmetric equilibrium, where all changers pick a new $\hat{p}$ from the same repricing distribution $R_{t+1}(\hat{p})$. As in Head et al. (2012), given $\sigma$, the unique symmetric equilibrium repricing distribution that generates $F_{t+1}(p)$ is:

$$R_{t+1}(p) = \begin{cases} \frac{F_t\left(\frac{p}{p_t}\right) - \sigma F_t\left(\frac{p}{p_{t+1}}\right)}{1 - \sigma + \sigma F_t\left(\frac{p}{p_{t+1}}\right)} & \text{if } p \in [p_{t+1}, \bar{p}_t) \\ \frac{F_t\left(\frac{\bar{p}_t}{p_t}\right) - \sigma [1 - F_t(p_{t+1})]}{1 - \sigma + \sigma F_t\left(\frac{\bar{p}_t}{p_{t+1}}\right)} & \text{if } p \in [\bar{p}_t, \bar{p}_{t+1}] \end{cases}$$

(15)

Varying $\sigma$ generates a wide range of price dynamics, as demonstrated below, certainly wide enough to line up broadly with facts. Hence, for present purposes,
we content ourselves with stationary symmetric equilibrium given $\sigma$, and let data inform us as regards appropriate values for $\sigma$. It is then routine to compute statistics from the model and compare these to the data. While in future work it may be interesting to examine $F_t(p)$ directly, in the present exercise the concentration is on the distribution of price changes, $(p_{t+1} - p_t)/p_t$, because that is very much the topic of discussion in the research to which we are trying to contribute.

7 Quantitative Results

Given an interest in the dynamics of nominal pricing, it seems natural to bring to bear discipline from monetary observations. Therefore, we try to match the fractions of money and credit usage in the micro payment data, plus some statistics derived from a standard empirical notion of money demand. As in Lucas (2000), this notion is $L_i = \hat{z}_i/Y$, where $Y = x + (\alpha_1 + \alpha_2) \mathbb{E}Hq$ is output aggregated over AD and BJ. We pick $U(x) = \log(x)$, so that $x^* = 1$.\textsuperscript{13} Explicit formulae for $L_i$ and its elasticity $\eta_i$ are given in the Appendix, and we target these at $\mathbb{E}i$ in the data. Another key statistic is the average BJ markup $\mathbb{E}Gq = \gamma$ computed using posted prices (although the results are very similar using transaction prices). This is a key statistic because, as is well known, BJ equilibrium delivers anything being monopoly and marginal-cost pricing as $\alpha_1/\alpha_2$ vary. Two moments from price-change data are also used: the average absolute change; and the average duration between changes, which pins down $\sigma$ in the tie-breaking rule.\textsuperscript{14}

\textsuperscript{13}Obviously $x^* = 1$ is a normalization. Also, in general, utility is $\log(x) + \mu \mathbb{1}(y) - \psi \ell$, with $\mu$ capturing the relative importance of BJ vs AD consumption and $\psi$ the importance of leisure vs AD consumption. While $\psi$ can be set to match average hours in the data $\mathbb{E}\ell$, as in standard business cycle theory, the results below are independ of $\mathbb{E}\ell$, and hence we do not report $\psi$.

\textsuperscript{14}For the record, our method differs from Head et al. (2012), where the analog of $\sigma$ is calibrated along with other parameters to fit the $p$-change distribution. In principle, we thought there was more discipline in calibrating $\sigma$ to duration and then evaluating model performance by its match with the $p$-change distribution; in practice, it did not matter much.
7.1 Data

We focus on 1988-2004, given the price-change data are from that period, although in principle longer series can be used to calibrate some parameters. For money the best available data is the M1J series in Lucas and Nicolini (2012) that adjusts M1 for money-market deposit accounts, similar to the way M1S adjusts for sweeps as discussed in Cynamon et al. (2006). Lucas-Nicolini provide an annual series from 1915-2008 and a quarterly series from 1984-2013, and make the case that there is a stable relationship between these series and (3-month T-Bill) nominal interest rates. We use their quarterly series, because we prefer a higher frequency for reasons mentioned below, and because the years better correspond to the price-change sample. In these data, the average annualized nominal rate is $E_i = 0.048$, which implies $L_{Ei} = 0.279$ and $\eta_{Ei} = -0.149$.\footnote{In the longer annual sample, $E_i = 0.038\%$, $L_{Ei} = 0.257$ and $\eta_{Ei} = -0.105$. Using this instead does not affect the results much. We also tried truncating the data in 2004, to better match the pricing sample and eliminate the recent crisis; that did not affect the results much either.}

Following Aruoba et al. (2011), markup data comes from the U.S. Census Bureau Annual Retail Trade Report 1992-2008. In these survey data, at the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target is for the gross margin is 1.3, in the middle of these numbers. A gross margin of 1.3 implies a markup 1.39, exactly as in Bethune et al. (2014). While this number is above what some people used to use (see the discussion in Lagos and Wright 2005), it is very much consistent with the micro data analyzed by Stroebel and Vavra (2015). Moreover, the exact value does not matter too much over a reasonable range, similar to the findings in Aruoba et al. (2011). Although their markup comes from bargaining, not posting, Aruoba et al. suggest an intuition that we also offer here: any markup above a fairly low number leads to sellers having enough market power to be decidedly different from competitive price takers.
On the fraction of transactions using money and credit, there are various micro data sources. First, in terms of concept, we follow much of the literature by interpreting monetary transactions broadly to include cash, check and debit card purchases. As a rationale, first, checks and debit cards use demand deposits, which very much like currency are quite liquid and pay basically no interest, and it is irrelevant for the theory here whether your money is in your pocket or checking account. Second, a key feature of credit is that it allows buyers to pay for BJ goods by working in the next AD market, while cash, check and debit purchases require working in the previous AD market, and this can matter a lot because BJ transactions are random. Third, this notion of money in the micro data is consistent with the use of M1J in the macro data.

Older calibrations of monetary models (Cooley 1995, chapter 7) target of 16% for credit purchases, but much more information is now available. In detailed grocery-store data from 2001, Klee (2008) finds credit cards account for 12% of purchases, but that is just for groceries. In 2012 Boston Fed data, discussed by Bennett et al. (2014) and Schuh and Stavins (2014), credit cards account for 22% of purchases in the survey and 17% in the diary sample. In Bank of Canada data, discussed by Arango and Welte (2012), the number is 19%. While there are some differences across studies, the comprehensive Boston Fed and Bank of Canada data are actually close, and suggest a target of 20%. Also note that this number does not change very much over time, where the bigger evolution has been into debit cards, out of checks and to some extent out of currency.16

For price-change data we mainly use Klenow and Kryvtsov (2008), and benchmark their average duration of 8.6, but alternatives are also considered, since there

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16These numbers are shares of credit transactions by volume. In Canadian data the fraction by value is 40%, about double, since as theory predicts credit is used for larger purchases: “Cash [accounts] for 76 per cent of all transactions below $15, and for 49 per cent of those in the $15 to $25 range ... whereas credit cards clearly dominate payments above $50” (Arango and Welte 2012). But in Boston Fed data, fractions by value and volume are 16% and 17% in the diary sample. In conversations with analysts at the two institutions, there was no consensus as to why American and Canadian data differ on volume but in any case, we use volume, where they agree.
are differences across and within studies depending on details. Their average absolute price change is 11.3%, above average inflation due to many negative changes. Also, since the Klenow-Kryvtsov data are monthly, the model period is a month, too, and model-generated money demand data are aggregated to quarterly to line up with Lucas-Nicolini. A month is also natural because it is typically the period during which credit card debt can be paid without interest charges. As described below, different observations pin down different parameters, but they are actually determined jointly by minimizing the distance between model and data. While we cannot hit the targets exactly, except when indicated we get very close.

7.2 Findings

Calibration results are in Table 1. Consider first the fixed-cost model, which hits all of the targets except the fraction of credit trade, because parameters choices are constrained to stay in the region where MME exists. In particular, trying to set \( \delta \) small enough to get 20% credit transactions in BJ implies MME does not exist at \( \mathbb{E}i = 0.045 \). Hence, we use the smallest \( \delta \) consistent with MME at \( \max i = 0.085 \), which only gives 8% credit transactions. Notice the transaction cost parameter \( \delta \) is less than 2% of the BJ utility parameter \( \mu \), where the latter intuitively comes from matching average real balances. The value of \( \gamma \), about half of \( \mu \), comes primarily from the markup. The probability of sampling one price (two prices) in BJ is \( \alpha_1 = 0.18 \) (\( \alpha_2 = 0.23 \)), somewhat lower than calibrations of related models because here the period is 1 month.\(^{17}\)

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \gamma )</th>
<th>( \delta / \tau )</th>
<th>( pr(n = 1) )</th>
<th>( pr(n = 2) )</th>
<th>tie-break</th>
</tr>
</thead>
<tbody>
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<td>Fix</td>
<td>104.06</td>
<td>57.37</td>
<td>1.643</td>
<td>0.18</td>
<td>0.23</td>
</tr>
<tr>
<td>Var</td>
<td>21.47</td>
<td>11.53</td>
<td>0.063</td>
<td>0.12</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 1: Baseline Calibration

\(^{17}\)As usual a convenient feature of search models is that they can be fit to different frequencies simply by scaling parameters like arrival and discount rates.
The variable-cost model has no problem hitting the targets without endangering the existence of MME, and in particular easily gets 20% BJ credit. Note $\mu$ and $\gamma$ are lower than in the fixed-cost model, so BJ goods are now less important relative to AD goods, but $\gamma/\mu$ is basically the same. With $E_Hq$ around 15.4, the average transaction cost $\tau E_Hq$ is around 0.97, smaller than $\delta = 1.643$ in the other model, but we should scale these by BJ utility, resulting in $\tau E_Hq/\mu = 0.045$ and $\delta/\mu = 0.016$. To judge whether a calibrated $\tau = 0.063$ is reasonable, note the average US sales tax rate is virtually identical at 0.064. Adding interchange fees on credit cards, which average 1.5-2% without counting small fixed costs that can be added per transaction, we are still very much in the ballpark. Also notice $\alpha_1$ and $\alpha_2$ are now slightly lower than in the fixed-cost model. However, one result that is constant across specifications is the tie-breaking parameter $\sigma = 0.9$, implying that indifferent sellers reprice a mere 10% of the time.

Figure 6: Money Demand for Different Specifications

Figure 6 shows money demand, with the solid curve from the fixed-cost model and the dashed curve from the variable-cost model. The fit is obviously good in both cases, although the curves are somewhat different at low values of $i$. While this difference could be quite important for other issues – e.g., the welfare cost of inflation – it seems less relevant for our purposes. In terms of the price-change
evidence, Figure 7 shows the predicted distribution along with the one from the Klenow-Kryvtsov data. Both models more or less capture the overall shape, although the fit is obviously not perfect. Importantly, however, the models are broadly consistent with several observations deemed important in the literature.

![Figure 7: Distribution of Price Changes](image)

In particular, the average absolute change is 11.5% in the fixed-cost model, 12.3% in the variable-cost model and 11.3% in the data. While this is a calibration target, again, we only approximately match it. Important statistics not targeted include the fraction of small changes (below 5% in absolute value), which is 44% in the data, 31% in the fixed- and 30% in the variable-cost model. So, on this, the models are slightly off but not dramatically so.\(^{18}\) Similarly, the fraction of big changes (above 20% in absolute value) is 16% in the data, 20% in the fixed- and 23% in the variable-cost model, while the fraction of negative changes is 37% in the data and 43% in both models. So, on these, the models are not too far off. It is not trivial to match these facts in other models. Our setup does fairly

\(^{18}\)Eichenbaum et al. (2015) find a fraction of small (below 5%) prices changes much lower than other studies, and suggest that this is because others did not correct for measurement error. For this exercise we take the Klenow-Kryvtsov numbers at face value, but it is perhaps worth mentioning that their fraction of small changes may be and overestimate for this reason.
well, even without complications, like heterogeneity, idiosyncratic shocks or multi-product firms. Basically, a shifting $F_t$ and a tie-breaking rule calibrated to duration generate large average, many small, many big and many negative adjustments.

Figure 8 shows the hazard (probability of changing $p$ as a function of time since the last change). The left panel shows the hazard over 18 months from the data in Nakamura and Steinsson (2008), who argue this is interesting, and from the model with either a fixed or variable cost since they are basically identical. Theory does not generate enough action at low durations, evidently, but at least the model hazard slopes downward, something Nakamura and Steinsson say is hard to get in other models. Of course, one should not expect to explain every nuance, as presumably there is a lot going on in reality that is not in the model that could increase adjustments at low durations – e.g., experimentation by some sellers trying to learn market conditions. Even without such complications, our hazard decreases initially, before turning up at around 3 years, as shown in the right panel, where it is U-shaped over a longer horizon, since inflation makes any $p$ falls off the support eventually. Yet even at 10 years the model hazard is only up to 14%. Hence, some firms can stick to prices for a very long time, easily as long Cecchetti’s (1986) mucilaginous magazines mentioned in Section 2.
Figure 9: The Effect of Varying Duration

Figure 10: The Effect of Varying Inflation

Figure 9 shows results of varying duration in the variable-cost model, since reports on this differ across studies. The left panel shows results from a very low average duration of 1 month, corresponding to $\sigma \approx 0$. The right panel shows results from choosing $\sigma$ to minimize the distance between the model and empirical distribution, which implies $\sigma = 0.95$ and a corresponding duration of 16 months. Clearly the left panel fits worse and the right better than results from the benchmark duration of 8.6 months. After 16 months, the fit gets worse, and as $\sigma$ gets very big model performance becomes very bad – e.g., at $\sigma = 0.9999$, the fraction of negative changes drops to a mere 1.5%, quite far from the data. One conclusion is that matching the data is not automatic for arbitrary parameters, even if we do
fairly well at reasonable durations.

Figure 10 shows results of varying inflation in the variable-cost model. The left panel corresponds to \( \pi = 0 \) and the right to 22.5\%, which is the maximum inflation rate consistent with MME in this specification (more on this below). Naturally the fraction of negative changes falls with inflation, and it can also be checked that both frequency and size of price adjustments increase with inflation – which is not automatic, although, obviously, at least one of the two must increase. These are all features the model shares with the Klenow-Krysvstov data, while it should go without saying that standard Calvo models miss on at least the response of frequency to inflation. A key conclusion is that pricing behavior in this model is not invariant to policy, and it reacts in a realistic way.\(^\text{19}\)

Overall, while the models miss a few details they perform fairly well. It would be hard to say there is anything especially puzzling about the behavior of price changes in the data – it is pretty much what one should expect from elementary search theory. It would seem even harder to say there is anything especially informative about Mankiw costs or Calvo arrivals in these data, given the outcomes generated by a model with no such devices. We also emphasize the quantitative discipline imposed by the macro and micro data on money and credit, which at least the variable-cost model matches well. If we ignore these facts we can do even better, as shown in Figure 11, which comes from calibrating the variable-cost model without trying to match money demand. The fit is obviously good in this picture, but the model is way off in terms of money demand. The point is that it is not hard to capture sticky price observations, but we do more, by simultaneously capturing the credit share and money demand observations.

\(^{19}\text{Klenow and Krysvstov (2008) quantify these effects by regressing the size, frequency and fraction of negative changes on inflation. We content ourselves with qualitative statements on these effects, since running these regressions in the model requires additional assumptions and presumably more discussion of the stochastic environment.}\)
8 Participation and Policy

In the model with $k = 0$, all buyers enter the BJ market, and so the arrival rates $\alpha_n(\bar{b})$ are fixed exogenously. Since $y$ is indivisible BJ output in each transaction is fixed, too, and AD output is fixed by $U'(x) = 1$. Hence changes in the level of $M$, as well as changes in $\pi$ and $i$, have no impact on output: monetary policy is neutral, as well as superneutral. However, the specification can be amended in various ways to make output endogenous and show how the source of price stickiness might matter. A simple approach, following Liu et al. (2011), is to consider $k > 0$, so that the measure of BJ buyers $b$ is endogenous, with arrival rates adjusting until the marginal entrant is indifferent.

First note that if prices were sticky for Calvo or Mankiw reasons, a one-time unanticipated jump in $M$ can have real effects. This is because at least some firms could not (with Calvo) or would not (with Mankiw) adjust $p$, and so the nominal distribution $F(p)$ may not change enough to keep the same real distribution $G(q)$. If $M$ increases, one should expect the real distribution to turn in favor of buyers. This increases $b$, as households embark on a shopping spree, and this stimulates
output. By contrast, in our model, jumps in $M$ shift $F(p)$ but not $G(q)$, and hence they affect neither participation nor output. A policy pundit in our economy, seeing only a fraction of sellers adjusting $p$ each period while $Ep$ rises, may well conclude that changes in $M$ will have real effects. That would be a mistake. And recognizing this as just another example of the Lucas critique makes it no less relevant.

While $M$ is irrelevant, with endogenous entry, $\pi$ and $i$ are not. To see this, consider parameterizing the arrival rates by having BJ buyers attempt to solicit two price quotes, and succeed in each try with probability $s = s(b)$. Assume $s(0) = 1$, $s(b) = 0$, $s'(b) < 0$ and $s''(b) > 0$ to capture the standard congestion effects in search models. Then $\alpha_1(b) = 2s(b)[1 - s(b)]$ and $\alpha_2(b) = s(b)^2$. Inserting these into (12), we get

$$\hat{z}_i = \gamma + (2\delta)^{1/3} [1 - s(b)]^{2/3} (\mu - \delta - \gamma)^{2/3} \cdot i^{-1/3},$$

(16)

defining a relation between $\hat{z}_i$ and $b$ called the RB (real balance) curve. Similarly, (10) defines a relation called the FE (free entry) curve,

$$\kappa = s(b)^2 (\mu - \delta - \gamma) - \frac{\delta [1 - s(b)]^2 (\mu - \delta - \gamma)}{(\hat{z}_i - \gamma)^2} + \delta - i\hat{z}_i.$$

(17)
As shown in Figure 11, RB is increasing and convex, with \( \hat{z}_i = \gamma \) at \( b = 0 \), while FE is slopes up (down) to the left (right) of RB, with \( b \in (0, \bar{b}) \) at \( \hat{z}_i = 0 \). Hence, there is a unique equilibrium \((\hat{z}_i^*, b^*)\). An increase in \( i \), which ultimately comes from higher inflation and hence monetary expansion, shifts both curves toward the origin, so that output falls. Hence, monetary policy matters, but this has nothing to do with nominal rigidities – it is due to inflation taxing households’ participation in decentralized exchange. We think this is a good environment to study such effects because our consumers endogenously substitute between cash and credit as policy changes. However, at the risk of repetition, perhaps the bigger points are: (i) we do not claim here that money is or is not neutral, we only show observations of sluggish nominal prices do not constitute definitive evidence on the issue; and (ii) the underlying reason for sticky prices can make a huge difference.\(^{20}\)

### 9 Conclusion

A general contribution of search theorists is to show how some observations that appear anomalous from the perspective of “standard” theory can be more readily understood once frictions are incorporated. One example is price dispersion: deviations from the law of one price – something that must hold in frictionless economies – can emerge naturally in search equilibrium. The same is true for price rigidity. It is a serious puzzle for “standard” theory when sellers let their real prices vary in arbitrary ways by not responding to changes in economic conditions. Yet once one sees how to get price dispersion, stickiness is not so puzzling. Theory predicts that letting real prices vary over some range does not affect profits. This is not because of a knife-edge specification where profit functions just happen to be flat; it is because economic forces shape equilibrium so that profit is endogenously the

\(^{20}\)We can also calibrate the model with endogenous participation if we specify a matching technology, say \( s(b) = b^k \). This introduces a new parameter \( \chi \), plus there is the discounted entry cost \( \kappa \), but we also lose two parameters, because \( \alpha_1 \) and \( \alpha_2 \) become endogenous. However, note that if \((\chi, \kappa)\) lead to the same \((\alpha_1, \alpha_2)\) as in the direct calibration, the results will be identical.
same for any $p$ in some range.

One component of this project was to exploit that idea to build general equilibrium monetary models with sluggish nominal prices. Others papers mentioned above have previously made some similar points, including Caplin and Spulber (1987), Eden (1994) and Head et al. (2012). It may even be true that, based on those contributions, it is now well known that sticky prices imply neither nonneutrality nor the existence of menu costs. In any case, we tried go beyond the earlier papers. As insightful as the Caplin-Spulber may be, we do not regard it as a serious model of money, and neither they nor Eden make any attempt to match the data, as we do. Head et al. uses similar monetary theory, but is stuck between a rock and a hard place when it comes to either admitting indeterminacy of stationary monetary equilibrium or imposing the ad hoc restriction of linear menus. And while that exercise does a good job of matching price-change data, it cannot do this and match data on money and credit.

Moreover, in addition to studying pricing behavior, perhaps a bigger component of the project was to do this in a novel framework where money and credit coexist based on fixed or variable costs. Rigorous monetary theory is nontrivial, and indeed money is another anomaly for “standard” theory that can be better understood once frictions are modeled carefully. Our approach is based on lessons from the literature. We embedded a Burdett-Judd goods market in a Lagos-Wright monetary economy, then added costly credit because: (1) it resolves the indeterminacy problem; and (2) it is interesting for its own sake. With either a fixed or variable cost of credit, there emerge exact money demand functions that resemble classic results by Baumol and others, although the microfoundations are different.

In useful comments, Fernando Alvarez argued it is by now well known, but this was not always true. Consider Ball and Mankiw (1994): “We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky” and “As a matter of logic, nominal stickiness requires a cost of nominal adjustment” (emphasis added). Or Golosov and Lucas (2003): “menu costs are really there. The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing” (emphasis added).
and that fit the Lucas-Nicolini money demand data very well. It is important to highlight that it is in the context of theoretically and empirically reasonable models of money and credit that we fit the pricing data.\footnote{We do not repeat the details of this fit except to say the models are consistent with all 8 facts listed in fn. 5.}

An unanticipated finding is that a fixed-cost model performs less, mainly because it is harder to satisfy the parameter conditions for MME. So in calibrating that version, we gave up on matching credit’s share in micro data. To say it differently, if inflation increases much in that model, consumers switch entirely to credit. While some people see this as the wave of the future, for now money is very much in circulation.\footnote{Cash usage has decreased in some countries, as it like checking gets replaced by debit cards, but the ratio of currency in circulation to GDP is flat or even increasing in many places (see Jiang and Shao 2014\textit{a,b} and references therein). In any case, debit cards, cash and checks are close substitutes for our purposes.} We are not happy that fixed-cost models predict a flight from currency at moderate inflation, although this certainly may happen at very high inflation. But we do not take that prediction too seriously. In reality there are some buyers that always use cash and sellers that only take cash, for many of reasons not in the model, and may continue to do so at higher inflation. This suggests it would be useful to incorporate heterogeneity. While we emphasize that the variable-cost model does not have this problem, with money and credit coexisting for a much bigger range of parameters, it may be interesting to add heterogeneity in that version, too. This is left for future research.
Appendix

Derivation of (7): Consider $\delta > 0 = \tau$ to reduce notation. Then write

$$V(z) = W(z + T) + \alpha_1(b) \int_q^z (\mu - q) dG_1(q) + \alpha_1(b) \int_q^z (\mu - q - \delta) dG_1(q) + \alpha_2(b) \int_q^z (\mu - q) dG_2(q) + \alpha_2(b) \int_q^z (\mu - q - \delta) dG_2(q),$$

where $G_n(q) = 1 - \hat{G}(q)^n$ is the CDF of the lowest of $n$ draws from $G(q)$. The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with $q \leq z$, so only cash is used, times the expected surplus, which is simple because $W'(A) = 1$. The third is the probability of meeting a seller with $q > z$, so credit must be used, which adds cost $\delta$. The last two terms are similar except the buyer meets two sellers. The rest is algebra.

Proof of Lemma 3: (i) In NME, buyers’ BJ surplus is $\Sigma = \mu - q - \delta - \tau q$. Note $\Sigma = 0$ at $q = (\mu - \delta) / (1 + \tau)$, so no buyer pays more than this. If $\bar{q} < (\mu - \delta) / (1 + \tau)$ then the highest price seller has profitable deviation toward $(\mu - \delta) / (1 + \tau)$, which increases profit per unit without affecting sales. Hence $\bar{q} = (\mu - \delta) / (1 + \tau)$. (ii) In MME, for $q > z$, as it is near $\bar{q}$, $\Sigma = \mu - q - \delta - \tau(q - z)$. Note $\Sigma = 0$ at $q = (\mu - \delta + \tau z) / (1 + \tau)$, and repeat the argument for NME to show $\bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$. The definition of MME has $z < \bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$, which reduces to $z < \mu - \delta$. (iii) In PME, given buyers bring $z$ to BJ, they would pay $z$. Hence $\bar{q} \geq z$, as $\bar{q} < z$ implies the highest price seller has profitable deviation. We also have to be sure there is no profitable deviation to $q > z$, which requires buyers using some credit. The highest such $q$ a buyer would pay solves $\Sigma = \mu - q - \delta - \tau(q - z) = 0$, or $q = (\mu - \delta + \tau z) / (1 + \tau)$. There is no profitable deviation iff $(\mu - \delta + \tau z) / (1 + \tau) \leq z$, which reduces to $z \geq \mu - \delta$. ■

Proof of Proposition 1: (i) With fiat currency $\phi = 0$ is always self-fulfilling, so we simply set $G(q)$ according to (5), corresponding to equilibrium in the original BJ model.
(ii) From Figure 1, MME exists iff three conditions hold: (a) \( O_i^-(\bar{q}) < 0 \); (b) \( O_i^+(q) > 0 \); and (c) \( O_i(z_i) > O_i(0) \). Now (a) is equivalent to \( (\alpha_1 + \alpha_2) \delta H^-(\bar{q}) < i \), which holds iff \( i > \bar{i} \). Then (b) is equivalent to \( (\alpha_1 + \alpha_2) \delta H^+(q) > i \), which holds iff \( i < \bar{i} \) where

\[
\bar{i} = \frac{\delta (\alpha_1 + 2\alpha_2)^3}{2\alpha_1\alpha_2 (\mu - \delta - \gamma)} > \hat{i}.
\]

Also, (c) is equivalent to \( (\alpha_1 + \alpha_2) \delta H (z_i) - iz_i > (\alpha_1 + \alpha_2) \delta H (0) \), which holds iff \( \Delta (i) > 0 \) where

\[
\Delta (i) = -i\gamma + \frac{\delta (\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i\frac{\bar{i}^3}{2\alpha_1\alpha_2} (\mu - \delta - \gamma)^{\frac{1}{3}} (2^{-\frac{1}{3}} + 2^{-\frac{4}{3}}).
\]

Notice \( \Delta (0) > 0 > \Delta (\bar{i}) \) and \( \Delta' (i) < 0 \). Hence \( \exists! \bar{i} \) such that \( \Delta (\bar{i}) = 0 \), and \( \Delta (i) > 0 \) iff \( i < \bar{i} \). It remains to verify that \( \bar{i} > \hat{i} \), so that (a) and (c) are not mutually exclusive. It can be checked that this is true iff \( \delta < \bar{\delta} \). Hence a MME exist under the stated conditions. It is unique because \( \bar{q} = \mu - \delta \), which pins down \( G (q) \), and then \( \hat{z}_i = \arg\max_{z \in (\bar{q}, \hat{q})} O_i(z) \).

(iii) From Figure 1, PME exists iff three conditions hold: (a) \( O_i^- (\bar{q}) > 0 \); (b) \( O_i^+(q) > 0 \); and (c) \( O_i (\hat{q}) > O_i (0) \). Now (a) holds iff \( i < \bar{i} \) and (b) holds iff \( i < \hat{i} \). Condition (c) holds iff \( i < \hat{i} \). For \( \delta > \bar{\delta} \), it can be checked that \( \hat{i} < \bar{i} \) and \( \bar{i} < \hat{i} \), so the binding condition is \( i < \hat{i} \). For \( \delta < \bar{\delta} \), it is easily checked that \( \hat{i} > \bar{i} \), and \( \bar{i} < \hat{i} \), so the binding condition is \( i < \hat{i} \).

**Formulae for Calibration:** Consider the fixed-cost model (the variable cost case is similar but more algebraically intense). After inserting \( \bar{q} \) and \( q \), the real posted-price and transaction-price CDF’s reduce to

\[
G (q) = 1 - \frac{\alpha_1}{2\alpha_2} \frac{\mu - \delta - q}{q - \gamma},
\]

\[
H (q) = 1 - \frac{\alpha_1^2 (\mu - \delta - q) (\mu - \delta + q - 2\gamma)}{4\alpha_2 (\alpha_1 + \alpha_2) (q - \gamma)^2}.
\]
The fraction of monetary transactions and the markup are

\[ H(z_i) = \frac{(\alpha_1/2 + \alpha_2)^2 - [\alpha_1 \alpha_2 (\mu - \delta - \gamma)]^{2/3} i^{2/3}}{\alpha_2 (\alpha_1 + \alpha_2) (2\delta)^{2/3}} \]

\[ \frac{E_{Gq}}{\gamma} = 1 + \frac{\alpha_1 (\mu - \delta - \gamma) \log (1 + 2\alpha_2/\alpha_1)}{2\alpha_2 \gamma}. \]

The Lucas-style money demand function and its elasticity are

\[ L_i = \frac{\gamma + [\alpha_1^2 \delta (\mu - \delta - \gamma)^2 / 2\alpha_2]^{1/3} i^{-1/3}}{1 + \alpha_1 (\mu - \delta) + \alpha_2 \gamma} \]

\[ \eta_i = \frac{-1}{3 + 3\gamma [\alpha_1^2 \delta (\mu - \delta - \gamma)^2 / 2\alpha_2]^{-1/3} i^{1/3}}. \]
References


