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Monetary Equilibria with Indivisible Goods

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Abstract

This paper uses a New Monetarist framework to study the trade of indivisible goods with divisible money in a frictional market. We first derive conditions under which stationary equilibrium exists, and then show that if equilibrium exits, it is unique. The uniqueness result is due to the commitment and coordination nature of the pricing mechanisms. Money is superneutral in the model with generalized Nash bargaining, but not with competitive search. Because of the superneutrality of money, monetary equilibrium in the generalized Nash bargaining model only exists for low values of nominal interest rate. With competitive search, monetary equilibrium exists for all $i > 0$.

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1 Introduction

The original and seminal monetary model of Kiyotaki and Wright (1989) considers an environment with both divisible money and perishable goods. Green and Zhou (1998) are the first to consider the consequences of divisible money while maintaining the indivisibility of perishable goods. Within a purely random matching model, they show existence of a continuum of stationary monetary equilibria where all transactions occur at a single price.

Jean, Rabinovich, and Wright (2010) reconsider the environment of divisible money and indivisible goods. They apply the Lagos and Wright (2005) framework to study indivisible goods and price posting (akin to Curtis and Wright, 2004) in the decentralized market.[1] An important feature of Lagos-Wright that makes it comparable with the Green-Zhou framework is that, buyers and sellers in the decentralized market can switch identities across periods with a certain probability.[2] Jean et al (2010) find that if buyers and sellers move simultaneously by choosing money holdings and prices, respectively, there exists a continuum of steady state monetary equilibria, each indexed by a different real price hence a different real money holding of buyers. The continuum is mainly driven by the coordination failure of simultaneous moves, and any small deviation by agents in price or money holding causes a discrete drop in payoff. Therefore, as in Green and Zhou (1998), they also obtain the result of multiple monetary equilibria.

Jean et al (2010) provide a discussion on how to break the multiplicity of monetary equilibrium by assuming a finite number of agents and eliminating simultaneous moves. More specifically, either sellers move first by posting prices and buyers then respond by choosing money holdings, or vice versa. Given these assumptions, they find a unique equilibrium with a single price in the decentralized market.

In this note, we use the alternating market framework, as in Rocheteau and Wright (2005), in which agents participate in a decentralized and a centralized market sequentially in each period. Instead of modeling the divisible good in the decentralized market, we assume it is indivisible. Money in the model is nonetheless divisible. We consider two

[1] The Lagos-Wright framework has typically assumed both divisible goods and money, pairwise meetings, and generalized Nash bargaining in the decentralized market.

[2] This is in contrast with the Rocheteau and Wright (2005) framework we use in which buyers and sellers always remain who they are.

different pricing mechanisms in the decentralized market, which are commonly studied in the monetary literature, bargaining and competitive search. We find that, under certain parameter values, stationary monetary equilibrium exists and is unique.

First, we consider the price of indivisible goods in the decentralized market is determined by generalized Nash bargaining, as in Lagos and Wright (2005). Due to the indivisible nature of goods, trading quantity cannot adjust according to price or the buyer's real balance. With divisible goods, the quantity traded can adjust according to the total amount of money brought to the bargaining problem, and then different prices may arise. That is not true with indivisible goods, since the quantity traded is always one unit. Therefore, the buyer's decision on money holding reduces to two options: either bringing enough money to make a seller sell the good, or not bringing any money at all. As long as the nominal interest rate is not too high, $i < \bar{i}$, the buyer's surplus from trade, determined exogenously by bargaining power, can cover the cost of holding money, and there exists a unique stationary monetary equilibrium. Otherwise, if the cost of holding money is higher than the buyer's share of trade surplus, monetary equilibrium will cease to exist.

Equilibrium uniqueness comes from the committing nature of the generalized Nash bargaining problem. Before buyers and sellers match and negotiate, both parties can rationally expect the resulting price from the bargaining solution. Then, the bargaining outcome serves as a coordination device for both parties, and they commit to trade at this price by bringing the exact amount of money holding. So both equilibrium real price and real balance are uniquely determined by the bargaining solution, and they do not change for a range of i. Therefore, money is superneutral in the model with Nash bargaining.

Secondly, we use a competitive search approach to show the existence of a unique single price stationary equilibrium, when terms of trade in the decentralized market are determined by price posting with directed search. Compared to the case of bargaining, where monetary equilibrium only exists for $i < \bar{i}$, we show that, at any positive nominal interest rate, we can find a symmetric competitive search equilibrium. This is because with competitive search, the total surplus from trade is shared endogenously between a buyer and a seller, and the buyer is always guaranteed some surplus at any i.

On the other hand, for each price posted by a seller, the expected queue length at this seller endogenously adjusts in order to guarantee a market utility for buyers, which is the highest payoff they can get by visiting any other seller in the market. In the symmetric equilibrium, the queue length at every seller adjusts to match the buyer-seller ratio of the entire market, which then determines a unique optimal real price, hence a unique optimal real balance. Unlike bargaining, the equilibrium real price and real balance are functions of nominal interest rate. If holding money becomes more costly, buyers bring less money to trade at a lower price, and get a smaller surplus from trade. Therefore, money is not superneutral in the competitive search model.

Our findings of equilibrium uniqueness is consistent with the results demonstrated in Galenianos and Kircher (2012), who use a non-monetary model to show the uniqueness of symmetric directed search equilibrium, but in contrast with Green and Zhou (1998) and Jean et al (2010). Green and Zhou (1998), study indivisible goods and divisible money in a monetary environment where agents have to switch identities between buyers and sellers. They demonstrate a real indeterminacy of monetary equilibria, i.e. there exists a continuum of steady state single price equilibria.

In contrast to Jean et al (2010), we do not need to focus on a degenerate price distribution, nor do we need a finite number of agents to demonstrate a unique equilibrium. With competitive search, we provide a natural environment for one set of agents to move first and offer the terms of trade. Additionally, unlike Jean et al (2010), our equilibrium price does not automatically provide sellers with maximum surplus from trade. Instead, our equilibrium price is an endogenous share of total surplus, which adjusts according to the queue length and the cost of holding money.

Other papers have studied monetary economy featuring indivisible goods. Julien, Kennes, and King (2008) consider indivisible goods and indivisible money, and they show the uniqueness of monetary equilibrium using price posting and auctions with lotteries. Their results complement the findings in Berentsen, Molico, and Wright (2002). In the context of indivisible goods and divisible money, Galenianos and Kircher (2008) use a competitive search construct. However, their focus is on purely random search with auctions. They show the existence of monetary equilibrium with a nondegenerate price distribution, away from the Friedman rule. Because carrying more money increases the probability of trade and vice versa, they demonstrate that in equilibrium buyers are indifferent among a range of different money holdings and hence there is price dispersion. Liu, Wang, and Wright (2014) also present equilibrium price dispersion in a monetary model featuring price posting and random search with indivisible goods. They extend regular random search by allowing buyers to sample more than one price. The paper focuses on the behavior of sticky prices and the relationship between money and credit. With indivisible goods, their model can generate a unique equilibrium outcome for the price posting problem.

2 The Model

Time is discrete. A continuum of buyers and sellers, with measures 1 and n, live forever, implying a buyer-seller ratio of $\theta = 1/n$. In each period, all the agents participate in two markets which open consecutively. Agents discount between periods with factor $\beta \in (0, 1)$ but not across markets within a period.

The first market to open is a decentralized market (DM), and the second market is a frictionless centralized market (CM). Both buyers and sellers consume a divisible good in the CM, while only buyers consume a perishable indivisible good in the DM, which is produced at constant cost c by sellers. We assume agents in the DM are anonymous, lack record keeping technology, and have limited commitment. Therefore, buyers pay sellers with cash in the DM. Let M_t be the money supply per buyer at time t, with $M_t = \gamma M_{t-1}$ with γ constant. Changes in M occur in the CM via lump-sum transfers (taxes) if $\gamma > 1$ (if $\gamma < 1$). Assume $\gamma > \beta$, where γ is the inflation rate in stationary equilibrium. Nominal interest rate is given by the Fisher equation $1 + i = \gamma/\beta$. Thus, $\gamma > \beta$ iff $i > 0$, and the Friedman rule is the limiting case of $i \to 0$.

The buyer's preferences within a period are separable across (x_t,e_t,q_t), as given by $U(x_t) - e_t + u\mathbf{1}(q_t)$, where x_t is CM consumption, e_t is CM labor, q_t is DM consumption, $\mathbf{1}(q_t)$ is an indicator function giving 1 if $q_t = 1$ and 0 otherwise, and u is the utility from consuming one unit of DM good. Let x_t be the CM numeraire at t. For simplicity, we assume $U(x_t) = x_t$, which is produced one-to-one from labor e_t. Define ϕ_t as the CM price of money in terms of x_t, and $p_t = 1/\phi_t$ is the nominal price level. In stationary equilibrium, intertemporal change in prices is depicted as $p_{t+1}/p_t = \phi_t/\phi_{t+1} = \gamma$.

In the CM, every agent maximizes utility subject to a budget constraint. They also decide how much money to bring to the subsequent DM. In the DM, each seller maximizes revenue, taking as given the behavior of buyers and other sellers, and every buyer randomly meets a seller to trade. The probability of a successful match in the DM depends on market tightness. Given the buyer-seller ratio θ, the probability of a seller matching with a buyer is $\alpha(\theta)$, while the probability for a buyer is $\alpha(\theta)/\theta$. Assume $\alpha' > 0$, $\alpha'' < 0$, $\alpha(0) = 0$, and $\lim_{\theta\to\infty} \alpha(\theta) = 1$.

The CM and DM value functions are $W(m)$ and $V(m)$. Buyers who bring money m to the CM face the following problem

$$W_t^b(m) = \max_{x,e,m'} \left\{x - e + \beta V_{t+1}^b(m')\right\} \text{ st } x = \phi_t(m+T) + e - \phi_t m', \tag{1}$$

where m' is the money holding carried to the next DM and T represents net transfers received from the government. Eliminating e using the budget equation, we get

$$W_t^b(m) = \phi_t(m+T) + \max_{m'} \left\{-\phi_t m' + \beta V_{t+1}^b(m')\right\}. \tag{2}$$

Similarly, the CM payoff of a seller carrying money m_s is

$$W_t^s(m_s) = \phi_t(m+T) + \max_{m_s'} \left\{-\phi_t m_s' + \beta V_{t+1}^s(m_s')\right\}. \tag{3}$$

The specific form of DM value functions depends on how prices are determined. In the following, we consider two different pricing mechanisms to determine the DM price d, bargaining and competitive search.

3 Bargaining

We first consider the case when sellers commit to generalized Nash bargaining to determine the terms of trade in the DM. By committing to this pricing mechanism, sellers only trade at the price determined by the bargaining solution.[3] With Nash bargaining, buyers and sellers randomly form bilateral matches and negotiate only the price of the indivisible good d, not quantity. Production happens after a buyer and a seller reach an agreement on price.

[3] We have also studied the case where sellers do not fully commit to bargaining, i.e. if the buyer's money holding is less than the bargaining price, sellers still trade with buyers. In that case, the equilibrium money holding is $\phi m = \phi d = c$, and buyers get all the surplus from trade, even though their bargaining power is less than one. That is due to the buyer's first-mover advantage.

With bargaining, the buyer's payoff in the DM is

$$V_t^b(m) = \frac{\alpha(\theta)}{\theta}\left[u + W_t^b(m-d)\right] + [1 - \frac{\alpha(\theta)}{\theta}]W_t^b(m). \tag{4}$$

If the buyer gets to trade, he pays d for the indivisible good and gets u from consumption. Notice that the buyer's ability to pay is bound by his money holding m, and then $d \leq m$ has to satisfy in equilibrium. As in Lagos and Wright (2005), $\partial W^b/\partial m = \phi_t$ keeps the analysis tractable. This allows us to simplify (4) to

$$V_t^b(m) = W_t^b(m) + \frac{\alpha(\theta)}{\theta}(u - \phi_t d). \tag{5}$$

The seller's payoff can be similarly derived as

$$V_t^s(m_s) = W_t^s(m) + \alpha(\theta)(-c + \phi_t d), \tag{6}$$

where the seller collects d for producing the good at cost c.

Let $\eta \in (0,1)$ be the buyer's bargaining power, and the generalized Nash bargaining problem is

$$\max_d \left[u + W_t^b(m-d) - W_t^b(m)\right]^\eta \left[-c + W_t^s(m_s+d) - W_t^s(m_s)\right]^{1-\eta}, \tag{7}$$

where $u + W_t^b(m-d) - W_t^b(m)$ is the buyer's surplus from trade, and $-c + W_t^s(m_s+d) - W_t^s(m_s)$ is the seller's. Following Lagos and Wright (2005), we can simplify the bargaining problem by substituting the CM and DM value functions from (2), (3), (5), and (6). The problem then becomes

$$\max_d (u - \phi_t d)^\eta (-c + \phi_t d)^{1-\eta}, \tag{8}$$

with a unique optimal solution

$$\phi_t d^B = (1-\eta)u + \eta c, \tag{9}$$

where d^B is the price of the DM good with bargaining, assuming the buyer carries enough money to cover the price.

Because the quantity of the DM good cannot be adjusted, the total surplus from trade is constant, and the real price $\phi_t d^B$ does not depend on money holding or nominal interest rate. Hence, the seller's optimal money holding decision in the CM satisfies $m^s = 0$ if $i > 0$. Regarding the buyer's money holding decision, using (5) to substitute V_t^b in (2) yields the following aggregate CM value function

$$W_t^b(m) = \phi_t (m + T) + \beta W_{t+1}^b (0) + \max_{m' \geq 0} \beta \left[-i\phi_{t+1} m' + \mathbf{1} \left(m' \geq d^B \right) \frac{\alpha(\theta)}{\theta} \left(u - \phi_{t+1} d^B \right) \right], \tag{10}$$

where $\mathbf{1}\left(m' \geq d^B\right)$ is an indicator function giving 1 if $m' \geq d^B$. Taking into account the cost of holding money, buyers choose money holding to maximize the surplus from DM trade.

We know from the bargaining solution that, a buyer can trade with a seller only if he brings $m' \geq d^B$. Therefore, the buyer's optimal money holding decision in (10) reduces to bringing either $m' = d^B$ or $m' = 0$. Clearly, the buyer does not bring $m' > d^B$ if $i > 0$. If $m' < d^B$, the seller does not want to trade, and the buyer may as well decrease his money holding to zero. Define $\Phi(m') = -i\phi_{t+1}m' + \alpha(\theta)(u - \phi_{t+1}d^B)/\theta$. In order to have an equilibrium where money bears value, we need $\Phi(d^B) > \Phi(0)$, which implies the following condition on i

$$i < \frac{\eta (u - c) \alpha(\theta) / \theta}{(1 - \eta) u + \eta c} \equiv \bar{i}. \tag{11}$$

Therefore, as long as the nominal interest rate is not too large, that is, carrying money is not too costly, we have a unique steady state monetary equilibrium, and the buyer's optimal real balance and the real price in the DM are both characterized by (9).

Proposition 1 *For $i < \bar{i}$, there exists a unique steady state monetary equilibrium with generalized Nash bargaining in the DM.*

The real balance in equilibrium only depends on u, c, and the bargaining power η. Notice that $\bar{i} \to 0$ if $\eta \to 0$, and monetary equilibrium does not exist. If the buyer's bargaining power decreases to zero, all the surplus from trade goes to seller, and buyer has no incentive to bring money to the DM. $\bar{i}$ increases with the buyer's bargaining power. As buyer claims more surplus from the DM trade, he can bear a larger cost of holding money.

Because the DM good is indivisible, there is no benefit of carrying money at the intensive margin, i.e. the quantity of trade. The only gain of bringing money comes from the extensive margin, i.e. to trade or not to trade. Given the seller's commitment to bargaining, the buyer's money holding decision essentially becomes a discrete choice, and the real balance in monetary equilibrium is not affected by nominal interest rate, as long as (11) holds. Notice that in equilibrium $d^B = m_t = M_t$, and $\phi_t d^B = \phi_t m_t = \phi_t M_t = (1-\eta)u + \eta c$. Nominal interest rate has no effect on the DM real price, the buyer's real balance, or the real value of money. Therefore, money is superneutral in the model with bargaining.

This result differs from most of the New Monetarist literature, which generally features the neutrality of money but real allocations are affected by changes in inflation. Because of the indivisible nature of the DM good, the total surplus from trade is constant, and does not depend on nominal interest rate or money stock. The generalized Nash bargaining mechanism determines the buyer's share of surplus according to exogenous bargaining power, which then determines the unique optimal real balance. Monetary variables do not play a role in the determination of real variables, but only affects the price of money γ. Unlike Green and Zhou (1998) or Jean et al (2010), there is no real indeterminacy under bargaining. In those papers, real indeterminacy exists due to coordination failure between buyers and sellers. In the current environment, the unique solution generated by Nash bargaining works as a coordination device. Given the commitment nature of Nash bargaining, both buyer and seller know ex ante that $\phi_t d^B$ will be the price in DM trade.

4 Competitive Search

We now study competitive search equilibrium, and terms of trade are determined using price posting with directed search. In this setup, sellers post DM prices before buyers choose their money holdings in the CM. Each seller commits to selling the indivisible DM good at his posted price. After observing all the posted prices in the DM, each buyer chooses the price that gives him the maximum surplus from trade. If a seller is visited by multiple buyers, he chooses one of the buyers with equal probability to trade. The trading probability is determined by the ratio of buyers and sellers, Θ, who are willing to trade at the given price. Therefore, Θ represents the expected queue length for any seller offering that price. The probability of trade for a buyer is $\alpha(\Theta)/\Theta$ and the probability for a seller is $\alpha(\Theta)$. By posting a lower price, a seller attracts more buyers and increases his trading probability via a larger Θ.

The buyer's DM value function is now

$$V_t^b(m) = W_t^b(m) + \frac{\alpha(\Theta)}{\Theta}(u - \phi_t d). \tag{12}$$

From (2) and (12), a buyer faces

$$W_t^b(m) = \phi_t(m+T) + \max_{m',d',\Theta}\left\{-\phi_t m' + \beta\left[\frac{\alpha(\Theta)}{\Theta}\left(u - \phi_{t+1}d'\right) + W_{t+1}^b(m')\right]\right\}. \tag{13}$$

Let Ω be the equilibrium expected utility of a buyer in the DM. In order to attract buyers, a seller has to post a price which offers a payoff at least equal to the highest value they can get elsewhere. Therefore, to attract Θ buyers, a seller must offer price d satisfying

$$-i\phi_t d - \frac{\alpha(\Theta)}{\Theta}\phi_t d + \frac{\alpha(\Theta)}{\Theta}u = \Omega. \tag{14}$$

The seller then chooses Θ and d to maximize his payoff

$$\alpha(\Theta)(-c + \phi_t d), \tag{15}$$

subject to constraint (14). As before, the seller's choice of money holding in the DM is $m_s' = 0$, since it does not affect his payoff. Notice that as long as a buyer can trade in the DM, Ω does not depend on his money holding. Therefore, the buyer's optimal money holding satisfies $m = d$ in equilibrium with $i > 0$.

Using (14) to substitute $\phi_t d$ in (15) gives the seller an unconstrained optimization problem in Θ. The first-order condition implies the following characterization of equilibrium price in the DM

$$\phi_t d^C = \frac{\alpha(\theta)\{[1-\varepsilon(\theta)]u + \varepsilon(\theta)c\} + \theta\varepsilon(\theta)ic}{\alpha(\theta) + \theta\varepsilon(\theta)i}, \tag{16}$$

where d^C is the price of the indivisible DM good under competitive search, and $\varepsilon(\theta) = \alpha'(\theta)\theta/\alpha(\theta)$ is the elasticity of the matching function. Notice that we have imposed $\Theta = \theta$ since we focus on symmetric equilibrium, where homogeneous agents make the same optimal decision. Given θ, (16) determines a unique real price for the DM good, and hence a unique Ω for any $i > 0$. These objects together complete the equilibrium characterization.

Proposition 2 *For $i > 0$ there exists a unique symmetric steady state monetary equilibrium with competitive search in the DM.*

Note that this environment satisfies all the properties under which Galenianos and Kircher (2012) demonstrate uniqueness of equilibrium when terms of trade of an indivisible good are determined by price posting with directed search. To compare with Nash bargaining, the real DM price and the buyer's real balance under competitive search are affected by i, and money is not superneutral, while it is still neutral. Therefore, the buyer's surplus from trade under competitive search adjusts endogenously with inflation. As Berentsen, Rocheteau, and Shi (2007) point out, the surplus from trade is divided between a buyer and a seller, such that it compensates their search efforts. While under bargaining, the buyer's share of surplus is determined exogenously by the bargaining power, and does not adjust with nominal interest rate. As a result, we need $i < \bar{i}$ to guarantee equilibrium existence. In the case of competitive search, from the pricing equation (16), one can see that d^C decreases with i. If $i = 0$, holding money is costless. The DM price under competitive search then becomes

$$\phi_t d^C = [1 - \varepsilon(\theta)] u + \varepsilon(\theta) c,$$

which is the same as the bargaining solution given $\varepsilon(\theta) = \eta$. If i goes to infinity, $\phi_t d^C$ converges to c, and buyers take all the surplus from trade. Therefore, the price of the indivisible DM good under competitive search exists for any nominal interest rate.

Apart from the finding of equilibrium existence, our results differ quite substantially from those of Jean et al (2010). They consider price posting and random search, and show the existence of a continuum of equilibria, indexed by different real balances. Their result of multiple equilibria is driven by coordination failure of simultaneous moves by buyers and sellers. In order to obtain a unique equilibrium, they impose the assumptions of finite agents and sequential move. While with competitive search, we do not need those assumptions to get unique equilibrium. Because a buyer can observe all the prices in the market, he can direct his search to the seller who gives the highest expected payoff. Competition among sellers guarantees that a buyer gets Ω from DM trade. On the other hand, the buyer competes with other buyers who also want to trade with the preferred seller. Competition on both sides of the market endogenously generates the expected queue length at a seller, which then determines the probability of trade and the share of surplus for both sides. The expected queue length adjusts continuously with the posted price, and the market-clearing price in the DM is uniquely determined when the expected queue length equals the buyer-seller ratio of the entire economy θ. This adjustment mechanism does not exist under price posting and random search, since trading probabilities and the share of trade surplus are exogenously determined.

5 Conclusion

In this paper, we use a general equilibrium model, with an explicit role for money, to study the trade of indivisible goods with divisible money in a frictional market. In particular, two commonly used pricing mechanisms, bargaining and competitive search, have been considered, and they have not been previously studied with indivisible goods. We lay out the conditions under which stationary monetary equilibrium exists. Most importantly, we have shown that as long as equilibrium exists, it is also unique.

A key difference between indivisible good and divisible good is the adjustment of trading quantity. Since only one unit of good is traded, the total surplus from trade is a constant. Using generalized Nash bargaining, the trading price is uniquely determined by the exogenous bargaining power, and is not affected by the amount of money buyer brings to trade. The money holding decision then becomes a binary choice problem. As long as the total cost of holding money is not too large, all buyers carry the same real balance, which does no depend on nominal interest rate. So money is superneutral. While under competitive search, the rule of splitting surplus between buyer and seller is endogenously determined by nominal interest rate and market tightness. Given the overall buyer-seller ratio, the mechanism delivers a unique equilibrium price. When i changes, the buyer's share of total surplus adjusts to compensate for his cost of holding money. Money is then not superneutral.

The result of a unique single price equilibrium comes from two common features shared by the two pricing mechanisms. First, both bargaining and competitive search have a built-in commitment component. With bargaining, buyer and seller are committed to split the total surplus from trade between the two parties according to the bargaining power. With competitive search, seller commits to sell one unit of indivisible good, at the posted price, to one of the present buyers with equal probability. Second, both pricing mechanisms provide a coordination device between buyer and seller. An exogenous bargaining power uniquely determines an outcome of the bargaining problem. Before negotiation, both parties expect to agree on the bargained price, and then they act according to their expectations. As for competitive search, two sides of the market coordinate through adjusting market tightness. Seller then posts the right price to induce a market tightness equal to the exogenous overall buyer-seller ratio. There is no such coordination device in Green and Zhou (1998) or Jean et al (2010). Seller and buyer have to form their own beliefs about trading price, and hence multiple equilibrium prices exist.

Finally, our model does not include many other elements in the study of trading indivisible goods, such as the durability of good and the timing of production, which also play an important role in determining equilibrium existence, uniqueness, and allocation. Those topics are left for future research.

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