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Intra-Industry Reallocations and Long-run Impacts of Environmental Regulations

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Intra-Industry Reallocations and Long-run Impacts of Environmental Regulations[1]

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Abstract: We investigate the long-run impact of environmental regulations on the intra-industry distribution of firm-level productivity and the resulting aggregate variables. In a general-equilibrium model that accounts for endogenous entry/exit of heterogeneous firms, neither the average productivity of firms nor the mass of firms is independent of the choice of policy instruments (i.e. emissions tax vs. emissions trading) or permit allocation rules. The equilibrium price of permits under emissions trading is lower than the emissions tax rate that would support the same aggregate emissions. An incomplete emissions market results in a net increase in combined aggregate emissions.

JEL Codes: Q50, Q52, Q58
Key Words: Emissions Tax, Emissions Trading, Heterogeneous Firms, Endogenous Entry/Exit, Melitz Model, Incomplete Regulation, Emissions Leakage

1. Introduction

Since Rose-Ackerman (1973), economists have long been interested in the impacts of environmental policies on the long-run industry dynamics. Conventional wisdom suggests that an emissions tax would induce excessive exit of firms from a polluting industry by raising the firms' average costs whereas a green subsidy would induce excessive entry by lowering them (Baumol, 1988; Polinsky, 1979). The excessive entry (exit) then induces excessive (insufficient) aggregate emissions and output. This conventional theoretical premise, however, relies heavily on the assumption of perfect competition with identical firms.

A large number of empirical studies have substantiated the existence of large and persistent variation in firm-level productivity across firms (e.g. Cabral and Mata, 2003; Eaton *et al.*, 2011). When firms are heterogeneous, choice of environmental policies may affect different firms differently via their differentiated effects on the expected value of entry, resulting in the reallocation of firm-level variables. The intra-industry reallocation of these firm-level variables may, in turn, have ambiguous effects on aggregate industry emissions, output, and welfare. These effects may be even more ambiguous under an emissions trading (ET) policy, since some firms receive implicit subsidies while others receive implicit taxes in the form of the initial distribution of permits. Taken in this view, it is not so immediate that the simple economic intuition used in the conventional argument would work in the presence of heterogeneous firms.

This paper attempts to investigate these intra-industry effects of various market-based environmental policies in a general equilibrium model that accounts for entry and exit of heterogeneous firms. Our model starts with the Melitz-type economy (2003) consisting of a continuum of heterogeneous firms. The firms make endogenous entry, draw heterogeneous productivity shocks upon entry, and then monopolistically compete in the commodity markets. In the original Melitz model, there is only one input, *labor*, for production and there is no pollution. To analyze the impacts of environmental policies, we introduce *emissions* as an input in a manner analogous to Copeland and Taylor (1994).[2] Our model then embeds a suit of environmental policy instruments in this Melitz-type economy. The resulting model suggests a new venue for the theory of environmental

[2] In this sense, our basic setup is similar to that of Bajona and Missios (2010), Kreickemeier and Richter (2012), or Yokoo (2009). We, however, diverge from these studies by specifically considering an emissions trading policy with and without complete coverage. Analysis of the emissions trading policy turns out to be substantially more involved than that of the tax policy, because in the former, both the commodity markets and the emissions market clear simultaneously. Furthermore, analysis of the incomplete emissions market also requires the benchmark case with no regulation (i.e. with a zero emissions tax rate), which none of these papers considers.

regulations — by explicitly considering heterogeneity and distribution of firm-level variables, the model allows us to describe the long-run impacts of environmental regulations in terms of the intensive and extensive margins: i.e. their effects on the reallocation of firm-level variables as well as the equilibrium mass of firms.

The paper considers not only the emissions tax and the emissions market but also alternative allocation rules for the emissions market. A standard theory predicts that the equilibrium outcome of an efficient emissions market is independent of the initial distribution of permits. However, a number of studies have substantiated the dependence of the equilibrium outcome on the initial permit allocation under various conditions, such as imperfect competition in the emissions market (Hahn, 1984), transaction costs (Stavins, 1995; Montero, 1997), and imperfect competition in the commodity markets (Fowlie and Perloff, 2012). To investigate this issue, our paper considers three alternative allocation rules: (i) uniform allocation (UA), where all incumbent firms receive an equal amount of permits, (ii) emissions-based allocation (EBA), where firms receive permits based on their emissions share in the aggregate emissions, and (iii) rate-based allocation (RBA), where firms receive permits based on a uniform emissions rate. The last two allocation schemes have been applied under the U.S. Acid Rain Program and EU Emissions Trading System.

Our model is also useful in evaluating the effects of an incomplete emissions market. By incompleteness, we mean that some economic entities are excluded from the coverage of regulation despite the fact that their polluting activities are the object of the regulation and that these unregulated entities can voluntarily participate in the emissions market, either as the sellers or buyers of the permits. One important example is Clean Development Mechanism (CDM) under the Kyoto Protocol.[3] Under CDM, firms in developing countries can implement climate mitigation projects for credits to be sold in the international carbon emissions markets. These firms are generally not mandated to reduce carbon emissions by their domestic regulation, but can earn credits for projects that would reduce carbon emissions below some business-as-usual (BAU) baselines. The CDM-related carbon emissions reduction is not small — CDM projects are expected to reduce GHG emissions by more than 2,900 million tons (CO_2e) by the end of 2012 (Zhang and Wang, 2011).

One obvious benefit of CDM is that it induces voluntary abatement to take place in

[3] Aldy and Stavins (2011) call such a policy an emission-reduction-credit (ERC) system. We define it more broadly, however, to include systems that allow non-regulated entities' purchase of permits from regulates sources. There are indeed many examples besides CDM. In some watersheds in U.S., water-quality trading policies involve sales of credits from agricultural sources that are not subject to pollution control standards to regulated sources. Another example is the stipulation that allows any non-regulated party to buy sulfur dioxides allowances under the U.S. Acid Rain program.

developing countries where the cost of abatement is presumably low. Thus, CDM is expected to mitigate potential emissions leakages from developed countries by allowing the CDM credits to increase the supply of permits and thereby lowering their price in sponsoring developed countries. However, precisely because of this, the overall emissions from the host and sponsoring countries can increase if the amount of CDM credits generated exceeds the amount of emissions reduction in the host countries relative to no CDM policy. In this context, there are several pathways through which CDM may increase aggregate emissions and undermine economic efficiency. First, because CDM credits work as implicit subsidies, they may induce excessive entry of firms. Second, the CDM policy may increase emissions from firms that choose optimally not to participate in CDM by shifting market power to these firms. Our model can account for all of these pathways that are important for evaluating the CDM policy.[4]

The foremost important contribution of the paper lies in a number of findings we establish that are rather surprising and counter-intuitive. Both the equivalence between tax and emissions-trading policies and the invariance of emissions-trading outcomes with respect to initial allocation of permits fail in a manner that was not foreseen in the existing literature. Interestingly, the choice of policy instruments may have opposing impacts on the firm-level variables and the industry size, suggesting a new and important venue for the theory of environmental regulations. Specifically, we find:

1. The emissions tax does not favor any particular firm, productive or unproductive, because the tax increases *all* firms' marginal cost the same way, and *all* firms respond to it by increasing the markup the same way. Hence, the stationary-equilibrium distribution of productivity is the same with or without the emissions tax (regardless of the tax rate). In contrast, the productivity distribution under ET depends on the initial distribution of permits: the average productivity of active firms is lower (the same as, and higher than) under ET with the uniform allocation (emissions-based allocation, and rate-based allocation) schemes than under no regulation or the emissions tax. This occurs because under the EBA, the permit distribution is proportional to the no-regulation equilibrium emissions whereas under the UA (RBA) rule, firms with higher productivity receive disproportionately less (more) permits than firms with lower productivity. As a result, the UA, EBA, and RBA rules turn

[4]There are some economic studies that investigated emissions leakages under the incomplete emissions market. Fowlie (2009), for example, developed a partial equilibrium model of incomplete regulation under the Cournot oligopoly, with an application to California's electricity market, and found that incomplete regulation can be welfare increasing relative to complete regulation. Fowlie *et al.* (2012) developed a dynamic industry model with trade exposure to unregulated foreign markets with an application to the Portland cement industry and analyzed implications of emissions leakages.

out to be progressive, neutral, and regressive, respectively, in the initial distribution of permits.

2. As expected, the equilibrium mass of active firms is generally larger under the ET policy than under the tax policy. An exception is with the RBA rule, in which case more firms may enter under the tax policy depending on the distribution of potential entrants. Counter-intuitively, the equilibrium mass of firms is larger under the tax policy than no regulation. This result occurs because firms under the tax policy reduce output and employ less labor relative to no regulation. Hence, under the full employment assumption, more firms enter in equilibrium.

3. For a given price of pollution (either as the tax rate or the price of permits), the emissions tax results in more aggregate emissions than the ET does regardless of the permit allocation rule, despite our finding above that the ET policy tends to induce more entry (except under the RBA rule). We emphasize here that we obtain this counter-intuitive result *not* because of the full employment or other artificial assumptions of the model, rather because our model accounts for how the tax revenue is spent. Firms under the tax policy have an incentive to produce more and employ more emissions inputs to make up for the tax payments. The increased consumer demand due to the lump-sum transfer of tax revenue allows firms to absorb this incentive. In contrast, firms do not face this incentive at least in aggregate terms under the grandfathered ET policy.

4. The welfare impact of environmental regulation can be generally decomposed into the effects on (i) the overall entry, (ii) the average productivity, and (iii) the output per firm of each productivity level. Because these effects tend to go in the opposite directions, which policy instrument achieves higher social welfare is in general ambiguous.

5. Under the CDM policy, we analyze two baseline rules: (i) the uniform baseline (UB), where the *average* emissions that would result in the absence of regulation is used as the uniform baseline for CDM credits for all firms and (ii) the emission-based baseline (EBB), where the baseline for each firm is the emission level that would result in the absence of regulation for *that* firm. Both are alternative interpretations of the additionality condition.

    - Under the UB rule, less productive firms participate in CDM whereas under the EBB rule, either all active firms participate or no firms participate in CDM. The intuition behind this result is similar to our finding on the ET policy.

- CDM unambiguously reduces aggregate emissions relative to no regulation regardless of the baseline rule. Yet, the stationary-equilibrium CDM emissions may or may not be higher than under the ET with complete coverage. Under both baseline rules, CDM is shown to result in a net increase in the combined emissions from the host and the sponsoring countries under certain conditions. Thus our analysis provides a way to explain the empirical finding by Zhang and Wang (2011) that CDM did not reduce prefecture-level CO_2 emissions in China statistically significantly.

The paper is organized as follows. The next section describes our basic model setup. In **Section 3**, we establish the existence and uniqueness of equilibrium for no regulation, tax, and ET with alternative allocation schemes, and compares equilibrium outcomes across the three policy regimes. **Section 4** further extends the model to analyze the incomplete emissions market, or CDM, with varying baselines. The last section concludes.

2. Basic Model Setup

Our basic model setup builds upon Melitz (2003). Consider an economy characterized by monopolistic competition (Dixit and Stiglitz, 1977), with an unbounded pool of potential entrants.[5] To enter, each firm first pays the fixed entry cost $f_e > 0$, which represents the unrecoverable investment cost of intangible and tangible resources devoted prior to entry. Upon entry, the firm observes its productivity level ϕ, drawn from a common distribution G that has a positive support over $(0, \infty)$ with a density g. Because the firm is uncertain as to its productivity level prior to entry, the firm enters if and only if its expected value of entry is higher than the fixed cost of entry. Each firm with successful entry produces a unique commodity according to its productivity level, and engages in monopolistic competition. The firms that make zero or negative profits exit the markets immediately.

2.A. Demand

[5] Hopenhayn (1992) considers perfect competition (without international trade) while Melitz (2003) considers monopolistic competition (with international trade). We consider monopolistic competition for two reasons. First, the existing literature finds that imperfect competition is an important source of the entry-exit problem for environmental policies. Second, the Melitz-type economy yields theoretical predictions that are roughly consistent with empirical regularities (e.g. Helpman *et al.*, 2008; Eaton *et al.*, 2011).

The preferences of a representative consumer are given by the Dixit-Stiglitz CES utility with an additional disutility from aggregate pollution:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}} - Lh(Z), \tag{1}$$

where ω is an index of commodities, Ω the measure of the set of available goods, L is the population size, and h is a convex function of aggregate emissions Z. The parameter ρ represents the elasticity of substitution between commodities. We assume that $\rho \in (0, 1)$, i.e. the commodities are substitutes. As in Melitz, we consider per capita utility of the aggregate consumer as the measure of social welfare: $W \equiv U/L = Q/L - h(Z)$.

The standard two-step procedure as in Dixit and Stiglitz (1977) yields the following aggregate price index:

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \tag{2}$$

where $\sigma \equiv \frac{1}{1-\rho} > 1$. Assuming that individual consumers ignore the term $h(Z)$ in making the consumption decision,[6] we obtain the following standard formulas for consumer demand and expenditures:

$$q(\omega) = Q \left[\frac{p(\omega)}{P} \right]^{-\sigma} \text{ and } r(\omega) = R \left[\frac{p(\omega)}{P} \right]^{1-\sigma}, \tag{3}$$

where $r(\omega) = p(\omega) q(\omega)$, $Q = [\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega]^{\frac{1}{\rho}}$ is the aggregate quantity index, and $R = PQ$ is the economy's total expenditures.

2.B. Production and Abatement

To analyze the impacts of environmental policies, we introduce emissions as an input for production in a manner analogous to Copeland and Taylor (1994). Each firm is endowed with productivity level $\phi \in [0, \infty)$, with higher ϕ modeled as higher productivity for expositional simplicity. Production requires only one input, *labor*, which we assume, as in Melitz (2003), is inelastically supplied at the aggregate level L. The labor cost consists of a variable component wl as well as a fixed overhead cost wf. Unlike in Melitz (2003), firms generate emissions as a byproduct of production. Firms have access

[6] This assumption is justified by assuming the representative consumer consists of a continuum of consumers.

to abatement technologies, which use labor to reduce emissions. Following Copeland and Taylor (1994), the joint production function is written as:

$$q = \begin{cases} \phi z^{\alpha} l^{1-\alpha} & \text{if } z < \lambda l, \\ \phi A l & \text{otherwise,} \end{cases} \tag{4}$$

where $\lambda > 0$ is the bound on the substitution possibility between labor and pollution inputs and $A = \lambda^{\alpha}$.[7]

As shall be explained below, we compare the impacts of environmental regulations on the stationary equilibrium. Therefore, even though our model is inherently a dynamic one, both the regulatory regime and regulatory variable (emissions tax rate $\tau > 0$ under the tax policy or emissions cap $Z^s > 0$ under the emissions trading (ET) policy) are set at the outset once and for all periods, and firms observe them prior to making entry decisions.[8] This approach is identical to that of Melitz (2003) in his analysis of the impact of international trade.

The firm operating under each policy regime thus maximizes profits, taking into account the effect of the policy:

$$\max \; p^r q - wl - wf - s_i(z), \tag{5}$$

where p^r is a residual demand curve with constant elasticity σ for its variety and s_i represents the costs of emissions associated with regulatory regime i, which takes the following form:

$$s_i(z) = \begin{cases} 0 & \text{if } i = n \text{ (no regulation)}, \\ \tau z & \text{if } i = t \text{ (tax)}, \\ \tau(z - z_e^s(\phi)) & \text{if } i = e \text{ (ET)}, \end{cases} \tag{6}$$

where τ is the price of pollution (i.e. the tax rate under the tax policy or the permit price under the ET policy). Under the ET policy, z_e^s represents the initial allocation of permits, which may or may not vary across firms, depending on the allocation rule used.[9]

[7]Because output must be bounded above for a given level of labor input, the substitution possibility must be bounded by some $\lambda > 0$. See Copeland and Taylor (1994) for a detailed discussion on this production function.

[8]In this paper, we use the terms "cap-and-trade (CAT)," "emissions trading (ET)," and "permit trading" interchangeably. By "grandfathered emissions trading," we only mean the ET without auctioning initial permits, although the term "grandfathering" could sometimes mean a specific allocation rule.

[9]We could also consider varying the baseline emissions level z_t^s under the tax policy, beyond which firms are required to pay emissions taxes and below which they receive subsidies or nothing. Previous studies find that this baseline has important implications for entry/exit of firms and optimality of taxation (e.g. Baumol, 1988; Pezzey, 2003). In most practical applications, however, z_t^s is set to equal zero, and therefore, is assumed so in this paper.

Note that when τ is zero (no regulation) or sufficiently low, the firm would attempt to substitute more pollution for labor, eventually reaching the maximum substitution possibility between labor and pollution. Therefore, the firm's cost-minimizing choice of l and z must satisfy $z = \lambda l$ if input price ratio w/τ exceeds the marginal rate of technical substitution along the ray $z = \lambda l$. With w normalized to 1, the condition can be written $\tau \leq \alpha/(1-\alpha)\lambda$. From here on, we assume that τ is large enough to induce emissions reduction beyond the no-regulation level: $\tau > \alpha/(1-\alpha)\lambda$. The case with no regulation or $\tau \leq \alpha/(1-\alpha)\lambda$ is discussed more fully in **Appendix 1**.

With $\tau > \alpha/(1-\alpha)\lambda$, the firm chooses the optimal combination of inputs so that the input price ratio equals the marginal rate of technical substitution, which yields a cost function given by:

$$c(q) = \frac{q}{\phi B} \quad \text{where} \quad B = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\tau^{\alpha}}. \tag{7}$$

Thus firm's production technology is characterized by a constant marginal cost. Note that $A > B$ because the marginal cost of production is higher with than without regulation. Maximizing (5) with (7) yields firm's optimal markup:

$$p_i(\phi) = (\rho \phi B)^{-1}. \tag{8}$$

Given the constant marginal cost, each firm's supply is perfectly elastic. Thus using (3) and (4), we can readily derive firm's output quantity, revenues, and emissions.

$$q_i(\phi) = Q\,(P\rho\phi B)^{\sigma}, \; r_i(\phi) = R\,(P\rho\phi B)^{\sigma-1}, \; z_i(\phi) = \frac{\alpha\rho}{\tau} R\,(P\rho\phi B)^{\sigma-1}. \tag{9}$$

Though these variables are written without explicit reference to τ for notational simplicity, their dependence on τ must be understood.

Using expressions in (9), the ratios of any two firms' outputs, revenues, and emissions can be conveniently expressed as the functions of ratios of their productivity levels for all policy regimes. That is, for $i = n, t$, and e:

$$\frac{q_i(\phi_1)}{q_i(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{\sigma}, \; \frac{r_i(\phi_1)}{r_i(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{\sigma-1}, \; \frac{z_i(\phi_1)}{z_i(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{\sigma-1}. \tag{10}$$

Thus more productive firms are larger, not only in output and revenues, but also in emissions. This result is consistent with Melitz (2003).

Interestingly, the ratio of any two firms' emissions intensities (i.e. emissions per unit of output) is an inverse of the ratio of the two firms' productivity levels. That is, for $i = n, t$, and e:

$$\frac{z_i(\phi_1)/q_i(\phi_1)}{z_i(\phi_2)/q_i(\phi_2)} = \frac{\phi_2}{\phi_1}. \tag{11}$$

Thus more productive firms emit more in absolute terms, yet emit less per unit of output and thus are more environmental friendly. As many of our subsequent discussions hinge on these results, it is worthwhile to discuss their empirical validity. Though it seems quite intuitive that more productive firms tend to be larger in all firm-level variables, it is not necessarily obvious why more productive firms need to have less emissions intensities. But this follows directly from the Copeland-Taylor framework. Because firms use emissions as an input for production *and* because more productive firms can produce more given any input levels, more productive firms emit less for a given output level, including the profit-maximizing output level. Indeed, the empirical evidence on the relationship between sectoral productivity and emissions intensity suggests this result is consistent with observed firm behaviors (Cole *et al.*, 2005, Mazzanti and Zoboli, 2009).

Lastly, the variable part of the profit equals $(p - 1/\phi B)q$ for $i = t$ or e, provided that τ is sufficiently high. Therefore, under all policy regimes, we can rewrite firm's profit as:

$$\pi_i(\phi) = \frac{r_i(\phi)}{\sigma} - d_i(\phi). \tag{12}$$

where d_i is the fixed-cost component of the firm's profit given by $d_i(\phi) = f - \tau z_i^s(\phi)$ where $\tau z_i^s(\phi) = 0$ under no regulation or under tax. Because firm's revenue is increasing in ϕ, firm's profit is also increasing in ϕ per equation (12) under all policy regimes (given that $z_e^s(\phi)$ does not decrease with ϕ). The following Lemma summarizes the important properties of the extended model we obtained so far.

Lemma 1 *For all policy regimes $i = n, t$, and e, more productive firms produce more, earn more revenues and profits, and emit more. That is, $q_i(\phi_1) > q_i(\phi_2)$, $r_i(\phi_1) > r_i(\phi_2)$, $\pi_i(\phi_1) > \pi_i(\phi_2)$, and $z_i(\phi_1) > z_i(\phi_2)$ for $\phi_1 > \phi_2$. Yet, more productive firms have lower emissions intensities per output: $z_i(\phi_1)/q_i(\phi_1) < z_i(\phi_2)/q_i(\phi_2)$ for $\phi_1 > \phi_2$.*

2.C. Allocation Rules under the Emissions Trading Policy

Up until now, our model is a straightforward extension of the Melitz model in a way that accounts for firms' emissions. Our modeling setup, however, differs substantially from that of Melitz in the case of the ET policy, for then the emissions market must clear simultaneously with the commodity markets. Because in our model, emission levels vary across firms in the absence of regulation, different firms face different marginal (opportunity) costs of abatement depending on the initial distribution of permits. We thus consider three alternative allocation rules, all of which emerged in the literature.

Uniform Allocation (UA) Rule: All active firms receive the same amount of permits irrespective of their production or emissions size: $z_e^s(\phi) = z_e^s$ for all active firms.

Emissions-based Allocation (EBA) Rule: Each firm's allowance allocation is proportional to its "business-as-usual (BAU)" or no-regulation baseline. In our model, this would mean that a firm with productivity ϕ receives the amount of allowances equaling a fraction θ of the emissions it would have emitted with no regulation: More specifically, for all active firms,

$$z_e^s(\phi) = \theta z_n(\phi) \quad \text{with} \quad 0 < \theta < 1. \tag{13}$$

Note that if a firm's productivity is such that the firm would not be active in the absence of regulation, the firm would receive no permits. Moreover, EBA is identical to an allocation rule where each firm's allocation is proportional to the share of its BAU emissions in the total BAU industry emissions.

Rate-based Allocation (RBA) Rule: Regulatory authority allocates permits according to a uniform emissions rate θ^s and the BAU output level: $z_e^s(\phi) = \theta^s q_n(\phi)$ for all active firms. This allocation rule is similar to the one used in the U.S. SO_2 Allowance Program where each regulated unit received allowances roughly based on the fixed emissions rates (i.e. 2.5 lbs/mmBtu in Phase I and 1.2 lbs/mmBtu) and its historical fuel use (which has roughly one-to-one relationship to its electricity output), with some unit-specific bonus reserves. Substituting $q_n(\phi) = Q_n(P_n\rho\phi A)^\sigma$ (see **Appendix 1**), this allocation rule can be rewritten as

$$z_e^s(\phi) = \theta\phi^\sigma \quad \text{with} \quad \theta \equiv \theta^s Q_n(P_n\rho A)^\sigma. \tag{14}$$

Note that RBA is identical to an allocation rule where each firm's allocation is proportional to the share of its BAU output in the total BAU industry output, because such an allocation rule would specify $z_e^s(\phi) = (q_n(\phi)/Q_n) \times Z^s$. But we can rewrite this as $z_e^s(\phi) = \theta^s q_n(\phi)$, with $\theta^s = Z^s/Q_n$ being the industry-wide emissions intensity target.

We clarify two assumptions concerning how permit allocation is treated in our model. Given the emissions cap Z^s, the amount of permits each firm receives depends critically on aggregate equilibrium conditions — in particular, the emissions cap must equal the total amount of permits allocated *for firms that enter and stay active in equilibrium*. To ensure this, we first assume that firms take all aggregate economic variables as exogenous with perfect foresight about them, so that they can perfectly anticipate their own permit allocations z_e^s prior to entry given the knowledge of Z^s and the allocation rule. By this, we are implicitly assuming that firms only anticipate how many permits they would receive upon entering the market, and that firms *do not* expect either their entry/exit or their output/emissions to influence the distribution of permits. This behavioral assumption is employed in the study of the impact of permit allocation rules by Fowlie *et al.* (2012), and is also consistent with virtually all economic analyses of perfectly competitive markets concerning the equilibrium prices. Second, we also assume that firms lose their permits upon exit (though they receive permits upon entry) as specified in the new entrant and closure provisions of European Union Emission Trading Scheme (Ellerman and Buchner, 2007). Footnote 12 discusses the case where permit allocation is permanent regardless of their entry/exit status.

One caveat here is that there is another common allocation mechanism called the "output-based allocation (OBA)" of emission allowances where each firm's allocation is proportional to its share in the total industry output *that would occur in equilibrium*. Such an allocation rule was proposed in Waxman-Markey and Lieberman-Warner bills in the United States, and has thus been the object of several recent studies (e.g. Fischer and Fox, 2007, Fowlie *et al.*, 2012). Under such an allocation rule, firms should behave strategically because the amount of permits z_e^s depends on firm-level variables. We call such an allocation rule "endogenous." In this paper, we focus on three "exogenous" allocation rules, yet discuss what might happen under "endogenous" allocation rules in **Subsection 3.C**.

2.D. Stationary Equilibrium

As in Melitz, the paper focuses on the steady-state and stationary equilibrium in which all aggregate variables P, Q, R, and Π as well as the mass M and distribution μ of incumbent firms stay constant over time. Because all aggregate variables are functions of the mass and distribution of firms, we are interested in the determination of them in such an equilibrium.

Given regulatory regime i, a potential entrant enters the commodity market if and only if the expected present value of profit can recover the fixed cost of entry f_e. The entering firm drawing ϕ will immediately exit if $\pi_i(\phi) < 0$. Let ϕ_i^* be the cutoff productivity level such that $\pi_i(\phi_i^*) = 0$. Since firm's profit is increasing in ϕ, all firms with $\phi < \phi_i^*$ will exit the market and never produce. The distribution of incumbent firms then is determined by the initial distribution G of productivity shocks, conditional on successful entry:

$$\mu_i(\phi) = \begin{cases} \frac{g(\phi)}{1-G(\phi_i^*)} & \text{if } \phi \geq \phi_i^* \\ 0 & \text{o.w.} \end{cases} . \tag{15}$$

Hence, the cutoff ϕ_i^* uniquely defines the distribution of firm-level productivity, which in our model also uniquely defines the distributions of production and abatement levels per (10) and (11).[10]

Since we are interested in the steady-state/stationary equilibrium, $\pi_{t,i} = \pi_i$ and $\mu_{t,i} = \mu_i$ for all periods t. Thus we can simplify the entry and exit conditions as follows. First, using firm's profit (12), we see that $\pi_i(\phi^*) = 0$ implies $r_i(\phi_i^*) = \sigma d_i$ for all policy regimes i. Using (10), we can conveniently write firm's revenues, profit, and emissions as in **Appendix 2**. Taking the conditional average of all firms' profits, we see that the average profit $\bar{\pi}_i$ satisfies:

$$\bar{\pi}_i = \begin{cases} \left[\left(\frac{\tilde{\phi}_i}{\phi_i^*}\right)^{\sigma-1} - 1\right] f & \text{if } i = n, t, e^{EBA} \\ \left[\left(\frac{\tilde{\phi}_i}{\phi_i^*}\right)^{\sigma-1} - 1\right] (f - \tau z_i^s) & \text{if } i = e^{UA} \\ \left[\left(\frac{\tilde{\phi}_i}{\phi_i^*}\right)^{\sigma-1} - 1\right] f + \tau\theta(\hat{\phi}_i^{\sigma} - \tilde{\phi}_i^{\sigma-1}\phi_i^*) & \text{if } i = e^{RBA} \end{cases} , \tag{ZCP}$$

where $\hat{\phi}_i$ and $\tilde{\phi}_i$ are weighted average productivities defined by:

$$\hat{\phi}_i \equiv \left[\int \phi^{\sigma} \mu_i(\phi) d\phi\right]^{1/\sigma}, \tag{16}$$

$$\tilde{\phi}_i = \left[\int \phi^{\sigma-1} \mu_i(\phi) d\phi\right]^{\frac{1}{\sigma-1}}. \tag{17}$$

Note that the ZCP condition describes the relationship between the cutoff productivity ϕ_i^* and the average profit $\bar{\pi}_i$ implied by firms' exit behaviors.

On the other hand, free entry implies that entry occurs until all net expected profits are exhausted. Because a successful entrant with productivity ϕ earns $\pi_i(\phi)$ for all periods,

[10] This stationary equilibrium concept has been employed in Melitz (2003) and other subsequent studies for several reasons. First, the empirical literature finds the size distribution of firms that persists over time. Second, the theoretical literature suggests that a history of firm-specific independent shocks in a dynamic process can generate such a stationary distribution of productivity (e.g. Luttmer, 2007). Third, while it is possible to incorporate an evolution of size distributions over time in the spirit of Ericson and Pakes (1995) or Hopenhayn (1992), such models tends to be substantially less tractable.

its discounted sum of profits is $\pi_i(\phi)/\delta$, where $\delta > 0$ is a discount factor. The expected (discounted) profit prior to entry then is $E[\pi_i(\phi)] = p_{in}(\bar{\pi}_i/\delta)$, where $p_{in} = 1 - G(\phi_i^*)$. Thus entry occurs until:

$$(1 - G(\phi_i^*))\frac{\bar{\pi}_i}{\delta} - f_e = 0. \tag{FE}$$

Because the free entry condition (FE) is identical to that of Melitz (2003) across all policy regimes, important variation in the equilibrium cutoff productivity ϕ^* and average profit $\bar{\pi}_i$ arise from variation in (ZCP) due to differences in environmental regulations.

Under the ET policy, the aggregate emissions Z_e must equal the total supply of permits Z^s (assuming perfect regulatory enforcement). Hence, the emissions market clears with a permit price $\tau > 0$ such that:[11]

$$Z^s = \int z_e^s(\phi) M_e \mu_e(\phi) d\phi = \int z_e(\phi, \tau) M_e \mu_e(\phi) d\phi. \tag{18}$$

Condition (18) has different implications for different allocation rules. First, under the UA rule, using the expression $z_e = (\alpha\rho/\tau) r_e$ in (9), the average revenue $\bar{r} = \sigma(\bar{\pi} + f - \tau z_e^s)$, and $z_e^s = Z^s/M_e$ in (18), we obtain:

$$\tau z_e^s = \frac{\alpha(\sigma - 1)(\bar{\pi}_e + f)}{1 + \alpha(\sigma - 1)}. \tag{EMC}$$

Clearly, τz_e^s depends on $\bar{\pi}_e$, which in turn depends on τz_e^s. Hence, τ, $\bar{\pi}_e$, and z_e^s are jointly determined in equilibrium. Second, under the EBA rule, condition (18) turns out to hold trivially, yet has an important implication for the size of θ, as shall be discussed in **Subsection 3.B**. Lastly, under the RBA rule, condition (18) simply implies that θ must satisfy $\theta = Z^s/M_e\hat{\phi}_e^{\sigma}$.

2.E. Aggregate Conditions

Our interest lies in identifying the impacts of environmental regulations on the distribution of firm-level variables as well as on the aggregate variables in the long run. For the former, we can use (15) once the equilibrium cutoff productivity ϕ_i^* is identified. For the latter, we can make use of the important property of the Melitz model, which still holds in our extended model (except under the CDM policy). That is, all aggregate variables of interest are characterized by the mass of incumbent firms M_i and the productivity

[11]Note that for a sufficiently large cap Z^s, the permit price τ would be zero or less than $\alpha/(1-\alpha)\lambda$. A sufficient condition for $\tau > \alpha/(1-\alpha)\lambda$ is $Z^s < (1-\alpha)\lambda\rho L$. We assume this condition holds throughout the paper.

distribution μ_i for each policy regime i as follows:

$$P_i = M_i^{\frac{1}{1-\sigma}} p_i(\tilde{\phi}), \quad Q_i = M_i^{\frac{1}{\rho}} q_i(\tilde{\phi}), \quad R_i = P_i Q_i = M_i r_i(\tilde{\phi}), \quad \Pi_i = M_i \pi_i(\tilde{\phi}), \quad Z_i = M_i z_i(\tilde{\phi}), \tag{19}$$

Because the weighted average of the firm's productivity levels $\tilde{\phi}$ is independent of the number of firms M, an industry comprised of M with any distribution that yields the same average productivity $\tilde{\phi}$ behaves the same way as an industry with M representative firms having the same productivity $\phi = \tilde{\phi}$. Hence, *the impacts of environmental regulations on aggregate variables can be conveniently analyzed as if they impact only the mass of firms and the average behavior of the firms, despite the fact that they may influence different firms differently*.

The question then is how to pin down the equilibrium mass of firms. To do so, we need to consider aggregate conditions under different policy regimes. For no regulation ($i = n$), the aggregation conditions coincide with those in Melitz. For the sake of exposition for policy regimes $i = t$ or e, let us first restate Melitz' exposition for the case of no regulation.

The labor is used either in production (including abatement) or investment by new entrants. The total supply of labor L, which also represents the size of the country, thus equals the labor used for production L_p and that for investment L_e. The aggregate revenues R must equal payments to the labor: $R = L$. The aggregate payments to the labor used for production must also equal the difference between the aggregate revenue and the aggregate profit: $L_p = R - \Pi$. The aggregate payments to the labor used for investment must also equal the the costs incurred by the new entrants: $L_e = N_e f_e$ where N_e is the mass of new entrants. In the stationary equilibrium, the mass of new entrants is such that the mass of successful entrants equals the mass of incumbent firms that exit: $(1 - G(\phi^*))N_e = \delta M$. Using the fact that $M\bar{r}_n = R_n = L$ and $\bar{r}_n = \sigma(\bar{\pi}_n + f)$, we see:

$$M_n = \frac{L}{\sigma(\bar{\pi}_n + f)}. \tag{20}$$

Now, in the case of Pigouvian tax ($i = t$), the aggregation conditions depend on how the government spends the tax revenues. We shall assume, as in the conventional macroeconomics literature, that the government makes a lump-sum transfer to consumers. The lump-sum payments should not alter either consumers' or firms' behaviors. The aggregate profits in this case satisfy $\Pi = R - L_p - \tau Z$ instead of $\Pi = R - L_p$. Substituting $\Pi = L_e$, the aggregate payments to labor must equal the aggregate revenue net of tax payments: $L = R - \tau Z$. On the other hand, using expressions for $r(\phi)$ and $z(\phi)$ from (9), we see that $Z = \alpha\rho R/\tau$ for $\tau > \alpha/(1-\alpha)\lambda$. Substituting $Z = \alpha\rho R/\tau$ into $L = R - \tau Z$

and manipulating, we obtain:

$$R_t = \frac{1}{1-\alpha\rho} L. \tag{21}$$

This equality is consistent with Bajona and Missios (2010). Moreover, because $M\bar{r}_t = R_t$ and $\bar{r}_t = \sigma(\bar{\pi}_t + f)$, the equilibrium mass of firms is given by:

$$M_t = \frac{R_t}{\sigma(\bar{\pi}_t + f)} = \frac{L}{\sigma(\bar{\pi}_t + f)(1-\alpha\rho)} \tag{22}$$

Comparing (20) and (22), we see that $M_t > M_n$: i.e. the equilibrium mass of firms is larger under the tax policy than under no regulation. One may find this result puzzling, as it stands in contrast to the conventional wisdom. As we shall see below, this occurs because each firm of the same productivity adjusts its production and employs less labor input under the tax policy while the set of firms ϕ is exactly the same between the two policy regimes (see Proposition 2).

Under emissions trading ($i = e$) regardless of the allocation rules, the market clearing in the emissions market requires that total payments for permits equal the value of all permits: $\tau Z = \tau Z^s$. Thus the aggregate profits satisfy: $\Pi = R - L_p - \tau(Z - Z^s) = R - L_p$. Using the same logic as under no regulation, we see that

$$R_e = L. \tag{23}$$

Substituting (9) and $R = L$ into (18), we obtain:

$$\tau = \frac{\alpha\rho L}{Z^s} > 0. \tag{24}$$

It is clear from (24) that the equilibrium price τ only depends on Z^s and exogenous parameters of the model. Using the fact that $M\bar{r}_e = R_e$ and $\bar{r}_e = \sigma(\bar{\pi}_e + f - \tau\bar{z}_e^s)$, we obtain

$$M_e = \frac{R_e}{\sigma(\bar{\pi}_e + f - \tau\bar{z}_e^s)} = \frac{(1+\alpha\rho\sigma)L}{\sigma(\bar{\pi}_e + f)}. \tag{25}$$

where the last equality follows because $\tau\bar{z}_e^s \equiv \tau Z^s/M_e = \tau Z_e/M_e = \alpha\rho L/M_e$.

3. Tax versus Cap-and-Trade Policies

3.A. Existence of Equilibrium

Under the policy regime $i = n$ or t, the tax rate is exogenously given by the regulator ($\tau = 0$ or $\tau > 0$). Thus the equilibrium is determined via the zero cutoff profit condition

(ZCP) and the free entry condition (FE) only. All other aggregate variables M, P, Q, R and Π follow once the average profit $\bar{\pi}_i$ and the cutoff productivity ϕ_i^* are determined, as discussed above. As we shall discuss below, under the EBA and RBA schemes, the ET policy also results in analogous ZCP conditions, with the same FE condition. Thus the proof of existence of the steady-state equilibrium under these allocation rules are the same as, or analogous to, that for the no-regulation or tax policy.

With the UA rule, however, both the commodity market and the emissions market must clear jointly. On one hand, the zero cutoff profit and free entry conditions pin down the cutoff ϕ_e^* and the average profit $\bar{\pi}_e$ for a given price of permits τ. On the other hand, the emissions market must clear, given the distribution of firms μ_e and the cap on aggregate emissions Z^s. Unlike other allocation rules, the uniform emissions allowance $z_e^s = Z^s/M_e$ must be determined endogenously as well. Thus in the joint market equilibrium, $(\bar{\pi}_e^*, \phi_e^*, \tau^*, z_e^{s*})$ must be determined jointly.

Proposition 1 *(i) For the policy regime $i = n, t$, or e^{EBA}, for a given tax rate ($\tau = 0$ or $\tau > 0$), there exists a unique steady-state equilibrium of the economy characterized by a pair $(\bar{\pi}_i^*, \phi_i^*)$ such that the following two conditions are satisfied:*

$$\bar{\pi}_i^* = \left[\left(\frac{\tilde{\phi}_i}{\phi_i^*} \right)^{\sigma-1} - 1 \right] f, \tag{ZCP}$$

$$\bar{\pi}_i^* = \frac{\delta}{1 - G(\phi_i^*)} f_e, \tag{FE}$$

(ii) For the policy regime $i = e^{UA}$, suppose $0 < Z^s \leq (1-\alpha)\lambda\rho L$. Suppose that $\alpha(\sigma-1)\delta f_e < f$ holds. Then there exists a unique steady-state equilibrium of the economy with the emissions market characterized by $(\bar{\pi}_e^*, \phi_e^*, \tau^*, z_e^{s*})$ such that $f > \tau^* z_e^{s*}$ and the following four conditions are satisfied:

$$\bar{\pi}_e^* = \left[\left(\frac{\tilde{\phi}_e}{\phi_e^*} \right)^{\sigma-1} - 1 \right] (f - \tau^* z_e^s), \tag{ZCP}$$

$$\bar{\pi}_e^* = \frac{\delta}{1 - G(\phi_e^*)} f_e, \tag{FE}$$

$$\tau^* z_e^{s*} = \frac{\alpha(\sigma-1)(\bar{\pi}_e^* + f)}{1 + \alpha(\sigma-1)}, \tag{EMC}$$

$$\tau^* = \frac{\alpha\rho L}{Z^s}.$$

(iii) For the policy regime $i = e^{RBA}$, suppose $0 < Z^s \leq (1-\alpha)\lambda\rho L$. Then there exists a unique

steady-state equilibrium of the economy characterized by a pair ($\bar{\pi}_i^*, \phi_i^*$) such that the following two conditions are satisfied:

$$\bar{\pi}_e^* = \left[\left(\frac{\tilde{\phi}_e}{\phi_e^*}\right)^{\sigma-1} - 1\right] f + \tau\theta(\hat{\phi}_e^{\sigma} - \tilde{\phi}_e^{\sigma-1}\phi_e^*), \qquad \text{(ZCP)}$$

$$\bar{\pi}_e^* = \frac{\delta}{1 - G(\phi_e^*)} f_e. \qquad \text{(FE)}$$

3.B. Equilibrium Comparisons

We now examine the long-run impacts of alternative environmental policies on intra-industry allocations and aggregate variables. Let us first examine how the choice of policy instruments affects the size distribution of firms or equivalently the cutoff productivity level.

Proposition 2 *Suppose the price of pollution τ (either via tax or via price of permits) is such that $\tau > \alpha/(1-\alpha)\lambda > 0$. Then the cutoff productivity levels have the following relationships across policy regimes:*

$$\phi_{eUA}^* < \phi_n^* = \phi_t^* = \phi_{eEBA}^* < \phi_{eRBA}^*.$$

That is, the emissions market with the uniform allocation rule induces entry of less productive firms and thus decreases the average profit of active firms relative to the tax system whereas the rate-based allocation induces entry of more productive firms and thus increases the average profit. Furthermore, these cutoff productivity are the same regardless of the size of the tax rate τ or the cap Z^s.

Figure 1 describes the idea of the proof of Propositions 1 and 2. The result that $\phi_n^* = \phi_t^*$ (and that ϕ_t^* does not depend on τ) is counter-intuitive, as it implies, first, that the *average* firm in the long run earns exactly the same profit with or without the emissions tax despite the fact that *all* firms face an additional cost of production under the tax regime, and second, that a naive conjecture — that the emissions tax may induce exit of less productive firms and entry of more productive firms — fails here. The key to understanding this result is to see that firm's profit can be completely written as the sole function of firm's revenues as in (12). Because *all* firms face increased marginal cost of production due to the emissions tax *and* can readjust their markup prices in proportion to their productivity levels, the presence of the tax affects *all firms the same way*, including those entrants who decided to exit the market. Hence, the emissions tax does not favor any particular firm, either productive or unproductive, and thus, does not have any impact on industry-wide allocation of firm-level variables. As a result, exactly the same type of firms stay in the market, each with a higher price, a lower output quantity, and a lower labor input level due to increased marginal costs of production. It thus follows that the equilibrium mass of firms is larger under the emissions tax than under no regulation, $M_t > M_n$, as seen in (20) and (22).

[Figure 1. Existence and Uniqueness of Equilibrium under Alternative Policy Regimes]

Why then do we have $\phi^*_{eUA} < \phi^*_t = \phi^*_{eEBA} < \phi^*_{eRBA}$? First, the result that $\phi^*_{eUA} < \phi^*_t$, combined with the reasoning above, suggests that compared to the tax system, the UA rule does not affects all firms the same way, and instead, favors *less productive firms*. The key to understanding this is to recognize that firms receive implicit subsidies (taxes) if firms are distributed permits in such a way that enables them to be net sellers (buyers) of the permits in the market equilibrium. Under the UA, all firms receive the same amount of permits whereas their equilibrium emissions are increasing in productivity. Thus firms with lower (higher) productivity levels tend to be net sellers (buyers) of the permits. This means that the UA rule is progressive in distributing implicit subsidies (taxes). The progressive nature of the allocation rule thus encourages less productive firms to enter the market. On the other hand, under the EBA, firms receive the permits based on the emissions share. Because firms' emissions increase proportionately with productivity, the EBA results in the neutral distribution of permits and thus it does not distort the cutoff productivity. In fact, it is so neutral that no trade occurs. This result on the EBA rule mirrors that of Böhringer and Lange (2005) who find that allocating permits proportionally to past emissions allows firms to "face the same marginal benefits from emissions... in subsequent periods." How about the RBA rule? Recall Lemma 1 that the emissions intensity is inversely related to the productivity level. That is, more productive firms lower emissions intensities per output. Thus the allocation rule based on the uniform emission intensity would give out disproportionately more permits to more productive firms. Because the RBA rule is regressive, it discourages entry of less productive firms and induces more (less) productive firms to be the net seller (buyers) of permits.[12]

[12]This proposition rests on our assumption of the closure provision where firms do not retain their permits upon exit. If permit distribution is instead permanent, then the equilibrium cutoff productivity level under emissions trading is equal to ϕ^*_n regardelss of the permit allocation rule. This result occurs because the (net) value of economic rents on distributed permits is zero for all firms and thus does not distort firms' entry/exit decisions, at least in the long run (Ellerman, 2008). Proposition 5 continues to hold, however, even if we remove this closure assumption.

Proposition 3 *Suppose that the government implements the emissions trading policy. Then (i) if the uniform allocation rule is used, there exists a productivity* ϕ_e^u *such that firms with* $\phi < \phi_e^u$ *are net sellers of permits and firms with* $\phi > \phi_e^u$ *are net buyers of permits. (ii) If the emissions-based allocation is used, no permits are traded in equilibrium. That is,* $z_e(\phi) = z_e^s(\phi)$ *for all* ϕ*. (iii) If the rate-based allocation rule is used, there exists a productivity* ϕ_e^u *such that firms with* $\phi > \phi_e^u$ *are net sellers of permits and firms with* $\phi < \phi_e^u$ *are net buyers of permits.*

Because the UA rule encourages entry of less productive firms relative to the EBA rule, it also induces larger overall entry of firms into the market. On the other hand, the RBA rule discourages entry of less productive firms relative to the EBA rule. Hence, it induces smaller overall entry. Moreover, the ET policy gives out economic rents in terms of the initial distribution of permits, and thus it induces larger overall entry of firms in general than the tax policy does. However, this intuition does not apply for the RBA rule. Indeed, the equilibrium mass of firms under the RBA scheme can be higher or lower than that under the tax policy.

Proposition 4 *Suppose the price of pollution* τ *(either via tax or via price of permits) is such that* $\tau > \alpha/(1-\alpha)\lambda > 0$*. Then the equilibrium mass of firms have the following relationships across policy regimes:*

$$
\begin{aligned}
(i) \quad M_{e^{UA}} &> M_{e^{EBA}} > M_{e^{RBA}}; \\
(ii) \quad M_{e^{UA}} &> M_{e^{EBA}} > M_t > M_n; \\
(iii) \quad M_{e^{RBA}} &\gtrless M_t > M_n \gtrless M_{e^{RBA}}.
\end{aligned}
$$

Why can the mass of firms be lower under the ET-RBA than under the tax policy or even no regulation? To see this, suppose that the prior distribution of firms g is highly disperse so that a larger mass of productive firms exist in the pool of potential entrants. Upon entry, these productive firms tend to be more monopolistic, charging lower prices and employing more labor than others. Hence, high dispersion tends to reduce overall entry. As discussed above, the RBA rule favors these productive firms by giving them implicit subsidies. Thus the RBA rule can exacerbate the effect of high dispersion to the degree that $M_n > M_{e^{RBA}}$ occurs.

There is another important implication of this proposition for the size of θ in (13) under the EBA rule. Suppose the government intends to cut aggregate emissions by a fraction $(1-\psi)$ relative to no regulation: $Z^s = \psi Z_n$. The government would then have to distribute permits in such a way that $\theta < \psi$, in order to achieve this emissions target. That is, *if the policy goal is to meet a $(1-\psi)\%$ reduction, the government must allocate permits to each firm less than $\psi\%$ of the no-regulation baseline*. To understand this, note that because $Z_i = M_i z_i(\tilde{\phi}_i)$, $Z^s = Z_e = \psi Z_n$ if and only if $M_e z_e(\tilde{\phi}_e) = \psi M_n z_n(\tilde{\phi}_n)$. But Proposition 2 says that $\phi_e^* = \phi_n^*$, which in turn implies $\tilde{\phi}_e = \tilde{\phi}_n$ under the EBA rule. Provided that $z_e(\phi) = \theta z_n(\phi)$, we can rewrite $z_e(\tilde{\phi}_e) = \theta z_n(\tilde{\phi}_n)$. Then $M_e z_e(\tilde{\phi}_e) = \psi M_n z_n(\tilde{\phi}_n)$ if and only if $\theta M_e z_n(\tilde{\phi}_n) = \psi M_n z_n(\tilde{\phi}_n)$ or $\theta/\psi = M_n/M_e$. Using (20) and (25), $\theta/\psi = M_n/M_e = 1/(1+\alpha\rho\sigma) < 1$.

We now turn to one of the most surprising results.

Proposition 5 *Suppose the price of pollution τ is such that $\tau > \alpha/(1-\alpha)\lambda > 0$. Suppose that the government makes a lump-sum transfer of tax revenues to consumers. Then (i) the aggregate emissions in the stationary equilibrium of the closed economy across different policy regimes satisfy the following relationships:*

$$Z_n > Z_t > Z_e,$$

and (ii) if an aggregate emissions cap is set such that $Z^s = Z_t(\tau)$, where $Z_t(\tau)$ is the aggregate emissions in equilibrium of the economy under tax with tax rate τ, the associated stationary equilibrium $(\bar{\pi}_e^*, \phi_e^*, \tau_e^*)$ under the ET policy must be such that $\tau_e^* = \tau(1-\alpha\rho) < \tau$. Both (i) and (ii) hold regardless of allocation rules.

This result is rather surprising, as it says not only that the equivalence between the emissions tax and the emissions market does not hold in our economy, but also that contrary to the established literature, the emissions tax induces more (not less) aggregate emissions than the emissions market. The key to understanding our result is to recognize that $Z_i = M_i z_i(\tilde{\phi}_i)$. Because $M_e > M_t$ (except under the RBA rule), we must have $z_t(\tilde{\phi}_t) > z_e(\tilde{\phi}_e)$. Why does the emissions tax increase individual firms' emissions on average relative to the ET policy?

The answer lies in the income effect of public expenditures. Both under the tax and the ET, firms face the same price of pollution τ, which raises the marginal cost of production. Because each firm equates marginal revenue with marginal cost of production, both policies reduce output of each firm relative to no regulation in the same manner. The difference, however, is that the "average" firm faces no implicit subsidy or tax under the ET policy (because the value of sales of permits must equal the value of purchase of permits) whereas the "average" firm needs to pay taxes under the tax policy. Thus the ET policy does not influence the average cost of the "average" firm while the tax policy raises it. Thus the average firm profit would have been lower under the tax than under the ET.

If the model fails to specify how the tax payments would be spent, the tax policy would induce less overall entry of firms relative to no regulation, and thus, less aggregate emissions relative to the ET. However, in our model, the tax payments are transferred back to consumers in a lump-sum manner. Because consumers increase demand in response to this lump-sum transfer, firms under the tax policy can increase output and emissions to make up for the profit loss relative to the ET. As a result, the emissions tax induces more emissions on average per firm: $z_t(\tilde{\phi}_t) > z_e(\tilde{\phi}_e)$.[13] We emphasize here that our full-employment assumption is critical in deriving the result $M_e > M_t$, but is *not* in determining the relationship $z_t(\tilde{\phi}_t) > z_e(\tilde{\phi}_e)$. The former is consistent with the standard theory, but the latter is not.

A natural question then is, how does the social welfare differ across different policy regimes? Because the welfare per capita is given by $W_i(Z) = Q_i/L - h(Z)$, we know that for a given aggregate emissions target Z, $W_t < W_e$ if and only if $Q_t < Q_e$. Because $Q_i = M_i^{\frac{1}{\rho}} q_i(\tilde{\phi}_i)$ and $M_t < M_e$ (except under the RBA rule) per Proposition 4, $Q_t < Q_e$ if $q_t(\tilde{\phi}_t) < q_e(\tilde{\phi}_e)$. It is tempting to infer that $q_t(\tilde{\phi}_t) < q_e(\tilde{\phi}_e)$ because the result that $\tau_e = (1 - \alpha\rho)\tau_t$ for a given aggregate emissions target Z implies that firms under the emissions trading policy face lower marginal costs of production due to the lower price of pollution. However, because firms increase output in response to the lump-sum transfer of tax revenues and because the emissions trading policy can induce entry of less productive firms, the average output can be higher or lower under the tax policy than under the emissions trading policy. The following proposition pins down the general condition for $W_t > W_e$.

Proposition 6 *Given any level of the aggregate emissions target Z, we have $W_t(Z) > W_e(Z)$ if and only if*

$$\frac{\tilde{\phi}_t}{\tilde{\phi}_e} > (1 - \alpha\rho)^{1-\alpha} \left(\frac{M_e}{M_t}\right)^{\frac{1}{\sigma-1}}. \tag{26}$$

Condition (26) demonstrates that the welfare per capita is in general ambiguous, and depends on the three distinct effects on (i) the average productivity, (ii) the overall entry, and (iii) the output per firm of the same productivity level. Per Proposition 2, $\phi^*_{eUA} <$

[13]This, however, does not necessarily imply that *each incumbent firm* under the tax policy inceases emissions more than under the CAT policy under the UA rule. The necessary and sufficient condition for it is:

$$\phi^*_e \left[1 + \alpha(\sigma - 1)\left(\frac{\tilde{\phi}_e(\phi^*_e)}{\phi^*_e}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} > \phi^*_t.$$

$\phi_t^* = \phi_{eEBA}^* < \phi_{eRBA}^*$, which in turn implies $\tilde{\phi}_t/\tilde{\phi}_e \lesseqgtr 1$ depending on the allocation rule. Per Proposition 4, $M_{eUA} > M_{eEBA} > M_t$ and $M_{eRBA} \gtreqless M_t$, which again implies $M_e/M_t \lesseqgtr 1$ depending on the allocation rule. Moreover, as the proof demonstrates, the factor $(1-\alpha\rho)^{1-\alpha}$ represents the net effect of the tax (relative to the emission trading) on the output per firm of the same productivity, via the increased demand due to the lump-sum transfer of tax revenues (i.e. $(1-\alpha\rho)^{-1}$) and the decreased supply due to the higher price of pollution (i.e. $(1-\alpha\rho)^{-\alpha}$). Because this factor is larger than 1, the net effect tends to increase the average output.

3.C. Endogenous vs. Exogenous Allocation Schemes

Up until here, we assumed that firms take θ as an exogenous policy variable and cannot influence their baseline emissions or output. This assumption would be justifiable under a regulatory regime where the regulator can use, as the basis for determining the relevant baselines, information on historical emissions or output levels *before* firms know how the regulator uses firm-level information, for then firms would be unable to retrospectively influence $\theta, z_n(\phi)$, or $q_n(\phi)$. However, in practice, the regulatory authority may update any of these policy variables based on information obtained *after* implementation of the policy. Fisher and Fox (2007) and Fowlie *et al.* (2012) considered such an endogenous output-based allocation (OBA) rule.

In our model, a counterpart of the endogenous OBA rule could be written as follows.[14]

$$z_e^s(\phi) = \begin{cases} \frac{q_e(\phi)^\rho}{Q_e^\rho} Z^s & \text{if} \quad \phi \geq \phi_e^*, \\ 0 & \text{if} \quad \phi < \phi_e^*. \end{cases} \tag{27}$$

Note that unlike in (13) or (14), all expressions are written with subscript e. Hence, each firm receives permits based on its share in the aggregate output *that would arise in the equilibrium under this allocation rule* rather than its historical share (or what would have happened) under no regulation.

It turns out that this endogenous allocation rule gives rise to a neutral tax/subsidy so that (i) no trade occurs in equilibrium, $z_e(\phi) = z_e^s(\phi)$, and (ii) the equilibrium cutoff productivity and average profit are the same with those under the tax or no regulation, $\phi_e^* = \phi_n^* = \phi_t^*$ and $\bar{\pi}_e^* = \bar{\pi}_n^* = \bar{\pi}_t^*$. This neutrality follows, because firm's optimal output under this allocation rule also increases with productivity at a rate σ so that the amount

[14] Note that the shares add up to the aggregate emissions target: $\int z^s(\phi) M_e \mu(\phi) d\phi = \frac{\int q_e(\phi)^\rho M_e \mu(\phi) d\phi}{Q_e^\rho} Z_e = \frac{\int q_e(\phi)^\rho M_e \mu(\phi) d\phi}{\int q_e(\phi)^\rho M_e \mu(\phi) d\phi} Z_e = Z_e$.

of permits increases with productivity at a rate $\sigma \times \rho = \sigma - 1$, i.e. the same rate as the no-regulation BAU emissions. As a result, the same results as under EBA follow.

However, one interesting outcome occurs under the endogenous rule — firms end up increasing output more than under the exogenous allocation rule because firms know that they can increase the initial receipt of permits by increasing its output share. This perverse incentive in turn increases the demand for permits and thus the price of permits more than under the exogenous rule for a given emissions cap Z^s. This increase in output is welfare-improving for consumers. This result is consistent with Fisher and Fox (2007) and Fowlie *et al.* (2012).

Proposition 7 *(i) The cutoff productivity level under the endogenous OBA rule (27) is the same as the exogenous EBA rule (13):* $\phi^*_{eOBA} = \phi^*_{eEBA}$. *(ii) For a given level of the emissions target Z, the price of permits* τ *is higher under the endogenous OBA rule (27) than under the exogenous EBA rule (13).*

4. Incomplete Emissions Market

We demonstrate that our model is also well suited for the analysis of an incomplete emissions market. Incomplete emissions markets possess two properties in general.

Incomplete Coverage: *Some economic entities are excluded from the coverage of regulation despite the fact that their polluting activities are the object of regulation.* The incompleteness of regulatory coverage can be geographic or economic. For example, only power plants and other large combustion sources in the eastern United States are mandated to reduce NO_x emissions under the U.S. NO_x Budget Trading Program. In another example, in many watersheds, wastewater treatment plants are legally mandated to reduce their nutrient loads while agricultural farmers are not. As Fowlie (2009) shows, the incomplete coverage may result in pollution leakage via reallocation of economic activities and market power.

Voluntary Market Participation: *Unregulated economic entities may voluntarily participate in the emissions market.* The voluntary participants could be either sellers or buyers of the permits. For example, under the U.S. SO_2 Allowance Program, consumers who are not subject to the regulation can buy the SO_2 allowances to reduce the supply of permits (and some did so). In another example, the unregulated agricultural farmers may reduce nutrient loads and sell their credits to the regulated point sources of the nutrients in many U.S. water quality trading systems.

Indeed, virtually all existing emissions markets satisfy these two properties and thus are in essence incomplete. One notable example is the clean development mechanism

(CDM) under the Kyoto Protocol. Under the CDM, firms in developing countries are not mandated to reduce CO_2 emissions, yet may voluntarily reduce emissions below some agreed-upon baseline and sell the carbon credits in the outside emissions markets. For concreteness of our discussions, we use CDM as the example of the incomplete emissions market below.

A theoretical premise of CDM is that by allowing for voluntary abatement to take place in developing countries where the (opportunity) cost of abatement is presumably low, it can minimize the costs of achieving the emissions targets in developed countries. However, such voluntary participation can also endogenously increase the total supply of permits in developed countries. Thus the CDM policy would be efficient *if* the amount of credits generated to increase the supply of permits in developed countries is less than or equal to the amount of emissions reduction in developing countries. This is a big "if" that requires a careful investigation.

Indeed, the amount of CDM credits may exceed the amount of emissions reduction, because the overall impact of CDM relies on several competing effects. First, the CDM induces more abatement investment for firms that can optimally sell credits in the outside emissions market. Second, the CDM may induce more emissions from firms that optimally choose not to participate in the emissions market, by shifting the market power to these firms from the firms that choose to abate. Third, the CDM may have an entry-inducing effect. The CDM credits work like a green subsidy, lowering CDM participants' average cost curves, and therefore, the CDM may induce more entry. The first effect tends to reduce industry-wide emissions whereas the last two tend to increase it. Thus CDM may have in general an ambiguous effect on the aggregate emissions of the host country. Our model is well-suited for analyzing these pathways.

In the analysis below, we assume that firms do not trade internationally in the commodity markets and that the host country's CDM market is small relative to the outside emissions market. Therefore, unlike the ET policy, the price of credits τ is determined in the international emissions market and thus is treated exogenous. This admittedly simple assumption is made for the purpose of pinning down the behavior of firms under CDM without worrying about the complexity of joint determination of commodity and permit prices. The model that fully incorporates international trade in both commodity and emissions markets where prices of both markets are determined jointly would be our future research.

4.A. The Model under the CDM Policy[15]

[15]This subsection only sketches our model and results under the CDM policy. A more detailed note

Firms operating under CDM maximize profits (5) with the policy-specific component $s_c(z)$:

$$s_c(z) = \min\{0, \tau(z - z_c^s)\},$$

where τ is the price of permits as in the ET policy and z_c^s represents the agreed-upon BAU baseline in the case of CDM. In the literature, CDM requires that firms must meet so-called an *additionality condition*. That is, to receive credits, the firms must demonstrate to the CDM auditor that their emissions reduction is *in addition to what would have occurred under the business-as-usual baseline scenario*.[16] We consider two alternative interpretations of this additionality condition.

Uniform Baseline (UB) Rule: As a benchmark, we consider the uniform baseline rule where the *average* emissions that would result in the absence of regulation is used as the uniform baseline for CDM credits for all firms:

$$z_c^s(\phi) = \begin{cases} z_c^s = \bar{z}_n & \text{if} \quad \phi \geq \phi_c^*, \\ 0 & \text{if} \quad \phi < \phi_c^*. \end{cases} \tag{28}$$

In essence, this rule may be considered an analogue of the sectoral baseline rule discussed in Aldy and Stavins (2009).

Emissions-based Baseline (EBB) Rule: Another way to interpret the additionality condition is that the baseline z_c^s differs across firms with different productivity levels and is set to equal the equilibrium emissions that would have happened in the absence of environmental regulation. Specifically, we define z_c^s:

$$z_c^s(\phi) = \begin{cases} z_n(\phi) & \text{if} \quad \phi \geq \phi_n^*, \\ 0 & \text{if} \quad \phi < \phi_n^*. \end{cases} \tag{29}$$

With either rule, each firm will choose to abate and sell credits if it is more profitable to do so than otherwise. Thus firm's profit-maximization program can be proceeded in two steps. First, we solve this program as if the firm solves (5) with $s_e(z) = \tau(z - z_c^s)$. Firm's profit is identical to $\pi_e(\phi)$ in this case. Second, solve this program as if the firm solves (5) with $s_n(z) = 0$. Firm's profit is then identical to $\pi_n(\phi)$. Firm's profit is then

describing the CDM equilibrium is available in the online technical appendix.

[16]How to set baseline z_c^s is often the central problem in the CDM literature. For example, Zhang and Wang (2011) find that CDM does not have a statistically significant effect in lowering sulfur dioxide emissions (and therefore carbon emissions) in China, casting doubt on CDM projects' additionality. Indeed, the issue is not unique to CDM — it has been discussed in all of the examples of incomplete emissions markets discussed above.

given by $\pi_c(\phi, \tau) = \max\{\pi_e(\phi, \tau), \pi_n(\phi)\}$. Note that it is *not* correct to conjecture that the firm would *not* participate in CDM if the solution z^* to the first optimization happens to exceed z_c^s — the firm could still make a higher profit by abating and selling credits than by choosing to emit $z^* > z_c^s$.

Proposition 8 *(i) Suppose that the government uses the uniform baseline (28) under the CDM policy. Suppose that $f > \{f - \tau z^s\}\left\{(A/B)^{\sigma-1} - 1\right\}$. Then there exists $\phi^u > \phi_c^*$ such that for all $\phi \in [\phi_c^*, \phi^u)$, $\pi_c(\phi) = \pi_e(\phi)$ and for $\phi \in [\phi^u, \infty)$, $\pi_c(\phi) = \pi_n(\phi)$ and ϕ^u is increasing in ϕ_c^*. (ii) Suppose that the government sets the emissions-based baseline (29) under the CDM policy. Then either all incumbent firms participate or no firms participate in CDM in equilibrium.*

Given the exogeneity of the credit price τ, the equilibrium is determined solely by the zero cutoff condition and the free entry condition. The free entry condition for either rule is the same as under no regulation. It turns out that the ZCP curve under the EBB rule is the same as that under no regulation whereas the ZCP curve under the UB rule may lie either above or below the ZCP curve under no regulation in the $(\phi, \bar{\pi})$ space. Hence, $\phi_{cEBB}^* = \phi_n^*$ (and $\bar{\pi}_{cEBB}^* = \bar{\pi}_n^*$) while $\phi_{cUB}^* \lesseqgtr \phi_n^*$ (and $\bar{\pi}_{cUB} \lesseqgtr \bar{\pi}_n$).

To see why this happens, note that the CDM market generally has three competing effects on firms' profits. First, it reduces the average cost of production for participating firms and thereby induces entry of less productive firms. Second, the CDM market allows participating firms to earn extra profits from the sales of CDM credits. Third, firms that choose not to participate in CDM face lower marginal costs of production, and thus produce and earn more than otherwise. The first effect tends to reduce the average profit while the latter two tend to increase it. Thus the overall impact of CDM on the average profit depends on which of the three effects dominates.

Because each firm participating in CDM receives monetary transfers equaling $\tau(z_c^s(\phi) - z_c(\phi))$, we have

$$R_c = L - T, \tag{30}$$

where $T \equiv \tau M_c^P(\bar{z}_c^s - \bar{z}^P)$ is the total CDM payment received from developed countries, M_c^P is the mass of participating firms, and $\bar{z}^P$ is the average emissions by participating firms. Hence, the equilibrium mass of firms M_c is given by:

$$M_c = \frac{L - T}{\sigma(\bar{\pi}_c + f - \tau s^P \bar{z}_c^{sP})}, \tag{31}$$

where s^P is the share of firms participating in CDM and $\bar{z}_c^{sP}$ is the average emissions baseline for participating firms. Note that $s^P \bar{z}_c^{sP} = s^P \bar{z}_n$ under the UB rule and $s^P \bar{z}_c^{sP} = \bar{z}_n$ under the EBB rule.

4.B. The Effect of CDM on Aggregate Emissions

As discussed above, the CDM may have in general an ambiguous effect on the aggregate emissions. Yet, the following proposition establishes that CDM unambiguously reduces aggregate emissions relative to no regulation, and somewhat counter-intuitively, may do so even relative to the ET policy, at least in the host country.

Proposition 9 *(i) For $i = c^{UB}$, suppose the price of pollution τ is such that $\tau > \alpha/(1-\alpha)\lambda > 0$ and that some firms participate in CDM (i.e. $T > 0$). Then the aggregate emissions in the stationary equilibrium of the closed economy across different policy regimes satisfy the following relationships:*

$$Z_n > Z_c \gtrless Z_e.$$

(ii) For $i = c^{EBB}$, suppose that firms participate in CDM. Then the aggregate equilibrium emission under CDM is lower than that under ET:

$$Z_n > Z_e > Z_c.$$

That $Z_n > Z_c$ or $Z_e > Z_c$ does not imply, however, that CDM necessarily reduces the *combined* aggregate emissions from home and sponsoring countries. Because CDM credits generated in the home country are used to *i*ncrease the emissions cap in the sponsoring country, CDM may still result in a net increase in the combined aggregate emissions if the amount of emissions credits generated exceeds the emissions reduction in the home country.

Suppose that all of the credits generated from the host country are used to satisfy the emissions cap in the sponsoring country and that the aggregate emissions cap of the sponsoring country is binding. Then the combined aggregate emissions would increase relative to no CDM if and only if

$$Z_c^H + Z_e^S + T/\tau > Z_n^H + Z_e^S \quad \text{or} \quad T/\tau > Z_n^H - Z_c^H. \tag{32}$$

Proposition 10 *(i) For $i = c^{UB}$, suppose the price of pollution τ is such that some firms participate in CDM (i.e. $T > 0$) and $\bar{\pi}_c(\tau) \leq \bar{\pi}_n$. Then CDM results in a net increase in the combined aggregate emissions from home and sponsoring countries. (ii) For $i = c^{EBB}$, suppose that firms participate in CDM. Then a sufficient condition for CDM to result in a net increase in the combined aggregate emissions from home and sponsoring countries is that $\tau < 1/\lambda\rho(\sigma - 1)$.*

A few remarks are in order. First, as discussed above, under the UB rule, the relationship between $\bar{\pi}_c$ and $\bar{\pi}_n$ is ambiguous and depends on the international price of permits in general. Hence, it is possible that the CDM policy results in the net decrease in the combined aggregate emissions.[17] Second, the condition for Proposition 10-(ii) that $\tau < 1/\lambda\rho(\sigma - 1)$ must trivially hold for an interior CDM equilibrium where $z_c(\phi) > 0$ for all ϕ under the EBB rule. To see this, note that $z_c(\phi) = (\alpha\rho/\tau)(\phi/\phi_c^*)^{\sigma-1}\sigma(f - \tau z_c^s(\phi_c^*))$. But $\phi_c^* = \phi_n^*$ implies that $z_c^s(\phi_c^*) = z_n(\phi_n^*) = \lambda\rho\sigma f$. Thus $z_c(\phi)$ can be rewritten as $z_c(\phi) = (\alpha\rho/\tau)(\phi/\phi_c^*)^{\sigma-1}\sigma f(1 - \tau\lambda\rho)$. It then follows that $z_c(\phi) > 0$ if and only if $\tau < 1/\lambda\rho$, which is smaller than $1/\lambda\rho(\sigma - 1)$. Therefore, the CDM policy unambiguously increases the combined aggregate emissions under the EBB rule. Third, it may sound puzzling that CDM may increase the combined emissions despite that *each* firm in the host country has to reduce its emissions relative to the no-regulation baseline. But this occurs because a larger mass of firms enter under CDM relative to no regulation (due to implicit subsides in sales of credits) *and* all these incumbent firms emit some positive amount.

5. Concluding Remarks

This paper examined the long-run intra-industry effects of market-based environmental policies in the Melitz-type economy. We augmented the Melitz model by incorporating emissions as an input for production in the spirit of Copeland and Taylor and by embedding a suit of market-based environmental policies. The model offers several important innovations. First, it solves for a joint (stationary) equilibrium of the commodity markets and the emissions market under the tradable permit policy, with three alternative allocation schemes. Second, it solves for a market equilibrium under CDM (or an incomplete permit trading policy). The augmented model then allows us to identify the long-run effects of these alternative policy instruments, explicitly considering the heterogeneity and distribution of firm-level variables and distinguishing the policy impacts on the extensive and intensive margins of the aggregate variables of interest.

[17] In a numerical example with the Pareto distribution, however, the UB rule resulted in the net increase for a wide range of parameter values.

Highlights of our findings are as follows. First, neither the distribution of firms nor the aggregate variables (such as aggregate emissions and welfare per capita) are independent of the choice of policy instruments or the allocation schemes. Specifically, the average productivity of active firms under the complete emissions-trading policy is lower than (the same as, and higher than) under the tax policy if initial allocation of permits is uniform (emissions-based, and rate-based) allocations. Yet, the tax policy results in more aggregate emissions than the emissions-trading policy for a given price of pollution, regardless of the allocation schemes. Second, environmental policies can influence the mass of firms and the average behavior of firms in the opposite direction. For example, the emissions trading policy induces a larger mass of firms than the equivalent tax policy in general (except the RBA rule), yet induces less average emissions of firms. Third, the incomplete emissions market is shown to result in a net increase in combined aggregate emissions from home and sponsoring countries regardless of allocation schemes under certain conditions. All of these results suggest important pathways for future studies on the theory of environmental regulations.

There are two important limitations of the paper. First, we analyzed only a closed economy, leaving out the effect of international trade in both commodity and emissions market. The environmental effects of CDM are likely to be two folds, one in the host country and the other in the sponsoring country. The CDM policy considered so far treats the price of credits as an exogenous variable. We only examined the effect of CDM on the aggregate emissions in the host country only. Presumably, the supply of credits created from the host country would lower the international price of credits and may reduce incentives for polluting firms to abate less. This important effect is left out in our current model. Second, we followed Melitz in modeling the nature of competition in commodity markets as Dixit-Stiglitz's monopolistic competition. One major limitation of the Dixit-Stiglitz model is that the price (and markup) of any brand is exogenously fixed by the symmetric elasticity of substitution. Different conclusions might arise with other types of imperfect competition.

Appendix 1. Production and Abatement under No Regulation

In this case, firms do not engage in any mandated abatement. Firms thus operate along the maximum substitution possibility of pollution for labor, $z < \lambda l$, and therefore, produce with the technology $q = \phi A l$ per equation (4). Note that this production function is identical to that of Melitz except for the multiplicative factor A. Using this production function in (5) and maximizing it with respect to q, we obtain optimal pricing rule:

$$p_n(\phi) = \frac{1}{\rho\phi A}. \tag{A1}$$

Because the production technology is characterized by constant marginal costs, each firm producing variety ϕ supplies the good as exactly demanded by the consumers. Thus using (3), we obtain firm's optimal quantity and revenue:

$$q_n(\phi) = Q\left(P\rho\phi A\right)^{\sigma} \quad \text{and} \quad r_n(\phi) = R\left(P\rho\phi A\right)^{\sigma-1}. \tag{A2}$$

These expressions in (A1) and (A2) are identical to those in Melitz except, again, by the factor A. Note that even in the absence of environmental regulation, firms still engage in *some* abatement because the input substitution possibility between labor and emissions is bounded. It thus follows that:

$$z_n(\phi) = \lambda\rho R\left(P\rho\phi A\right)^{\sigma-1}. \tag{A3}$$

Analogous expressions for (A2) and (A3) when the price of pollution τ is sufficiently small (i.e. $\tau \leq \alpha/(1-\alpha)\lambda$) are given as follows: For $i = t$ or e,

$$q_i(\phi) = Q\left(\frac{P\rho\phi A}{1+\tau\lambda}\right)^{\sigma} \quad \text{and} \quad z_i(\phi) = \frac{\lambda\rho}{1+\tau\lambda} r_i(\phi).$$

Appendix 2. Firm-level Variables

It is sometimes convenient to write per-firm revenues, profits, and emissions as functions of the zero cutoff productivity ϕ_i^*. For policy regime $i \neq c^{UB}$, using the fact that $r_i(\phi_i^*) = \sigma d_i$ ($d_n, d_t = f$ and $d_e, d_c = f - \tau z^s$) and (10), we have:

$$r_i(\phi) = \left(\frac{\phi}{\phi_i^*}\right)^{\sigma-1} \sigma d_i, \tag{A4}$$

$$\pi_i(\phi) = \begin{cases} \left[\left(\frac{\phi}{\phi_i^*}\right)^{\sigma-1} - 1\right] f & \text{if} \quad i = n, t, e^{EBA}, c^{EBB} \\ \left[\left(\frac{\phi}{\phi_i^*}\right)^{\sigma-1} - 1\right] \left(f - \tau z_i^s\right) & \text{if} \quad i = e^{UA} \\ \left[\left(\frac{\phi}{\phi_i^*}\right)^{\sigma-1} - 1\right] f + \tau\theta\phi^{\sigma-1}(\phi - \phi_i^*) & \text{if} \quad i = e^{RBA} \end{cases}, \tag{A5}$$

$$z_i(\phi) = \begin{cases} \frac{\lambda\rho}{1+\lambda\tau}\left(\frac{\phi}{\phi_i^*}\right)^{\sigma-1}\sigma d_i & \text{if } \tau \le \frac{\alpha}{(1-\alpha)\lambda} \\ \frac{\alpha\rho}{\tau}\left(\frac{\phi}{\phi_i^*}\right)^{\sigma-1}\sigma d_i & \text{if } \tau > \frac{\alpha}{(1-\alpha)\lambda} \end{cases}. \tag{A6}$$

For regime $i = c^{UB}$, see Technical Appendix.

Appendix 3. Proofs of the Propositions

Proof of Proposition 1 (*Existence of Stationary Equilibrium under Tax and Emissions Trading Policies*). (i) In this case, it suffices to show that there exists a pair $(\bar{\pi}_i^*, \phi_i^*)$ such that for a given tax rate τ, ZCP and FE are jointly satisfied. It is clear, however, that these equations do not depend on τ and are identical to those in Melitz. Thus the proof of the existence follows from that of Melitz.

(ii) Given any product of τ and z_e^s such that $\tau z_e^s < f$, there exists a unique pair $(\bar{\pi}_e^*(\tau z_e^s), \phi_e^*(\tau z_e^s))$ that solves the ZCP and the FE. The FE curve is increasing in ϕ_e and is cut by the ZCP curve only once from above in the $(\phi, \bar{\pi})$ space, as shown in Melitz. The ZCP curve is decreasing in τz_e^s while the FE curve does not depend on τz_e^s. Both the cutoff $\phi_e^*(\tau z_e^s)$ and the average profit $\bar{\pi}_e^*(\tau z_e^s)$ must be decreasing in τz_e^s. In particular, the curve $\bar{\pi}_e^*(\tau z_e^s)$ is decreasing in τz_e^s such that $\bar{\pi}_e^* = \bar{\pi}_t^*$ when $\tau z_e^s = 0$ and $\lim_{\tau z^s \to f+} \bar{\pi}_e^*(\tau z_e^s) = \delta f_e$. To see the latter, suppose τz_e^s approaches f from above. Then the recurring fixed cost for incumbent firms approaches zero, which implies the measure of incumbent firms with non-negative profits approaches 1. This in turn implies that the ex ante probability of successful entry $1 - G(\phi_i^*)$ approaches 1. It then follows from the FE condition that $\bar{\pi}_e^*$ approaches δf_e. On the other hand, the EMC curve is increasing in $\bar{\pi}_e$ such that the EMC curve gives $\tau z_e^s = \frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)} f > 0$ when $\bar{\pi}_e = 0$ and $\tau_t z_e^s = \frac{\alpha(\sigma-1)(\bar{\pi}_t+f)}{1+\alpha(\sigma-1)} > 0$ when $\bar{\pi}_e = \bar{\pi}_t$. Thus the EMC curve is cut only once from above in the $(\bar{\pi}, \tau z_e^s)$ space if

$$\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}(f + \delta f_e) < f \quad \text{or} \quad \alpha(\sigma-1)\delta f_e < f.$$

Note that if $\alpha(\sigma-1)\delta f_e \ge f$, the $\bar{\pi}_e^*(\tau z_e^s)$ curve and the EMC curve would not intersect in $(\bar{\pi}, \tau z^s)$ space, in which case $\bar{\pi}_e^* = \delta f_e$ and $\pi_e(\phi) > 0$ for all ϕ. Moreover, it follows from $\alpha(\sigma-1)\delta f_e < f$ that $\tau z_e^s < f$.

(iii) The proof is analogous to that of Melitz. Substituting (FE) into (ZCP) for the case of RBA and rearranging terms, we have $\delta f_e / f = j(\phi)$ where

$$j(\phi) = [1 - G(\phi)]\,[t(\phi) + s(\phi)],$$

and

$$t(\phi) = \left(\frac{\tilde{\phi}}{\phi}\right)^{\sigma-1} - 1 \quad \text{and} \quad s(\phi) = \frac{\tau\theta}{f}(\hat{\phi}^{\sigma} - \tilde{\phi}^{\sigma-1}\phi).$$

Because $\delta f_e/f$ is some finite positive number, a sufficient condition for the existence and uniqueness of the equilibrium pair $(\bar{\pi}_e, \phi_e^*)$ is that $j(\phi)$ is monotonically decreasing from infinity to zero on $(0, \infty)$. The derivative of j is:

$$j'(\phi) = -\frac{(\sigma-1)[1-G(\phi)]}{\phi}\left(\frac{\tilde{\phi}_e}{\phi}\right)^{\sigma-1} - \frac{\tau\theta}{f}\left[\int_{\phi}^{\infty}\xi^{\sigma}g(\xi)d\xi\right] < 0.$$

Moreover, the elasticity of $j(\phi)$ is negative and bounded away from zero because the elasticities of $[1-G(\phi)]\,t(\phi)$ and $[1-G(\phi)]\,s(\phi)$ with respect to ϕ are:

$$\begin{aligned}
\epsilon_t &= -(\sigma-1)\left(1+\frac{1}{t(\phi)}\right) < -(\sigma-1), \\
\epsilon_s &= -\frac{1}{1+\hat{\phi}^{\sigma}/(\tilde{\phi}^{\sigma-1}\phi)} < -1,
\end{aligned}$$

where the last inequality follows $\hat{\phi}^{\sigma} > \tilde{\phi}^{\sigma-1}\phi$ by definition. Because $j(\phi)$ is nonnegative and its elasticity is negative and bounded away from zero, it must be decreasing to zero as ϕ goes to infinity. Lastly, it is clear that $\lim_{\phi\to 0} j(\phi) = \infty$, because $\lim_{\phi\to 0}[1-G(\phi)]\,t(\phi) = \infty$ and $[1-G(\phi)]\,s(\phi) > 0$ by definition. *Q.E.D.*

Proof of Proposition 2 (*Cutoff Productivities under Tax and Emissions Trading Policies*). It is trivial to see that $\phi_t^* = \phi_n^* = \phi_{eEBA}^*$, because the ZCP curve and the FE curve are identical under all three policy regimes (see ZCP).

To show $\phi_{eUA}^* < \phi_t^*$: Recall from the proof of Proposition 1 (ii) that given any product of τ and z_e^s such that $\tau z_e^s < f$, there exists a unique pair $(\bar{\pi}_e^*(\tau z_e^s), \phi_e^*(\tau z_e^s))$ that solves the ZCP and the FE, that both the cutoff $\phi_e^*(\tau z_e^s)$ and the average profit $\bar{\pi}_e^*(\tau z_e^s)$ are decreasing in τz_e^s, and that $\phi_e^*(0) = \phi_t^*$ and $\bar{\pi}_e^*(0) = \bar{\pi}_t^*$. To see ϕ_e^* is independent of Z^s, substitute (EMC) into (ZCP) to obtain

$$\bar{\pi}_e^* = \frac{\left[\left(\frac{\tilde{\phi}}{\phi_e^*}\right)^{\sigma-1} - 1\right] f}{1+\alpha(\sigma-1)\left(\frac{\tilde{\phi}}{\phi_e^*}\right)^{\sigma-1}}. \tag{A7}$$

This together with (FE) identifies ϕ_{eUA}^*, yet both conditions are independent of Z^s or τ.

To show $\phi_t^* < \phi_{eRBA}^*$: Using (10) in (12), we see that firm's profit is given by

$$\begin{aligned}\pi_e(\phi) &= [f - \tau z_e^s(\phi_e^*)] \left(\frac{\phi}{\phi_e^*}\right)^{\sigma-1} - [f - \tau z_e^s(\phi)], \\ &= \left[\left(\frac{\phi}{\phi_e^*}\right)^{\sigma-1} - 1\right] f + \tau \left[z_e^s(\phi) - z_e^s(\phi_e^*)\left(\frac{\phi}{\phi_e^*}\right)^{\sigma-1}\right].\end{aligned}$$

Note that the expression inside the brackets of the second term is

$$z_e^s(\phi) - z_e^s(\phi_e^*)\left(\frac{\phi}{\phi_e^*}\right)^{\sigma-1} = \begin{cases} \theta\phi^{\sigma-1}(\phi - \phi_e^*) & \text{if } \phi_e^* > \phi_n^*, \\ \theta\phi^{\sigma} \text{ or } 0 & \text{if } \phi_e^* \leq \phi_n^*. \end{cases}$$

for all $\phi > \phi_e^*$. Hence, the ZCP condition in this case is:

$$\bar{\pi}_e = \left[\left(\frac{\tilde{\phi}}{\phi_e^*}\right)^{\sigma-1} - 1\right] f + \begin{cases} \tau\theta(\hat{\phi}^{\sigma} - \tilde{\phi}^{\sigma-1}\phi_e^*) & \text{if } \phi_e^* > \phi_n^*, \\ \tau\theta\hat{\phi}^{\sigma} & \text{if } \phi_e^* \leq \phi_n^*. \end{cases}$$

Because the second term is always positive regardless of whether we have $\phi_e^* > \phi_n^*$ or not, the ZCP curve in this case would lie above that under no regulation in the (ϕ, π) space. It then follows that $\phi_e^* > \phi_n^*$. Furthermore, this condition is independent of Z^s or τ. *Q.E.D.*

Proof of Proposition 3 (*Trading Outcomes under the Emissions Trading Policy*). (i) Under the uniform allocation rule, the difference between the amount of initial permits and the equilibrium emissions for firm with productivity ϕ is:

$$\begin{aligned}z_e^s - z_e(\phi) &= \frac{Z^s}{M_e} - \frac{\alpha\rho R}{\tau}(P_e\rho B_e)^{\sigma-1}\phi^{\sigma-1}, \\ &= \frac{Z^s}{M_e} - Z^s(M_e^{\frac{1}{1-\sigma}}\tilde{p}_e\rho B_e)^{\sigma-1}\phi^{\sigma-1} = \frac{Z^s}{M_e}\left(1 - \frac{\phi^{\sigma-1}}{\tilde{\phi}^{\sigma-1}}\right).\end{aligned}$$

This expression is positive if and only if $\phi_e^u \equiv \tilde{\phi} > \phi$.

(ii) Using the expression (A3) in (13), the amount of permits allocated for firm with ϕ is given by

$$z_e^s(\phi) = \theta\lambda\rho R_n(P_n\rho A)^{\sigma-1}\phi^{\sigma-1},$$

while the equilibrium emissions for the firm is

$$z_e(\phi) = \frac{\alpha\rho R_e}{\tau}(P_e\rho B)^{\sigma-1}\phi^{\sigma-1}.$$

For the emissions market to clear, we must have

$$Z_e - Z^s = \int_0^\infty z_e(\phi) M_e \mu_e(\phi) d\phi - \int_0^\infty z_e^s(\phi) M_e \mu_e(\phi) d\phi = 0.$$

Substituting the expressions for $z_e^s(\phi)$ and $z_e(\phi)$ on the left hand side of this equation,

$$\begin{aligned} Z_e - Z^s &= \left[\theta\lambda\rho R_n (P_n \rho A)^{\sigma-1} - \frac{\alpha\rho R_e}{\tau}(P_e \rho B)^{\sigma-1}\right] \int_0^\infty \phi^{\sigma-1} M_e \mu_e(\phi) d\phi \\ &= \left[\theta\lambda\rho R_n (P_n \rho A)^{\sigma-1} - \frac{\alpha\rho R_e}{\tau}(P_e \rho B)^{\sigma-1}\right] \tilde{\phi}_e^{\sigma-1}. \end{aligned}$$

where the weighted average firm productivity $\tilde{\phi}_e^{\sigma-1} > 0$. Thus the market clears if and only if $\theta\lambda\rho R_n (P_n \rho A)^{\sigma-1} - \frac{\alpha\rho R_e}{\tau}(P_e \rho B)^{\sigma-1} = 0$, which implies $z_s(\phi) - z_e(\phi) = 0$.
(iii) The difference between the amount of permits and the equilibrium emissions for firm with productivity ϕ is given by

$$\begin{aligned} z_e^s(\phi) - z_e(\phi) &= \theta\phi^\sigma - \frac{\alpha\rho R}{\tau}(P_e \rho B_e)^{\sigma-1}\phi^{\sigma-1}, \\ &= \frac{Z^s}{M_e \hat{\phi}^\sigma}\phi^\sigma - Z^s (M_e^{\frac{1}{1-\sigma}} \tilde{p}_e \rho B_e)^{\sigma-1}\phi^{\sigma-1} = \frac{Z_e}{M_e}\frac{\phi^{\sigma-1}}{\hat{\phi}^\sigma \tilde{\phi}^{\sigma-1}}(\tilde{\phi}^{\sigma-1}\phi - \hat{\phi}^\sigma). \end{aligned}$$

This expression is positive if and only if $\phi > \hat{\phi}^\sigma / \tilde{\phi}^{\sigma-1} \equiv \phi_e^u$. *Q.E.D.*

Proof of Proposition 4 (*Mass of Firms under Tax and Emissions Trading Policies*). See Technical Appendix. *Q.E.D.*

Proof of Proposition 5 (*Aggregate Emissions under Tax and Emissions Trading Policies*). (i) For no regulation ($i = n$), applying $\tau = 0$ in (A2), the aggregate emissions level is given by:

$$Z_n = \int_0^\infty \lambda\rho r_n(\phi) M_n \mu_n(\phi) d\phi = \lambda\rho R = \lambda\rho L,$$

where the last inequality follows from the aggregation condition $R = L$ under no regulation. Under tax ($i = t$), using (9), the aggregate emissions level is given by:

$$Z_t = \int_0^\infty \frac{\alpha\rho}{\tau} r_t(\phi) M_t \mu_t(\phi) d\phi = \frac{\alpha\rho}{\tau} R = \frac{\alpha\rho}{\tau(1-\alpha\rho)} L,$$

where the last inequality follows from the aggregation condition $R = L/(1-\alpha\rho)$ under tax (21). Under permit trading ($i = e$), again using (9), the aggregate emissions level is given by:

$$Z_e = \int_0^\infty \frac{\alpha\rho}{\tau} r_e(\phi) M_e \mu_e(\phi) d\phi = \frac{\alpha\rho}{\tau} R = \frac{\alpha\rho}{\tau} L,$$

where the last inequality follows from the aggregation condition $R = L$ under permit trading (23). Thus we have $Z_n > Z_t > Z_e$ if and only if

$$\lambda\rho L > \frac{\alpha\rho}{\tau(1-\alpha\rho)}L > \frac{\alpha\rho}{\tau}L.$$

The first inequality follows from the assumption $\tau > \alpha/(1-\alpha)\lambda$ and the second inequality from $\alpha, \rho \in (0.1)$.
(ii) The market clearing implies that $Z^s = Z_e = Z_t(\tau)$. Using the expressions for $Z_t(\tau)$ and Z_e above, it follows that:

$$\frac{\alpha\rho}{\tau(1-\alpha\rho)}L = \frac{\alpha\rho}{\tau^*}L \quad \text{or} \quad \tau^* = \tau(1-\alpha\rho).$$

Q.E.D.

Proof of Proposition 5 (*Social Welfare under Tax and Emissions Trading Policies*). The aggregate output under the emissions tax satisfies

$$Q_t = L\left(1-\alpha\rho\right)^{-1}P_t^{-1} = L\left(1-\alpha\rho\right)^{-1}\rho\tilde{\phi}_t\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\tau^\alpha}M_t^{\frac{1}{\sigma-1}},$$

whereas the aggregate output under the emissions trading satisfies

$$Q_e = LP_e^{-1} = L\left(1-\alpha\rho\right)^{-\alpha}\rho\tilde{\phi}_t\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\tau^\alpha}M_t^{\frac{1}{\sigma-1}}.$$

It thus follows that $Q_t > Q_e$ if and only if

$$Q_t/Q_e = (1-\alpha\rho)^{\alpha-1}\left(\frac{\tilde{\phi}_t}{\tilde{\phi}_e}\right)\left(\frac{M_t}{M_e}\right)^{\frac{1}{\sigma-1}} > 1.$$

Q.E.D.

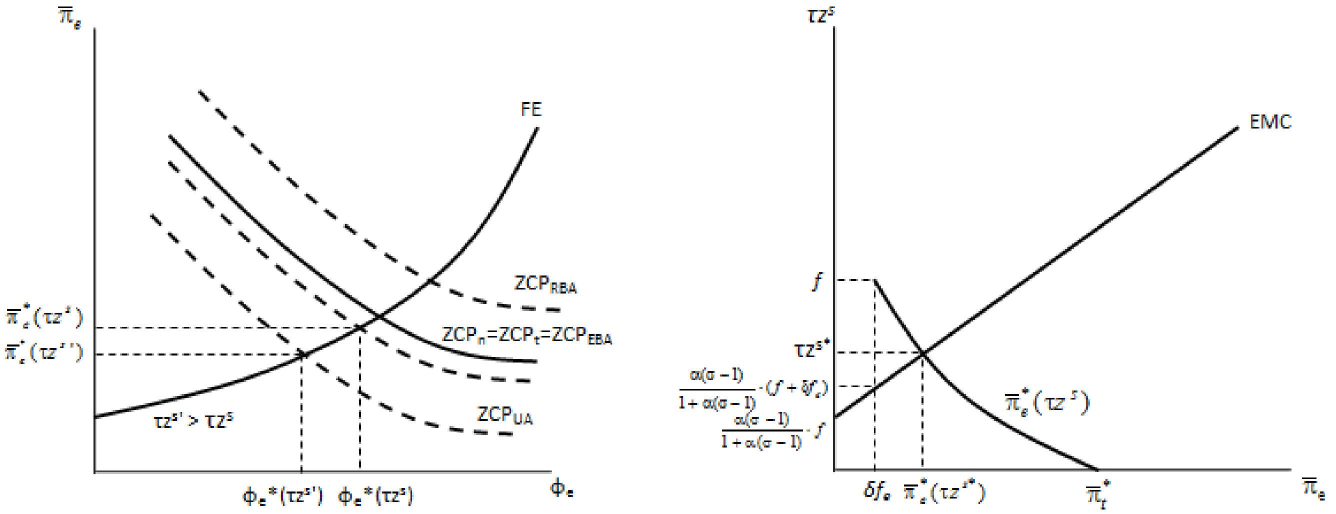
Proof of Proposition 7 (*Equilibrium under the Endogenous Output-Based Allocation Rule*). See Technical Appendix. *Q.E.D.*

Proof of Proposition 8 *(CDM Participation)*. See Technical Appendix. *Q.E.D.*

Proof of Proposition 9 (*Aggregate Emissions under the Clean Development Mechanism Policy).* See Technical Appendix. *Q.E.D.*

Proof of Proposition 10 (*Combined Emissions from Host and Sponsoring Countries under the Clean Development Mechanism Policy*). See Technical Appendix. *Q.E.D.*

Figure 1. Existence and Uniqueness of Equilibrium under CAT



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Technical Appendix for Ouline Publication

A. Proofs of Propositions Omitted in the Main Text

Proof of Proposition 4 (*Mass of Firms under Tax and Emissions Trading Policies*). (i) We have already seen $M_t > M_n$. To see $M_{eUA} > M_{eEBA} > M_{eRBA}$, simply use the result, $\bar{\pi}_{eRBA} > \bar{\pi}_n = \bar{\pi}_{eEBA} > \bar{\pi}_{eUA}$, in (25).

$$\begin{aligned} M_{eUA} &= \frac{(1+\alpha\rho\sigma)L}{\sigma(\bar{\pi}_{eUA}+f)} > \frac{(1+\alpha\rho\sigma)L}{\sigma(\bar{\pi}_{eEBA}+f)} = M_{eEBA}, \\ M_{eEBA} &= \frac{(1+\alpha\rho\sigma)L}{\sigma(\bar{\pi}_{eEBA}+f)} > \frac{(1+\alpha\rho\sigma)L}{\sigma(\bar{\pi}_{eRBA}+f)} = M_{eRBA}. \end{aligned}$$

(ii) To show $M_{eEBA} > M_t$, using (22) and (25), we have $M_{eEBA} > M_t$ if

$$M_{eEBA} = \frac{(1+\alpha\rho\sigma)L}{\sigma(\bar{\pi}_{eEBA}+f)} = \frac{(1+\alpha\rho\sigma)L}{\sigma(\bar{\pi}_t+f)} > \frac{L}{\sigma(\bar{\pi}_t+f)(1-\alpha\rho)} = M_t.$$

Because $\bar{\pi}_t = \bar{\pi}_{eEBA}$, the last inequality holds if and only if

$$(1+\alpha\rho\sigma)(1-\alpha\rho) > 1.$$

Manipulating the left hand side of this expression, with $\sigma = 1/(1-\rho)$, we obtain

$$(1+\alpha\rho\sigma)(1-\alpha\rho) = 1+\alpha\rho(\sigma^2-1),$$

which is greater than 1 by assumption (i.e. $\alpha\rho > 0$ and $\sigma > 1$).
(iii) We prove this by construction of a numerical example. Suppose that firm-level productivity ϕ follows the Pareto distribution with cumulative distribution function G:

$$G(\phi) = 1 - \left(\frac{b}{\phi}\right)^k \quad \text{for all } \phi \geq b,$$

where $b > 0$ and $k \geq \sigma - 1 > 0$. For all policy regimes, the weighted average productivity $\tilde{\phi}_i$ is given by:

$$\tilde{\phi}_i = \left(\frac{k}{k-(\sigma-1)}\right)^{\frac{1}{\sigma-1}} \phi_i^*.$$

For $i = n$ or t, using (A9) in (ZCP), we obtain:

$$\bar{\pi}_n^* = \bar{\pi}_t^* = \left[\frac{\sigma-1}{k-(\sigma-1)}\right] f.$$

If the government uses the RBA rule, solving for $\bar{\pi}_e^*$ gives us

$$\bar{\pi}^*_{e^{RBA}} = \frac{\sigma - 1 + \alpha\rho\sigma^2}{k - (\sigma - 1)} f.$$

Substituting (A11) and (A13) in the expressions for the equilibrium mass of firms under respective policy regimes, (20), (22), and (25), we obtain the equilibrium mass of firms under each policy scenario:

$$M_n = \frac{k - (\sigma - 1)}{\sigma k f} L, \quad M_t = \frac{k - (\sigma - 1)}{\sigma k f (1 - \alpha\rho)} L, \quad M_{e^{RBA}} = \frac{(1 + \alpha\rho\sigma)\{k - (\sigma - 1)\}}{\sigma(k + \alpha\rho\sigma^2) f} L. \tag{A1}$$

Note that from (A1), $M_{e^{RBA}} > M_t$ if and only if

$$\frac{k(1 + \alpha\rho\sigma)(1 - \alpha\rho)}{k + \alpha\rho\sigma^2} > 1.$$

Substitute the following values: $\alpha = 0.5, \rho = 0.4, \sigma = 1/(1 - \rho), k = 10$. Then the left hand side becomes $\approx 1.0105 > 1$. Next, from (A1), we see that $M_n > M_{e^{RBA}}$ if and only if

$$1 > \frac{k + \alpha\rho\sigma k}{k + \alpha\rho\sigma^2}.$$

Substitute the following values: $\alpha = 0.5, \rho = 0.4, \sigma = 1/(1 - \rho), k = 0.7 > \sigma - 1$. Then the right hand side becomes $\approx 0.7434 < 1$. *Q.E.D.*

Proof of Proposition 7 (*Equilibrium under the Endogenous Output-Based Allocation Rule*). (i) Substituting (7) and (27) into firm's profit, the firm with productivity ϕ under the endogenous OBA solves

$$\max_{q \geq 0} pq - \frac{q}{\phi B} + \tau \frac{q^\rho}{Q^\rho} Z^s,$$

where $p(q) = Q^{\frac{1}{\sigma}} P q^{-\frac{1}{\sigma}}$ from (3). The necessary and sufficient condition for optimum is

$$\rho p - \frac{1}{\phi B} + \tau\rho \frac{q^{\rho - 1}}{Q^\rho} Z^s = 0.$$

Consolidating terms and solving for p, we have

$$p_e(\phi) = (\rho\phi B\gamma)^{-1}, \quad q_e(\phi) = Q_e \left(P_e \rho\phi B\gamma\right)^\sigma, \quad r_e(\phi) = R_e \left(P_e \rho\phi B\gamma\right)^{\sigma - 1},$$

where $\gamma \equiv 1 + \tau(Z^s/R)$. Thus the ratio of any two firms' outputs and revenues are proportional to the ratio of the firms' productivity. Thus using (12), we can write the firm's profit as

$$\pi_e(\phi) = \left\{ \left(\frac{\phi}{\phi_e^*} \right)^{\sigma-1} - 1 \right\} f - \tau \left\{ z_e^s(\phi_e^*) \left(\frac{\phi}{\phi_e^*} \right)^{\sigma-1} - z_e^s(\phi) \right\},$$

where the expression inside the second braces is

$$\frac{q_e(\phi^*)^\rho}{Q_e^\rho} Z^s \left(\frac{\phi}{\phi^*} \right)^{\sigma-1} - \frac{q_e(\phi)^\rho}{Q_e^\rho} Z^s$$
$$= Z^s \{P_e \rho \phi_e^* B(1+\tau\theta)\}^{\sigma-1} \left(\frac{\phi}{\phi_e^*} \right)^{\sigma-1} - Z^s \{P_e \rho \phi B(1+\tau\theta)\}^{\sigma-1} = 0.$$

Hence, the profit of the firm with productivity ϕ is the same as that of no regulation or the emissions trading with the EBA rule.

(ii) First, note that we have

$$z_e(\phi) = \frac{\alpha}{\tau} \frac{q_e(\phi)}{\phi B} = \frac{\alpha\rho}{\tau} r_e(\phi)\gamma.$$

Aggregating this expression for all firms and replacing $\gamma = 1 + \tau(Z^s/R)$ and $R_e = L$, we obtain

$$Z_e = \frac{\alpha\rho}{\tau} R_e \gamma = \frac{\alpha\rho L}{\tau} + \alpha\rho Z_e.$$

Solving this for τ for a given level of emissions cap Z^s, the equilibrium permit price under the endogenous OBA is

$$\tau_{e^{OBA}} = \frac{\alpha\rho L}{(1-\alpha\rho)Z^s} > \frac{\alpha\rho L}{Z^s} = \tau_{e^{EBA}}.$$

Q.E.D.

Proof of Proposition 8 *(CDM Participation)*. (i) The proof is for the case where $\pi_c(\phi_c^*) = \pi_e(\phi_c^*)$. We know that $\pi_c(\phi) = \pi_e(\phi)$ if $\pi_e(\phi) > \pi_n(\phi)$ and $\pi_c(\phi) = \pi_n(\phi)$ otherwise. Using expressions for profits, $\pi_e(\phi) > \pi_n(\phi)$ if and only if

$$\left[\left(\frac{\phi}{\phi_c^*} \right)^{\sigma-1} - 1 \right] (f - \tau z_c^s) > \left(\frac{A}{B} \frac{\phi}{\phi_c^*} \right)^{\sigma-1} (f - \tau z_c^s) - f.$$

This inequality holds if and only if

$$\phi^u \equiv \left[\frac{f}{\{f - \tau z_c^s\}\{(A/B)^{\sigma-1} - 1\}} \right]^{\frac{1}{\sigma-1}} \phi_c^* > \phi.$$

By assumption, $\sigma > 1$ and $f > \{f - \tau z_c^s\}\left\{(A/B)^{\sigma-1} - 1\right\}$. It thus follows that $\phi^u > \phi_c^*$. Furthermore, ϕ^u *is increasing in* ϕ_c^* because $f > 0$ and $\{f - \tau z_c^s\}\left\{(A/B)^{\sigma-1} - 1\right\} > 0$
(ii) Firm's profit when not participating in CDM is:

$$\pi_c^{NP}(\phi) = \frac{r_c^{NP}(\phi)}{\sigma} - f,$$

while when participating is:

$$\pi_c^P(\phi) = \frac{r_c^P(\phi)}{\sigma} - f + \tau z_c^s(\phi).$$

Thus the firm participates in CDM if and only if

$$\pi_c^P(\phi) - \pi_c^{NP}(\phi) = \frac{r_c^P(\phi)}{\sigma} + \tau z_c^s(\phi) - \frac{r_c^{NP}(\phi)}{\sigma} > 0.$$

Substituting the EBB and recalling $r_c^{NP}(\phi) = R\,(P\rho\phi A)^{\sigma-1}$ and $r_c^P(\phi) = R\,(P\rho\phi B)^{\sigma-1}$, the firm participates if and only if

$$\begin{aligned}
\pi^P - \pi^{NP} &= \frac{R_c}{\sigma}(P_c\rho B)^{\sigma-1}\phi^{\sigma-1} + \tau\lambda\rho R_n(P_n\rho A)^{\sigma-1}\phi^{\sigma-1} - \frac{R_c}{\sigma}(P_c\rho A)^{\sigma-1}\phi^{\sigma-1}, \\
&= \left[\frac{R_c}{\sigma}(P_c\rho B)^{\sigma-1} + \tau\lambda\rho R_n(P_n\rho A)^{\sigma-1} - \frac{R_c}{\sigma}(P_c\rho A)^{\sigma-1}\right]\phi^{\sigma-1} > 0.
\end{aligned}$$

The sign of the expression inside the square brackets is independent of ϕ. Hence, either all active firms participate or no firms at all. *Q.E.D.*

Proof of Proposition 9 (*Aggregate Emissions under the Clean Development Mechanism Policy*).
(i) Recall from the proof of Proposition 5 that the aggregate emissions level is given by $Z_n = \lambda\rho L$ under no regulation ($i = n$), and $Z_e = \alpha\rho L/\tau$ under emissions trading ($i = e$).

On the other hand, under CDM ($i = c^{UB}$), the aggregate emissions level is given by:

$$\begin{aligned} Z_c &= \int_0^{\phi^u} \frac{\alpha\rho}{\tau} r_c(\phi) M_c \mu_c(\phi) d\phi + \int_{\phi^u}^{\infty} \lambda\rho r_c(\phi) M_c \mu_c(\phi) d\phi \\ &= M_c \left[\frac{\alpha\rho}{\tau} r_c(\tilde{\phi}_c^P) + \lambda\rho r_c(\tilde{\phi}_c^{NP}) \right] \\ &< M_c \left[\lambda\rho r_c(\tilde{\phi}_c^P) + \lambda\rho r_c(\tilde{\phi}_c^{NP}) \right] \\ &= \lambda\rho R_c = \lambda\rho (L - T) < \lambda\rho L = Z_n \end{aligned}$$

where the third inequality follows from the assumption $\tau > \alpha/(1-\alpha)\lambda$ and the second-to-last equality follows from the aggregation condition $R = L - T$ under CDM (30). To see $Z_c \gtreqless Z_e$, observe that as $\phi^u \to \phi_c^*$, $\lim_{\phi^u \to \phi_c^*} T = 0$ and

$$\begin{aligned} \lim_{\phi^u \to \phi_c^*} Z_c &= \lim_{\phi^u \to \phi_c^*} \frac{1}{1 - G(\phi_c^*)} \left[\int_{\phi_c^*}^{\phi^u} \frac{\alpha\rho}{\tau} r_c(\phi) M_c g(\phi) d\phi + \int_{\phi^u}^{\infty} \lambda\rho r_c(\phi) M_c g(\phi) d\phi \right] \\ &= \frac{1}{1 - G(\phi_c^*)} \int_{\phi_c^*}^{\infty} \lambda\rho r_c(\phi) M_c g(\phi) d\phi = \lambda\rho R_c = \lambda\rho L = Z_n > Z_e. \end{aligned}$$

On the other hand, as $\phi^u \to \phi_c^*$, $T > 0$ and

$$\begin{aligned} \lim_{\phi^u \to \infty} Z_c &= \lim_{\phi^u \to \infty} \frac{1}{1 - G(\phi_c^*)} \left[\int_{\phi_c^*}^{\phi^u} \frac{\alpha\rho}{\tau} r_c(\phi) M_c g(\phi) d\phi + \int_{\phi^u}^{\infty} \lambda\rho r_c(\phi) M_c g(\phi) d\phi \right] \\ &= \int_{\phi_c^*}^{\infty} \frac{\alpha\rho}{\tau} r_c(\phi) M_c g(\phi) d\phi = \frac{\alpha\rho}{\tau} R_c = \frac{\alpha\rho}{\tau}(L - T) < Z_e. \end{aligned}$$

(ii) As proved in Proposition 8, all active firms participate in CDM if some participate. Moreover, as shown above, when all active firms participate in CDM, the aggregate emissions level is given by

$$Z_c = \frac{\alpha\rho}{\tau}(L - T),$$

where $T = \tau(\bar{Z}_c - Z_c) > 0$ because

$$\begin{aligned} \bar{Z}_c &\equiv \int_0^{\infty} z_c^s(\phi) M_c \mu_c(\phi) d\phi = \int_0^{\infty} z_n(\phi) M_c \mu_c(\phi) d\phi = M_c \bar{z}_n \qquad \text{(A2)} \\ &> M_c \bar{z}_c = Z_c, \end{aligned}$$

where $\bar{z}_n > \bar{z}_c$ by assumption. Thus $Z_c = \frac{\alpha\rho}{\tau}(L - T) < \frac{\alpha\rho}{\tau} L = Z_e$. *Q.E.D.*

Proof of Proposition 10 (*Combined Emissions from Host and Sponsoring Countries under the Clean Development Mechanism Policy*). (i) Recall that by definition, $T/\tau \equiv M_c^P(\bar{z}_n - \bar{z}_c^P)$

under the UB rule. Hence, $T/\tau > Z_n - Z_c$ if and only if

$$\bar{z}_n M_c^P - \bar{z}_c^P M_c^P > \bar{z}_n M_n - \bar{z}_c M_c \quad \text{or} \quad \bar{z}_c M_c - \bar{z}_c^P M_c^P > \bar{z}_n M_n - \bar{z}_n M_c^P.$$

The LHS of the inequality equals $\bar{z}_c^{NP} M_c^{NP}$ by definition. Thus the inequality can be rewritten as

$$\bar{z}_c^{NP} M_c^{NP} + \bar{z}_n M_c^P > \bar{z}_n M_n.$$

By definition, $\bar{z}_c^{NP} > \bar{z}_n = z_c^s$. Therefore, the LHS of this inequality must be greater than $\bar{z}_n M_c^{NP} + \bar{z}_n M_c^P$ or $\bar{z}_n M_c$. It follows that a sufficient condition for the above inequality to hold is $M_c > M_n$. Now using $M_c^P = s^P M_c$ and $\bar{z}_c^{sP} = z_c^s = \bar{z}_n$ in (31), we have

$$M_c = \frac{L - \tau s^P M_c (\bar{z}_n - \bar{z}_c^P)}{\sigma(\bar{\pi}_c + f - \tau s^P \bar{z}_n)}.$$

Solving for M_c, we obtain

$$M_c = \frac{L}{\sigma(\bar{\pi}_c + f) - \tau s^P \left[(\sigma - 1)\bar{z}_n) - \bar{z}_c^P\right]}.$$

On the other hand,

$$M_n = \frac{L}{\sigma(\bar{\pi}_n + f)}.$$

Thus $M_c > M_n$ if

$$\sigma(\bar{\pi}_c + f) - \tau s^P \left[(\sigma - 1)\bar{z}_n) - \bar{z}_c^P\right] < \sigma(\bar{\pi}_n + f). \tag{A3}$$

Using the expressions for Z_n and M_n, the average baseline emissions $\bar{z}_n$ is given by

$$\bar{z}_n = Z_n / M_n = \lambda \rho \sigma(\bar{\pi}_n + f).$$

Substituting this for $\bar{z}_n$ in (A3), we see that (A3) if and only if

$$\sigma(\bar{\pi}_c + f) - \tau s^P < \left[\sigma + \lambda(\sigma - 1)^2\right] (\bar{\pi}_n + f),$$

which holds if $\bar{\pi}_c \leq \bar{\pi}_n$ because $\lambda(\sigma - 1)^2 > 0$ by assumption.
(ii) CDM results in a net increase in the combined aggregate emissions if and only if $T/\tau > Z_n - Z_c$ as discussed above. Using (33), $T/\tau = \bar{z}_n M_c - Z_c$. Thus $T/\tau > Z_n - Z_c$ if and only if

$$\bar{z}_n M_c - Z_c > \bar{z}_n M_n - Z_c \quad \text{or} \quad M_c > M_n.$$

To see $M_c > M_n$, first note that using (33) and $\bar{\pi}_c = \bar{\pi}_n$ and $\bar{z}_c^s = \bar{z}_n$ in (31), we have

$$M_c = \frac{L - \tau(\bar{z}_n M_c - Z_c)}{\sigma(\bar{\pi}_n + f - \tau\bar{z}_n)}.$$

Collecting terms and solving for M_c, we obtain

$$M_c = \frac{L + \tau Z_c}{\sigma(\bar{\pi}_n + f - \tau\bar{z}_n) + \tau\bar{z}_n}. \tag{A4}$$

On the other hand, using the expressions for Z_n and M_n, the average baseline emissions $\bar{z}_n$ is given by

$$\bar{z}_n = Z_n/M_n = \lambda\rho\sigma(\bar{\pi}_n + f).$$

Substituting this into (A4) and collecting terms, it follows that

$$M_c = \frac{L + \tau Z_c}{\sigma(\bar{\pi}_n + f)\left\{1 - \tau\lambda\rho(\sigma - 1)\right\}}.$$

Hence

$$\begin{aligned} M_c - M_n &= \frac{L + \tau Z_c}{\sigma(\bar{\pi}_n + f)\left\{1 - \tau\lambda\rho(\sigma - 1)\right\}} - \frac{L}{\sigma(\bar{\pi}_n + f)} \\ &= \frac{\tau Z_c + \tau\lambda\rho(\sigma - 1)L}{\sigma(\bar{\pi}_n + f)\left\{1 - \tau\lambda\rho(\sigma - 1)\right\}} > 1, \end{aligned}$$

if $1 - \tau\lambda\rho(\sigma - 1) > 0$ or $\tau < 1/\lambda\rho(\sigma - 1)$. *Q.E.D.*

B. Equilibrium under the CDM Policy

The stationary equilibrium under the CDM policy with the UB rule is more involved than under other policy regimes. This appendix offers some of the derivations and proofs omitted in the paper. Under the UB rule, there exists $\phi^u \geq \phi_c^*$ such that firms with ϕ above ϕ^u participate in the CDM policy and firms with ϕ below ϕ^u do not, as in Proposition 8.

Moreover, using the fact that $r_c(\phi_c^*) = \sigma d_c = f - \tau z^s$, we have:

$$r_i(\phi) = \begin{cases} \left(\frac{\phi}{\phi_e^*}\right)^{\sigma-1} \sigma d_c & \text{if } \phi \leq \phi^u \\ \left(\frac{A}{B}\frac{\phi}{\phi_c^*}\right)^{\sigma-1} \sigma d_c & \text{o.w.} \end{cases}, \tag{A5}$$

$$\pi_i(\phi) = \begin{cases} \left[\left(\frac{\phi}{\phi_c^*}\right)^{\sigma-1} - 1\right] d_c & \text{if } \phi \leq \phi^u \\ \left(\frac{A}{B}\frac{\phi}{\phi_c^*}\right)^{\sigma-1} d_c - f & \text{o.w.} \end{cases}, \tag{A6}$$

$$z_i(\phi) = \begin{cases} \frac{\alpha\rho}{\tau}\left(\frac{\phi}{\phi_c^*}\right)^{\sigma-1} \sigma d_c & \text{if } \phi \leq \phi^u \\ \lambda\rho\left(\frac{A}{B}\frac{\phi}{\phi_c^*}\right)^{\sigma-1} \sigma d_c & \text{o.w.} \end{cases}. \tag{A7}$$

Given our assumption on the exogeneity of the credit price τ, the equilibrium is determined solely by the zero cutoff condition and the free entry condition. The free entry condition for either rule is the same as under no regulation. Using this result, we can write the average profit $\bar{\pi}_c$ as follows:

$$\bar{\pi}_c = \left[\left(\frac{\tilde{\phi}_c^P}{\phi_c^*}\right)^{\sigma-1} + \left(\frac{A}{B}\frac{\tilde{\phi}_c^{NP}}{\phi_c^*}\right)^{\sigma-1} - \frac{G(\phi^u) - G(\phi_c^*)}{1 - G(\phi_c^*)}\right](f - \tau z_c^s) - \frac{1 - G(\phi^u)}{1 - G(\phi_c^*)} f, \tag{ZCP-UB}$$

where $d_c = f - \tau z_c^s$, $\tilde{\phi}_c^P = \left[\int_0^{\phi^u(\phi_c^*)} \phi^{\sigma-1}\mu_c(\phi)d\phi\right]^{\frac{1}{\sigma-1}}$, and $\tilde{\phi}_c^{NP} = \left[\int_{\phi^u(\phi_c^*)}^{\infty} \phi^{\sigma-1}\mu_c(\phi)d\phi\right]^{\frac{1}{\sigma-1}}$. Note that $\tilde{\phi}_c^P$ and $\tilde{\phi}_c^{NP}$ are *not* defined as conditional means.

Proof: Assuming that some firms participate in CDM,

$$\begin{aligned}
\bar{\pi}_c &\equiv \int_0^\infty \pi_c(\phi)\mu_c(\phi)d\phi = \int_0^{\phi^u} \pi_c(\phi)\mu_c(\phi)d\phi + \int_{\phi^u}^\infty \pi_c(\phi)\mu_c(\phi)d\phi \quad \text{(by definition)} \\
&= \int_0^{\phi^u} \left[\left(\frac{\phi}{\phi_c^*}\right)^{\sigma-1} - 1\right] d_c\mu_c(\phi)d\phi + \int_{\phi^u}^\infty \left[\left(\frac{A}{B}\frac{\phi}{\phi^*}\right)^{\sigma-1} d_c - f\right]\mu_c(\phi)d\phi \quad \text{(using A2)} \\
&= \left(\frac{1}{\phi_c^*}\right)^{\sigma-1}\int_0^{\phi^u}\phi^{\sigma-1}d_c\mu_c(\phi)d\phi - \int_0^{\phi^u} d_c\mu_c(\phi)d\phi + \left(\frac{A}{B}\frac{1}{\phi^*}\right)^{\sigma-1}\int_{\phi^u}^\infty \phi^{\sigma-1}d_c\mu_c(\phi)d\phi - \int_{\phi^u}^\infty f\mu_c(\phi)d\phi \\
&= \left(\frac{1}{\phi_c^*}\right)^{\sigma-1} d_c\left\{\left[\int_0^{\phi^u}\phi^{\sigma-1}\mu_c(\phi)d\phi\right]^{\frac{1}{\sigma-1}}\right\}^{\sigma-1} - \int_0^{\phi^u} d_c\mu_c(\phi)d\phi \\
&\qquad + \left(\frac{A}{B}\frac{1}{\phi^*}\right)^{\sigma-1} d_c\left\{\left[\int_{\phi^u}^\infty \phi^{\sigma-1}\mu_c(\phi)d\phi\right]^{\frac{1}{\sigma-1}}\right\}^{\sigma-1} - \int_{\phi^u}^\infty f\mu_c(\phi)d\phi \\
&= \left[\left(\frac{\tilde{\phi}_c^P}{\phi_c^*}\right)^{\sigma-1} + \left(\frac{A}{B}\frac{\tilde{\phi}_c^{NP}}{\phi_c^*}\right)^{\sigma-1}\right] d_c - \int_0^{\phi^u} d_c\mu_c(\phi)d\phi - \int_{\phi^u}^\infty f\mu_c(\phi)d\phi \\
&= \left[\left(\frac{\tilde{\phi}_c^P}{\phi_c^*}\right)^{\sigma-1} + \left(\frac{A}{B}\frac{\tilde{\phi}_c^{NP}}{\phi_c^*}\right)^{\sigma-1} - \frac{G(\phi^u) - G(\phi_c^*)}{1 - G(\phi_c^*)}\right] d_c - \frac{1 - G(\phi^u)}{1 - G(\phi_c^*)} f
\end{aligned}$$

where $d_c = f - \tau z_c^s$. *Q.E.D.*

Equation (ZCP-UB) gives us the zero cutoff condition for the CDM policy (when some firms participate). As shown in Proposition 8, ϕ^u is increasing in the cutoff productivity ϕ_c^*. Therefore, $\tilde{\phi}_c^P$ and $\tilde{\phi}_c^{NP}$ also depend on ϕ_c^*. It turns out this ZCP-UB curve may lie either above or below the ZCP curve under no regulation in the $(\phi, \bar{\pi})$ space (See the Pareto example below). Hence, $\phi_{c^{UB}}^*$ can be lower or higher than ϕ_n^* and $\bar{\pi}_c$ can be lower or higher than $\bar{\pi}_n$.[18]

Using this expression, we can show the existence and uniqueness of the equilibrium under the CDM with the UB rule.

Proposition A1. *For the policy regime $i = c^{UB}$, for a given price of CDM credits $\tau > 0$, there exists a unique stationary equilibrium of the economy with the CDM market characterized by a*

[18]On the other hand, the expression for average profit $\bar{\pi}_c$ under the EBB rule is the same as under no regulation, whether no firms participate or all firms participate in CDM:

$$\bar{\pi}_c = \left[\left(\frac{\tilde{\phi}_c}{\phi_c^*}\right)^{\sigma-1} - 1\right] f. \qquad \text{(ZCP-EBB)}$$

Because the ZCP and FE conditions under the EBB rule are exactly the same as the case with no regulation, we have $\phi_{c^{EBB}}^* = \phi_n^* = \phi_t^*$ and $\bar{\pi}_{c^{EBB}}^* = \bar{\pi}_n^* = \bar{\pi}_t^*$. The proof of this is analogous to that for the EBA rule under the CAT policy.

pair ($\bar{\pi}_c^*, \phi_c^*$) *such that the following two conditions are satisfied:*

$$\bar{\pi}_c^* = \left[\left(\frac{\tilde{\phi}_c^P}{\phi_c^*}\right)^{\sigma-1} + \left(\frac{A}{B}\frac{\tilde{\phi}_c^{NP}}{\phi_c^*}\right)^{\sigma-1} - \frac{G(\phi^u)-G(\phi_c^*)}{1-G(\phi_c^*)}\right](f-\tau z_c^s) - \frac{1-G(\phi^u)}{1-G(\phi_c^*)}f, \quad \text{(ZCP)}$$

$$\bar{\pi}_c^* = \frac{\delta}{1-G(\phi_c^*)}f_e, \quad \text{(FE)}$$

Proof: The proof follows closely that of Melitz and is for the case in which there exists $\phi^u > \phi_c^*$. Substituting (FE) into (ZCP-UB) and rearranging terms, we have $\delta(f_e/d_c) + [1-G(\phi^u)](f/d_c) = j(\phi)$ where

$$j(\phi) = [1-G(\phi)]\,k(\phi),$$

and

$$k(\phi) = \left(\frac{\tilde{\phi}^P}{\phi}\right)^{\sigma-1} + \left(\frac{A}{B}\frac{\tilde{\phi}^{NP}}{\phi}\right)^{\sigma-1} - \frac{G(\phi^u)-G(\phi)}{1-G(\phi)}.$$

Because the LHS takes some finite positive number in $(\delta(f_e/d_c), \delta(f_e/d_c) + f/d_c)$, a sufficient condition for the existence and uniqueness of the equilibrium pair $(\bar{\pi}_c, \phi_c^*)$ is that $j(\phi)$ is monotonically decreasing from infinity to zero on $(0,\infty)$. The derivative of k is:

$$\begin{aligned} k'(\phi) &= \frac{g(\phi)}{1-G(\phi)}k(\phi) - \frac{(\sigma-1)}{\phi}\left[k(\phi) + \frac{G(\phi^u)-G(\phi)}{1-G(\phi)}\right] \\ &\quad - \frac{g(\phi^u)}{1-G(\phi)}\phi^{u\prime}\left[1 + \left(\frac{A}{B}\frac{\phi^u}{\phi}\right)^{\sigma-1} - \left(\frac{\phi^u}{\phi}\right)^{\sigma-1}\right]. \end{aligned}$$

Therefore, the derivative of j is:

$$\begin{aligned} j'(\phi) &= -\frac{(\sigma-1)[1-G(\phi)]}{\phi}\left[k(\phi) + \frac{G(\phi^u)-G(\phi)}{1-G(\phi)}\right] \\ &\quad - g(\phi^u)\phi^{u\prime}\left[1 + \left(\frac{A}{B}\frac{\phi^u}{\phi}\right)^{\sigma-1} - \left(\frac{\phi^u}{\phi}\right)^{\sigma-1}\right] < 0. \end{aligned}$$

Moreover, its elasticity with respect to ϕ is:

$$\frac{j'(\phi)\phi}{j(\phi)} = -(\sigma-1)\left(1 + \frac{G(\phi^u)-G(\phi)}{k(\phi)}\right) - c < -(\sigma-1),$$

where c is some positive number. Because $j(\phi)$ is nonnegative and its elasticity is negative and bounded away from zero, it must be decreasing to zero as ϕ goes to infinity. Lastly, it is clear that $\lim_{\phi \to 0} j(\phi) = \lim_{\phi \to 0} \frac{1}{\phi^{\sigma-1}} \left[\int_{\phi}^{\phi^u(\phi)} \xi^{\sigma-1} g(\xi) d\xi + \frac{A}{B} \int_{\phi^u(\phi)}^{\infty} \xi^{\sigma-1} g(\xi) d\xi \right] - G(\phi^u(\phi)) + G(\phi) = \infty.$ *Q.E.D.*

C. Analytical Example with the Pareto Distribution

All of the analytical properties up to now have been derived without assuming a functional form for productivity distribution G. Some analytical properties cannot be obtained, however, without closed-form equilibrium solutions. We use a Pareto distribution as an example. The empirical literature suggests that the Pareto distribution is a reasonable approximation to firm distributions (Cabral and Mata, 2003). Due to its tractability, the Pareto distribution has been used in several applications (e.g. Helpman, Melitz, and Yeaple, 2004; Baldwin, 2005).

Suppose that firm-level productivity ϕ follows the Pareto distribution with cumulative distribution function G:

$$G(\phi) = 1 - \left(\frac{b}{\phi}\right)^k \quad \text{for all } \phi \geq b, \tag{A8}$$

where $b > 0$ and $k \geq \sigma - 1 > 0$. The shape parameter k offers a natural measure of dispersion or heterogeneity. The lower k implies a higher level of dispersion of firm-level productivity. For all policy regimes, the weighted average productivity $\tilde{\phi}_i$ is given by:

$$\tilde{\phi}_i = \left(\frac{k}{k - (\sigma - 1)}\right)^{\frac{1}{\sigma-1}} \phi_i^*. \tag{A9}$$

Thus the weighted average productivity is an explicit function of the cutoff productivity ϕ_i^* and exogenous parameters of the model. Because all aggregate variables are characterized by average productivity (except under CDM), many of the analytical properties of interest rest on the analytical solution for ϕ_i^*. Using (A8) in (FE), we see that

$$\phi_i^* = \left(\frac{\bar{\pi}_i^*}{\delta f_e}\right)^{1/k} b. \tag{A10}$$

Thus solving for the equilibrium cutoff ϕ_i^* requires the solution for the average profits $\bar{\pi}_i^*$, under all policy regimes.

Tax and Cap-and Trade Policies: Because the other equilibrium conditions differ across different policy regimes, the analytical solution for $\bar{\pi}_i^*$ also varies. For $i = n$ or t, using

(A9) in (ZCP), we obtain:

$$\bar{\pi}_n^* = \bar{\pi}_t^* = \left[\frac{\sigma - 1}{k - (\sigma - 1)}\right] f. \tag{A11}$$

For $i = e$, under the uniform allocation (UA) rule, solving the ZCP and EMC conditions simultaneously using (A9), we obtain:

$$\bar{\pi}_{e^{UA}}^* = \left[\frac{\sigma - 1}{k\{1 + \alpha(\sigma - 1)\} - (\sigma - 1)}\right] f. \tag{A12}$$

Because $\alpha > 0$ and $\sigma - 1 > 0$ by assumption, $1 + \alpha(\sigma - 1) > 1$. Thus $\bar{\pi}_n^* = \bar{\pi}_t^* > \bar{\pi}_e^*$, which also implies $\phi_n^* = \phi_t^* > \phi_e^*$ per (A10). We know that $\bar{\pi}_{e^{EBA}}^* = \bar{\pi}_n^* = \bar{\pi}_t^*$ under the emissions-based allocation (EBA) rule. If, instead, the government uses the RBA rule, solving for $\bar{\pi}_e^*$ gives us

$$\bar{\pi}_{e^{RBA}}^* = \frac{\sigma - 1 + \alpha\rho\sigma^2}{k - (\sigma - 1)} f. \tag{A13}$$

Substituting (A11), (A12), and (A13) in the expressions for the equilibrium mass of firms under respective policy regimes, we obtain the equilibrium mass of firms under each policy scenario:

$$M_n = \frac{k - (\sigma - 1)}{\sigma k f} L, \tag{A14}$$

$$M_t = \frac{k - (\sigma - 1)}{\sigma k f(1 - \alpha\rho)} L, \tag{A15}$$

$$M_{e^{UA}} = \frac{k\{1 + \alpha(\sigma - 1)\} - (\sigma - 1)}{\sigma k f} L, \tag{A16}$$

$$M_{e^{EBA}} = \frac{(1 + \alpha\rho\sigma)\{k - (\sigma - 1)\}}{\sigma k f} L, \tag{A17}$$

$$M_{e^{RBA}} = \frac{(1 + \alpha\rho\sigma)\{k - (\sigma - 1)\}}{\sigma(k + \alpha\rho\sigma^2) f} L. \tag{A18}$$

CDM Policy: Under the CDM policy, the case with the emissions-based baseline (EBB) (when all firms participate in CDM) is analytically identical to the CAT policy with the emissions-based allocation rule, except the equilibrium mass of firms $M_{c^{EBB}}$. Using $\bar{z}_{c^{EBB}} = z_{c^{EBB}}(\tilde{\phi}_{c^{EBB}})$, $\bar{\pi}_n^* = \bar{\pi}_{c^{EBB}}^*$ with (A11), and (A9) in (31) and solving for $M_{c^{EBB}}$, we have

$$M_{c^{EBB}} = \frac{k - (\sigma - 1)}{\sigma k f\{1 - \alpha\rho + \tau\lambda\rho(1 - \alpha)(\sigma - 1)\}} L. \tag{A19}$$

Comparing this expression with (33), we see that $M_t > M_{c^{EBB}}$ because $(1 - \alpha)(\sigma - 1) > 0$.

Analytical solutions for the CDM policy under the uniform baseline (UB) rule are more involved. We restrict our attention to the case in which some firms participate in CDM, i.e. $\phi^u > \phi_c^*$. We know that the marginal firm who participates in CDM has the productivity ϕ^u such that:

$$\phi^u = \left[\frac{f}{\{f - \tau z_c^s\} \{(A/B)^{\sigma-1} - 1\}} \right]^{\frac{1}{\sigma-1}} \phi_c^* \equiv F\phi_c^*,$$

where $F > 1$ is a non-monotonic function of τ, since B is a decreasing function of τ. Using this in (ZCP) above and manipulating, we obtain:

$$\bar{\pi}_c^* = \frac{1}{k-(\sigma-1)} \left[(\sigma - 1) + \left\{ \left(\frac{A}{B} \right)^{\sigma-1} - 1 \right\} \frac{1}{F^{k-(\sigma-1)}} \right] (f - \tau z_c^s) - \left(\frac{1}{F} \right)^k \tau z_c^s. \quad \text{(A20)}$$

Note that because F is a non-monotonic function of τ, $\bar{\pi}_c^*$ is also a non-monotonic function of τ in general. However, as τ gets sufficiently small and below $\alpha/(1-\alpha)\lambda$, participating firms will operate along the maximum substitution possibility. Therefore, as τ approaches zero, all incumbent firms operate with marginal cost $1/\phi A$ in place of $1/\phi B$ (so A/B is replaced with $A/A = 1$) and τz_c^s approaches zero. Thus substituting these values in (A20), we see that $\bar{\pi}_c^* = \bar{\pi}_n^*$ at a sufficiently low price of permits. Because $\bar{\pi}_c^*$ is also a non-monotonic function of τ, ϕ_c^* can be higher or lower than ϕ_n^*.

To identify aggregate quantities, we need to identify $M_c = R_c/\bar{r}_c$ where $R_c = L - T = L - \tau M_c^p(z_c^s - \bar{z}_c^p)$, M_c^p the mass of firms participating in CDM, and $\bar{z}_c^p$ the average emissions by the firms participating in CDM. Because $\pi_c(\phi) = r_c(\phi)/\sigma - d_c$, we have:

$$\bar{\pi}_c^* = \frac{\bar{r}_c}{\sigma} - \int_0^\infty d_c(\phi)\mu_c(\phi)d\phi = \frac{\bar{r}_c}{\sigma} - \left(\int_0^{\phi^u} (f - \tau z^s)\mu_c(\phi)d\phi + \int_{\phi^u}^\infty f\mu_c(\phi)d\phi \right) = \frac{\bar{r}_c}{\sigma} - J,$$

where $J \equiv s^p(f - \tau z_c^s) + (1 - s^p)f$. Hence, $\bar{r}_c = \sigma(\bar{\pi}_c^* + J)$. Substitution this into $M_c = R_c/\bar{r}_c$, we have:

$$M_c = \frac{L - \tau s^p M_c(z^s - \bar{z}^p)}{\sigma(\bar{\pi}_c^* + J)} \quad \text{or} \quad M_c = \frac{L}{\sigma(\bar{\pi}_c^* + J) + \tau s^p(z^s - \bar{z}_c^p)}.$$

where s^p is the share of the firms participating in CDM so that $M_c^p = s^p M_c$. The values of s^p and $\bar{z}^p$ can be obtained as follows:

$$s^p = \int_0^{\phi^u} \mu(\phi)d\phi = \int_{\phi_c^*}^{F\phi_c^*} \frac{g(\phi)}{1 - G(\phi_c^*)}d\phi = \left(\frac{\phi_c^*}{b} \right)^k \left[-b^k\phi^{-k} \right]_{\phi_c^*}^{F\phi_c^*} = 1 - \frac{1}{F^k},$$

and

$$
\begin{aligned}
\bar{z}^p &= \int_0^{\phi^u} z_c(\phi)\mu_c(\phi)d\phi/s^p \\
&= \int_0^{\phi^u} \frac{\alpha\rho}{\tau}\left(\frac{\phi}{\phi_c^*}\right)^{\sigma-1}\sigma(f-\tau z^s)\mu_c(\phi)d\phi/s^p \\
&= \frac{\alpha\rho}{\tau}\left(\frac{1}{\phi_c^*}\right)^{\sigma-1}\sigma(f-\tau z^s)\int_{\phi_c^*}^{\phi^u}\left(\frac{\phi_c^*}{b}\right)^k kb^k\phi^{\sigma-k-2}d\phi/s^p \\
&= \frac{\alpha\rho}{\tau}\left(\frac{1}{\phi_c^*}\right)^{\sigma-1}\sigma(f-\tau z^s)\left(\frac{\phi_c^*}{b}\right)^k\left[\frac{kb^k}{\sigma-k-1}\phi^{\sigma-k-1}\right]_{\phi_c^*}^{F\phi_c^*}/s^p \\
&= \frac{\alpha\rho}{\tau}\sigma(f-\tau z^s)\frac{k}{k-(\sigma-1)}\left[1-\frac{1}{F^{k-(\sigma-1)}}\right]/s^p.
\end{aligned}
$$