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Emission Taxes and Border Tax Adjustments for Oligopolistic Industries

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Abstract

We examine the welfare consequence of emissions tax with and without a Border Tax Adjustment for an imperfectly competitive industry, where intra-industry trade arises between countries. BTA allows a government to impose a pollution-content tariff on imports and refund an emission tax for export sales. We analyze the structure of an optimal emission tax with BTA when a government chooses its emission tax rate to maximize its national welfare. We show that the optimal emission tax policy with BTA achieves greater national welfare and higher environmental quality than the optimal policy without BTA.

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1 Introduction

Climate change policy has been entangled with trade issues in recent years. The United States refused to ratify the Kyoto Protocol partly because it gives an unfair advantage to manufacturers in nations such as China and India that are not required to cut greenhouse gas emissions (Revkin, 2009). More recently, at the United Nations Climate Change Convention, 15th Conference of the Parties (UNFCCC, COP 15) held in Copenhagen, the United States pushed for the right to impose border adjustments in a draft deal, i.e. "tariffs" on certain goods from countries such as China and India that did not act to limit their greenhouse gas emissions[1]. Some argue that such trade policy is an indirect measure for those countries to reduce their greenhouse gas emissions so that the global nature of climate change is taken into account (World Trade Organization, 2009). However, others warn that there could be a backlash from those trading partners that could, in the end, trigger tit-for-tat actions that would hurt exports of the developed countries (Revkin, 2009).

The idea of border adjustments is not new.[2] Under the definition of General

[1]A group of developing countries supported a draft having provisions that restrict the use of unilateral trade measures as a part of climate change policies. European Union, together with other developed countries including Japan, firmly opposed any provisions that would question the parties' right to apply trade measures in the climate change context. As a result, no references to trade are made in *Copenhagen Accord*, but trade related proposals were included in the Chair's draft text.

This conflict between the developed and developing countries has not been over. In September 2011, India asked the United Nations to table a proposal to ban climate related trade measures including border adjustments at the Durban conference. However, the developed countries commented that the issue should be addressed at the WTO (Helm et al., 2012).

[2]In the context of climate change policy, the term Border Adjustments refers to the measure that takes the form of a tax or a regulation imposed at the border aiming at equal treatment of the embedded carbon content of like foreign and domestic products (Horn and Mavroidis, 2010). Horn and Mavroidis use the term Border Carbon Adjustments instead of Border Adjustments. If the measure of a Border Adjustment takes the specific form of a tax, then it is called a "Border Tax Adjustment (BTA)."

In 1960s and 1970s, there was an extensive discussion concerning the legality of a Border Tax Adjustment scheme as a means for correcting different forms of indirect taxation. The interest in the role of border tax adjustments resulted in an extensive literature. This research suggests that a BTA is neutral from a trade point of view, i.e., implementation of a BTA transforms an origin-base tax system to a destination-base tax system and it does not distort trade under certain conditions. See Horn and Mavroidis (2010) for a recent review of this literature. For the relevance of this early literature to the current debate on border adjustments motivated by environmental objectives, see

Agreement on Tariffs and Trade and World Trade Organization (GATT/WTO) rules, a border tax adjustment (BTA) consists of two provisions: (i) the imposition of a tax on imported products, corresponding to a tax born by similar domestic products, and/or (ii) the refund of domestic taxes when the products are exported (WTO, 2009). BTAs are commonly used with respect to domestic taxes on the sale or consumption of goods. Recently, there has been an extensive legal debate over the eligibility of domestic carbon/energy taxes for BTAs (Hufbauer et al., 2009). In trade policy circles, researchers have often expressed a fear that border adjustments could be imposed to protect national commercial interests, thereby being used as an instrument for protectionism (Fischer and Horn, 2010). Economists argued that border adjustments based on climate change policy could be justified. Nobel-Prize winning trade economist Paul Krugman stated "what the economics really says is that incentives should reflect the marginal cost of greenhouse gases in all goods, wherever produced - which in this case happens to imply border adjustments (Krugman, 2009)."

How does a BTA affect an emission tax policy of a country when a welfare maximizing government chooses its emission tax rate? What is the impact of a BTA on the welfare of trading countries and the global environment? We examine these issues by using a framework of an international trade model with imperfect competition. In our setting with two countries (a home country and a foreign country), intra-industry trade arises due to imperfect competition in an oligopolistic sector. Production in this sector generates pollution emissions such as greenhouse gases that cause cross-border externalities. The government of each country can impose an emission tax on this sector under two different policies, an emission tax policy with and without a BTA. The former is a destination (consumption) base scheme, under which an emission tax on domestic products and a pollution-content tariff on like foreign products are imposed at equal rates and the emission tax on exports of domestic products are exempted.[3] The latter is an origin (production) base scheme,

Lockwood and Whalley (2008).

[3]Copeland (1996) uses the idea of a pollution-content tariff in a different context.

in which a border tax adjustment is applied neither to imports nor to exports so that an emission tax is imposed on domestic products at equal rates regardless of sales destination, and a pollution-content tariff is exempted.

We analyze the structure of an optimal emission tax in each policy when a home country government chooses its emission tax rate to maximize its national welfare given the foreign country's tax rate. In the case with the BTA, a positive emission tax necessarily reduces global emissions even though other distortional effects such as market power and profit shifting influence the optimal level of the emission tax rate. However, in the case without the BTA, a positive emission tax would be harmful to the global environment if a technology gap in emission coefficients (emissions per unit of output) between the countries is sufficiently large. These results suggest that the BTA can be justified from the environmental point of view even though it has a side effect in protecting domestic industries from foreign competition.

We also compare the emission tax policies in terms of the welfare of the home country and the environment. We show that the optimal emission tax of the home country achieves greater home country welfare and higher environmental quality in the emission tax policy with the BTA as compared to the policy without the BTA if a home firm has cleaner technology than a foreign firm and the technology gap is small. Furthermore, even when the home country government maximizes its national welfare, its optimal emission tax with the BTA can benefit the foreign country. This suggests that the emission tax policy with the BTA is not necessarily a beggar-thy-neighbor policy.

There is a large literature on border adjustments in the context of trade and the environment. One branch of this literature quantitatively investigated border adjustment measures and their implications for trade and welfare. The work used Computable General Equilibrium (CGE) models and examined the quantitative impacts of border adjustment measures as a part of unilateral regulation on greenhouse gas emissions, including Matoo et al. (2009), McKibbin and Wilcoxen (2009), Boehringer et al. (2010) among others.[4] More recently, by using input-output analysis, Atkinson

[4]Fischer and Fox (2012) examined the effects of a border tax adjustment policy on competitiveness and leakage in a given sector by using a partial equilibrium model parameterized by simulations

et al. (2011) calculated 'virtual' carbon content of imports and showed that a tax on carbon would lead to substantial effective tariff rates on imports from most of the carbon-intensive developing nations.

While the above studies considered unilateral implementation of climate change policy with BTAs, another branch of this literature examined strategic aspects of border adjustments, including Horn and Mavroidis (2010), Tarui et al. (2010), and Helm et al. (2012).[5] Horn and Mavroidis (2010) pointed out that one of the purposes of BTAs was to induce other countries to adopt comparable environmental policy and examined several different types of strategic effects of BTAs. Tarui et al. (2010) used a competitive partial equilibrium model and showed that an import tariff could not necessarily induce an exporting country to impose a higher emission tax. Helm et al. (2012) considered a model with a political economy aspect and concluded that border carbon adjustments could induce an exporting country to adopt a carbon adjustment to exports.

These existing studies on BTAs assumed perfect competition in the regulated industries. However, in climate change policies proposed in developed countries, BTAs were targeted on carbon/energy intensive industries including chemicals, paper, steel, and cement, most of which have features of oligopolistic industries.[6] In addition, policy implications of imperfect competitive models differ from those of competitive settings. The literature on trade policy in oligopoly suggests that an export subsidy can improve welfare of an exporting country in Cournot oligopoly although, in a standard competitive setting, an export tax benefits an exporting country through an improvement in its terms of trade. This property of the oligopoly model plays a role in our analysis since the BTA influences an incentive for a policymaker to set an

of a CGE model. Gros (2009) also used a partial equilibrium setting to examine the effects of border measures motivated by climate change policy from a global welfare point of view.

[5]See Chow (2011) for a general review of economic approaches, including those based on game theory, to study global environmental issues.

[6]For instance, the American Clean Energy and Security Act was supposed to set up a cap and trade scheme in the U.S., and planned to implement unilateral border measures to impose duties on certain energy intensive foreign goods. This bill was passed by the House in 2009 but rejected by the Senate.

optimal emission tax rate by allowing refund of an emission tax for export sales.[7]

Our work is also related to recent studies on incomplete environmental regulation such as Fowlie (2009) and Ritz (2009). They considered a situation in which environmental regulation is incomplete, i.e., some firms are regulated and others are not. In such a situation, they examined how carbon leakage could arise from regulated to unregulated firms. Our work is similar to theirs in that we use a Cournot oligopoly model. However, they did not explicitly consider international trade, and thus neither of them examined the effects of a border tax adjustment.[8]

The rest of this paper is organized as follows. In Section 2, we develop a framework of a reciprocal market model of international trade for examining an emission tax policy with and without the BTA. In Section 3, we analyze the optimal emission tax for the home country under each scheme. In Section 4, we compare these two optimal policies in terms of home country welfare and environmental quality. In Section 5, we examine the welfare impact of the home country's optimal emission tax policy on the foreign country. In Section 6, we discuss implications of abatement technology for the optimal emission tax policy with the BTA. In Section 7, we close this paper with concluding remarks.

[7] The welfare effects of an emission tax in a competitive sector are totally different from those in an oligopolistic sector. In a competitive setting, an emission tax provides a benefit for an exporting country through an improvement in its terms of trade. In our setting with Cournot oligopoly, this benefit does not arise since an emission tax reduces profits of exporting firms. Thus, it is worth analyzing the effect of the BTA on an optimal emission tax policy in a setting with imperfect competition.

[8] There is an extensive literature on strategic environmental policy, including Kennedy (1994), Conrad (1996), Burguet and Sempere (2003), and Lai and Hu (2008) among others. This paper is related to these studies in that they examined environmental policy by using intra-industry trade models with imperfect competition. Also, in the CGE literature on trade and the environment, Babiker (2005) used a Cournot oligopoly framework of international trade to examine the impact of a unilateral greenhouse gas regulation on carbon leakage. However, none of them examined a border tax adjustment motivated by environmental concerns.

2 The Model

There are two countries, labelled home and foreign. The industry is an oligopoly with a given number of firms. Firms are located in each country, produce homogeneous goods, and compete à la Cournot. They do not incur transport costs, but such costs for arbitragers are prohibitively high, so the demand curves in the two markets are independent. The technology for each firm is described by a fixed cost and a constant marginal cost. Production of firms generates emissions of pollution such as greenhouse gases that cause cross-border external costs.[9]

Let n denote the number of firms located in the home country, c the constant marginal cost, and f the fixed cost of each firm. Let x denote the sales of each firm to its domestic market and y the sales of each to the other country's market, and $z = x + y$ total output. Let the corresponding variables in the foreign country be denoted by asterisk, $*$. Then, total sales of the home market are $q = nx + n^*y^*$. The inverse demand in the home country is given by $p = p(q)$. For the foreign country, total sales are $q^* = n^*x^* + ny$ with inverse demand $p^* = p^*(q^*)$.

The policy instruments include emission taxes, refunds for emission taxes on export sales, and pollution-content tariffs. Let the home country's emission tax be denoted by τ, its refund for the emission tax on foreign sales by s, and its pollution-content tariff on imports by t. All of them are specific forms. The corresponding instruments for the foreign country are τ^*, s^*, and t^*.

Production of firms emits pollution. Emissions of each firm are proportional to its total output. Let each home firm's emission coefficient be denoted by e, and a corresponding coefficient of each foreign firm by e^*. Then, emissions of each home firm are $e(x + y)$ and those of each foreign firm are $e^*(x^* + y^*)$. In the following analysis, we will consider a case in which emission coefficients differ between each home firm and each foreign firm due to a technology gap between the countries. Let

[9] We extend a framework of a reciprocal market model à la Brander and Krugman (1983) to a setting with cross-border production externalities and use it to analyze an emission tax policy with and without the BTA.

π and π^* denote the profits of the home and foreign firms, respectively:

$$\pi = [p(q) - \tau e]x + [p^*(q^*) - e(\tau - s + t^*)]y - c(x+y) - f,$$
$$\pi^* = [p^*(q^*) - \tau^* e^*]x^* + [p(q) - e^*(\tau^* - s^* + t)]y^* - c^*(x^* + y^*) - f^*.$$

With Cournot behavior, each firm maximizes its profit regarding sales of other firms as fixed. The Cournot equilibrium conditions in the home market are

$$p(q) + xp'(q) = c + e\tau, \tag{1}$$
$$p(q) + y^*p'(q) = c^* + e^*(\tau^* - s^* + t). \tag{2}$$

Similarly, in the foreign market,

$$p^*(q^*) + x^*p^{*\prime}(q^*) = c^* + e^*\tau^*, \tag{3}$$
$$p^*(q^*) + yp^{*\prime}(q^*) = c + e(\tau - s + t^*). \tag{4}$$

We assume that the second order conditions are satisfied in each market.[10] Notice that the equilibrium conditions in the home market are independent of those in the foreign market. With $q = nx + n^*y^*$, (1) and (2) determine x and y^* in market equilibrium. Similarly, with $q^* = n^*x^* + ny$, (3) and (4) determine x^* and y in equilibrium.

Let $u(q)$ denote the gross benefit of home consumers,

$$u(q) = \int_0^q p(z)dz.$$

Home consumer surplus is $u(q) - p(q)q$. Let $E = n(x+y)e$ be the home country's total pollution emissions and $E^* = n^*(x^* + y^*)e^*$ the corresponding emissions of the foreign country. Let h denote the external cost of pollution emissions for home consumers,

$$h = h(E + \mu E^*),$$

[10] The second order conditions are $2p' + xp'' < 0$ and $2p' + y^*p'' < 0$ in the home market. Similarly, in the foreign market, $2p^{*\prime} + x^*p^{*\prime\prime} < 0$ and $2p^{*\prime} + yp^{*\prime\prime} < 0$.

where μ is a parameter for the degree of cross-border pollution and we assume that $0 < \mu \leq 1$, $h' > 0$ and $h'' \geq 0$. The home government's budget surplus is

$$enx\tau + eny(\tau - s) + e^*n^*y^*t.$$

The total home country welfare is given by

$$w = u(q) - p(q)q + n\left\{p(q)x + [p^*(q^*) - et^*]y - c(x+y) - f\right\} + e^*n^*y^*t - h(E + \mu E^*). \tag{5}$$

3 Emission Tax Policy

We will examine a home country's optimal emission tax under two alternative schemes: an emission tax policy with and without a BTA. In the former, the home country's government imposes the emission tax on home firms and exempts them from the emission tax for foreign export sales by setting $s = \tau$. In addition, the home country's government imposes the pollution-content tariff on imports from the foreign country at the same rate as the emission tax rate, $t = \tau$.[11]

In the latter, the home country's government imposes the emission tax on home firms and does not refund the emission tax for foreign export sales, $s = 0$. Also, the home country's government does not impose the pollution-content tariff on imports from the foreign country, $t = 0$.

In sum, the adjustment transforms the emission tax policy from an origin base scheme to a destination base scheme. In the following analysis, we will consider a situation, in which the foreign country's government does not impose taxes on emissions of pollution, $\tau^* = s^* = t^* = 0$, and the degree of cross-border pollution is perfect, $\mu = 1$.[12]

[11] The existing work on BTAs such Matoo et al. (2009) and Fischer and Fox (2012) also examined full border adjustments, i.e., both import tariffs and export refunds.

[12] It seems to be relevant to assume the absence of an emission tax in the foreign country because border adjustment policies proposed in developed nations were unilateral measures against countries that did not act to limit greenhouse gas emissions. The assumption of the perfect cross-border pollution is also relevant for an environmental issue such as climate change.

3.1 Emission Tax Policy with the BTA

The total welfare of the home country can be described as a function of the policy instruments,

$$w_i = w(\tau, s, t), \ (i = B, N),$$

where a subscript i denotes each policy, B indicates the policy with the BTA and N indicates the policy without the BTA. We can easily see that $w_B = w(\tau, \tau, \tau)$ and $w_N = w(\tau, 0, 0)$. In this section, we will consider the home country's emission tax policy with the BTA. It is convenient to rewrite (5) with the use of (2) as[13]

$$w_i = u(q) - cnx + n(p^* - c)y + n^* y^{*2} p' - c^* n^* y^* - nf - h(E + E^*).$$

Note that we use the condition $\tau^* = s^* = t^* = 0$. By substituting $nx = q - n^* y^*$ and $n^* x^* = q^* - ny$ and treating q, y^*, q^* and y^* as the independent variables, we find[14],

$$\begin{aligned} dw_i = {} & (p - c - eh' + n^* y^{*2} p'')dq \\ & + n^*[c + eh' - (c^* + e^* h') + 2p' y^*]dy^* \\ & + n(p^* - c - eh' + e^* h')dy + (nyp^{*\prime} - h' e^*)dq^*. \end{aligned} \quad (6)$$

In the policy with the BTA, the home country's government chooses τ to maximize the total home country welfare under the constraint that $s = t = \tau$. Then, the effect of the home country's emission tax on its welfare is

$$\frac{\partial w_B}{\partial \tau} = (p - c - eh' + n^* y^{*2} p'')\frac{\partial q}{\partial \tau} + n^*[c + eh' - (c^* + e^* h') + 2p' y^*]\frac{\partial y^*}{\partial \tau}. \quad (7)$$

We will illustrate the results with a case in which the demand curves and external cost functions are linear, i.e., $p(q) = a - bq$ and $h(E + E^*) = \theta \cdot (E + E^*)$.[15] In the linear setting, we can derive an optimal emission tax rate in the emission tax policy

[13] We apply a procedure developed by Dixit (1984) to the analysis of an optimal emission tax policy with the BTA.

[14] See Appendix A for the derivation.

[15] See Appendix B for the details of the linear setting.

with the BTA as,[16]

$$\widetilde{\tau}_B = \Gamma\{en[e(n^*+1) - e^*n^*]\theta - enb[(n^*+1)x_0 + n^*y_0^*] \\ + e^*n^*[e^*(n+1) - en]\theta + e^*n^*b[nx_0 + (n+1)y_0^*]\}, \quad (8)$$

where $\Gamma = (n + n^* + 1)/\{(en + e^*n^*)^2 + 2n^*[en - e^*(n+1)]^2\} > 0$, θ is a constant parameter of a marginal external cost of pollution emissions for home consumers, and b is a parameter of the slope of the home country's inverse demand curve. Note that x_0 denotes sales of each home firm to the home market and y_0^* denotes sales of each foreign firm to the home market, respectively, in the absence of the emission tax, the pollution-content tariff, and the refund, $\tau = t = s = 0$.[17]

There are two channels through which the emission tax policy affects the home country's total welfare. The first one can be called a "domestic emission tax channel," which arises due to the emission tax on each home firm's sales to the home market. The first and second terms describe this channel since they are multiplied by the emission coefficient of each home firm. The second one can be called an "import emission tax channel," which arises due to the pollution-content tariff on imports from the foreign country. The third and fourth terms that are multiplied by each foreign firm's emission coefficient describe this channel.

In each channel, there is an "externality effect", the first term for the domestic emission tax channel and the third term for the import emission tax channel on the RHS of (8). The externality effects reflect external costs for home consumers due to emissions caused by production of home and foreign firms for their sales to the home market. First, let us look at the externality effect in the domestic emission tax channel. The emission tax on each home firm's sales to the home market reduces its emissions by $-e\frac{\partial x}{\partial \tau} = ee(1+n^*)/b(n+n^*+1)$.[18] At the same time, the emission tax induces a reduction in each home firm's outputs, which leads to an increase in each foreign firm's export sales. This reallocation of outputs causes "emission leakage,"

[16] See Appendix C for the derivation.

[17] There are feasibility constraints for an emission tax rate. In the policy with the BTA, τ must satisfy $x > 0$, $y^* > 0$, and $p > 0$ in equilibrium. See Appendix D for the details of these constraints.

[18] We can easily derive this by using (36).

i.e., emissions of each foreign firm increase by $e^*\frac{\partial y^*}{\partial \tau} = e^*en/b(n+n^*+1)$. Thus, a change in total emissions equals

$$-ne\frac{\partial x}{\partial \tau} - n^*e^*\frac{\partial y^*}{\partial \tau} = \frac{en[e(n^*+1) - e^*n^*]}{b(n+n^*+1)}. \tag{9}$$

The numerator is exactly the same as the coefficient of the externality effect in the domestic emission tax channel, the first term on the RHS of (8). Therefore, the externality effect of the domestic emission tax channel is positive if a reduction in the emissions of home firms exceeds an increase in those of foreign firms.[19] This condition holds if and only if $e(n^*+1) > e^*n^*$.

Lemma 1 *The externality effect of the domestic emission tax channel is positive if and only if* $e > e^*n^*/(1+n^*)$.

Next, let us turn to the externality effect in the import emission tax channel. The third term is the externality effect that reflects external costs for home consumers because of emissions generated by production of foreign firms for their exports to the home market. Again, this term is related to emission leakage. Imposing the pollution content tariff on foreign firms leads to a reallocation of outputs from foreign to home firms. As a result, emissions of foreign firms decline but those of home firms increase. Total emissions generated by home and foreign firms' production for their sales to the home market change by

$$-n^*e^*\frac{\partial y^*}{\partial t} - ne\frac{\partial x}{\partial t} = \frac{e^*n^*[e^*(n+1) - en]}{b(n+n^*+1)}. \tag{10}$$

The numerator is the same as the coefficient of the externality effect, i.e., the third term on the RHS of (8). Thus, the externality effect of the import emission tax channel is positive if a reduction in the emissions of foreign firms is larger than a rise in those of home firms.[20] This condition is satisfied if and only if $e^*(n+1) > en$.

[19] Note that the externality effect of the domestic emission tax channel would be negative and thus it would suggest an emission subsidy if a reduction in the emissions of foreign firms is greater than an increase in the emissions of home firms.

[20] Note that the externality effect would be negative and thus it would suggest a subsidy to

Lemma 2 *The externality effect of the import emission tax channel is positive if and only if* $e^* > en/(n+1)$.

We can provide a more intuitive explanation for the externality effects. With the use of (9) and (10), we can derive

$$-ne\frac{\partial x}{\partial \tau} - n^*e^*\frac{\partial y^*}{\partial \tau} = -ne\left(\frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial t}\right), \tag{11}$$

$$-n^*e^*\frac{\partial y^*}{\partial t} - ne\frac{\partial x}{\partial t} = -n^*e^*\left(\frac{\partial y^*}{\partial t} + \frac{\partial y^*}{\partial \tau}\right), \tag{12}$$

$$-ne\left(\frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial t}\right) - n^*e^*\left(\frac{\partial y^*}{\partial \tau} + \frac{\partial y^*}{\partial t}\right) = \frac{ne^2 + nn^*(e-e^*)^2 + n^*e^{*2}}{b(n+n^*+1)}. \tag{13}$$

The equation (11) implies that the externality effect of the domestic emission tax channel is positive if and only if the total emissions of the home firms falls due to the emission tax with the BTA. As we have shown in Lemma 1, the sign of the externality effect depends on the gap in the emission coefficients between the countries. If each home firm has a smaller emission coefficient than each foreign firm but its gap is small so that $e > e^*n^*/(n^*+1)$ holds, then the externality effect of the domestic emission tax channel is positive. Then, the optimal emission tax policy suggests that imposing a positive emission tax on home firms reduces emissions of the home country. However, if the gap is large and $e < e^*n^*/(n^*+1)$ holds, then imposing a positive emission tax on home firms raises the emissions of the home country.

The equation (12) suggests that the externality effect in the import emission tax channel is positive if and only if the emission tax policy reduces the total emissions of the foreign firms. As shown in Lemma 2, the externality effect is positive if and only if $e^* > en/(n+1)$. This condition implies that a positive pollution-content tariff necessarily reduces the emissions of the foreign country as long as each home firm has a smaller emission coefficient than each foreign firm.

The equation (13) indicates that the total externality effect of the emission tax policy with the BTA is necessarily positive. In other words, a positive emission tax

imports if a reduction in the emissions of home firms is greater than an increase in those of foreign firms.

reduces the total emissions of the home and foreign countries regardless of the gap in emission coefficients between the countries. We can summarize the results in the following proposition.

Proposition 1 *Suppose that the home government implements the emission tax policy with the BTA. The optimal emission tax of the home country has the following properties.*

1. *If the externality effect of the domestic emission tax channel is positive, i.e.,* $n^*e^*/(n^*+1) < e$*, then a positive emission tax on home firms reduces the emissions of the home country.*

2. *If the externality effect of the import emission tax channel is positive, i.e.,* $e < e^*$*, then a positive pollution-content tariff reduces the emissions of the foreign country.*

3. *The total externality effect is positive regardless of the technology gap in the emission coefficients between the countries. Thus, a positive emission tax with the BTA necessarily reduces the total emissions of the world.*

The pollution-content tariff on imports could be justified if each home firm has cleaner technology than each foreign firm. The domestic emission tax channel suggests that a positive emission tax could be harmful to the environment if each home firm's technology is too "cleaner" than each foreign firm's, i.e., if $e < n^*e^*/(n^*+1)$. Nonetheless, as long as a pollution-content tariff is applied at the same rate as an emission tax and each home firm is exempted from an emission tax for export sales, the emission tax policy with the BTA is always justified as a policy to reduce global emissions of greenhouse gases.

Other distortions influence the optimal emission tax of the home country. In the domestic emission tax channel, the second term on the RHS of (8) includes two different effects, a "market power effect" and a "profit shifting effect." This term has a negative impact on $\widetilde{\tau}_B$ because reducing the emission tax for each home firm improves the total welfare of the home country through two effects, a reduction of the market power distortion and an increase in the profits of home firms at the expense of foreign firms. This profit shifting effect can be regarded as an "ecological dumping effect" because the home country's government lowers the domestic emission tax rate below the level determined by the externality effect in order to provide a competitive advantage for each home firm as compared to each foreign firm.

In the import emission tax channel, the fourth term on the RHS of (8) indicates a profit shifting effect. This term is positive because levying the tariff on imports shifts the profits from foreign to home firms. Even in the absence of home firms, $n = 0$, a profit shifting arises as an increase in the tariff revenue. One of the reason for emerging market countries to oppose the imposition of carbon tariffs by developed countries is that the tariffs can be used as a means of disguised protection for developed countries' firms. The profit shifting effect can justify this concern since it suggests that a level of the pollution-content tariff can be higher than the one based on the externality effect.

In sum, distortions such as the market power effect and the profit shifting effect influence the optimal level of the emission tax. However, the total externality effect is necessarily positive, which implies that a positive emission tax with the BTA reduces global emissions regardless of the technology gap. In this sense, the emission tax with the BTA can be justified from the environmental point of view.

3.2 Emission Tax Policy without the BTA

Next, we turn to an optimal emission tax policy without the BTA. In this policy, the home country's government chooses τ to maximize the total home country welfare under the constraints that $s = 0$ and $t = 0$. With the use of (6), we can derive the effect of the emission tax on the total welfare of the home country as

$$\frac{\partial w_N}{\partial \tau} = (p - c - eh' + n^* y^{*2} p'')\frac{\partial q}{\partial \tau} + n^*[c + eh' - (c^* + e^* h') + 2p' y^*]\frac{\partial y^*}{\partial \tau} + n[p^* - c - eh' + e^* h']\frac{\partial y}{\partial \tau} + (nyp^{*\prime} - h'e^*)\frac{\partial q^*}{\partial \tau}. \quad (14)$$

As before, we will illustrate the results with the setting in which the demand curves and external cost functions are linear. In the policy without the BTA, the home country's optimal emission tax rate is[21]

$$\widetilde{\tau}_N = \Lambda \left\{ en[e(n^*+1) - e^*n^*]\theta/b - en[(n^*+1)x_0 + n^*y_0^*] \right. \\ \left. + en[e(n^*+1) - e^*n^*]\theta/b^* - en(n^*+1-n)y_0 \right\}, \quad (15)$$

where $\Lambda = (n+n^*+1)/e^2n^2[2(n^*+1)/b^* + (2n^*+1)/b] > 0$. Note that y_0 denotes export sales of each home firm to the foreign market in the absence of the emission tax, $\tau = 0$.[22]

In the policy without the BTA, the home country's emission tax affects the total home country welfare through two channels, a "domestic emission tax channel" and an "export emission tax channel." The domestic emission tax channel is similar to the one in the policy with the BTA. It arises due to the emission tax on each home firm's production for its sales to the home market. It consists of the first two terms on the RHS of (15). Recall that the first term is the externality effect and the second term includes both the market power effect and the profit shifting effect. The economic interpretations of these effects are exactly the same as those in the policy with the BTA.

Unlike the emission tax policy with the BTA, the import emission tax channel does not exist since the home country does not impose the pollution-content tariff on imports. Instead, an "export emission tax channel" exists since there is no refund of the emission tax for foreign export sales. This channel includes the third and fourth terms on the RHS of (15). The third term is an externality effect that reflects external costs for home consumers due to emissions caused by production of home firms for their export sales. The emission tax on each home firm's production for its export sales results in a reallocation of outputs between home and foreign firms. A fall in each home firm's export sales reduces its emissions by $-e\frac{\partial y}{\partial \tau} = ee(n^*+1)/b^*(n+n^*+1)$

[21] See Appendix C for the derivation.

[22] In the scheme without the BTA, the emission tax rate must satisfy the following feasibility constraints, $x > 0$, $y^* > 0$, $x^* > 0$, and $y > 0$ in equilibrium. See Appendix D for the details of these constraints.

and a rise in each foreign firm's sales to the foreign market increases its emissions by $e^* \frac{\partial x^*}{\partial \tau} = e^* en/b^*(n + n^* + 1)$. As a result, a total reduction in emissions is

$$-ne\frac{\partial y}{\partial \tau} - n^* e^* \frac{\partial x^*}{\partial \tau} = \frac{en[e(n^*+1) - e^* n^*]}{b^*(n+n^*+1)}. \tag{16}$$

Therefore, the externality effect of the export emission tax channel is positive if a fall in the emissions of home firms is larger than a rise in those of foreign firms. This condition holds if $e(n^* + 1) > e^* n^*$.

Lemma 3 *In the optimal emission tax policy without the BTA, the externality effect of the export emission tax channel is positive if and only if* $e^* n^*/(n^* + 1) < e$.

With (9) and (16), we can derive the total externality effect,

$$-ne\frac{\partial x}{\partial \tau} - n^* e^* \frac{\partial y^*}{\partial \tau} - ne\frac{\partial y}{\partial \tau} - n^* e^* \frac{\partial x^*}{\partial \tau} = \frac{en[e(n^*+1) - e^* n^*]}{n+n^*+1}\left(\frac{1}{b} + \frac{1}{b^*}\right).$$

The total externality effect consists of a change in the emissions of each country. A reduction in emissions of the home country is

$$-ne\left(\frac{\partial x}{\partial \tau} + \frac{\partial y}{\partial \tau}\right) = \frac{e^2 n(n^*+1)}{n+n^*+1}\left(\frac{1}{b} + \frac{1}{b^*}\right).$$

The emission tax without the BTA causes emission leakage, which leads to an increase in emissions of the foreign country,

$$ne\left(\frac{\partial x^*}{\partial \tau} + \frac{\partial y^*}{\partial \tau}\right) = \frac{ene^* n^*}{n+n^*+1}\left(\frac{1}{b} + \frac{1}{b^*}\right).$$

Thus, the total externality effect is positive if and only if the total emissions of the world decline, i.e., $e(n^* + 1) - e^* n^* > 0$.

Proposition 2 *Suppose that the home government implements the emission tax policy without the BTA. The optimal emission tax of the home country has the following properties.*

1. *If the total externality effect is positive, i.e.,* $e^* n^*/(n^*+1) < e$, *then a positive emission tax without the BTA reduces the emissions of the home country but increases the emissions of the foreign country. The worldwide emissions decline.*

2. *If the total externality effect is negative, i.e.,* $e < e^* n^*/(n^*+1)$, *then a positive emission tax without the BTA results in emission leakage that raises the emissions of the world.*

This result implies that a positive emission tax without the BTA would be harmful to the global environment if each home firm has sufficiently cleaner technology than each foreign firm, i.e., $e < e^* n^*/(n^*+1)$. Unlike the policy with the BTA, the emission tax policy without the BTA is not necessarily justified from the environmental point of view.

The fourth term on the RHS of (15) is a profit shifting effect in the export emission tax channel. The sign of this term depends on the number of firms. A fall in the emission tax leads to a rise in export sales of home firms, but at the same time, an increase in export sales has a negative impact on the price in the foreign market. If the effect of an increase in export sales overwhelms the effect of a decline in the price, a lower emission tax would increase the profits of home firms in the foreign market. This possibility arises when the number of home firms is equal to or smaller than the number of foreign firms, i.e., $n \leq n^*$[23]. Therefore, under this condition, the profit shifting effect has a negative impact on $\widetilde{\tau}_N$ and thus it induces the home country's government to lower the emission tax below the level based on the externality effect. If the number of home firms is larger than the number of foreign firms, i.e., $n > n^*$, then the profit shifting effect would be positive and thus it positively affects the optimal emission tax rate.

If the technology gap in emission coefficients is sufficiently large, then the total externality effect implies that a negative emission tax rate, a subsidy to emissions of each home firm, would be environmentally optimal. In addition, if the number of home firms is equal to or smaller than the number of foreign firms, then the profit

[23]Given that the number of firms is discrete, we can state $n \leq n^*$ instead of $n < n^* + 1$.

shifting effect of the export emission tax channel is also negative. Then, it is optimal to subsidize emissions of home firms since the market power effect and the profit shifting effect of the domestic emission tax channel are negative as well.

Proposition 3 *If each home firm has a sufficiently smaller emission coefficient than each foreign firm, $e < e^* n^*/(n^* + 1)$, and the number of home firms is equal to or smaller than that of foreign firms, $n \leq n^*$, then the home country's optimal emission tax rate, $\widetilde{\tau}_N$, in the policy without the BTA is negative, i.e., it is optimal to subsidize emissions of home firms.*

The subsidy to emissions is not harmful to the global environment. It raises emissions by home firms but reduces those of foreign firms. Since the goods produced by home firms are much cleaner than those of foreign firms, the total emissions of the world decline.

4 Welfare Comparison between Emission Tax Policy with and without the BTA

In this section, we will compare the policy with and without the BTA, in terms of the total home country welfare and the environmental quality. We will consider a situation in which the home country's government sets an emission tax rate at the optimal level in each policy given that the foreign country's tax rate is zero. We assume that the number of firms equals one in each country, $n = n^* = 1$, and the slopes of the inverse demand curves are the same, $b = b^* = \overline{b}$, in the countries. These assumptions enable us to neutralize the effect of differences in the number of firms and the size of market. They also allow us to derive the results analytically.

It is convenient to rewrite the home country's optimal emission tax rates under the current assumptions.

$$\begin{aligned}\widetilde{\tau}_B = \Gamma[&e(2e - e^*)\theta - e\overline{b}(2x_0 + y_0^*) \\ &+ e^*(2e^* - e)\theta + e^*\overline{b}(x_0 + 2y_0^*)], \end{aligned} \tag{17}$$

$$\widetilde{\tau}_N = \Lambda e[2(2e - e^*)\theta/\bar{b} - (2x_0 + y_0^* + y_0)], \tag{18}$$

where $\Gamma = 3/[(e + e^*)^2 + 2(e - 2e^*)^2] > 0$ and $\Lambda = 3\bar{b}/7e^2 > 0$. We also assume that $2e > e^* > e$, which implies that the home firm has the smaller emission coefficient than the foreign firm and the technology gap, $e^* - e$, is smaller than e. This assumption guarantees that the total externality effect in the policy without the BTA is positive. The following lemma shows that the optimal emission tax rates are positive if the marginal external cost of emissions for home consumers is sufficiently large.[24]

Lemma 4 *Suppose that* $2e > e^* > e$. *If* $\theta > \bar{b}(2x_0 + y_0^* + y_0)/(2e - e^*)$, *then* $\widetilde{\tau}_B > 0$ *and* $\widetilde{\tau}_N > 0$.

Let the total home country welfare under each optimal emission tax policy be denoted by $\widetilde{w}_i$ $(i = B, N)$, and let the total emissions of home and foreign countries under each optimal policy be denoted by $\widetilde{E}_i$ and $\widetilde{E}_i^*$ $(i = B, N)$, respectively. Then, we can derive the following proposition.[25]

Proposition 4 *Suppose that* $4e/(\sqrt{13}-1) \geq e^* > e$. *If* $\theta > \bar{b}(2x_0+y_0^*+y_0)/(2e-e^*)$, *then (i)* $\widetilde{\tau}_B > \widetilde{\tau}_N > 0$, *(ii)* $\widetilde{w}_B > \widetilde{w}_N$, *and (iii)* $\widetilde{E}_N + \widetilde{E}_N^* > \widetilde{E}_B + \widetilde{E}_B^*$.

If the home firm has cleaner technology than the foreign firm and the marginal external cost of home consumers is sufficiently large, then the emission tax policy with the BTA could result in a higher optimal tax rate in the home country than the policy without the BTA. Furthermore, the home country's optimal policy with the BTA achieves greater total welfare of the home country and lower total emissions as compared with the optimal policy without the BTA.

Welfare implications can be explained as follows. Home consumers incur more of losses in the presence of the BTA because the higher emission tax and the imposition of the pollution content tariff lead to a higher price in the home market. For the home firm, the policy with the BTA is preferred to the policy without the BTA in

[24] We can easily prove this result by using (17) and (18).

[25] See Appendix E for the proof.

terms of its profit obtained in the foreign market since the BTA exempts the home firm from the emission tax on its export sales. As to the home firm's profit from its sales to the home market, it is ambiguous which optimal policy is more preferred for the home firm. In the optimal policy with the BTA, the pollution-content tariff provides gains for the home firm by the profit shifting effect. However, the higher emission tax rate reduces more of profits for the home firm as compared to the policy without the BTA.

In terms of the environment, the optimal emission tax with the BTA generates a better outcome because the marginal impact on total emissions of a rise in the tax rate is greater and the higher optimal tax rate leads to more of reductions in total emissions as compared to the policy without the BTA. Finally, it is ambiguous which emission tax policy leads to the larger tax revenue for the home country government. The revenue from the pollution content tariff would arise only for the policy with the BTA and the emission tax on foreign export sales generates the revenue only for the policy without the BTA. This difference causes an ambiguous result on the comparison of the emission tax revenue between the two schemes.

5 The Impact on the Foreign Welfare

We can examine the impact of the home country's emission tax policy on total foreign country welfare. The total foreign country welfare can be described as a function of the home country's policy instruments,

$$w_i^* = w^*(\tau, s, t),\ (i = B, N).$$

Note that $w_B^* = w^*(\tau, \tau, \tau)$ and $w_N^* = w^*(\tau, 0, 0)$.

First, let us consider the policy with the BTA. Under the assumptions that $n = n^* = 1$ and $b = b^* = \bar{b}$, we can evaluate the impact of the home country's emission tax on the total foreign country welfare at $\tau = 0$ as follows,[26]

$$\left.\frac{\partial w_B^*}{\partial \tau}\right|_{\tau=0} = \frac{2e\bar{b}y_0^*}{3\bar{b}} - \frac{4e^*\bar{b}y_0^*}{3\bar{b}} + \frac{[e^2 + (e - e^*)^2 + e^{*2}]\theta^*}{3\bar{b}} \tag{19}$$

The home country's emission tax affects the foreign country welfare through two channels, a domestic emission tax channel and an import emission tax channel. The first term on the RHS of (19) is a profit shifting effect in the domestic emission tax channel. It is positive since the home country's emission tax increases the foreign firm's profit obtained in the home market. The second term is another profit shifting effect in the import emission tax channel. It has a negative effect since the pollution-content tariff reduces the foreign firm's profit in the home country's market. The third term is a total externality effect, which positively affects the welfare of the foreign country since the emission tax policy with the BTA improves the global environment.

It is easy to see that the absolute value of the positive profit shifting effect in the domestic emission tax channel is smaller than the absolute value of the negative profit shifting effect in the import emission tax channel under the assumption that $e^* > e$. This implies that the home country's optimal emission tax policy may negatively affect the welfare of the foreign country. However, due to the positive externality effect, the home country's optimal emission tax policy with the BTA can raise the total welfare of the foreign country. Let the total foreign country welfare evaluated at $\widetilde{\tau}_i$ be denoted as $\widetilde{w}_i^*$, $(i = B, N)$. We can derive the following proposition.[27]

Proposition 5 *Suppose that* $2e > e^* > e$ *and* $\theta > \bar{b}(2x_0 + y_0^* + y_0)/(2e - e^*)$*, i.e.,* $\widetilde{\tau}_B > 0$*. If* $\theta^* > 2(2e^* - e)\bar{b}y_0^*/[e^2 + (e - e^*)^2 + e^{*2}]$*, then* $\widetilde{w}_B^* > w_0^*$*.*[28]

This result suggests that imposing a pollution-content tariff under the emission tax policy with the BTA is not necessarily a beggar-thy-neighbor policy.

[26] See Appendix F for the derivation.

[27] See Appendix G for the proof.

[28] Note that w_0^* denotes the total welfare of the foreign country in the absence of the emission tax, the pollution-content tariff, and the export refund.

Next, let us turn to the policy without the BTA. Under the assumptions that $n = n^* = 1$ and $b = b^* = \bar{b}$, we can evaluate the effect of the home country's emission tax on the total foreign welfare at $\tau = 0$ as[29]

$$\left.\frac{\partial w_N^*}{\partial \tau}\right|_{\tau=0} = \frac{2e\bar{b}y_0^*}{3\bar{b}} + \frac{e\bar{b}(x_0^* - y_0)}{3\bar{b}} + \frac{2e(2e - e^*)\theta^*}{3\bar{b}} \tag{20}$$

There are two channels through which the home country's emission tax policy affects the total foreign country welfare, the domestic and export emission tax channels. The first term on the RHS of (20) is a profit shifting effect in the domestic emission tax channel and it is exactly the same as the one in the policy with the BTA. The second term is an effect in the export emission tax channel, which is positive if an increase in the profit of the foreign firm outweighs a loss in foreign consumer surplus. The third term is a total externality effect. It has a positive effect on the foreign welfare since a positive emission tax reduces the total emissions of the world under the current assumption that the technology gap between the countries is small, $2e > e^*$.

Unlike the policy with the BTA, all the effects are positive if the profit shifting gain overwhelms the consumer loss in the export emission tax channel. If fact, this holds if the foreign firm has a smaller marginal production cost than the home firm, $c^* < c$. This suggests that the home country's optimal emission tax policy could benefit the foreign country regardless of the size of the marginal external costs of foreign consumers.[30]

Proposition 6 *Suppose that* $2e > e^* > e$ *and* $\theta > \bar{b}(2x_0 + y_0^* + y_0)/(2e - e^*)$, *i.e.,* $\widetilde{\tau}_N > 0$. *Then, if* $c^* < c$, *then* $\widetilde{w}_N^* > w_0^*$.

6 Abatement Technology

In this section, we discuss how the introduction of abatement technology to the present setting modifies our result on the optimal emission tax with the BTA. Let us consider a situation in which each firm can choose a different emission coefficient

[29] See Appendix F for the derivation.
[30] See Appendix G for the proof.

in its production for each market. Each home firm can choose e_x in its production for the home market sales and e_y in its production for its export market sales. The cost for each home firm's abatement is

$$g(e_x, e_y) = \frac{k}{1+\gamma}\{[(e - e_x)x]^{1+\gamma} + [(e - e_y)y]^{1+\gamma}\},$$

where e is each home firm's emission coefficient without any abatement of pollution emissions. We assume that $k > 0$ and $\gamma \geq 1$. Given the absence of the emission tax in the foreign country, each home firm chooses e_x and e_y to minimize the sum of tax burden and abatement costs,

$$\tau e_x x + (\tau - s)e_y y + \frac{k}{1+\gamma}\{[(e - e_x)x]^{1+\gamma} + [(e - e_y)y]^{1+\gamma}\}.$$

The first order conditions for the cost minimization are

$$\tau = k[(e - e_x)x]^{\gamma}, \tag{21}$$

$$\tau - s = k[(e - e_y)y]^{\gamma}. \tag{22}$$

Each emission coefficient is chosen to equalize the emission tax rate with the marginal abatement cost. Similarly, for each foreign firm,

$$0 = k^*[(e^* - e_x^*)x^*]^{\gamma^*}, \tag{23}$$

$$t = k^*[(e^* - e_y^*)y^*]^{\gamma^*}. \tag{24}$$

Since $\tau^* = 0$, each foreign firm does not make any abatement in production for its sales to the foreign market. The pollution-content tariff imposed by the home country government induces each foreign firm to make abatement in production for its sales to the home market.

This abatement cost function has a nice property in that the choice of each emission coefficient is independent of the decision of output.[31] Thus, the profit

[31] This was pointed out by Ritz (2009).

maximization conditions for each firm's sales to the home and foreign markets are exactly the same as (1)-(4). This property allows us to show how the abatement technology introduces additional elements into the structure of the optimal emission tax of the home country.

Let us consider the emission tax policy with the BTA. We can evaluate the impact of a rise in the emission tax rate on the total welfare at $\widetilde{\tau}_B$, i.e., the optimal emission tax rate in the absence of the abatement technology,[32]

$$\left.\frac{\partial w_B^A}{\partial \tau}\right|_{\tau=\widetilde{\tau}_B} = \frac{(\theta-\widetilde{\tau}_B)(e-e_x)nx}{\gamma\widetilde{\tau}_B} + \frac{(\theta-\widetilde{\tau}_B)(e^*-e_y^*)n^*y^*}{\gamma^*\widetilde{\tau}_B} - (e^*-e_y^*)n^*y^*. \quad (25)$$

The first two terms on the RHS capture the effects of reductions in emission coefficients. The first term implies that the optimal emission tax rate must be higher than $\widetilde{\tau}_B$ if $\widetilde{\tau}_B < \theta$, i.e., the tax rate is lower than the marginal external cost of home consumers. The reason is clear, i.e., an abatement activity by each home firm is insufficiently low at $\widetilde{\tau}_B$. Similarly, if $\widetilde{\tau}_B < \theta$, then the second term implies that a higher emission tax rate improves the total welfare of the home country by inducing an abatement activity of each foreign firm.

The last term on the RHS captures the effect of tariff revenue due to the pollution-content tariff. A higher tariff rate leads to a lower emission coefficient of each foreign firm, which would reduce the tariff revenue. The tariff revenue effect implies that the optimal emission tax rate must be lower than $\widetilde{\tau}_B$. We can show that the effect of a reduction in the emission coefficient of each foreign firm outweighs the tariff revenue effect if $\widetilde{\tau}_B < \theta/(1+\gamma^*)$. Then, the optimal emission tax rate is higher in a situation in which the abatement technology is available as compared to a situation without the abatement technology.

If the emission tax rate $\widetilde{\tau}_B$ is greater than the marginal external cost θ, then the effects of reductions in emission coefficients negatively affect the total welfare of the home country. This is because abatement activities of each home firm and each foreign firm are excessively high at $\widetilde{\tau}_B$. Since the tariff revenue effect is negative as

[32] See Appendix H for the derivation.

well, the availability of the abatement technology leads to a lower optimal emission tax rate.

Proposition 7 *If the optimal emission tax rate in the absence of the abatement technology is sufficiently smaller than the marginal external cost of the home country, i.e.,* $\widetilde{\tau}_B < \theta/(1+\gamma^*)$*, then the optimal emission tax rate must be higher in the presence of the abatement technology as compared to the one without the technology. If the optimal emission tax rate in the absence of the abatement technology is larger than the marginal external cost of the home country, i.e.,* $\theta < \widetilde{\tau}_B$*, then the availability of the technology leads to a lower optimal emission tax rate.*

7 Concluding Remarks

We have analyzed the optimal emission tax in the policy with and without the BTA when the home country's government maximizes its national welfare by choosing the emission tax rate. In each policy, the optimal emission tax rate consists of three key effects—the market power effect, the profit shifting effect, and the externality effect—on domestic production, export, and import. Under emission tax with the BTA, the total externality effect is necessarily positive. This implies that a positive emission tax with the BTA has a beneficial effect to the global environment even though the market power and profit shifting effects distort the level of the optimal emission tax rate. Under emission tax without the BTA, the total externality effect can be negative and thus a positive emission tax is not necessarily justified from the view point of the global environment.

This analysis on the structure of the optimal emission tax is useful in deriving the welfare implications of the optimal emission tax policy. If the home firm has cleaner technology than the foreign firm and the technology gap is small, then the total externality effect would be positive in either optimal policy. Under the positive externality effect, the optimal emission tax policy with the BTA achieves the higher emission tax rate, the higher home country welfare, and the lower total emissions as compared to the optimal policy without the BTA. In addition, the optimal emission tax of the home country can improve the total welfare of the foreign country even in the policy with the BTA.

There are possible extensions of the present work. As we pointed out in Introduction, one of the purposes of BTAs is to induce other countries to adopt comparable climate change policy. It is worth analyzing the strategic effects of BTAs with our model. We can consider a situation in which the foreign country's government chooses its emission tax in response to the BTA policy of the home country's government. In our setting with Cournot oligopoly, strategic effects of the BTAs could be different from those of existing studies. Unlike the competitive market, the foreign country's government could prefer a lower emission tax rate to improve its welfare because it promotes exports. If the marginal external cost of the foreign country is sufficiently low, then the negative impact of the BTA on the profit of foreign firms would outweigh the positive effect due to a reduction in total emissions. In such a situation, the foreign country's government may agree to adopt a policy for reducing its emissions in order to avoid the BTA that could hurt the foreign firms.

We have shown that a unilateral increase in the emission tax rate in the home country leads to emission leakage through international trade and the leakage does not lead to an increase in total emissions under the optimal emission tax policy of the home country. It is well known that there is another channel through which emission leakage arises, foreign direct investment, i.e., a firm may move its production abroad to avoid a burden of the emission tax. We can extend the present framework to a setting with a possibility that a firm relocates its production to a country with a lax emission tax policy. In such an extended setting, the BTA would have an effect of preventing emission leakage caused through foreign direct investment. It is worth comparing the welfare and environmental effects of the emission tax policy with the BTA and those of the policy without the BTA in the presence of a possibility of foreign direct investment.

These tasks are beyond the scope of the present paper and left for our future work.

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Appendices

A: Total Welfare

Taking the total derivatives of w_i, we have

$$\begin{aligned}
dw_i &= pdq - ncdx + n(p^* - c)dy + nyp^{*\prime}dq^* + n^*y^{*2}p''dq \\
&\quad + (2n^*p'y^* - n^*c^*)dy^* - dh \\
&= (p - c + n^*y^{*2}p'')dq + n^*(c - c^* + 2p'y^*)dy^* \\
&\quad + n(p^* - c)dy + nyp^{*\prime}dq^* - dh, \qquad (26)
\end{aligned}$$

and

$$dh = h'[edq + \mu e^*dq^* + n^*(\mu e^* - e)dy^* + n(e - \mu e^*)dy]. \qquad (27)$$

Under the assumption that the degree of cross-border pollution is perfect, $\mu = 1$, by using (26) and (27), we find (6).

B: Linear Setting

We will consider a setting in which demand curves and external cost functions are linear. For the home country, the inverse demand function is

$$p(q) = a - bq, \qquad (28)$$

where $a, b > 0$, $a > c$, and $a > c^*$. The home country's external cost function is

$$h(E + E^*) = \theta(E + E^*), \qquad (29)$$

where $\theta > 0$ is a parameter of a marginal external cost of pollution emissions for home consumers. Similarly, for the foreign country, the inverse demand function is

$$p^*(q^*) = a^* - b^*q^*, \qquad (30)$$

where $a^*, b^* > 0$, $a^* > c^*$, and $a^* > c$. The foreign country's external cost function is

$$h^*(E^* + E) = \theta^*(E^* + E), \tag{31}$$

where $\theta^* > 0$ is a parameter of a marginal external cost of pollution emissions for foreign consumers.

We assume that $\tau^* = t^* = s^* = 0$ since the foreign country does not impose taxes on emissions. In the scheme with the BTA, the constraint that $\tau = t = s$ holds. In the scheme without the BTA, the constraint that $s = t = 0$ holds. The Cournot equilibrium conditions for the home market are

$$a - b(n+1)x - bn^*y^* = c + e\tau, \tag{32}$$

$$a - bnx - b(n^*+1)y^* = c^* + e^*t. \tag{33}$$

For the foreign market,

$$a^* - b^*(n^*+1)x^* - b^*ny = c^*, \tag{34}$$

$$a^* - b^*n^*x^* - b^*(n+1)y = c + e(\tau - s). \tag{35}$$

Solving (32) and (33) simultaneously, we have equilibrium outputs for the home market,

$$x = \frac{1}{b(n+n^*+1)}[a - c + n^*(c^* - c) - e(n^*+1)\tau + e^*n^*t], \tag{36}$$

$$y^* = \frac{1}{b(n+n^*+1)}[a - c^* + n(c - c^*) - e^*(n+1)t + en\tau]. \tag{37}$$

Similarly, for the foreign market,

$$x^* = \frac{1}{b^*(n+n^*+1)}[a^* - c^* + n(c - c^*) + en(\tau - s)], \tag{38}$$

$$y = \frac{1}{b^*(n+n^*+1)}[a^* - c + n^*(c^* - c) - e(n^*+1)(\tau - s)]. \tag{39}$$

The total sales in the home and foreign markets are respectively given by

$$q = \frac{1}{b(n+n^*+1)}[(a-c-e\tau)n + (a-c^*-e^*t)n^*], \tag{40}$$

$$q^* = \frac{1}{b^*(n+n^*+1)}\left\{(a^*-c^*)n^* + [a^*-c-e(\tau-s)]n\right\}. \tag{41}$$

The prices in the home and foreign markets are

$$p = \frac{1}{(n+n^*+1)}[a+(c+e\tau)n+(c^*+e^*t)n^*], \tag{42}$$

$$p^* = \frac{1}{(n+n^*+1)}\left\{a^*+c^*n^*+[c+e(\tau-s)]n\right\}. \tag{43}$$

The profits of each home firm and each foreign firm are respectively

$$\pi = bx^2 + b^*y^2 - f, \tag{44}$$

$$\pi^* = b^*x^{*2} + by^{*2} - f^*. \tag{45}$$

The total home country welfare and the total foreign country welfare are

$$w = \frac{bq^2}{2} + n\pi + nex\tau + e^*n^*y^*t + eny(\tau-s) - \theta[en(x+y) + e^*n^*(x^*+y^*)], \tag{46}$$

$$w^* = \frac{b^*q^{*2}}{2} + n^*\pi^* - \theta^*[e^*n^*(x^*+y^*) + en(x+y)]. \tag{47}$$

C: Optimal Emission Tax

It is convenient to derive the coefficient of each term in (6) by using the linear setting. First, we can derive the coefficient of dq with the use of (28), (29), and (42),

$$p - c - eh' + n^*y^{*2}p'' = bx_0 - e\theta + \frac{en\tau + e^*n^*t}{n+n^*+1}. \tag{48}$$

Recall that the subscript 0 indicates that $\tau = s = t = 0$. Second, we can obtain the coefficient of $dy^*$by using (28), (29), and (37),

$$n^*[c + eh' - (c^* + e^*h') + 2p'y^*] = n^*\{-b(x_0 + y_0^*) + (e - e^*)\theta - \frac{2[en\tau - e^*(n+1)t]}{n + n^* + 1}\}. \tag{49}$$

Third, we can derive the coefficient of dy with the use of (29) and (43),

$$n(p^* - c - eh' + e^*h') = n[b^*y_0 - (e - e^*)\theta + \frac{en(\tau - s)}{n + n^* + 1}]. \tag{50}$$

Fourth, we can obtain the coefficient of dq^* by using (29), (30), and (39),

$$nyp^{*\prime} - e^*h' = -n[b^*y_0 - \frac{e(n^* + 1)(\tau - s)}{n + n^* + 1}] - e^*\theta. \tag{51}$$

Under the constraint of the BTA, $t = s = \tau$, we can derive $\partial q/\partial\tau = -(en + e^*n^*)/b(n + n^* + 1)$ and $\partial y^*/\partial\tau = [en - e^*(n+1)]/b(n + n^* + 1)$. Using these partial derivatives with (48) and (49), we can rewrite (7) as

$$\begin{aligned}
\frac{\partial w_B}{\partial\tau} &= -[bx_0 - e\theta + \frac{(en + e^*n^*)\tau}{n + n^* + 1}] \times \frac{en + e^*n^*}{b(n + n^* + 1)} \\
&+ n^*\{-b(x_0 + y_0^*) + (e - e^*)\theta - \frac{2[en - e^*(n+1)]\tau}{n + n^* + 1}\} \times \frac{en - e^*(n+1)}{b(n + n^* + 1)} \\
&= \frac{en}{b(n + n^* + 1)}\{[e(n^* + 1) - e^*n^*]\theta - b[(n^* + 1)x_0 + n^*y_0^*]\} \\
&+ \frac{e^*n^*}{b(n + n^* + 1)}\{[e^*(n + 1) - en]\theta + b[nx_0 + (n + 1)y_0^*]\} \\
&- \frac{\{(en + e^*n^*)^2 + 2n^*[en - e^*(n+1)]^2\}\tau}{b(n + n^* + 1)^2}.
\end{aligned} \tag{52}$$

Solving $\partial w_B/\partial\tau = 0$ for τ, we can obtain $\widetilde{\tau}_B$. We can easily see that the second order condition $\partial^2 w_B/\partial\tau^2 < 0$ is satisfied.

Similarly, we can derive the optimal emission tax rate under the scheme without the BTA. Under the constraints of this scheme, $t = s = 0$, we can derive $\partial q/\partial\tau =$

$-en/b(n+n^*+1)$, $\partial y^*/\partial\tau = en/b(n+n^*+1)$, $\partial y/\partial\tau = -e(n^*+1)/b^*(n+n^*+1)$, and $\partial q^*/\partial\tau = -en/b^*(n+n^*+1)$. Using these partial derivatives with (48), (49), (50), and (51), we can rewrite (14) as

$$\begin{aligned}\frac{\partial w_N}{\partial\tau} &= -[bx_0 - e\theta + \frac{en\tau}{n+n^*+1}] \times \frac{en}{b(n+n^*+1)} \\ &+ n^*\{-b(x_0+y_0^*) + (e-e^*)\theta - \frac{2en\tau}{n+n^*+1}\} \times \frac{en}{b(n+n^*+1)} \\ &- n[b^*y_0 - (e-e^*)\theta + \frac{en\tau}{n+n^*+1}] \times \frac{e(n^*+1)}{b^*(n+n^*+1)} \\ &- \{-n[b^*y_0 - \frac{e(n^*+1)\tau}{n+n^*+1}] - e^*\theta\} \times \frac{en}{b^*(n+n^*+1)} \\ &= \frac{en}{b(n+n^*+1)}\{[e(n^*+1) - e^*n^*]\theta - b[(n^*+1)x_0 + n^*y_0^*]\} \\ &+ \frac{en}{b^*(n+n^*+1)}\{[e(n^*+1) - e^*n^*]\theta - b^*(n^*+1-n)y_0\} \\ &- \frac{e^2n^2}{(n+n^*+1)^2}[\frac{2n^*+1}{b} + \frac{2(n^*+1)}{b^*}]\tau. \end{aligned} \tag{53}$$

Solving $\partial w_N/\partial\tau = 0$ for τ, we can derive $\tilde{\tau}_N$. Clearly, the second order condition holds, $\partial^2 w_N/\partial\tau^2 < 0$.

D: Feasibility Constraints for Emission Tax Rates

There are feasibility constraints for the emission tax rate. In the scheme with the BTA, τ must satisfy $x > 0$, $y^* > 0$, and $p > 0$ in equilibrium. In the linear setting, by using (36), (37), and (42), we can derive these constraints as follows. If $e^* > e$ and $e(n^*+1) - e^*n^* > 0$, then

$$-\frac{(n+n^*+1)p_0}{en+e^*n^*} < \tau < \max\left\{\frac{b(n+n^*+1)x_0}{e(n^*+1)-e^*n^*}, \frac{b(n+n^*+1)y_0^*}{e^*(n+1)-en}\right\}.$$

If $e^* > e$ and $e(n^*+1) - e^*n^* < 0$, then

$$\frac{b(n+n^*+1)x_0}{e(n^*+1)-e^*n^*} < \tau < \frac{b(n+n^*+1)y_0^*}{e^*(n+1)-en}.$$

In the scheme without the BTA, τ must satisfy the constraints that $x > 0$, $y^* > 0$, $x^* > 0$, and $y > 0$ in equilibrium. With the use of (36), (37), (38), and (39), these constraints can be rewritten as,

$$\max\left\{-\frac{b(n+n^*+1)y_0^*}{en}, -\frac{b^*(n+n^*+1)x_0^*}{en}\right\} < \tau < \min\left\{\frac{b(n+n^*+1)x_0}{e(n^*+1)}, \frac{b^*(n+n^*+1)y_0}{e(n^*+1)}\right\}.$$

E: Proof of Proposition 4

Proof of (i): Since $2 > 4/(\sqrt{13}-1)$, Lemma 4 implies that $\widetilde{\tau}_B > 0$ and $\widetilde{\tau}_N > 0$. Taking a difference by using (17) and (18), we have

$$\begin{aligned}\frac{\widetilde{\tau}_B}{\Gamma} - \frac{\widetilde{\tau}_N \bar{b}}{\Lambda} &= [e^*(2e^* - e) - e(2e - e^*)]\theta + \bar{b}e^*(x_0 + 2y_0^*) + \bar{b}ey_0 \\ &= 2(e^* - e)(e^* + e)\theta + \bar{b}e^*(x_0 + 2y_0^*) + \bar{b}ey_0 > 0,\end{aligned} \tag{54}$$

because $e^* > e$. Since $\Gamma > 0$ and $\widetilde{\tau}_N > 0$, we have

$$\frac{\widetilde{\tau}_B}{\widetilde{\tau}_N} > \frac{\bar{b}\Gamma}{\Lambda}.$$

By using $\Gamma = 3/[(e+e^*)^2 + 2(e-2e^*)^2] > 0$ and $\Lambda = 3\bar{b}/7e^2 > 0$, we have

$$\frac{\bar{b}\Gamma}{\Lambda} = \frac{7e^2}{3(e^2 - 2e^*e + 3e^{*2})} \geq 1,$$

if $e \geq \left(\sqrt{13}-1\right)e^*/4$. Using this result with (54), we can obtain the desired result.

Proof of (ii): Under the assumptions that $n = n^* = 1$ and $b = b^* = \bar{b}$, we can evaluate (52) at $\tau = 0$,

$$\frac{\partial w_B}{\partial \tau} = \frac{e}{3\bar{b}}[(2e - e^*)\theta - \bar{b}(2x_0 + y_0^*)] + \frac{e^*}{3\bar{b}}[(2e^* - e)\theta + \bar{b}(x_0 + 2y_0^*)] > 0.$$

We can also evaluate (53) at $\tau = 0$,

$$\frac{\partial w_N}{\partial \tau} = \frac{e}{3\bar{b}}[(2e - e^*)\theta - \bar{b}(2x_0 + y_0^*)] + \frac{e}{3\bar{b}}[(2e - e^*)\theta - \bar{b}y_0] > 0.$$

Using these partial derivatives, we have

$$\begin{aligned}\frac{\partial w_B}{\partial \tau} - \frac{\partial w_N}{\partial \tau} &= \frac{1}{3\bar{b}}\{[e^*(2e^* - e) - e(2e - e^*)]\theta + \bar{b}e^*(x_0 + 2y_0^*) + \bar{b}ey_0\} \\ &= \frac{1}{3\bar{b}}[2(e^* - e)(e^* + e)\theta + \bar{b}e^*(x_0 + 2y_0^*) + \bar{b}ey_0] > 0,\end{aligned}$$

because $e^* > e$. Thus, we have $\partial w_B/\partial \tau > \partial w_N/\partial \tau$ at $\tau = 0$. Notice that $w_B = w_N$ in the absence of emission taxes and refunds, $\tau = t = s = 0$. Combining these results with $\partial^2 w_B/\partial \tau^2 < 0$, $\partial^2 w_N/\partial \tau^2 < 0$, and $\widetilde{\tau}_B > \widetilde{\tau}_N > 0$, we can obtain the desired result.

Proof of (iii): Under the assumptions that $n = n^* = 1$ and $b = b^* = \bar{b}$, by using (36), (37), (38), and (39), we can derive

$$\begin{aligned}\frac{\partial E_B}{\partial \tau} + \frac{\partial E_B^*}{\partial \tau} &= -\frac{1}{3\bar{b}}[e(2e - e^*) + e^*(2e^* - e)] < 0, \\ \frac{\partial E_N}{\partial \tau} + \frac{\partial E_N^*}{\partial \tau} &= -\frac{2}{3\bar{b}}[e(2e - e^*)] < 0.\end{aligned}$$

Taking a difference of the absolute values, we have

$$\left|\frac{\partial E_B}{\partial \tau} + \frac{\partial E_B^*}{\partial \tau}\right| - \left|\frac{\partial E_N}{\partial \tau} + \frac{\partial E_N^*}{\partial \tau}\right| = \frac{2(e^* - e)(e^* + e)}{3\bar{b}} > 0.$$

Combining this condition with $E_B + E_B^* = E_N + E_N^*$ at $\tau = t = s = 0$ and $\widetilde{\tau}_B > \widetilde{\tau}_N > 0$, we can obtain the desired result.

F: The Impact on the Foreign Welfare

The total welfare of the foreign country is

$$w_i^* = u^*(q^*) - p^*q^* + n^*[p^*x^* + (p - e^*t)y^* - c^*(x^* + y^*) - f^*] - h^*(E^* + E).$$

By using (4), we can rewrite w_i^* as

$$w_i^* = u^*(q^*) - c^*n^*x^* + n^*(p - e^*t - c^*)y^* + ny^2p^{*\prime} - [c + (\tau - s)e]ny - n^*f^* - h^*(E^* + E^*).$$

Then, we find

$$
\begin{aligned}
dw_i^* = & (p^* - c^* - e^*h^{*\prime} + ny^2 p^{*\prime\prime})dq^* \\
& + n\{c^* + e^*h^{*\prime} - [c + e(\tau - s) + eh^{*\prime}] + 2p^{*\prime}y\}dy \\
& + n^*(p - c^* - e^*t - e^*h^{*\prime} + eh^{*\prime})dy^* + (n^*y^*p' - eh^{*\prime})dq \\
& - e^*n^*y^*dt - eny(d\tau - ds).
\end{aligned} \tag{55}
$$

By using the linear setting, we can derive each coefficient of (55) as follows.

$$
p^* - c^* - e^*h^{*\prime} + ny^2p^{*\prime\prime} = b^*x_0^* - e^*\theta^* + \frac{en(\tau - s)}{n + n^* + 1}, \tag{56}
$$

$$
\begin{aligned}
n\{c^* + e^*h^{*\prime} - [c + e(\tau - s) + eh^{*\prime}] + 2p^{*\prime}y\} = & \; n[-b^*(x_0^* + y_0) + (e^* - e)\theta^* \\
& + \frac{(n^* + 1 - n)e(\tau - s)}{n + n^* + 1}],
\end{aligned} \tag{57}
$$

$$
n^*(p - c^* - e^*t - e^*h^{*\prime} + eh^{*\prime}) = n^*[by_0^* - (e^* - e)\theta^* - \frac{e^*(n+1)t - en\tau}{n + n^* + 1}], \tag{58}
$$

$$
n^*y^*p' - eh^{*\prime} = -n^*[by_0^* - \frac{e^*(n+1)t - en\tau}{n + n^* + 1}] - e\theta^*, \tag{59}
$$

$$
e^*n^*y^* = \frac{e^*n^*[b(n + n^* + 1)y_0^* - e^*(n+1)t + en\tau]}{b(n + n^* + 1)}, \tag{60}
$$

$$
eny = \frac{en[b^*(n + n^* + 1)y_0 - e(n^* + 1)(\tau - s)]}{b^*(n + n^* + 1)}. \tag{61}
$$

Under the constraint of the BTA, by using (37) and (40), we can obtain the effect of home country's emission tax on total sales and export sales as $\partial q/\partial \tau = -(en + e^*n^*)/b(n + n^* + 1)$ and $\partial y^*/\partial \tau = -[e^*(n+1) - en]/b(n + n^* + 1)$. Combining these partial derivatives with (58), (59), and (60), we can derive the effect of the home country's emission tax on the total foreign country welfare as

$$\frac{\partial w_B^*}{\partial \tau} = n^*(p - c^* - e^*t - e^*h^{*\prime} + eh^{*\prime})\frac{\partial y^*}{\partial \tau} + (n^*y^*p' - eh^{*\prime})\frac{\partial q}{\partial \tau} - e^*n^*y^*$$
$$= \frac{en}{b(n+n^*+1)}\{2bn^*y_0^* + [e(n^*+1) - e^*n^*]\theta^*\}$$
$$+ \frac{e^*n^*}{b(n+n^*+1)}\{-2b(n+1)y_0^* + [e^*(n+1) - en]\theta^*\}$$
$$- \frac{2n^*[e^*(n+1) - en]^2}{b(n+n^*+1)^2}\tau.$$

Under the assumptions that $n = n^* = 1$ and $b = b^* = \bar{b}$, we can obtain (19).

In the scheme without the BTA, with the use of (37), (39), (40), and (41), we can derive $\partial q/\partial \tau = -en/b(n+n^*+1)$, $\partial y^*/\partial \tau = en/b(n+n^*+1)$, $\partial y/\partial \tau = -e(n^*+1)/b^*(n+n^*+1)$, and $\partial q^*/\partial \tau = -en/b^*(n+n^*+1)$. By using these partial derivatives with (56), (57), (58), (59), and (61), we can obtain the effect of the home country's emission tax on the total foreign welfare as

$$\frac{\partial w_N^*}{\partial \tau} = (p^* - c^* - e^*h^{*\prime} + ny^2p^{*\prime\prime})\frac{\partial q^*}{\partial \tau}$$
$$+ n\{c^* + e^*h^{*\prime} - [c + e(\tau - s) + eh^{*\prime}] + 2p^{*\prime}y\}\frac{\partial y}{\partial \tau}$$
$$+ n^*(p - c^* - e^*t - e^*h^{*\prime} + eh^{*\prime})\frac{\partial y^*}{\partial \tau} + (n^*y^*p' - eh^{*\prime})\frac{\partial q}{\partial \tau} - eny$$
$$= \frac{en}{b^*(n+n^*+1)}\{b^*(n^*x_0^* - ny_0) + [e(n^*+1) - e^*n^*]\theta^*\}$$
$$+ \frac{en}{b(n+n^*+1)}\{2bn^*y_0^* + [e(n^*+1) - e^*n^*]\theta^*\}$$
$$+ \frac{e^2n^2[(2n^*+1)/b^* + 2n^*/b]}{(n+n^*+1)^2}\tau.$$

We can derive (20) under the assumptions that $n = n^* = 1$ and $b = b^* = \bar{b}$.

G: Proofs of Proposition 5 and 6

First, we will prove Proposition 5. We can rewrite (19) as

$$\left.\frac{\partial w_B^*}{\partial \tau}\right|_{\tau=0} = \frac{1}{3\bar{b}}\{2(e-2e^*)\bar{b}y_0^* + [e(2e-e^*)+e^*(2e^*-e)]\theta^*\},$$
$$= \frac{1}{3\bar{b}}\{2(e-2e^*)\bar{b}y_0^* + [e^2+(e-e^*)^2+e^{*2}]\theta^*\} > 0$$
if and only if
$$\theta^* > 2(2e^*-e)\bar{b}y_0^*/[e^2+(e-e^*)^2+e^{*2}].$$

Lemma 4 guarantees that $\widetilde{\tau}_B > 0$. Thus, the desired result is obtained.

Second, we will prove Proposition 6. With (38) and (39), we can easily show that $y_0 < x_0^*$ if and only if $c^* < c$. Clearly, $\partial w_N^*/\partial\tau|_{\tau=0} > 0$ if $y_0 < x_0^*$. Lemma 4 guarantees that $\widetilde{\tau}_N > 0$. Thus, $\partial w_N^*/\partial\tau|_{\tau=0} > 0$ implies that $\widetilde{w}_N^* > w_0^*$.

H: Optimal Emission Tax in the Presence of Abatement Technology

In the presence of the abatement technology, the total welfare of the home country and the profit of each home country's firm can be written respectively as,

$$w_i^A = u(q) - pq + n\pi^A + e_x nx\tau + e_y n^* y^* t + e_y ny(\tau - s) - h(E+E^*),$$
$$\pi^A = px + p^*y - c(x+y^*) - e_x x\tau - e_y y(\tau - s) - f - g(e_x, e_y),$$

where $i = B, N$. We can rewrite the total welfare as

$$w_i^A = u(q) - cnx + n(p^*-c)y + n^*\{p'y^{*2} - c^*y^* - (e^*-e_y^*)ty^*\}$$
$$- ng(e_x, e_y) - nf - h(E+E^*).$$

Taking a total derivative of the total welfare, we have

$$\begin{aligned}
dw_i^A &= dw_i - (-g_{e_x} - h')(e - e_x)dq \\
&+ n^*[(-g_{e_x} - h')(e - e_x) - (-g^*_{e_y} - h')(e^* - e^*_y)]dy^* \\
&- n[(-g_{e_y} - h')(e - e_y) + (e^* - e^*_x)h']dy \\
&+ (e^* - e^*_y)h'dq^* - n^*(e^* - e^*_y)y^*dt \\
&+ (-g_{e_x} - h')nxde_x + (-g_{e_y} - h')nyde_y \\
&+ (-g^*_{e_y} - h')n^*y^*de^*_y - h'n^*x^*de^*_x,
\end{aligned}$$

where dw_i is given by (6) and $g_{e_j} = \partial g(e_x, e_y)/\partial e_j$, $(j = x, y)$. Let us consider the policy with the BTA, i.e., $\tau = s = t$. Then, we can derive

$$\begin{aligned}
\frac{\partial w_B^A}{\partial \tau} &= \frac{\partial w_B}{\partial \tau} - (-g_{e_x} - h')(e - e_x)\frac{\partial q}{\partial \tau} \\
&+ n^*[(-g_{e_x} - h')(e - e_x) - (-g^*_{e_y} - h')(e^* - e^*_y)]\frac{\partial y^*}{\partial \tau} \\
&- n^*(e^* - e^*_y)y^* + (-g_{e_x} - h')nx\frac{\partial e_x}{\partial \tau} + (-g^*_{e_y} - h')n^*y^*\frac{\partial e^*_y}{\partial \tau}.
\end{aligned}$$

Taking partial derivatives of (21) and (24), we have

$$\begin{aligned}
\frac{\partial e_x}{\partial \tau} &= \frac{e - e_x}{x}\frac{\partial x}{\partial \tau} - \frac{e - e_x}{\gamma \tau}, \\
\frac{\partial e^*_y}{\partial \tau} &= \frac{e^* - e^*_y}{y^*}\frac{\partial y^*}{\partial \tau} - \frac{e^* - e^*_y}{\gamma^* \tau}.
\end{aligned}$$

In the setting with the linear demand curve and the linear external cost function, by using these partial derivatives with (37) and (40), we can derive

$$\begin{aligned}
\frac{\partial w_B^A}{\partial \tau} &= \frac{\partial w_B}{\partial \tau} - \frac{(\tau - \theta)(e - e_x)nx}{\gamma \tau} - \frac{(\tau - \theta)(e^* - e^*_y)n^*y^*}{\gamma^* \tau} \\
&- (e^* - e^*_y)n^*y^*.
\end{aligned}$$

The second order condition is satisfied

$$\begin{aligned}\frac{\partial^2 w_B^A}{\partial \tau^2} = \frac{\partial^2 w_B}{\partial \tau^2} &- \frac{(e-e_x)[\tau+(\gamma-1)\theta]nx}{\gamma\tau^2} \\ &- \frac{(e^*-e_y^*)[2\tau+(\gamma^*-1)\theta]n^*y^*}{\gamma^*\tau^2} < 0,\end{aligned}$$

since $\gamma \geq 1$ and $\gamma^* \geq 1$. By evaluating $\partial w_B^A/\partial\tau$ at $\tau = \widetilde{\tau}_B$, we have (25).