UNIVERSITY of HAWAI'I®
MĀNOA

University of Hawai`i at Mānoa Department of Economics Working Paper Series

Saunders Hall 542, 2424 Maile Way,
Honolulu, HI 96822
Phone: (808) 956 -8496
www.economics.hawaii.edu

Working Paper No. 13-1

Multi-Unit Auctions

By

Anthony M. Kwasnica and Katerina Sherstyuk

January 2013

Multi-Unit Auctions*

Anthony M. Kwasnica and Katerina Sherstyuk†

January 2013

Abstract

We survey experimental research on multi-unit auctions with an emphasis on topics that may be of a unifying interest to experimental, as well as theoretical and empirical economists. Topics include static and dynamic multi-unit auctions; combinatorial auctions and efficient auction design; simultaneous and sequential auctions; bidder asymmetry and endogenous entry, and collusion in auctions. We also discuss behavioral regularities observed in multi-unit auction experiments.

Key words: multi-unit auctions; experiments

1 Introduction

In a multi-unit auction multiple identical (homogeneous) or distinct (heterogeneous) objects are to be allocated to bidders. Multi-unit auctions are widely used in both the public and private sector for agricultural products, construction procurement, electromagnetic spectrum, environmental projects, electricity distribution and generation, gas leases, government securities, real estate, and timber among others. The growing practical relevance of multi-unit auctions has motivated substantial theoretical, empirical and experimental research on the topic. The purpose of this survey is to provide an overview of experimental research on multi-unit auctions with an emphasis on the topics or themes that may be of a unifying interests to experimental, as well as theoretical and empirical economists. A number of common research themes arise across methodologies and include: static multi-unit auctions (including pay-your-bid and uniform-price auctions), dynamic auctions, combinatorial auctions and efficient auction design, bidder asymmetry, common values, endogenous entry, and

*We would like to thank Charles Noussair, Steven Tucker, John Kagel and Charles Plott for valuable comments and suggestions. Sherstyuk is grateful for the support by the Ministry of Education and Science of Russia, project 14.B37.21.0263.

†Kwasnica: Smeal College of Business, The Pennsylvania State University, 332 Business, University Park, PA 16802, email: kwasnica@psu.edu. Sherstyuk: Department of Economics, University of Hawaii at Manoa, 2424 Maile Way, Honolulu, HI 96822, email: katyas@hawaii.edu

collusion in auctions (see Ausubel (2008), Perrigne and Vuong (2008), and Bajari (2008)). We explore many of these areas in this survey.

There are a number of earlier surveys of experimental auctions. Kagel (1995) surveys early (pre-1995) research and Kagel and Levin (2011) examine experimental auction research since 1995; Roth (2012) discusses the history and the influence of auction experiments on the design of the Federal Communications Commission (FCC) electromagnetic spectrum auctions. Sherstyuk (2008) reviews demand reduction and bidder collusion is ascending multi-unit auctions. Normann and Ricciuti (2009) review experiments on economic policy making, including auctions and emission permits. Bajari and Hortacsu (2004) and Ockenfels, Reiley and Sadrieh (2006) survey internet auctions. Given the existing surveys, we do not review environmental and online auctions, as well as electricity, water and gas markets specifically. For the topics that are reviewed but have overlap with the existing surveys, such as comparison of auction formats, auction design, demand reduction and collusion, we try to minimize the overlap and provide a different perspective on the research.

We begin by reviewing a few basic multi-unit auction institutions. As with single-unit auctions, auction formats differ in whether the auction is static (or sealed bid), or dynamic. In static sealed bid auctions, the bids are submitted only once, and the prices and allocations are then immediately determined. Dynamic auctions involve multiple rounds of price and demand adjustments, or continuous price changes, with a specified stopping rule. Dynamic auctions may be either ascending (increasing price) or descending (decreasing price). The auction formats further differ in how the prices for the units are determined once the bidding stops. The most commonly discussed multi-unit auction formats are generalizations of English, Dutch, first-price sealed bid (FPSB), and Vickrey second-price sealed bid (SPSB) auctions. Suppose k units of a homogeneous good are offered for sale. Under the ascending uniform-price English clock (EC) auction, the price starts low, and then rises, either continuously or in certain increments, with bidders having an option to irreversibly reduce their demand (drop out on units) at each price. The clock stops with the last drop out when the total number of units demanded reduces to k, the number supplied. The price where the clock stops determines the uniform selling price for all k units. Smith (1967) refers to this pricing rule as "competitive" as such a price would presumably prevail in a competitive market. Examples of real-world clock auctions are auctions for diamonds, electricity, gas and emission allowances in Europe (Cramton, Filiz-Ozbay, Ozbay and Sujarittanonta (2012)). A sealed bid analog of this institution is the uniform-price sealed bid auction where k highest bids are accepted, and the price equals to the highest rejected bid; this corresponds to the Vickrey SPSB auction in the single-unit case. There is also the uniform-price sealed bid auction where the price equals to the lowest accepted bid. Such a pricing rule is presently

used in the U.S. Treasury auctions. In a k-unit discriminative sealed bid auction (the multi-unit analog of the FPSB auction), the k highest bidders buy the goods at the prices equal to their submitted bids; this auction procedure is used by the European Central Bank and many other countries to sell government securities, and was used by the U.S. Treasury pre-1992 (Abbink, Brandts and Pezanis-Christou 2006). The dynamic analog of the discriminative auction is the simultaneous Dutch descending price auction, where the prices start high on each of the units offered for sale, and then decrease on all units at the same pace; the bidder who is the first to stop the clock on a given unit buys the unit at the corresponding price. Further, when goods are heterogeneous, they may be sold in separate auctions run either simultaneously or sequentially. Many more auction institutions emerge when there are synergies or other interdependencies in buyer values across the goods. We will introduce these auction formats as we discuss them later in the paper.

In most studies, the auction institutions are evaluated and compared against several performance criteria, the most prominent of which are allocative efficiency and seller revenue. Efficiency is typically measured as the percentage of the sum of the values of the winning units relative to the maximal feasible sum of the values of the units. Other performance criteria include bidder profits and auction duration.

The rest of the survey is organized as follows. Section 2 reviews auctions where each bidder demands only one unit. Section 3 discusses auctions with multi-unit demand with additive valuations. Section 4 takes up the issues of auctions with synergies and package bidding. Specific institutional issues relevant to many multi-unit auction environments are discussed in Section 5. Section 6 reviews experiments examining collusion. Section 7 discusses sequential as opposed to simultaneous auctions. Finally, we present conclusions and some open issues in Section 8.

2 Multi-unit auctions, single-unit demand

We first study settings where there are several objects offered for sale, but each bidder is constrained to buy at most one unit. As Krishna (2002) discusses, the fact that bidders demand only a single-unit greatly reduces the difficulties of the auction design process from a strategic standpoint and, therefore, many results from standard (single-unit) auction theory can be extended to this context. One of the earliest studies in this setting is Cox, Smith and Walker (1984) who are the first to report that bidding tends to be greater than the risk neutral Nash equilibrium level in multi-unit discriminative auctions, thus extending the similar finding from single-unit, first-price sealed auctions. Cox et al. (1984) also point out that the extent of overbidding relative to the risk neutral Nash equilibrium prediction

depends on the number of bidders relative to the number of units, with a higher ratio of bidders to units leading to more overbidding. Experimental evidence from many studies that followed suggests that this ratio is indicative of auction competitiveness and affects how aggressively bidders behave across a wide range of auction environments, as we discuss below.

2.1 Comparison of auction formats motivated by spectrum sales

Government electromagnetic spectrum sales gave a boost to experimental research on multi-unit auctions. Several studies are motivated by the 3G spectrum auctions in countries such as Great Britain and the Netherlands, where each bidder was constrained to buy at most one license. Much of the motivation comes from Klemperer (2002) who argues that encouraging entry and competition are the key elements in good auction design. Klemperer further suggests that adding the element of a first-price sealed bid auction to an ascending auction may increase competition in 'uncompetitive' environments by giving new entrants a greater chance of winning, and further help bidders to avoid the winner's curse in settings with a common value component. In view of these arguments, several experimental papers compare the performance of simultaneous ascending auctions with first-price sealed bid, descending auction formats, or hybrids.

Abbink, Irlenbusch, Pezanis-Christou, Rockenbach, Sadrieh and Selten (2005) report on experiments that were conducted in preparation for the 2000 British 3G auctions. In the experiments, four homogeneous units were auctioned off to four incumbent (strong) bidders, and four new entrant (weak) bidders, with the incumbents more likely to have their value drawn from a distribution with a higher support. The underlying bidder values included both private and common values components. Three auction mechanisms were compared: discriminatory and uniform-price variants of the Anglo-Dutch auction, and a traditional English auction. The Anglo-Dutch format proceeded as the English auction until only five (of the original eight) bidders remain; the last stage then proceeded as a sealed bid, with the four highest bidders winning the units. In the uniform-price auction, all winning bidders paid the fourth highest bid, whereas in the discriminatory auction, the bidders paid their own bids. In the English auction, the price paid by the four highest bidders equaled the fifth highest bid. The authors report very little difference among the three auction formats with respect to efficiency (which was around 70% under all auction formats), revenue, avoidance of the winner's curse, and chances of winning by new entrants. The new entrants (weak bidders) bid somewhat more aggressively in the English auction than in either of the hybrids, giving them a slightly better chance of winning under this format; however, some entry was

inefficient.[1] This lack of differences between formats may be attributed to the relatively high competitiveness of the environment, with eight bidders competing for four licenses.[2]

Goeree, Offerman and Schram (2006) study an 'uncompetitive' setting motivated by the Dutch spectrum auctions, with three heterogeneous units offered for sale to only four bidders. In a simultaneous ascending auction (SAA), all units are offered for sale simultaneously. At each price, bidders indicate which unit they want to buy, and the price rises on units which are demanded by more than one bidder; as the price rises, bidders can switch their bid to different objects. The SAA is compared with three first-price auction formats: (1) simultaneous first-price sealed bid (FPSB) auction, where all bidders simultaneously submit bids for every unit; (2) sequential FPSB auction, where units are sold one at a time; and (3) simultaneous descending auction, which is a Dutch clock where the clock starts at the same price and drops at equal speed for all units, and a bidder who is the first to stop the clock for a given unit buys this unit at the price where they stopped the clock. Each auction format is studied across four environments of different complexity: symmetric or asymmetric bidders, and private values or values with an added uncertain common value component. Under asymmetry, there are three strong bidders (incumbents) and one weak bidder (entrant); the weak bidder's values are drawn from a distribution with a lower support. There is also a treatment with an endogenous entry decisions of the weak bidder. The SAA is found to be the most efficient (resulting in up to 98% efficiency), but yielded the revenue which was around 10% lower, and also more variable, than that in the first-price auctions. The revenue was the highest in the sequential FPSB auctions where the most valuable item was auctioned first ("the best foot forward" strategy). Consistent with Klemperer's (2002) claims, the simultaneous FPSB auction offered weak bidders a higher chance to win, and resulted in higher entry; it also showed the lowest incidence of the winner's curse.[3] The authors conclude that in low-competition environments, the choice between auction formats

[1] In the epilogue, the authors report that the elements of the first-price auction were not implemented in the British 3G auctions, which proceeded under the simultaneous ascending auction format; a fifth license was added and reserved exclusively for new entrants.

[2] Sutter, Kocher and Strauss (2009) adopt the values design of Abbink et al. (2005) to consider whether teams may bid differently than individuals in an ascending multi-round, uniform-price auction. They report that teams behaved more competitively than individuals, and, consequently, made smaller profits and suffered more often from the winner's curse. The authors explain this by the prevalence of the 'winning' motive in teams. Interestingly, such competitive behavior also resulted in higher auction efficiency with teams. However, the authors do not report on the demographics of the subject pool; it is possible that the observed higher competitiveness of teams is specific for college-aged participants, but may be mitigated in teams of more seasoned experts bidding in real-world auctions.

[3] This observation is curious since, in the context of single-unit auctions, Levin, Kagel and Richard (1996) demonstrate that ascending auctions have advantages over the sealed bid formats in alleviating the winner's curse because of the informativeness of drop-out prices. In Goeree et al. (2006), however, because of the low competitiveness, the auction ends with the first drop-out, and hence the drop-out price information cannot be of use to the remaining three bidders.

depends on whether efficiency or revenue is a more important performance criterion.

2.2 Other single-unit demand auctions

Olson and Porter (1994) is an early study that compares sealed bid Vickrey (Leonard 1983) and ascending DGS (Demange, Gale and Sotomayor 1986) auctions, along with two non-monetary mechanisms, to solve the assignment problem where each bidder is constrained to buy at most one of a set of heterogeneous objects. The Vickrey-Leonard (VL) is the second-price sealed bid auction extended to the heterogeneous goods environment, whereas the DGS is unlike the SAA explained above, in that it requires bidders to indicate all their most desired items at a current vector of prices; the prices are then increased on the items that are 'over-demanded.' Both low-contention ('easy') and high-contention ('hard') value environments are considered. Olson and Porter (1994) find that, unlike the non-monetary mechanisms, both the VL and the DGS auctions result in close to full efficiency, as most bidders reveal their demands truthfully (although they are less likely to report all indifferences between objects under the DGS than under the VL auction). In contrast, in a recent study Andersson, Andersson and Andersson (2012) report that the DGS auction performs worse than the VL auction in terms of truthful demand revelation, efficiency, and seller revenue. Andersson et al.'s (2012) findings largely contradict the rest of the literature, which shows that dynamic auctions tend to outperform sealed bids in terms of efficiency; their results merit robustness checks, as the authors consider only one value setting, repeated across only ten periods.

Barut, Kovenock and Noussair (2002) compare all-pay and winner-pay sealed bid auctions in multiple identical units settings with two or four objects. Unlike winner-pay auctions where only winning bidders pay, in all-pay auctions bidders pay whether they win a unit or not. The authors suggest allocation of A's on a grading curve, or assigning of identical jobs in a hierarchy, as examples of multi-unit all-pay auctions. The recent emergence of "penny auctions" online can also be seen as an example of all-pay auctions. Barut et al. (2002) report that, as in the single-unit case, all-pay sealed bid auctions lead to over-dissipation of rent, thus generating higher revenue than the Nash equilibrium prediction. They find similar revenue in the all-pay and winner-pay sealed bid auctions, but higher efficiency in winner-pay auctions,[4] providing a rationale why all-pay auctions are rarely used in the field to sell goods.

Damianov, Oechssler and Becker (2010) study auctions with variable supply, where the seller decides how many customers with unit demand to serve after observing their bids.

[4]For four-object auctions, Pareto efficient allocations were achieved in 46.8% of all-pay (AP) auctions, as compared to 67.1% for winner-pay (WP) auctions; the average efficiency was 96.6% in AP as compared to 97.9% in WP.

The authors suggest that the variable supply auctions are quite frequent in reality and include auctions for electricity, Treasury bills, and initial public offerings. There are two bidders who have a commonly known value for the good, but who are uncertain about the seller's constant marginal cost. Damianov et al. (2010) compare a uniform (lowest accepted bid) and a discriminatory-price auction theoretically and experimentally. In their experiment, the seller is computerized and is programmed to sell all units that, given bids, yield a non-negative profit. The data from both auctions are consistent with the theoretical predictions: the uniform-price auction raises more revenue and is more efficient. Intuitively, bids are higher in the uniform-price than in the discriminatory auction, as raising a bid in the discriminatory auction ultimately leads to a higher transaction price for that bidder; in the uniform-price auction, a bidder is able to raise his chances of winning, but not necessarily the price he pays.

A related earlier study, Shogren, Margolis, Koo and List (2001), compares demand-revealing properties of the second-price sealed bid auction with those of the "random n-th price" auction. The latter may be interpreted as a uniform-price auction with random supply (realized after the bids are submitted). In this auction, the uniform cutoff price n is randomly determined and may be equal to any number from two up to the total number of bidders. The authors find, in a private values setting, that both auction formats were demand-revealing in aggregate. However, the random n-th price auction leads to more sincere bidding by "off-margin" bidders (with extremely high or low valuations), since everyone has a chance to win, but less so for "on-margin" bidders. The comparison with the second-price auction is constrained to the demand-revealing properties of the institutions as the differing supply conditions of the second-price and random n-th price auction make revenue and efficiency non-comparable.

Merlob, Plott and Zhang (2012) consider an auction designed by the Centers for Medicare and Medicaid Services (CMS) to procure medical supplies and equipment. The CMS auction is a uniform-price auction that employs two unorthodox rules. First, the selling price is set at the median of the winning bids. Second, bids are non-binding, meaning bidders can withdraw their bids once the prices are determined; the median price is not recalculated. Merlob et al. (2012) compare the CMS auction with the standard uniform-price auction where the price equals to the first rejected bid, and bids are binding. The standard first rejected bid auction performs well, leading to high efficiency (93.7%), perfect procurement (all seven units were procured in every period) and close to competitive prices. In contrast, the CMS auction created incentives for bidders to "lowball" bids, leading to median prices so low (as low as 33.4% of the competitive level in some treatments) that many sellers could not afford selling and eventually withdrew their bids. This resulted in low procurement (on

average, between two and four units out of seven) and low overall efficiency of the auction (as low as 37.2%). By systematically varying particular CMS auction rules, the authors further show that both the first rejected bid pricing and the binding bids rules are important to guarantee efficiency, and thus the CMS auction cannot be easily remedied by changing only one of these unorthodox rules. This study presents a fine example of a policy experiment that demonstrates how misguided auction design may lead to poor economic outcomes.

Goeree, Plott and Wooders (2004) and Efiaz, Offerman and Schotter (2008) study bidders' choice auctions that are popular in the sales of real estate such as condominiums, as well as antiques and jewelry. These are multi-stage auctions of heterogeneous goods, in which the highest bidder in each stage wins the right to choose the good first. Goeree et al. (2004) consider sequences of ascending bidders' choice auctions, comparing them to standard simultaneous auctions, whereas Efiaz et al. (2008) compare sequential second-price bidder choice auctions with standard sequential second-price sealed bid auctions. In both studies, each bidder had only one preferred object, with other objects having very little or no value to him. In Goeree et al. (2004), there were two objects and four bidders, and bidders were equally likely to prefer each object. In Efiaz et al. (2008), there were four objects and eight bidders, but each object was desired by only two bidders. While the Revenue Equivalence Theorem implies that, under risk-neutrality, bidders' choice auctions should yield the same expected revenue as their studied alternatives, both studies show that bidders' choice auctions resulted in revenues higher by 10 to 25%, thus effectively "creating competition out of thin air." Efiaz et al. (2008) show that this increased competition is due to the bidders behaving as if they faced more competition than they actually did. Salmon and Iachini (2007) study a "pooled" auction as an alternative to the simultaneous ascending price auction (SAA).[5] The former is a version of a multiple-unit sealed bid auction and is similar to a bidders' choice auction in that the highest bidder wins the right to choose the good. The key difference is that it is a single-stage mechanism, with the second-highest bidder picking after the first one and so on; all winning bidders pay their bids. Salmon and Iachini (2007) consider a relatively competitive environment with seven bidders competing for five objects, and all bidders having the same preference rankings over the objects. They report very aggressive bidding resulting in up to 40% higher revenue in the pooled auction than in the SAA. The pooled auction also yielded persistent losses for the bidders. Salmon and Iachini (2007) explain such aggressive over-bidding by attentional bias, with the subjects focusing their attention on their best two or three items, and ignoring the possibility of winning other items.

In sum, research on multi-unit auctions with single-unit demand has mainly focused on

[5]Salmon and Iachini (2007) point out that the SAA (used by the FCC to allocate spectrum) may take prohibitively long to run in many settings, and consider the pooled auction as an alternative format.

the comparison of various auction formats in terms of revenue and efficiency. The studies show that ascending auctions are often more efficient, but first-price sealed bid auctions yield higher revenue, which is a finding that will reemerge in other, more complex settings. Using misguided pricing rules or allowing non-binding bids may lead to performance failures in both revenue and efficiency. On the other hand, bidder biases in perception or attention may be used to increase competition in uncompetitive environments.

3 Multi-unit demand, no synergies

In multi-unit demand settings, a bidder has value for more than one unit of a homogeneous or heterogeneous good. In this section we review settings where bidder valuations for the goods are additively separable, i.e., the value of acquiring several goods is equal to the sum of the goods' valuations. Motivating real-world examples are Treasury bill auctions, initial public offerings, and spectrum auctions in countries, such as the US, where each bidder is allowed by the rules to acquire more than one license. While in spectrum auctions, bidder values for multiple licenses may exhibit synergies, they often exhibit decreasing returns as well (see Kagel and Levin (2001)). We consider the additively separable values case in this section. An alternative setting (where synergies may exist) is reviewed in Section 4.

The first experiment with multi-unit demands was Smith (1967). Motivated by Treasury auctions, he compared the behavior of discriminative and uniform-price auctions for a homogeneous good. Smith hypothesized that under a uniform-price auction with the highest rejected bid pricing rule, the average bid, the variance of bids, and seller revenue would exceed those under a discriminative auction in an identical environment. He found that the variance of bids under the uniform-price rule was indeed consistently greater than the variance of discriminative bids. However, whether the bids and revenue were higher in uniform-price auctions depended on the proportion of rejected bids, that is, the ratio of excess demand beyond the quantity offered for sale to the sale quantity. The bids and revenue increased with the proportion of rejected bids in discriminative auctions, but not in uniform-price auctions.

3.1 Demand reduction under uniform-price auctions and institutional remedies

Uniform-price auctions that are frequently used to sell homogeneous goods, such as Treasury bills, are theoretically demand-revealing under single-unit demand setting (Vickrey 1961). Yet, under multi-unit demands the uniform-price rule gives rise to demand reduction incentives: a bidder may want to reduce her bids on lesser-valued units in an attempt to lower the

price at which she buys higher-valued units. Smith (1967) did not observe demand reduction attempts in his experiment as he studied competitive settings with a large number of bidders (13 to 17). Because demand reduction may lead to inefficient allocations and low revenue in less competitive environments, in recent years experimentalists have carefully addressed two questions. First, does demand reduction occur under uniform-price open and sealed bid auctions? Second, do alternative institutions safeguard against this phenomenon and lead to more efficient allocations and higher seller revenues? As many experimental studies on demand reduction have been reviewed in detail in earlier surveys (Sherstyuk 2008, Kagel and Levin 2011), we give a relatively short summary of these studies, and focus on the latest contributions. Unless mentioned otherwise, all the studies below assume private values, and investigate simple environments with two-unit demand, where the equilibrium predictions can be derived for all institutions of interest, and where demand reduction incentives may also be more transparent to experimental participants.

The first experimental study on demand reduction is Alsemgeest, Noussair and Olson (1998), who compare multi-unit English clock (EC) auctions and sealed bid (SB) auctions with lowest-accepted-bid pricing, in both single-unit and two-unit demand environments. Consistent with the theoretical predictions, they observed a considerable amount of under-revelation on the lower-valued unit in EC auctions. However, they observed no under-revelation in the SB auctions. Kagel and Levin (2001) compare uniform price EC and SB auctions with dynamic Vickrey, or Ausubel auctions (Ausubel 2004), which are theoretically demand-revealing.[6] In their design, a human subject with a flat demand for two units competed with a number of computer rivals with single-unit demands. Kagel and Levin observe substantial demand reduction in both EC and SB auctions, but more so in the EC than in the SB: only 11.4% of all second-unit bids affected market price in EC, as compared to 31% in the SB (with the equilibrium prediction of 0%); the authors demonstrate that observing the drop-out prices of other bidders helped bidders in EC to learn demand reduction strategies. The Ausubel auction eliminated demand reduction incentives and yielded higher efficiency (over 99% in Ausubel as compared to around 97% in SB and EC), but resulted in less revenue than the uniform-price SB auction due to frequent overbidding under the latter.

List and Lucking-Reiley (2000) compare two-bidder uniform-price sealed bid auctions (UPSB) and Vickrey SB auctions in a field experiment involving sports trading cards, and report significant underbidding on the second unit in the UPSB auction as compared to the

[6]The dynamic Ausubel auction works similarly to the EC ascending price auction, except winning bidders in the Ausubel auction do not pay a common price, but the price at which they have "clinched" an item (see Ausubel (2004) for details). Thus a bidder in the dynamic Ausubel auction cannot affect the price he pays for one unit by misrepresenting demand for another unit. In equilibrium, the auction results in full demand revelation and full efficiency.

Vickrey auction. Engelbrecht-Wiggans, List and Reiley (2006) show that this phenomenon persists in three- and five-bidder auctions.[7] Porter and Vragov (2006) compare Vickrey and uniform-price sealed bid (SB) with the English clock (EC) auctions in a setting with two human bidders. They also find that EC results in more demand reduction than UPSB, but also observe significant overbidding under both Vickrey and UPSB. Engelmann and Grimm (2009) compare uniform EC, uniform SB, Ausubel, Vickrey SB, and FPSB (discriminative) auctions. They also find the most demand reduction under uniform EC, higher efficiency under Ausubel, and significant overbidding on the first unit (and consequently higher revenue) under all sealed bid formats.

Several papers compare the Vickrey SB auction with its ascending clock version, the Ausubel auction. Theoretically, both auction institutions are demand-revealing. Manelli, Sefton and Wilner (2006) compare Vickrey and Ausubel mechanisms in a three-unit supply environment with three bidders in both private value and common value component environments. They report significant overbidding on the first unit in both Vickrey and Ausubel auctions under private values, with more overbidding and higher revenues in Vickrey. Efficiency was comparable between the two institutions (around 85%) and neither auction was close to full efficiency, as bidders under Ausubel sometimes bid on all three units lowering efficiency.[8] Kagel and Levin (2009) compare Vickrey SB with dynamic Ausubel auctions in a setting with four human bidders with a supply of two or three units. They find substantially more sincere bidding under the Ausubel auction, and significantly more overbidding on both units under the Vickrey SB format. The Ausubel auction also resulted in higher efficiency, which averaged over 98%, as compared to around 96% under SB. The authors further study the modified Ausubel auction without drop-out price information, and conclude that the superior performance of the Ausubel auction over the Vickrey SB is due to the superior feedback (on the drop-out prices) inherent in dynamic auctions.

The above studies give a consistent picture: uniform-price auctions are confirmed to exhibit demand reduction behavior where it is predicted in equilibrium, but more so under clock auctions than under sealed bid variants. Further, sealed bid auctions are characterized by a substantial amount of overbidding on the first unit as compared to the equilibrium prediction. This is true for Vickrey SB as well as UPSB auctions, and leads to higher

[7] An informative, if unanticipated finding in the latter study concerns the first-unit bids. Whereas List and Lucking-Reiley (2000) observed significant overbidding on the first unit in the two-bidder uniform-price as compared to the Vickrey auctions, Engelbrecht-Wiggans et al. (2006) show that this effect disappears with three and five bidders; thus increased competition causes behavior to move closer to equilibrium predictions. Bidding true value on the first unit is a weakly dominant strategy under both Vickrey and uniform price auction formats. Therefore, no differences in bid levels should be expected on the first unit.

[8] Such efficiency-disturbing aggressive behavior under Ausubel is not reported in any other study and may deserve further investigation.

revenue under SB than under clock auctions. In fact, Engelmann and Grimm (2009) claim that "in clear contrast to the theory, the auctioneer's revenues do not primarily depend on the pricing rule but on whether the auction is open or sealed-bid" (p. 877). The dynamic Ausubel auction performs best in terms of demand revelation and efficiency, as it eliminates demand reduction incentives in theory, and is also more transparent for bidders than the (theoretically demand-revealing) Vickrey SB auction. Thus a closer conformity to equilibrium behavior under dynamic ascending clock than under sealed bid auctions is observed to generalize from single-unit to multi-unit auction settings.[9]

3.2 Extensions to asymmetric bidders and common values

Goeree, Offerman and Sloof (forthcoming) compare a uniform-price clock auction with a discriminatory (first-price) sealed bid auction in a setting with three bidders each having a flat demand for three units and a supply of six units. The authors enrich the well-studied two-unit demand setting with symmetric bidders in several non-trivial ways. First, the bidders are asymmetric: there are two incumbents and one entrant, who differ only in that the incumbents suffer a fixed externality cost (representing increased competition in the product market) if the entrant is successful in buying any number of units, whereas the entrant does not bear any such cost. Thus, the entrant has an ex-ante cost advantage over the incumbents.[10] Further, the symmetric bidders setting (zero externality cost) is compared with those with moderate and high externality cost. The ascending auctions in this setting allow for two types of equilibria: the demand reduction equilibrium, where all three bidders reduce their demands and buy two units each at zero prices, resulting in zero revenue; and the preemptive bidding equilibrium where the incumbents engage in aggressive bidding in an attempt to keep the entrant from buying any units. The discriminative SB auction does not have a symmetric demand reduction equilibrium. Goeree et al. (forthcoming) report that demand reduction in ascending auctions was quite prevalent under all levels of the externality cost, thus leading to low and more variable revenue (with a mode of zero for all levels of externality costs) and lower efficiency (around 88% in ascending auctions as compared to 96% in discriminative SB under zero and low externality costs); the revenue in discriminative auctions was significantly higher and consistent with the preemptive equilibrium prediction. The study presents clear evidence that in ascending auctions, bidders are likely to coordinate

[9] See Kagel (1995) for a discussion of closer conformity of open auctions than of sealed bids to equilibrium in single-unit auctions, and Kagel and Levin (2009) for generalizations to multi-unit settings.

[10] This is unlike Abbink et al. (2005) and Goeree et al. (2006) who model entrants as weak bidders, with their values drawn from a distribution with a lower support than those of the incumbents (see Section 2). In the current study, all bidders' independent private values are drawn from the same distribution, and the only difference between the incumbents and the entrant is the fixed externality cost that the incumbents bear if the entrant enters.

on a higher-payoff tacit collusion equilibrium rather than on the low-payoff competitive preemptive bidding equilibrium. This conclusion is very much in line with Kwasnica and Sherstyuk (2007) (to be discussed in Section 6 below).[11]

In contrast to the above findings on uniform-price ascending auctions, there is little evidence that uniform-price SB auctions may result in low revenue. Motivated by Treasury auctions, Abbink et al. (2006) consider multi-unit demand auctions in a common values setting.[12] Abbink et al. (2006) compare revenue-raising properties of three SB auction formats that are used in Treasury auctions: uniform-price, discriminatory and a "Spanish" auction, which is a hybrid of the first two auctions. Under the latter format, adopted by the Bank of Spain, the winning bidders are charged the average winning bid for all bids above this average, while all winning bids that fall below the average are fully paid. In the experiments, both the uniform-price and the Spanish auction raised significantly more revenue than the discriminatory auction, allowing the authors to suggest that the uniform-price and the Spanish auctions may be "interesting alternatives" to the widely-used discriminatory auction format (p. 298).[13] Further, on average, winning bidders in this experiment do not bid above value, and thus do not fall prey to the winner's curse. This may be attributed to the less competitive environment than the usual single-unit common values settings: eight bidders, with two-unit demand each, competed for seven objects offered for sale. In fact, the authors suggest that in the early rounds of experiment, the bidders tried to suppress competition, which then increased towards the later rounds. Further, the bidders submitted downward-sloping demands on their two units, which may be interpreted as evidence of (apparently unsuccessful) demand reduction attempts.

Superior revenue-raising properties of the uniform-price SB auction are also documented by Zhang (2009), who compares fixed-price offerings with uniform-price SB auctions in an experimental common values setting motivated by initial public offerings. There is a large

[11] Although Goeree et al. (forthcoming) do not give direct evidence that coordinating on demand reduction equilibria resulted in increased payoffs for bidders (as compared to the preemptive equilibrium predictions, or to the actual payoffs in the discriminative auctions), they do mention that "the weak appeal of the preemptive bidding equilibrium might be related to the fact that this equilibrium potentially results in the worst-case scenario for an incumbent." The latter would occur if preemptive bidding is unsuccessful and the incumbent ends up buying units at above cost and paying the externality cost caused by the entrant. However, such loss-aversion argument does not explain the prevalence of demand reduction equilibria under zero externality cost (the symmetric bidder case), where losses cannot occur in the competitive equilibrium as bidding stops at value. The prevalence of the demand reduction equilibrium is thus more likely explained by the payoff-dominance argument.

[12] A common value framework is modeled in the standard way: bidders are uncertain about the common value, but each receives a private signal about the value (see Kagel (1995)).

[13] These findings on the comparison between uniform-price and discriminatory auctions are, to a large extent, consistent with Smith (1967), who conducted such a comparison 39 years earlier; see the beginning of Section 3 for a review of Smith (1967).

number of homogeneous units available for sale to three "informed" buyers (who receive a signal about the value of the good), and one "uninformed" buyer (who does not receive a signal). Under the fixed-price offering, the price is exogenously set so that all units are sold in equilibrium. Under the uniform-price auction, bidders submit (possibly multiple) bids that are price-quantity pairs, and the price is determined by the highest price where demand exceeds supply. Uniform-price auctions are characterized by a multiplicity of equilibria, including "tacit collusion equilibria" (TCE), which are similar in spirit to demand reduction equilibria, and result in all bidders splitting the quantity supplied at a low price. Zhang (2009) finds no evidence that such TCE are played in the experiment. The uniform-price auctions lead to significantly higher selling prices and revenues than fixed-price offerings.

In summary, demand reduction is a well-documented phenomenon in uniform-price ascending bid auctions with a small number of bidders (two); in contrast, sealed bids result in higher revenue in such environments. However, the revenue-raising properties of uniform-price auctions, as compared to other formats, in environments with a higher number of bidders still warrant further investigation.

4 Synergies and package bidding

When bidders demand multiple units their valuations may exhibit "synergies" meaning the value for the combination of the objects is greater than the sum of the valuations for the individual components.[14] When this is the case, the auctioneer may want to allow bidders to place bids for packages of objects where the bid indicates that they are to be allocated all or none of the items in the package. Auctions that allow such bidding are known as combinatorial auctions and the complexity of the auction problem is greatly increased for both the auctioneer and the bidders. The designer faces new choices of auction rules that were previously irrelevant, and the bidders may find the task of selecting a good bid cognitively difficult.

With the exception of the simplest environments, theory is difficult and frequently untractable so the experimental laboratory has played a critical role. In many cases, theory is used as a 'guide' where the theoretical results from simple environments are examined in the laboratory to see whether they also apply to the more complex, practical settings.

Combinatorial auction designs have been proposed and experimentally examined for a variety of applications such as airport landing slots (Rassenti, Smith and Bulfin 1982), space shuttle load allocation (Banks, Ledyard and Porter 1989), transportation services (Ledyard, Olson, Porter, Swanson and Torma 2002) and road painting (Lunander and Nilsson 2004).

[14]Synergies for homogeneous goods may be the result of increasing returns.

Most recent research on combinatorial auction design has been at least in part motivated by the FCC's highly publicized electromagnetic spectrum auctions.

Most of the experimental literature focuses on one or both of the following issues associated with auctions in the presence of synergies. First is the examination of the aggregate performance of various auction designs in environments often designed to mimic practical applications. Second is the examination of bidder behavior that may impact overall auction performance.

4.1 Vickrey-Clarke-Groves auctions

One auction with well-known theoretical properties is the sealed bid Vickrey-Clarke-Groves (VCG) auction, which is an extension of the second-price sealed bid auction to this more complex environment. As in simpler environments, it is a weakly dominant strategy for bidders to reveal their valuations and the auction outcomes should be efficient. A natural question is how well the VCG actually performs when synergies are present. Isaac and James (2000) examine the VCG auction in a simple environment with three bidders and two objects. Across a number of information conditions and valuation structures (sub-additive versus super-additive values) they find that bidders rarely truthfully report their valuations. Despite this, Isaac and James report that the VCG is more efficient than two simultaneous second-price sealed bid auctions achieving an average efficiency of 96% compared to 89% for the non-package bidding auction.

Chen and Takeuchi (2010) consider the VCG in an environment with three bidders and four objects. In many sessions single bidders compete against two computer-simulated bidders in order to mitigate difficulties potentially created by strategic uncertainty. The authors find that bidders frequently fail to bid on all objects they have values for in the VCG, and they are more likely to bid on higher valued objects. This might impact the efficiency results since, due to the complexity of the problem, it is possible that efficiency might dictate that a bidder win an object with a relatively low valuation. Further, as opposed to well-known results in second-price auctions for a single object, 73% of bidders tend to under-report their valuations. Scheffel, Pikovsky, Bichler and Guler (2011) examine the VCG in a range of environments, which included as few as three objects and as many as 18. As in Chen and Takeuchi (2010) they find that in the VCG there is both underbidding but also a substantial portion of the bidders who bid above valuation when they decide to bid. Further, many positively valued packages are not bid on, and the percentages of positively valued packages bid on decreases as the number of objects increases.

4.2 Exposure and threshold problems

As it became clear early on that, in practice, VCG auctions may not lead to full demand revelation, experimentalists considered alternative auction mechanisms. Two potential strategic issues have emerged and played a significant role in the development of auction designs in environments with synergies. Bykowsky, Cull and Ledyard (2000) provide a detailed explanation of the "exposure" and "threshold" problems. The exposure problem provides a rationale for the inclusion of package bidding; a bidder who's values exhibit synergies may, if package bidding is not allowed, be forced to bid above her value for an individual object thereby incurring a risk of losses if she fails to obtain other objects. This exposure to losses might ultimately encourage more conservative behavior and lower efficiency. The threshold problem is the counter to the exposure problem and suggests that package bidding may be avoided; if package bidding is allowed larger (global) bidders might enjoy a benefit over smaller (local) bidders since those bidders must coordinate to displace the bid of the large bidder. While it may be efficient for the small bidders to win, each bidder would prefer that the other bidder be the one that raises the bid thus creating a sort of "free-rider" problem[15] where the large bidder wins resulting in inefficiency.

Bykowsky et al. (2000) demonstrate that the performance of the simultaneous ascending auction,[16] adopted by the FCC, is negatively impacted by the exposure problem. They suggest an alternative auction, AUSM with a "stand by queue" to allow for small bidder coordination that seems to avoid the threshold problem.[17] A number of studies that followed provide a more systematic examination of the problems by examining simpler environments where these behaviors can be captured formally.

Kagel and Levin (2005) provide an important formalization of the potential impact of the exposure problem by considering an environment where all but one of the bidders value one unit of the homogeneous good and one global bidder values two units and has a synergy associated with obtaining a second unit.[18] They theoretically and experimentally examine both the uniform-price sealed bid auction and a uniform-price ascending (clock) auction.

[15]The threshold problem is frequently termed the free-rider problem, but as described by Bykowsky et al. (2000), is more like a battle of the sexes game where there are multiple equilibria and the difficulty in coordinating behavior may result in inefficiency.

[16]See a detailed description of the simultaneous ascending auction in Sections 2 and 4.3.

[17]The Adaptive User Selection Mechanism (AUSM) originally reported by Banks et al. (1989) is a continuous time ascending auction that allowed package bids but did not provide price feedback. The stand by queue is a feature that allows bidders to publicly propose currently non-winning bids (e.g. those that do not raise auction revenue by themselves). Other bidders then could propose their own bids that would pair with these bids and together become winning bids in the auction.

[18]Many papers consider environments closely related to that analyzed originally by Krishna and Rosenthal (1996) where one or more bidders are global bidders who value more than one object (possibly with synergies) and other bidders are local bidders who only value one object.

Both auctions exhibit competing incentives for the large bidder: as outlined in Section 3 there is a demand reduction incentive, but there is also the risk of losses. In an experiment where the single-unit bidders are computerized, they find that bidding behavior is more often consistent with theoretical predictions under the ascending auctions, but in both cases the potential impact of exposure to losses is a strong demand reduction effect. Bidder profits are 6.7%[19] higher on average in the ascending auction but are still lower than theory predicts, seller revenue is 9.6% higher on average in the sealed bid auction, and efficiency is not significantly impacted.

Englmaier, Guillén, Llorente, Onderstal and Sausgruber (2009) provide a straightforward examination of the exposure problem by studying two sealed bid auction formats. In their environment there are two bidders and three homogeneous objects and bidders have a strong value synergy for obtaining the second unit but no value for the third unit. In the sealed bid auction they term a "chop stick auction" (CSA) the highest bidder wins two units at the per unit price of their bid whereas the second bidder receives only one unit. Both the CSA and the well-known second-price sealed bid auction (where two objects are sold as a bundle) have an efficient and revenue equivalent equilibrium. In the CSA bidders face some risk of losses (the chance they are the lower bidder and win only one unit), but the second-price sealed bid does not force bidders to risk losses. Englmaier et al. (2009) find that efficiency is higher under the second-price sealed bid and revenue is greater than predicted by theory. Under the CSA, the revenue starts high but decreases over time presumably because bidders are taking more conservative strategies to avoid losses.

Katok and Roth (2004) compare the performance of a Dutch (descending clock) auction to an ascending auction without package bidding. Two homogeneous units are for sale in both auctions but one bidder has increasing returns for obtaining a second object; the other two bidders only value one unit. Since, in the Dutch auction, the auction can be ended for both objects simultaneously, the global bidder only faces an exposure problem in the ascending auction. The authors find that the Dutch auction always performs at least as well in terms of efficiency and revenue.

Chernomaz and Levin (2012) examine the threshold problem in the first-price sealed bid auction with and without package bidding. They consider a model with two objects, one global bidder who has a synergy for obtaining both objects, and two local bidders (Krishna and Rosenthal 1996). Theoretically, when package bidding is allowed, the global bidders should bid more aggressively than the local bidders who will, as a result of the threshold problem, place lower relative bids. in environments with and without synergies, Chernomaz

[19]Since the average human bidder profits are occasionally negative, we compare the profits differences as a percentage of the human bidder's value for one object, which was fixed in each treatment.

and Levin (2012) consider three different auctions: no package bidding is allowed, package bidding is allowed and the global bidder can only bid on the package, package bidding is allowed and the global bidder can bid on both the package and individual items. The efficiency results are qualitatively consistent with the theory. Allowing package bidding lowers efficiency (weakly) without synergies but increases it with synergies. Similar qualitatively consistent results are obtained for revenue and bidder profits. An interesting and important behavioral result is the differences between the two package bidding auctions. In theory, when standalone bids are allowed for the global bidder, he should figure out that it is optimal to place zero bids on these individual objects. However, experimentally, global bidders fail to realize this benefit and consistently bid on the individual items, which has the impact of mitigating some of the negative effects on seller revenue from allowing package bidding. In contrast to experiments on the VCG where bidders failed to bid on all packages as they should in equilibrium, here it is the global bidder's excessive bidding (relative to equilibrium) that makes the package bidding auction (with standalone bids) perform better than expected.

4.3 Comparison of alternative auction mechanisms

Since the FCC has primarily conducted simultaneous ascending auction (SAA), many studies use some variant of SAA as a baseline for comparison with alternative formats. Under SAA, package bidding is not allowed, but bidders are permitted to bid on each object repeatedly in an English-style auction that is conducted either iteratively (over multiple rounds)[20] or in continuous time.[21] The key feature is that bidders desiring to obtain a package of items must be the high bidder on all individual items in the package.

Porter, Rassenti, Roopnarine and Smith (2003) present and evaluate a simple package bidding auction that has become a commonly considered alternative to the SAA. The Combinatorial Clock (CC) auction allows package bidding but raises prices in automatic, set increments based upon demand for the objects. In short, if more than one bidder demands a particular object, the price goes up, otherwise, it remains fixed. As such, an attractive feature of the CC is that it might eliminate potential strategic complications associated with placing a specific bid on a package such as jump bidding (see Section 5) since bidders are simply deciding what packages are profitable given current prices. Noussair (2003) provides a description of how the CC may have more desirable strategic properties over other auctions such as AUSM. Porter et al. (2003) report increased efficiency from the the CC relative to

[20]Such an SAA is often referred to as the simultaneous multi-round (SMR) auction.

[21]Variations in the auction rules of the SAA can be achieved by changing rules related to auction ending rules, minimum bid increments, or bidding activity requirements and will only be discussed to the extent that they are relevant for the experimental results presented.

the SAA and an alternative combinatorial auction proposed by Charles River & Associates (CRA) to the FCC; the CC was allocatively efficient in all but 2 (of 22) auctions compared to 6 (of 10) for SAA and 18 (of 21) for CRA. Banks, Olson, Porter, Rassenti and Smith (2003) also examine the CRA auction in comparison to the SAA. They find that CRA generates similar results to the SAA in additive environments but higher efficiency and lower revenue in environments with synergies. However, average auction duration increases significantly under the CRA from 8.9 rounds per auction under SAA to 29.3 rounds under CRA. This is perhaps driven by the fact that the CRA provides little price guidance and does not restrict bidding.

The importance of information feedback in the CC is examined by Adomavicius, Curley, Gupta and Sanyal (2012). They consider the CC auction under three information environments which increase from simply observing bids placed, to also observing winning allocation and price information. They find that efficiency increases as more information is provided; however, the difference between the two most generous information conditions is not significant.

Kwasnica, Ledyard, Porter and DeMartini (2005) present an auction design known as the Resource Allocation Design (RAD) that allows package bidding, is iterative, and provides feedback to bidders via single-items prices. The main innovation of the RAD design is to suggest a method for determining single-item prices that mimic competitive equilibrium prices when possible (Bikhchandani and Ostroy (2002) show the impossibility of finding anonymous competitive equilibrium prices) and provide a guide for bidders to determine future winning bids and hopefully avoid the threshold problem. The RAD design was compared to the SAA as well as AUSM. When examining performance for the objects with synergies, both RAD and AUSM significantly outperform the SAA in terms of efficiency (90% for RAD, 94% for AUSM, 67% for SAA), bidder profits ($4.23, $5.68, -$7.73), and net revenue (revenue minus revenue from bidder losses) (74%, 69%, 61% of maximal revenue). For the additive objects, RAD, AUSM, and SAA achieve similar results. Under both environments, RAD auctions were shorter, averaging 3.32 rounds to completion, than the SAA, averaging 16.2 rounds, in contrast to earlier reported results with the CRA auction.

Brunner, Goeree, Holt and Ledyard (2010) build on the previous two studies and directly compare RAD and CC along with the SAA and an alternative package bidding auction that bares similarities to RAD but uses "XOR" bidding (exactly one package bid per bidder can win) and a different pricing rule that bases current iteration prices on previous iteration prices whereas RAD prices were determined anew every round and therefore could be non-monotonic. Relatively complex environments with eight bidders and 12 objects are considered with value structures that increase complementarities (synergies) and overlap be-

tween bidders so as to examine progressively more difficult settings. Overall, the authors find that RAD yields higher efficiency than the three other auctions. When synergies are high, the SAA only averages 84% efficiency whereas all three combinatorial auctions average efficiencies 90% or greater. The CC yields seller revenue that is 10-15% greater (as percent of maximal revenue) than the other institutions. Likewise, bidder profits are 13-15% lower under the CC.

Finally, Goeree and Holt (2010) present a new design known as the Hierarchical Package Bidding (HPB) which limits the types of packages that can be bid on thereby making computational issues less severe. This design builds on the important work of Rothkopf, Pekeč and Harstad (1998) who show that by limiting the set of admissible packages the winner determination problem for the auctioneer can be greatly simplified. Goeree and Holt (2010) demonstrate that such a simplification also makes the pricing problem, which is a central concern in designs like RAD, substantially easier. Goeree and Holt (2010) compare HPB to the SAA and an auction similar to RAD. While both package bidding auctions obtain higher efficiency than the 85.1% achieved by the SAA, the HPB performs even better, with average efficiency of 92.9% as compared to 89.7% for RAD. A similar result holds for seller revenues (as a percentage of maximum revenue) with 65.6% (SAA), 70.8% (RAD), and 76.6% (HPB). Bidder profits are more difficult to rank with similar profits for small (local) bidders across auctions but higher profits for the big (global) bidder under the package bidding institutions.

Chen and Takeuchi (2010) consider the iBundle Extend & Adjust (iBEA) auction originally proposed by Parkes and Ungar (2002). The iBEA auction is a clock-type auction in which it has been shown that a myopic best response strategy (similar to straightforward bidding where bidders place bid on packages that are most profitable given current prices) is an ex post Nash equilibrium. Assuming that bidders follow these equilibrium strategies, iBEA should be efficient. As noted earlier, they also examine the VCG in a simple environment with some bidders being simulated. Chen and Takeuchi (2010) find that while the VCG achieves approximately 14% higher seller revenue, the iBEA achieves approximately 24% greater bidder profits and is more likely to be efficient.

Scheffel et al. (2011) examine many of the auctions previously discussed in environments specifically chosen to test auction performance. In most treatments, four bidders participated in one of four auction designs: VCG, CC, iBEA, and RAD,[22] in environments with three to 18 objects. Overall, the authors find that there are very few significant differences in allocative efficiency between auctions and environments. However, seller revenue tends to be lower under the VCG (in contrast to Chen and Takeuchi (2010) discussed above) and

[22]Slightly different versions of RAD and iBEA were utilized for these experiments.

auction duration is longer under *i*BEA than both RAD and CC.[23]

What general results can be drawn from these studies? First, the theoretically attractive VCG auction does not perform as well as ascending combinatorial auctions. This is likely because of the need, due to the complexity of the bidding problem, for the auction to provide guidance for bidders about profitable opportunities.[24] Second, it is clear that most auction designs achieve similar results when there are no synergies. Likewise, while most reasonable combinatorial auctions seem to perform well in terms of efficiency, differences are most likely to come from either environmental differences that are better or worse for one design or the other or interest in other performance measures beyond efficiency such as seller revenue and auction duration.

On the behavioral side, bidder behavior is rarely consistent with theory when there are concrete predictions. Interestingly, underbidding (in terms of values and objects bid on) seems common but is also countered by times when bidders bid on objects that theory predicts they should not. It seems that bidders adopt heuristic strategies that are only occasionally consistent with theory. It would be worthwhile considering whether behavioral models that either account for bounded rationality or other non-traditional preferences can better predict behavior. Rothkopf (2007) goes as far as to suggest that decision analysis may be preferable to game theory.[25] While given the complexity of the environment it makes sense that bidders might resort to simpler strategies, bidders also show a remarkable level of sophistication in some settings as is outlined in Section 6 when considering bidder collusion.

Kagel, Lien and Milgrom (2010) provide a particularly interesting combination of behavioral theory and auction performance. They theoretically examine sufficient conditions on package bids to obtain efficient or core outcomes. Under the assumption that bidders will only bid on a subset of all profitable packages, they show that there are environments that should be more or less difficult for combinatorial auctions. They then validate these predictions by comparing performance of the CC to the SAA in environments of varying difficulty. The CC outperforms the SAA in terms of efficiency in the simpler setting (95.5% versus 82.9%), but the results are reversed when the environment is sufficiently difficult (90.3% versus 93.4%). While the environments were specifically designed to "stress test"

[23]The VCG is a sealed bid auction so auction duration comparisons are not particularly meaningful.

[24]The need for such guidance has been understood since the very early years of multi-unit auction design. Banks et al. (1989) state the their choice of iterative ascending bid procedure for the allocation problem with complementarities was motivated by "... the feeling, based on experimental evidence, that in an environment in which the basis for common knowledge are little understood and controlled, iterations with commitment allow subjects to 'feel their way...' "

[25]Santamaría (2012) reports theory and experiments that uses decision analysis to generate predicted behavior and arrives at results similar to those reported by Chernomaz and Levin (2012).

the CC, it provides an important proof of concept that relatively straightforward theory and simulations may help in the design process. In a subsequent paper, Kagel, Lien and Milgrom (2012) examine other features of the auction environment that might impact performance. For example, they find that whether the efficient combination of bids corresponds to the "named" packages of each bidder (those that each bidder naturally is expected to have a higher valuation for) can impact auction performance. Obvious next steps are to address whether or not the applied environment of interest is simple or hard and whether there are alternative price-guided combinatorial auctions that might perform better than the CC in these difficult settings.[26]

5 Role of specific institutional features

Fine institutional details that govern auction dynamics can significantly impact performance. Here we discuss activity and ending rules that can impact ascending auction performance across a range of settings, including single-unit or multi-unit demand, and environments with or without synergies.

5.1 Bid withdrawal and eligibility rules

A practice allowed in the SAA conducted by the FCC was the withdrawal of bids; by allowing bid withdrawal (with a penalty) the intention is to mitigate some of the adverse effects of the exposure problem in auctions without package bidding. Porter (1999) considers the impact of bid withdrawal in a continuous version of the SAA in two environments. Interestingly, allowing bid withdrawal increases auction efficiency and seller revenue but actually increases bidder losses and thus lowers overall bidder surplus.[27] This seems to be in contrast to some of the intended impact of bid withdrawal shielding bidders from the exposure problem; the author attributes this to the increased difficulty in bidder coordination (e.g. avoiding the threshold problem) caused by bid withdrawal. In comparison, in settings without synergies, the lack of commitment caused by allowing bid withdrawal after the auction close is documented to have pronounced negative impact on auction efficiency and revenue (see Merlob et al. (2012) discussed in Section 2).

Another feature of the FCC SAA auctions was that bidders had a certain "eligibility" which limits the number of objects that they can bid on. The purpose of such a rule is two-fold. First, as in procurement, eligibility could represent a judgment by the auctioneer

[26] Auctions such as RAD and HPB were not evaluated although the suggestion of this approach is that these environments will also be difficult for these auctions.

[27] It is worth noting that the only efficiency result that meets most standard definitions of statistical significance with a p-value below .1 is in the homogeneous environment.

as to the number of objects the bidder can service if they win. Second, eligibility is generally coupled with a "use-it-or-lose-it" rule whereby failure to bid on a sufficient number of objects in the previous round will result in a decline in eligibility in future rounds thereby encouraging early bidding by bidders despite the obvious cheap talk features. Banks et al. (2003) consider SAA with different variants of the eligibility rule under environments with and without synergies. Not surprisingly, eligibility has little impact on auction performance in additive environments, but in environments with synergies efficiency is increased by 8.2% to 15.4% depending on the treatment by allowing for a flexible eligibility rule that permits bidding on more units than the bidder is eligible for on occasion. Similarly, seller revenue is improved by both flexible eligibility rules and unequal eligibility that requires more eligibility for some objects. On the other hand, flexible eligibility tends to increase auction duration.

5.2 Ending rules

Ariely, Ockenfels and Roth (2005) study ending rules and "sniping" in single-unit auctions modeled after online auctions. They show that a "hard" closing rule where an auction ends after a fixed period of time can create incentives for bidders to wait for the last moments of the auction to bid whereas a soft closing rule always provides the opportunity for rebidding thereby reducing the incentive to snipe. Sherstyuk (2009) extends the analysis to auctions for multiple heterogeneous goods with independent valuations, with four objects and four bidders. She compares first-price sealed bid auctions (FPSB), SAA with the hard closing rule, and SAA with the soft closing rule. The FPSB auctions resulted in revenues 30% higher than the hard closing rule auction and 13% higher than the soft closing rule auction, as bidders tend to bid above the risk-neutral equilibrium predictions. On the other hand, just as discovered in Ariely et al. (2005) for single-unit settings, SAA with the the hard closing rule lead to the lowest and most variable revenues, and to approximately 5% lower efficiency than the other auction formats, due to a significant amount of late bidding caused by the hard closing rule.

In more complex multi-unit auctions with synergies, the choice of a closing rule can be important since the high number of objects may make auctions last for a very long time. On the other hand, the cost of delay, bidding, and winner determination may provide incentives for the auctioneer to select an ending rule that ensures that the auction ends in a reasonable period of time. In fact, in the pioneering study that developed, experimentally tested, and implemented a combinatorial auction for the procurement of transportation services, Ledyard et al. (2002) state: "A critical, if seemingly innocuous, part of the auction design is the stopping rule. The stopping rule for an auction is absolutely crucial to its performance, both in the final cost of acquisition and in the time to completion, because it affects the

incentives and the information of the bidders." Ledyard et al. (2002) report the successful use of a rule that ended the auction if revenue from the winning bids did not increase by a certain percentage between rounds. All subsequent experimental studies that we are aware of have opted for more conservative ending rules that do not end the auction until no new bids are placed or at least the seller's revenue does not increase. Unfortunately, there are no systematic studies of how these rules might impact auction performance and the seemingly small differences in ending rules between experimental studies are often relegated to an afterthought.

5.3 Jump bidding

In (non-clock) ascending auctions bidders may place jump bids, i.e., bids that are significantly greater than the minimum acceptable bid increment. This sort of bidding behavior is common in both the field and the laboratory; the potential for inefficiencies due to jump bidding has been used as justification for clock ascending auctions (Banks et al. 2003). The motivation for jump bidding can come from signaling, irrationality, strategic bidding, and impatience. Isaac, Salmon and Zillante (2005) provide a review of these competing theories and an experimental examination in a single-unit setting that suggests that strategic bidding and impatience appears to be the most likely motivations for jump bidding.[28]

In the multi-unit context, Plott and Salmon (2004) propose a model of bidding behavior in simultaneous ascending auctions, based on the principles of surplus maximization and bid minimization, that would allow the auctioneer to predict the length of the auction as well as final prices.[29] They test their model using both lab experiments and data from the 2000 British 3G auctions, using a private values framework. They confirm predicted convergence to equilibrium prices and high levels of efficiency in the lab auctions. This is despite the fact that they observe substantial jump bidding (7-32% of all bids in their experiments) in both settings, which is inconsistent with the model. Therefore, they conclude that jump bidding is most likely due to impatience and an attempt to speed the pace of the auction.

Isaac and Schnier (2005) consider both field and lab data on charity silent auctions. In such auctions, multiple, heterogeneous units are for sale simultaneously, the auctions all end at a fixed time, the auctions are somewhat geographically dispersed (thus increasing monitoring costs), and jump bidding is permitted and common. The authors conclude

[28]Kwasnica and Katok (2007) also show that jump bidding is impacted by timing concerns. Isaac et al. (2005) largely confirm the theory proposed by Isaac, Salmon and Zillante (2007) and find that jump bidding does not significantly impact auction efficiency or seller revenue. Interestingly, they find that only the prohibition of jump bidding impacts auction performance; when jump bidding is not allowed many bidders drop out of the auction before reaching their value.

[29]Such behavior is frequently referred to as straightforward bidding.

that, "there is persistent support for the conjecture that bidders jump bid because they are impatient" (p. 772). As opposed to single-unit auctions, auction efficiency is somewhat lower than expected, but the lack of full efficiency may also be impacted by the relatively limited bidding time and high number of objects.

While the experimental literature on jump bidding in multi-unit auctions is still somewhat limited, the results all seem to point toward bidder impatience as being the driving force behind observed jump bids, with minimal impact on auction performance. Yet, jump bidding in multi-unit settings may also be an indicator of signaling and retaliation by colluding bidders, as we discuss in the next section.

6 Collusion

Concerns about bidder collusion have been prominent in the studies of auctions for a long time, but when highly publicized spectrum auctions resulted in clear evidence of coordinated attempts by bidders to avoid raising prices (Klemperer 2002) these concerns became even more pressing. The notion of collusion is closely related to that of demand reduction, but is not restricted to uniform-price auctions, and may refer to a coordinated attempt by bidders to engage in anti-competitive behavior under a wide range of environments and institutions. Compared to a single-unit auction, the multi-unit nature of sale may facilitate collusion, as the bidders may split the objects, much like sellers may split the market in other industrial settings. However, the multi-object nature may also make the environment more complex and create new coordination problems for bidders in reaching explicit or tacit cooperative agreements. Experimental research investigates the following questions: Can collusion be successful in multi-object settings? Does collusion occur to the same degree under different auction formats? What types of collusive strategies are adopted by bidders? Does the presence of a large number of bidders preclude collusion? And, for auction formats that are susceptible to collusion, what institutional features may facilitate or safeguard against collusion?

Traditionally, collusion has implied conspiracies (i.e., explicit communication among bidders). Recently a significant amount of research has been drawn to tacit collusion, where bidders coordinate on low-revenue (and often lower-efficiency) outcomes without explicit communication but as the result of equilibrium play. We review studies of explicit conspiracies first, and then move to studies of tacit collusion.

6.1 Collusion with explicit communication

Communication greatly facilitates cooperation among economic agents in a variety of settings. When studying conspiracies in multi-unit auctions, experimental researchers focus on the effect of institutions and their collusion-facilitating and collusion-destroying features, and on the analysis of collusive agreements that emerge among bidders.

Many studies are motivated by real-world settings where concerns about bidder collusion exists. Phillips, Menkhaus and Coatney (2003) study collusive practices in sequential multi-unit English ascending auctions for homogeneous goods modeled after cattle auctions. Perhaps the most surprising finding is that the bidders were able to sustain collusion successfully in auctions with six, as well as with two bidders; the prices in these auctions were about half of the competitive norm, and consistently lower than in auctions without communication. Communication was the key collusion-facilitating feature in the six-bidder auctions, whereas knowing the quantity for sale was the key in the two-bidder auctions. One reason for successful collusion is the low competitiveness of the environment (modeled after the real-world setting of interest), with a large number of units (between 19 and 30) available for sale in each auction. With communication, the bidders successfully coordinated on simple bid rotation schemes to split the objects. "Our explanation for collusion being successful, when there are six players, is a simple bid sharing plan that lets bidders alternate taking the low bid is focal" (Phillips et al. (2003), p. 977).[30]

Sade et al. (2006) study Treasury auctions, employing both students and finance industry professionals as experimental participants. The authors compare discriminatory and uniform-price formats with communication in a multi-unit common publicly known values setting with five bidders competing for 26 units. Whereas the discriminatory auction has a unique equilibrium with competitive pricing, the uniform-price auction admits a lower-price equilibrium. In the experiments, however, the discriminatory auction was more susceptible to collusion and thus raised less revenue (by around 4%) than the uniform-price auction. This result is in agreement with the earlier findings of Smith (1967) and Abbink et al. (2006) on the revenue rankings of uniform-price and discriminative auctions without communication (see Section 3). The authors claim that these findings are consistent with the recent switch to uniform pricing by the U.S. Treasury. Further, Sade et al. (2006) demonstrate that the seller's ability to reduce supply after observing bidders' demands in uniform-price auctions leads to higher revenue as compared to the fixed supply setting.

[30] In a later paper, Phillips and Menkhaus (2009) study repeated English auctions with advanced production of the good by sellers. The sellers in this experiment often reduce units supplied to the auction in response to low bids by buyers, thus destabilizing conspiracies among bidders. This result confirms the observation by Sade, Schnitzlein and Zender (2006) (discussed below) that the seller's ability to reduce supply may increase auction revenue.

Burtraw, Goeree, Holt, Myers, Palmer and Shobe (2009) compare, in the context of environmental markets, three auction formats in terms of susceptibility to collusion: uniform-price (UPSB) and discriminatory sealed bid (SB) auctions, where bidders submitted sealed bids on blocks of pollution permits, and multi-round ascending-clock auctions, where bidders were asked to state the number of blocks demanded at each price. Six bidders competed for 30 homogeneous units, in auctions with or without communication; this is compared to the competitive baseline where 12 bidders competed for 60 units. The authors report that the prices were below the competitive benchmark both with and without communication, but communication further reduced auction revenues under all formats (by around 16%). The clock auction was the most collusive and produced the lowest revenue; with communication 38% of the clock auctions stopped at the reserve price of $2.00, yielding the average price of $2.29, as compared to $2.77 under UPSB, $2.83 under the discriminatory SB, and the competitive benchmark of $3.60. The authors suggest that the sequential nature of the clock auction appears to facilitate successful collusion, as it allows bidders to focus on one dimension of cooperation (quantity) rather than two dimensions (price and quantity) under the sealed bid formats. A similar observation is made by Sherstyuk and Dulatre (2008) who compare bidder conspiracies in simultaneous and sequential ascending multi-object auctions. They report that bidders were able to reach and sustain collusive agreements more often under the sequential format than the simultaneous format. They attribute the result to the lower complexity of sequential auctions, which allow bidders to focus on one object at a time. It is interesting to note that bidders tendency to focus on one or a few objects manifests itself across a variety of multi-object settings, but has different effects depending on the setting. It causes overly aggressive bidding in pooled auctions (see Salmon and Iachini (2007) discussed in Section 2), may result in bidding on only a subset of all profitable packages in combinatorial auctions (see Kagel et al. (2010) discussed in Section 4), and facilitates collusion in sequential auctions, as discussed here.

Mougeot, Naegelen, Pelloux and Rulliere (2011) adopt the the framework of Burtraw et al. (2009) to consider whether introducing speculators (bidders who have no value for the objects except that they can resell them) may be used as a collusion-breaking device, as bidding rings may be harder to establish with such "outsiders." They compare the uniform-price sealed bid (UPSB) auction with the ascending clock auction and find that, in the UPSB, the presence of speculators makes other bidders bid aggressively raising auction revenue. The effect is quite different in the more collusive clock auction; bidders accommodate speculators, letting them buy permits in the auction and buying their necessary permits on the secondary market. They also observe that opening the auction to speculators deteriorates efficiency (from close to 100% without speculators to around 86% with speculators).

A number of papers study collusion in procurement auctions. Davis and Wilson (2002) investigate conspiracies in multi-unit auctions under posted offer institution, where sellers specify quantity as well as the asking price, and find a strong effect of communication. Lunander and Nilsson (2006) consider procurement auctions for two goods with scale economies, and suggest that a combinatorial auction makes bidders less likely to collude than the standard sealed bid auction.

Kwasnica (2000) studies collusive mechanisms in five-object, five-person sealed bid auctions. He reports that bidders largely adopt collusive mechanisms that are incentive compatible[31] and payoff-superior to the simple random assignment scheme.[32] Further, Kwasnica (2000) observes that whether bidders restrict themselves to incentive compatible strategies depend on the informational feedback available in the auction, and on whether bidders are symmetric. In an environment with symmetric bidders and bids tied to bidder identities, bidders select linear bid reduction strategies that are payoff maximizing, but are not incentive compatible. However, asymmetry or decreased information leads bidders to choose incentive compatible collusion mechanisms.

6.2 Tacit collusion

In repeated or ascending price auctions, collusive low-price outcomes may be supported as equilibria even without communication, using signaling to allocate goods and the threat of retaliation to deter deviations. Experimental literature on tacit collusion, nearly all of which has emerged in the last 10-15 years,[33] investigates whether existing or newly-adopted real-world auction institutions, such as the simultaneous ascending auction (SAA), are susceptible to collusion, and how collusion may be remedied.

Kwasnica and Sherstyuk (2007) investigate bidder collusion in SAA modeled after the FCC spectrum auctions. Cramton and Schwartz (2002) provide evidence of collusion via signaling in FCC auctions, where bidders split markets at low prices. Brusco and Lopomo (2002) show that such low-price signaling outcomes may be supported as equilibria, and that such equilibria may be sustained even in the presence of large but common complementarities between objects. Kwasnica and Sherstyuk (2007) study experimental auctions for two goods with two or five bidders, and find a large amount of collusion in two-bidder markets without complementarities or with moderate complementarities; up to one-half of all markets with no

[31] A collusive mechanism is incentive compatible if it is in the best interest of each bidder to reveal their true valuation.

[32] In particular, the ranking mechanism of Pesendorfer (2000) and the serial dictator mechanism were adopted frequently.

[33] Kagel (1995) writes that "...outright collusion among bidders has not been reported under standard experimental procedures." A notable exception is Burns (1985) who observed some unstable collusion attempts in a multi-unit sequential English auction with three bidders.

completmentarities and about one-third of the markets with moderate complementarities had prices below 50% of the competitive prediction. Moreover, just as predicted by the theory, they observe both signaling among bidders, and retaliation in response to deviators, often in the form of jump bids. There is strong evidence that bidders chose collusive schemes that were more efficient and payoff-superior than randomly splitting markets. Without complementarities, signaling preferred objects was used to split the markets as efficiently as possible among the bidders; with complementarities, bid rotation across periods was used to capture the complementarity term. However, collusion was never observed in two-bidder markets with large complementarities, or in five-bidder markets, indicating that large complementarities, and a low object to bidder ratio, tend to hinder collusion.

Sherstyuk (1999) and Sherstyuk (2002) study the role of bid improvement rules in ascending auctions. If the bidders are allowed to match each other's bids in open auctions, they can sustain low-price collusive equilibria where all bidders submit low equal bids, and the goods are allocated randomly among the bidders. These equilibria are observed in such 'weakly ascending' experimental auctions under both common (known) values and under private values with three bidders with single-unit demand and two goods for sale. In the common value setting, the average price in the weakly ascending auction was 25.6 cents, as compared to 64.5 cents under the uniform-price sealed bid run in the identical environment, and the competitive prediction of 100 cents. Bid matching was observed in 89.7% of all weakly ascending auction outcomes; it served as an equivalent mechanism to random assignment, allowing all bidders an equal chance to buy the good at a low price. These results indicate that a strict improvement rule (disallowing tie bids) is a necessary element for enhancing competition.

Li and Plott (2009) study conditions for tacit collusion facilitation and prevention, and show that collusion may occur even with a large number of bidders. They create a "collusion incubator" environment which results in perfect collusion in experimental auctions under the standard SAA format with eight objects and eight bidders. The environment is characterized by "symmetrically folded and item-aligned preferences" (p. 425). Essentially, each bidder has a different best-preferred item, for which he has the highest value among all bidders, and faces serious competition for this item from only one other bidder. The preferences structure was publicly known, and the experimental auctions evolved towards perfectly collusive outcomes, with each buyer buying their preferred item at the reserve price, thus resulting in zero seller revenue but full efficiency.[34] Moreover, once collusion is established, it persists even when

[34]A curious observation is that such a preference structure is reminiscent of that studied in Efiaz et al. (2008) in the context of sequential bidder choice auctions, where the opposite result was observed, and the auction format created "competition out of thin air." They key differences between the auction institutions and the environment studied in Efiaz et al. (2008) and Li and Plott (2009) are: bidder choice auction as

bidder identifiers are removed, the information about preferences is no longer public (but the preference structure remains unchanged), and even if some items are removed from sale. Competition is only restored when the preference structure is changed (without public announcement) in a way that creates head-to-head competition between two agents. Brown, Plott and Sullivan (2009) further demonstrate that tacit collusion that emerges under the SAA is effectively destroyed by switching to the simultaneous descending auction (SDA). The reason is that, unlike the ascending auction, retaliation against deviators under the SDA is costly to other bidders: every bid in the SDA is final, and punishing deviators typically involves bidding at a loss. The punishment thus rarely occurs.

The above two papers establish the following important regularities. First, forming common beliefs about cooperative behavior is a key factor in establishing successful collusion. Once the common beliefs are established, some institutional features (such as displaying bidder identifiers) that are important in sustaining collusion in theory, become unimportant in practice.[35] The second important factor in establishing and sustaining collusion is the ability to punish deviators. If this ability is nonexistent or very costly, as under simultaneous descending auctions, collusion is destroyed and competitive bidding prevails.

In summary, experimental studies document that collusion can and does occur with communication, or when it is supported as a low-price equilibrium, enhanced by repeated play. A large number of bidders does not necessarily preclude collusion, as long as the number of objects is large enough to share among bidders. Bidders gravitate towards collusive strategies that are payoff-maximizing, rather than simple random assignment. Sequential and multi-round auctions appear to be especially conducive to bidder conspiracies. However, simultaneous ascending auctions are also susceptible to collusion, and the presence of synergies does not always eliminate the phenomenon. There is some (but not enough) evidence that allowing combinatorial bids may break collusion. In the homogeneous goods setting, open ascending clock auctions have also been documented to be subject to collusion in the form of demand reduction (Section 3). Competitive pressures (measured by the object-to-bidder ratio) appear to be one of the most important factors for breaking collusion. Sellers may also counter collusion by reducing supply in response to low pricing, which may lead to higher auction revenue.

compared to simultaneous auction; private as compared to public information; and lower object to bidder ratio (four objects for eight bidders as compared to eight objects for eight bidders.)

[35]Thus repetition with the same cohort of bidders appears essential for establishing these common beliefs, which is an observation that is also supported by the results of Phillips et al. (2003) and Kwasnica and Sherstyuk (2007).

7 Sequential bidding

We now briefly review experimental studies that consider sequential sales.

7.1 Auction formats and sequencing of sales

A number of papers compare simultaneous and sequential auction formats. Lunander and Nilsson (2004) report that when bidders are asymmetric and have non-linear average costs of winning more than one contract, sealed bid combinatorial auctions are more efficient and have lower procurement costs, as compared to either simultaneous or sequential auctions. In their experiment, combinatorial auctions resulted in 91% efficiency,[36] as compared to 70% under simultaneous FPSB and 68% under sequential FPSB auctions, and a 4% lower procurement cost. In contrast, in a setting with single-unit demands (see Section 2) Goeree et al. (2006) find that the revenue is higher, by about 6%, in the sequential FPSB auctions where the most valuable item is auctioned first ("The best foot forward" strategy), whereas efficiency is the highest in the simultaneous ascending auction, exceeding other auction formats by up to 15%. The simultaneous FPSB auctions give a higher chance of winning to weak bidders, and generate more entry on their part. Grether and Plott (2009) consider revenue-maximizing sequencing in a field experiment on sequential automobile auctions. Interestingly, they report that the worst performing sequence is for the seller to order vehicles from highest to lowest values. Unlike Goeree et al. (2006), Grether and Plott (2009) study a setting with seller competition, where several sellers simultaneously sell vehicles in sequential ascending price auctions.

Leufkens, Peeters and Vorsatz (2012) compare first-price and second-price sealed bid sequential auctions for two goods, when the winner of the first auction receives a positive synergy from the second good. However, the value of the second object is not known at the time of bidding for the first object; an example of such a setting is expertise that is created during the first project and may be used in the second project. In this environment, bidding in the first auction involves bidding not only for the first object itself, but also for the option value of being in the favorable position in the second auction. However, because the option value may not be realized, bidders face an exposure problem (as explained in Section 4). Leufkens et al. (2012) find that the auctions are not different in terms of efficiency, but the first-price auction yields about 4% more revenue than the second-price auction, and also results in less frequent ex-post bidder losses when positive synergies are present. These findings contradict the theoretical predictions, and are explained by bidders not fully incorporating the option value in the bid for the first object. The authors claim

[36] The reported efficiency measure is normalized to the random assignment efficiency benchmark.

that the "results ... support the common use of the first-price auctions in governmental and business-to-business procurements" (p. 25).

Brosig and Reiss (2007) study bidder strategies in first-price sealed bid sequential auctions for two units, when both bidders are constrained (due to capacity) to win only one object. Bidder costs are drawn independently for both units. Due to the capacity constraint, the presence of the second unit presents an opportunity cost for a bidder, as winning the first unit would preclude the bidder from winning the second unit. Theoretically, a bidder should not always participate in the auction for the first unit,[37] and if they do, they should bid less aggressively than they would in a singe-unit auction. The experimental results confirm both of these phenomena in the lab. For the second unit, theory predicts that both bidders should bid more aggressively, provided they still face competition from the other bidder (which happens if they both forego the first unit).[38] This second phenomenon is not observed in the lab, and bidding on the second unit is indistinguishable from bidding in single-unit auctions. The strength of this paper is an in-depth analysis of bidder behavior. An interesting addition would be to take a viewpoint of an auctioneer offering two units for sale, and compare the sequential and simultaneous auctions with respect to efficiency and revenue.

Several papers consider non-auction alternatives to sequential sales. Salmon and Wilson (2008) study auctions with second-chance offers, where a loser of a single-unit English auction is given a take-it-or-leave-it offer to buy another unit, and find that such an auction/bargaining hybrid generates more revenue than two sequential English auctions. Fevrier, Linnemer and Visser (2007) study two-unit, two-buyer sequential auctions with or without a buyer option, which allows the winner of the first unit to buy the second unit at the same price. Theoretically, whether the buyer option has an effect on bidding behavior depends on the auction format, and on whether the individual demands are increasing or decreasing. The authors compare four standard auction institutions: English, Dutch, first-price sealed bid and second-price sealed bid, in environments with decreasing, flat and increasing demands for two units, and report that the buyer option was used correctly in most cases. However, they do not compare revenues between auctions with and without the buyer option.[39]

[37] A bidder should skip bidding for the first unit if the cost of the first object is high enough relative to that of the second object.

[38] This is because the presence of both bidders in the second auction indicates that both have a significant cost advantage in the second unit over the first one.

[39] The authors do compare revenues across auction formats though, and report that the revenue ranking for the four auction institutions was the same as in single-unit auctions.

7.2 Price trends in sequential auctions

Several experimental studies explore price trends in sequential auctions. Milgrom and Weber (1982) show, theoretically, that if a number of identical units are sold one after the other to risk-neutral bidders, then the resulting expected prices should be constant. In contrast, many empirical studies document a "declining price anomaly" where prices decline for later items (Ashenfelter 1989). Keser and Olson (1996) observe declining price trends in sequential first-price auctions for homogeneous goods with single-unit demands. Neugebauer and Pezanis-Christou (2007) show that declining price trends may be exacerbated by supply uncertainty, as the latter causes more aggressive bidding in early stages in the auction, as compared to a setting where the supply is certain.[40] This finding is in agreement with the studies discussed in Section 6, which find that varying supply may be used by the auctioneers to counter bidder collusion. Deltas and Kosmopoulou (2004) suggest that increasing variance in bids and a decreasing probability of sale, observed in a field experiment with sequential book sales, may be due to a reduction in attention of mail-in bidders, rather than bidder strategic behavior.

In summary, sequential multi-unit auctions, that are quite frequent in real-world settings, have not been studied by experimentalists as much as simultaneous auctions. There is some evidence, consistent with findings on simultaneous auctions, that first-price sealed bid auctions raise more revenue than other auction formats. The presence of synergies, or option values, affects bidding in early rounds. Some studies suggest that sequential auctions may be particularly vulnerable to bidder collusion (see Section 6). Many issues deserve further investigation including: choice of the auction format; sequencing of sales for heterogeneous goods; comparison with simultaneous auctions, or with other alternatives (such as bargaining) when applicable; and explaining price and sales trends. Bidder bounded rationality (such as limited attention span) is likely to play a role in explaining bidder behavior, much as in other complex multi-object settings.

8 Conclusions and open questions

In conclusion, we identify a few themes that cut across the various sections as well as discuss some areas that, in our opinion, warrant further experimental examination.

[40] Neugebauer and Pezanis-Christou (2007) investigate longer series of repetitions than Keser and Olson (1996), and do not observe declining price trends in the treatment with certain supply, but only because subjects with high values wait until later stages to bid, whereas subjects with low values bid more aggressively, which is inconsistent with theory.

8.1 Comparison of auction formats and alternative mechanisms

By far the most common theme of experimental research on multi-unit auctions is the comparison of various auction formats. While the efficacy of one auction format over another depends critically on the environment and performance measure, a few general trends can be identified. Sealed bid auctions tend to generate higher seller revenue in environments with both single and multiple unit demand without synergies. In environments with synergies, the sealed bid auction, with the exception of VCG, has been studied much less, and the revenue results for the VCG are mixed depending on the study (Chen and Takeuchi 2010, Scheffel et al. 2011). On the other hand, ascending auction formats appear to be uniformly attractive in terms of efficiency. The dynamic, feedback rich nature of ascending auctions seems to be important to generate efficient outcomes in the more complex multi-unit setting. The downside of the ascending auction is that it might also foster collusion and demand reduction. Interestingly, it might be the ability of the ascending auction to allow bidders to coordinate their behavior that makes the ascending auction work so well in environments with synergies. However, it is also evident that specific design elements of ascending combinatorial auctions such as price-guidance (RAD) or limited bidding combinations (HPB) might be important to further simplify the complex decision process for bidders. Unfortunately, we do not know theoretically (and experimentally) why these features usually work.

In practice, auctions rarely exist in a vacuum. Most experimental research on multi-unit auctions has focused on auctions in isolation from other institutions. While this is important to allow us to understand issues of auction performance, it might limit the practicality of many experimental results. For example, in procurement settings, it is frequently the case that some procurement contracts are written outside of an auction whereas others are the result of an auction. Engelbrecht-Wiggans and Katok (2006) investigate a mechanism that combines auctions with such noncompetitive contracts. There are a number of papers that look at hybrids of auctions with other mechanism. Second chance offers and buyer options (Salmon and Wilson 2008, Fevrier et al. 2007) were discussed in Section 7. Evans, Vossler and Flores (2009) study a hybrid allocation mechanism that combines features of auctions and lotteries for publicly provided goods.

As is evidenced by the protracted debate over the adoption of package bidding for FCC auctions, the decision to conduct and design of auctions frequently exists within a political or business decision making context. The process of decision making and the preferences of those decision makers needs to be considered when examining ultimate auction outcomes. Likewise, auctions are just one possible allocation mechanism; Banks et al. (1989) provide a comparison of various allocation mechanisms in addition to auctions for a problem that

involved a traditionally bureaucratic decision.[41] Given the complexity of the multi-unit environment, it is worth considering the performance of alternative mechanisms that may greatly simplify the allocation process or satisfy different performance measures.[42]

8.2 Asymmetric bidders, entry, and competition among auctions

As discussed by Klemperer (2002), encouraging entry and participation is considered to be one of the key elements of successful auction design. In many applications, such as procurement auctions, it is reasonable to perceive entrants as "weak" bidders, having either cost or information disadvantages compared to larger or more established incumbents. Indeed, a number of experiments consider environments with asymmetric bidders, typically modeling weak bidders as having a stochastic cost disadvantage (Abbink et al. 2005, Goeree et al. 2006). In contrast, Goeree et al. (forthcoming) model entrants as having a fixed-cost advantage over incumbents. Studies of environments with synergies explore bidder asymmetries in terms of objects demanded where local bidders demand only one objects and global bidders demand multiple objects thereby affording global bidders a strategic advantage (Katok and Roth 2004, Kagel and Levin 2005, Chernomaz and Levin 2012). With the exception of Zhang (2009), informational asymmetries in the multi-unit auction setting remain largely unexplored.[43]

Very few laboratory studies on multi-unit auctions, with the exception of Goeree et al. (2006), consider endogenous entry decisions, where bidders bear a cost (direct or opportunity) for entering the auction. In a single-unit auction setting with bidder choice over auction format, Ivanova-Stenzel and Salmon (2004) and Ivanova-Stenzel and Salmon (2008) show that ascending clock auctions are preferred by bidders and result in more entry as compared to first-price sealed bid auctions.[44] Buchanan, Gjerstad and Porter (2012) show that uncertainty about the number of competitors (as well as about the number of units for sale) increases revenue in a multi-unit, uniform-price Dutch auction. Carpenter, Holmes and Matthews (2008) provide field evidence that entry and participation play an important role in the revenue-raising properties of auctions. They compare first-price and second-price sealed bid auctions with all-pay sealed bid auctions in a multi-object charity auction field experiment. The authors find that the first-price auction raises more revenue than second-price or all-pay auctions, largely because they lead to more participation than the other two

[41] See also Olson and Porter (1994).

[42] Such comparisons have appeared outside of experimental research. For example, Park, Lee and Choi (2011) empirically compare spectrum auctions with beauty contests.

[43] Andreoni, Che and Kim (2007) consider informational asymmetries about rival types that may exist between incumbents and new entrants in a single-unit auction setting.

[44] See Kagel and Levin (2011) for a broader discussion of experiments related to entry.

auction formats.[45]

Although we do not cover internet auctions in our survey, there are many issues in auction design, such as choice of auction format, encouraging entry and increasing competition, that are relevant to both non-internet and internet auctions, and we expect both strands of research to benefit from each other's findings more in the future. For example, it is common on the internet for there to be many auctions or auction sites offering essentially equivalent products. Using a field experiment, Ely and Hossain (2009) show that bidders may use squatting (early bidding), rather than sniping (late bidding), to deter entry of competitors.[46] Competition between auction sites in terms of charging policies (e.g. fees to sellers and bidders) remains largely unexplored experimentally.

Finally, auctions may be used to create competition for the right to participate in a subsequent economic activity, which may greatly enhance the participants' performance in this activity. Van Huyck, Battalio and Beil (1993) show that auctioning off the rights to play a median effort coordination game remedies coordination failure and leads to coordination on the efficient high-output equilibrium of the game. Offerman and Potters (2006) demonstrate that auctioning off entry licenses in an oligopoly industry leads to higher prices as it facilitates tacit collusion among the sellers. This powerful role of auctioning off participation rights in enhancing performance has not been investigated enough and deserves more attention.

8.3 Behavioral models of bidding

Many experiments in the multi-unit auction setting have identified cognitive costs and observed limitations to rationality as key features in auction outcomes. Some examples are impatience (Plott and Salmon 2004), cognitive costs of bidding for multiple items at the same time (Isaac and Schnier 2005, List and Lucking-Reiley 2002), attention fatigue (Deltas and Kosmopoulou 2004), attention bias (Salmon and Iachini 2007), and not bidding for all items or bidding for too many items (Olson and Porter 1994, Chen and Takeuchi 2010, Chernomaz and Levin 2012). These behavioral features are no doubt important to take into account in auction design, and they deserve further investigation.

Recently, there has been significant progress in applying "behavioral" models of bidder behavior to single-unit auctions. Crawford and Iriberri (2007) demonstrate how a level-k model of boundedly rational bidders can explain bidding behavior in multiple single-unit auction environments. Likewise, anticipated regret is frequently identified as a salient

[45]The authors estimate that "...ceteris paribus and relative to second-price auctions, bidders are 14% more likely to participate in first-price auctions and 24% less likely to participate in all-pay auctions" (p. 104).

[46]Davis, Katok and Kwasnica (2012) examines a similar type strategy, as identified by Bulow and Klemperer (2009), in a single-unit auction with sequential arrivals. They find that, while early arriving bidders place preemptive bids, these bids do not sufficiently deter competition.

feature in auctions and can explain overbidding in simple auctions (Engelbrecht-Wiggans and Katok 2008, Filiz-Ozbay and Ozbay 2007). While it has been suggested that, due to the inherently asymmetric outcomes (one winner) of auctions, the issues of equity and reciprocity may not matter as much as in the other settings (Bolton and Ockenfels 2000), the multi-unit environment may make equity and fairness concerns more relevant. For example, equity concerns might reinforce collusive behavior since, under some strategies, bidder payoffs are less unequal than under competition. The application of models of social preferences (both theoretically and experimentally) to the multi-unit setting might provide improved explanations of observed bidder behavior as well as new normative insights regarding auction design.

References

Abbink, K, B Irlenbusch, P Pezanis-Christou, B Rockenbach, A Sadrieh, and R Selten, "An experimental test of design alternative for the British 3G/UMTS auction," *European Economic Review*, FEB 2005, *49* (2), 505–530.

Abbink, Klaus, Jordi Brandts, and Paul Pezanis-Christou, "Auctions for government securities: A laboratory comparison of uniform, discriminatory and Spanish designs," *Journal Of Economic Behavior & Organization*, OCT 2006, *61* (2), 284–303.

Adomavicius, Gediminas, Shawn P. Curley, Alok Gupta, and Pallab Sanyal, "Effect of Information Feedback on Bidder Behavior in Continuous Combinatorial Auctions," *Management Science*, 2012, *58* (4), 811–830.

Alsemgeest, P., C. Noussair, and M. Olson, "Experimental Comparisons of Auctions Under Single-and Multi-Unit Demand," *Economic Inquiry*, 1998, *36* (1), 87–97.

Andersson, Christer, Ola Andersson, and Tommy Andersson, "Sealed bid auctions versus ascending bid auctions: an experimental study," *Review of Economic Design*, 2012, pp. 1–16. 10.1007/s10058-012-0129-3.

Andreoni, James, Yeon-Koo Che, and Jinwoo Kim, "Asymmetric information about rivals' types in standard auctions: An experiment," *Games And Economic Behavior*, May 2007, *59* (2), 240–259.

Ariely, D, A Ockenfels, and AE Roth, "An experimental analysis of ending rules in Internet auctions," *RAND Journal Of Economics*, Winter 2005, *36* (4), 890–907.

Ashenfelter, O., "How auctions work for wine and art," *The Journal of Economic Perspectives*, 1989, *3* (3), 23–36.

Ausubel, Lawrence M., "Auctions (theory)," in Steven N. Durlauf and Lawrence E. Blume, eds., *The New Palgrave Dictionary of Economics*, Basingstoke: Palgrave Macmillan, 2008.

Ausubel, L.M., "An efficient ascending-bid auction for multiple objects," *The American Economic Review*, 2004, *94* (5), 1452–1475.

Bajari, P and A Hortacsu, "Economic insights from Internet auctions," *Journal Of Economic Literature*, June 2004, *42* (2), 457–486.

Bajari, Patrick, "Auctions (applications)," in Steven N. Durlauf and Lawrence E. Blume, eds., *The New Palgrave Dictionary of Economics*, Basingstoke: Palgrave Macmillan, 2008.

Banks, J, M Olson, D Porter, S Rassenti, and V Smith, "Theory, experiment and the federal communications commission spectrum auctions," *Journal Of Economic Behavior & Organization*, July 2003, *51* (3), 303–350.

Banks, Jeffrey S., John O. Ledyard, and David P. Porter, "Jeffrey S. Banks and John O. Ledyard and David P. Porter," *RAND Journal of Economics*, 1989, *20* (1), 1–25.

Barut, Y, D Kovenock, and CN Noussair, "A comparison of multiple-unit all-pay and winner-pay auctions under incomplete information," *International Economic Review*, August 2002, *43* (3), 675–708. International Conference on New Developments in Experimental Economics, OSAKA UNIV, OSAKA, JAPAN, MAR 03-05, 1999.

Bikhchandani, S. and J. M. Ostroy, "The package assignment model," *Journal of Economic Theory*, 2002, *107* (2), 377–406.

Bolton, Gary E. and Axel Ockenfels, "ERC: A Theory of Equity, Reciprocity, and Competition," *American Economic Review*, March 2000, *90* (1), 166–193.

Brosig, Jeannette and J. Philipp Reiss, "Entry decisions and bidding behavior in sequential first-price procurement auctions: An experimental study," *Games and Economic Behavior*, January 2007, *58* (1), 50–74.

Brown, Alexander L., Charles R. Plott, and Heidi J. Sullivan, "Collusion Facilitating And Collusion Breaking Power of Simultaneous Ascending and Descending Price Auctions," *Economic Inquiry*, July 2009, *47* (3), 395–424.

Brunner, Christoph, Jacob K. Goeree, Charles A. Holt, and John O. Ledyard, "An Experimental Test of Flexible Combinatorial Spectrum Auction Formats," *American Economic Journal-Microeconomics*, February 2010, *2* (1), 39–57.

Brusco, S. and G. Lopomo, "Collusion via signalling in simultaneous ascending bid auctions with heterogeneous objects, with and without complementarities," *Review of Economic Studies*, 2002, *69* (2), 407–436.

Buchanan, Joy A., Steven Gjerstad, and David P. Porter, "Information Effects in Multi-Unit Dutch Auctions," Working Papers 12-08, Chapman University, Economic Science Institute April 2012.

Bulow, Jeremy and Paul Klemperer, "Why Do Sellers (Usually) Prefer Auctions?," *American Economic Review*, 2009, *99* (4), 1544–1575.

Burns, P., "Market structure and buyer behaviour: Price adjustment in a multi-object progressive oral auction," *Journal of Economic Behavior & Organization*, 1985, *6* (3), 275–300.

Burtraw, Dallas, Jacob Goeree, Charles A. Holt, Erica Myers, Karen Palmer, and William Shobe, "Collusion in auctions for emission permits: An experimental analysis," *Journal of Policy Analysis and Management*, 2009, *28* (4), 672–691.

Bykowsky, MM, RJ Cull, and JO Ledyard, "Mutually destructive bidding: The FCC auction design problem," *Journal Of Regulatory Economics*, May 2000, *17* (3), 205–228.

Carpenter, Jeffrey, Jessica Holmes, and Peter Hans Matthews, "Charity auctions: A field experiment," *Economic Journal*, JAN 2008, *118* (525), 92–113.

Chen, Yan and Kan Takeuchi, "Multi-object auctions with package bidding: An experimental comparison of Vickrey and iBEA," *Games And Economic Behavior*, March 2010, *68* (2), 557–579.

Chernomaz, Kirill and Dan Levin, "Efficiency and synergy in a multi-unit auction with and without package bidding: An experimental study," *Games and Economic Behavior*, 2012, *76* (2), 611 – 635.

Cox, James C., Vernon L. Smith, and James M. Walker, "Theory and Behavior of Multiple Unit Discriminative Auctions," *The Journal of Finance*, 1984, *39* (4), 983–1010.

Cramton, P. and J. Schwartz, "Collusive bidding in the fcc spectrum auctions," *Contributions to Economic Analysis & Policy*, 2002, *1* (1), 1078–1078.

Cramton, Peter, Emel Filiz-Ozbay, ErkutY. Ozbay, and Pacharasut Sujarittanonta, "Discrete clock auctions: an experimental study," *Experimental Economics*, 2012, *15*, 309–322.

Crawford, Vincent and Nagore Iriberri, "Level-k auctions: Can a nonequilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions?," *Econometrica*, November 2007, *75* (6), 1721–1770.

Damianov, Damian S., Joerg Oechssler, and Johannes Gerd Becker, "Uniform vs. discriminatory auctions with variable supply - experimental evidence," *Games and Economic Behavior*, January 2010, *68* (1), 60–76.

Davis, Andrew, Elena Katok, and Anthony M. Kwasnica, "A Laboratory Comparison of Auctions and Sequential Mechanisms," 2012. unpublished.

Davis, DD and BJ Wilson, "Collusion in procurement auctions: An experimental examination," *Economic Inquiry*, April 2002, *40* (2), 213–230.

Deltas, G and G Kosmopoulou, "'Catalogue' vs 'order-of-sale' effects in sequential auctions: Theory and evidence from a rare book sale," *Economic Journal*, January 2004, *114* (492), 28–54.

Demange, G., D. Gale, and M. Sotomayor, "Multi-item auctions," *The Journal of Political Economy*, 1986, pp. 863–872.

Efiaz, Kfir, Theo Offerman, and Andrew Schotter, "Creating competition out of thin air: An experimental study of right-to-choose auctions," *Games and Economic Behavior*, March 2008, *62* (2), 383–416.

Ely, Jeffrey C. and Tanjim Hossain, "Sniping and Squatting in Auction Markets," *American Economic Journal-Microeconomics*, August 2009, *1* (2), 68–94.

Engelbrecht-Wiggans, R and E Katok, "E-sourcing in procurement: Theory and behavior in reverse auctions with noncompetitive contracts," *Management Science*, April 2006, *52* (4), 581–596.

____, JA List, and DH Reiley, "Demand reduction in multi-unit auctions with varying numbers of bidders: Theory and evidence from a field experiment," *International Economic Review*, February 2006, *47* (1), 203–231.

Engelbrecht-Wiggans, Richard and Elena Katok, "Regret and Feedback Information in First-Price Sealed-Bid Auctions," *Management Science*, 2008, *54* (4), 808–819.

Engelmann, Dirk and Veronika Grimm, "Bidding Behaviour in Multi-Unit Auctions - An Experimental Investigation," *Economic Journal*, April 2009, *119* (537), 855–882.

Englmaier, Florian, Pablo Guillén, Loreto Llorente, Sander Onderstal, and Rupert Sausgruber, "The chopstick auction: A study of the exposure problem in multi-unit auctions," *International Journal of Industrial Organization*, 2009, *27* (2), 286 – 291.

Evans, Mary F., Christian A. Vossler, and Nicholas E. Flores, "Hybrid allocation mechanisms for publicly provided goods," *Journal of Public Economics*, February 2009, *93* (1-2), 311–325.

Fevrier, Philippe, Laurent Linnemer, and Michael Visser, "Buy or wait, that is the option: the buyer's option in sequential laboratory auctions," *RAND Journal of Economics*, Spring 2007, *38* (1), 98–118.

Filiz-Ozbay, Emel and Erkut Y. Ozbay, "Auctions with Anticipated Regret: Theory and Experiment," *American Economic Review*, 2007, *97* (4), 1407–1418.

Goeree, Jacob, Charles Plott, and John Wooders, "Bidders' Choice Auctions: Raising Revenues Through the Right to Choose," *Journal of the European Economic Association*, April,May 2004, *2* (2-3), 504–515.

Goeree, Jacob K. and Charles A. Holt, "Hierarchical package bidding: A paper & pencil combinatorial auction," *Games and Economic Behavior*, September 2010, *70* (1, SI), 146–169.

Goeree, Jacob, Theo Offerman, and Randolph Sloof, "Demand reduction and preemptive bidding in multi-unit license auctions," *Experimental Economics*, forthcoming, pp. 1–36. 10.1007/s10683-012-9338-4.

Goeree, JK, T Offerman, and A Schram, "Using first-price auctions to sell heterogeneous licenses," *International Journal of Industrial Organization*, May 2006, *24* (3), 555–581.

Grether, David M. and Charles R. Plott, "Sequencing strategies in large, competitive, ascending price automobile auctions: An experimental examination," *Journal of Economic Behavior & Organization*, August 2009, *71* (2), 75–88.

Huyck, John B. Van, Raymond C. Battalio, and Richard O. Beil, "Asset markets as an equilibrium selection mechanism: Coordination failure, game form auctions, and tacit communication," *Games and Economic Behavior*, 1993, *5* (3), 485–504.

Isaac, R. Mark and Duncan James, "Robustness of the incentive compatible combinatorial auction," *Experimental Economics*, 2000, *3* (31-53).

___ , Timothy C. Salmon, and Arthur Zillante, "A theory of jump bidding in ascending auctions," *Journal Of Economic Behavior & Organization*, JAN 2007, *62* (1), 144–164.

Isaac, RM and K Schnier, "Silent auctions in the field and in the laboratory," *Economic Inquiry*, October 2005, *43* (4), 715–733.

___ , TC Salmon, and A Zillante, "An experimental test of alternative models of bidding in ascending auctions," *International Journal of Game Theory*, JUN 2005, *33* (2), 287–313.

Ivanova-Stenzel, Radosveta and Timothy C. Salmon, "Bidder Preferences Among Auction Institutions," *Economic Inquiry*, 2004, *42* (2), 223 – 236.

___ and ___ , "Revenue equivalence revisited," *Games and Economic Behavior*, 2008, *64* (1), 171 – 192.

Kagel, JH and D Levin, "Multi-unit demand auctions with synergies: behavior in sealed-bid versus ascending-bid uniform-price auctions," *Games and Economic Behavior*, November 2005, *53* (2), 170–207.

Kagel, J.H. and D. Levin, "Auctions: A Survey of Experimental Research, 1995-2010," 2011. In preparation for Handbook of Experimental Economics 2nd edition.

Kagel, John H., "Auctions: A Survey of Experimental Research," in John H. Kagel and Alvin E. Roth, eds., *The Hanbook of Experimental Economics*, Princeton University Press, 1995, chapter 7, pp. 501–535.

___ and Dan Levin, "Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickrey Auctions," *Econometrica*, 2001, *69* (2), pp. 413–454.

___ **and** ___ , "Implementing efficient multi-object auction institutions: An experimental study of the performance of boundedly rational agents," *Games and Economic Behavior*, May 2009, *66* (1), 221–237.

___ **, Yuanchuan Lien, and Paul Milgrom**, "Ascending Prices and Package Bidding: A Theoretical and Experimental Analysis," *American Economic Journal-Microeconomics*, August 2010, *2* (3), 160–185.

___ **,** ___ **, and** ___ , "Ascending Prices and Package Bidding: Further Experimental Analysis," March 2012. unpublished.

Katok, E and AE Roth, "Auctions of homogeneous goods with increasing returns: Experimental comparison of alternative "Dutch" auctions," *Management Science*, August 2004, *50* (8), 1044–1063.

Keser, C. and M. Olson, "Experimental examination of the declining-price anomaly," *Contributions to Economic Analysis*, 1996, *237*, 151–176.

Klemperer, P, "What really matters in auction design," *Journal of Economic Perspectives*, Winter 2002, *16* (1), 169–189.

Krishna, Vijay, *Auction Theory*, Academic Press, 2002.

___ **and Robert W. Rosenthal**, "Simultaneous Auctions with Synergies," *Games and Economic Behavior*, November 1996, *17* (1), 1–31.

Kwasnica, AM, "The choice of cooperative strategies in sealed bid auctions," *Journal of Economic Behavior & Organization*, July 2000, *42* (3), 323–346.

___ **, JO Ledyard, D Porter, and C DeMartini**, "A new and improved design for multiobject iterative auctions," *Management Science*, March 2005, *51* (3), 419–434.

Kwasnica, Anthony and Elena Katok, "The Effect of Timing on Bid Increments in Ascending Auctions," *Production and Operations Management*, August 2007, *16* (4), 483–494.

Kwasnica, Anthony M. and Katerina Sherstyuk, "Collusion and equilibrium selection in auctions," *Economic Journal*, January 2007, *117* (516), 120–145.

Ledyard, John O., Mark Olson, David Porter, Joseph A. Swanson, and David P. Torma, "The First Use of a Combined Value Auction for Transportation Services," *Interfaces*, 2002, *32* (5), 4–12.

Leonard, H.B., "Elicitation of honest preferences for the assignment of individuals to positions," *The Journal of Political Economy*, 1983, pp. 461–479.

Leufkens, Kasper, Ronald Peeters, and Marc Vorsatz, "An Experimental Comparison of Sequential First- and Second-Price Auctions with Synergies," *B E Journal of Theoretical Economics*, 2012, *12* (1).

Levin, Daan, John H. Kagel, and Jean-Francois Richard, "Revenue Effects and Information Processing in English Common Value Auctions.," *American Economic Review*, 1996, *86* (3), 442 – 460.

Li, Jin and Charles R. Plott, "Tacit Collusion in Auctions And Conditions for its Facilitation and Prevention: Equilibrium Selection in Laboratory Experimental Markets," *Economic Inquiry*, July 2009, *47* (3), 425–448.

List, JA and D Lucking-Reiley, "Demand reduction in multiunit auctions: Evidence from a sportscard field experiment," *American Economic Review*, September 2000, *90* (4), 961–972.

___ **and** ___ , "Bidding behavior and decision costs in field experiments," *Economic Inquiry*, October 2002, *40* (4), 611–619.

Lunander, A and JE Nilsson, "Taking the lab to the field: Experimental tests of alternative mechanisms to procure multiple contracts," *Journal of Regulatory Economics*, January 2004, *25* (1), 39–58.

Lunander, Anders and Jan-Eric Nilsson, "Combinatorial Procurement Auctions: A Collusion Remedy?," *Rivista di Politica Economica*, January-February 2006, *96* (1), 65–90.

Manelli, Alejandro M., Martin Sefton, and Benjamin S. Wilner, "Multi-unit auctions: A comparison of static and dynamic mechanisms," *Journal of Economic Behavior & Organization*, October 2006, *61* (2), 304–323.

Merlob, Brian, Charles R. Plott, and Yuanjun Zhang, "The CMS Auction: Experimental Studies of a Median-Bid Procurement Auction with Nonbinding Bids," *Quarterly Journal of Economics*, May 2012, *127* (2), 793–827.

Milgrom, P.R. and R.J. Weber, "A theory of auctions and competitive bidding," *Econometrica: Journal of the Econometric Society*, 1982, pp. 1089–1122.

Mougeot, Michel, Florence Naegelen, Benjamin Pelloux, and Jean-Louis Rulliere, "breaking collusion in auctions through speculation: An experiment on co2 emission permit markets," *Journal of Public Economic Theory*, October 2011, *13* (5), 829–856.

Neugebauer, Tibor and Paul Pezanis-Christou, "Bidding behavior at sequential first-price auctions with(out) supply uncertainty: A laboratory analysis," *Journal of Economic Behavior & Organization*, May 2007, *63* (1), 55–72.

Normann, Hans-Theo and Roberto Ricciuti, "Laboratory experiments for economic policy making," *Journal of Economic Surveys*, July 2009, *23* (3), 407–432.

Noussair, Charles, "Innovations in the design of bundled-item auctions," *Proceedings of the National Academy of Sciences*, September 2003, *100* (19), 10590–10591.

Ockenfels, Axel, David H. Reiley, and Abdolkarim Sadrieh, "Online Auctions," Technical Report 12785, NBER Working Paper Deember 2006.

Offerman, T and J Potters, "Does auctioning of entry licences induce collusion? An experimental study," *Review of Economic Studies*, JUL 2006, *73* (3), 769–791.

Olson, M. and D. Porter, "An experimental examination into the design of decentralized methods to solve the assignment problem with and without money," *Economic Theory*, 1994, *4* (1), 11–40.

Park, Minsoo, Sang-Woo Lee, and Yong-Jae Choi, "Does spectrum auctioning harm consumers? Lessons from 3G licensing," *Information Economics and Policy*, March 2011, *23* (1), 118–126.

Parkes, David and Lyle Ungar, "An ascending-price generalized Vickrey auction," in "Proceedings of 2002 Stanford Institute for Theoretical Economics Workshop on The Economics of the Internet" Stanford, CA 2002.

Perrigne, Isabelle and Quang Vuong, "Auctions (empirics)," in Steven N. Durlauf and Lawrence E. Blume, eds., *The New Palgrave Dictionary of Economics*, Basingstoke: Palgrave Macmillan, 2008.

Pesendorfer, M., "A study of collusion in first-price auctions," *The Review of Economic Studies*, 2000, *67* (3), 381–411.

Phillips, OR, DJ Menkhaus, and KT Coatney, "Collusive practices in repeated English auctions: Experimental evidence on bidding rings," *American Economic Review*, June 2003, *93* (3), 965–979.

Phillips, Owen R. and Dale J. Menkhaus, "Maintaining Tacit Collusion in Repeated Ascending Auctions," *Journal of Law & Economics*, February 2009, *52* (1), 91–109.

Plott, CR and TC Salmon, "The simultaneous, ascending auction: dynamics of price adjustment in experiments and in the UK3G spectrum auction," *Journal of Economic Behavior & Organization*, March 2004, *53* (3), 353–383.

Porter, D. and R. Vragov, "An experimental examination of demand reduction in multi-unit versions of the Uniform-price, Vickrey, and English auctions," *Managerial and Decision Economics*, 2006, *27* (6), 445–458.

Porter, David P., "The effect of bid withdrawal in a multi-object auction," *Review of Economic Design*, 1999, *4*, 73–97.

Porter, David, Stephen Rassenti, Anil Roopnarine, and Vernon Smith, "Combinatorial auction deisgn," *Proceedings of the National Academy of Sciences*, 2003, *100* (19), 11153–11157.

Rassenti, Steven J., Vernon L. Smith, and R. L. Bulfin, "A Combinatorial Auction Mechanism for Airport Time Slot Allocation," *Bell Journal of Economics*, 1982, *13*, 402–417.

Roth, A., "Experiments in market design," 2012. In preparation for Handbook of Experimental Economics 2nd Edition.

Rothkopf, Michael H., "Decision Analysis: The Right Tool for Auctions," *Decision Analysis*, September 2007, *4* (3), 167–172.

___ , Aleksandar Pekeč, and Ronald M. Harstad, "Computationally Manageable Combinational Auctions," *Management Science*, August 1998, *44* (8), 1131–1147.

Sade, O, C Schnitzlein, and JF Zender, "Competition and cooperation in divisible good auctions: An experimental examination," *Review of Financial Studies*, Spring 2006, *19* (1), 195–235.

Salmon, Timothy C. and Bart J. Wilson, "Second chance offers versus sequential auctions: theory and behavior," *Economic Theory*, January 2008, *34* (1), 47–67.

___ **and Michael Iachini**, "Continuous ascending vs. pooled multiple unit auctions," *Games and Economic Behavior*, October 2007, *61* (1), 67–85.

Santamaría, Natalia, "Competing Against Oneself in Sealed-Bid Combinatorial Auctions," Working Papers, SSRN Working Paper March 2012.

Scheffel, Tobias, Alexander Pikovsky, Martin Bichler, and Kemal Guler, "An Experimental Comparison of Linear and Nonlinear Price Combinatorial Auctions," *Information Systems Research*, 2011, *22* (2), 346–368.

Sherstyuk, K., "Collusion without conspiracy: An experimental study of one-sided auctions," *Experimental Economics*, 1999, *2* (1), 59–75.

Sherstyuk, K, "Collusion in private value ascending price auctions," *Journal of Economic Behavior & Organization*, June 2002, *48* (2), 177–195.

Sherstyuk, K., "Some results on anti-competitive behavior in multi-unit ascending price auctions," *Handbook of Experimental Economics Results*, 2008, *1*, 185–198.

Sherstyuk, Katerina, "A comparison of first price multi-object auctions," *Experimental Economics*, March 2009, *12* (1), 42–64.

___ **and Jeremy Dulatre**, "Market performance and collusion in sequential and simultaneous multi-object auctions: Evidence from an ascending auctions experiment," *International Journal of Industrial Organization*, March 2008, *26* (2), 557–572.

Shogren, JF, M Margolis, C Koo, and JA List, "A random nth-price auction," *Journal of Economic Behavior & Organization*, December 2001, *46* (4), 409–421.

Smith, V.L., "Experimental studies of discrimination versus competition in sealed-bid auction markets," *Journal of Business*, 1967, pp. 56–84.

Sutter, Matthias, Martin G. Kocher, and Sabine Strauss, "Individuals and teams in auctions," *Oxford Economic Papers-New Series*, April 2009, *61* (2), 380–394.

Vickrey, W., "Counterspeculation, auctions, and competitive sealed tenders," *The Journal of Finance*, 1961, *16* (1), 8–37.

Zhang, Ping, "Uniform price auctions and fixed price offerings in IPOs: an experimental comparison," *Experimental Economics*, June 2009, *12* (2), 202–219.