

IMPATIENCE AND INTERGENERATIONAL EQUITY IN A MODEL OF SUSTAINABLE GROWTH

by

Lee Endress

Asia Pacific Center for Security Studies, Hawaii

Sittidaj Pongkijvorasin

Chulalongkorn University, Bangkok, Thailand

James Roumasset

Economics Department, University of Hawaii
Manoa, 2424 Maile Way, Honolulu, HI 96822, USA.
Tel.: 1 808 956 7496; Fax: 1 808 956 4347; jimr@hawaii.edu

Basharat Pitafi

Southern Illinois University, Carbondale

Working Paper No. 09-6

May 14, 2009

Abstract

We argue that intergenerational neutrality has been prematurely excluded from the dialogue on sustainable growth. By incorporating Burton's distinction between intragenerational and intergenerational discounting into a model suitable for analyzing sustainability issues, we are able to accommodate some of the underlying concerns. We show that in an economy with a renewable resource, eschewing intergenerational discounting leads to the implication of a sustained growth path, without the necessity of a sustainability constraint. We find that green net national product remains constant along the optimal approach path to golden rule consumption. This avoids the paradox that maximizing sustainable income leads to unsustainable consumption and income.

Keywords: Sustainable development, intergenerational equity, intra-generational discounting, renewable resources, green net national product

JEL Codes: Q56, Q41, Q01

IMPATIENCE AND INTERGENERATIONAL EQUITY IN A MODEL OF SUSTAINABLE GROWTH

If I am not for myself, then who is for me? If I am not for others, who is for me?
Rabbi Hillel¹

1. Introduction

Neoclassical theory of sustainable growth has required a positive utility discount rate. This is odd because sustainability has everything to do with intergenerational equity (see e.g. Solow, 1993). Moreover, intergenerational equity is often interpreted as requiring a zero rate of pure time preference. Ramsey (1928) is often cited for his forceful pronouncement that discounting is “ethically indefensible”. Rawls (1971) explicitly rejected applying his maxi-min principle to intergenerational equity, and Solow has opined from a Rawlsian impartial spectator perspective (“in solemn conclaves assembled,” Solow, 1974) that “we ought to act as if the social rate of time preference were zero.” Pigou (1920) stated that pure time preference implies that “our telescopic faculty is defective,” and Harrod (1948) said that “pure time preference is a polite expression for the conquest of reason by passion.” Koopmans expressed “an ethical preference for neutrality as between the welfare of different generations.” Moreover, the optimal growth trajectory with positive utility discounting in an economy with an initial endowment of a non-renewable resource is unsustainable in the sense that the optimal consumption trajectory is eventually declining (Dasgupta and Heal, 1979; Endress et al., 2005). Despite these advantages of intergenerational neutrality, the assumption has been challenged on technical, empirical and moral grounds.

The technical difficulty with a utility discount rate of zero is discussed for example by Heal (1993) and is sometimes referred to as the “cake-eating problem.” In a simple cake-

¹ This and the quotes in the first paragraph can be found in Arrow (1999, p. 14).

eating model of a finite resource, Heal (1993) shows that zero utility discounting in such an economy implies that any arbitrarily-small initial consumption level is dominated by a still lower initial consumption level. This gloomy prospect is not a logical conundrum – indeed a complete ordering of all consumption paths exists. It is rather the dismal consequence of contemplating an economy wherein consumption cannot be sustained. Advocating a positive discount rate in this case results in condemning all generations born beyond some future date to zero consumption – hardly consistent with intergenerational equity.

Olson and Bailey (1981) reject zero discounting on empirical grounds. Using a partial equilibrium model with an exogenous positive interest rate, they argue that an individual rate of pure time preference equal to zero would impoverish the present. People would reduce current consumption down to the subsistence level to provide for future generations. Since people in fact do not choose subsistence in the present, the individual rate of pure time preference must be positive. A social welfare function without utility discounting would therefore seem to be in violation of the requirement that individual preferences must matter to the benevolent social planner.

Arrow (1999) and Dasgupta (2001) make a similar argument but emphasize the moral unacceptability of draconian savings in the first period. They show that in a Ramsey (1928) optimal growth framework with zero utility discounting and an elasticity of marginal “felicity” of consumption of 1.5 or less, the savings rate in generation one must be two thirds or more.² This would appear to discriminate unfairly against the present generation and violate the moral principle of *universalizability*.³ There are two highly restrictive assumptions hidden in this

² For details of the calculation, see Dasgupta (2001), p. 255.

³ Arrow cites the empirical studies in IPCC (1995) as justification. One difficulty in applying the universalizability criterion, however, is that the ethical grounds for the criterion, starting with Rabbi Hillel, derive from discussions

calculation, however. One is that said elasticity has been estimated from revealed preference. As Arrow et al. have later acknowledged, however, the relevant elasticity concerns the *planner's* tolerance for intergenerational inequality, not that of an individual generation. The other problem is that intergenerational neutrality has been taken to imply zero utility discounting even within a generation, an assumption that we see below is unnecessary.

Intergenerational equity, and sustainability more generally, are inherently normative inquiries. Consumer impatience does not necessarily preclude a social planner from applying neutral weighting across generations as a matter of normative policy. But how can a social welfare function embody intergenerational neutrality and be simultaneously based on individual lifetime utilities and the personal rates of time preference that they embody? A way out of the dilemma can be found by following Burton (1993).

Burton shows that inter and intragenerational discount rates affect the optimal resource harvesting decisions in different ways. He observes that the standard approach in resource economics of maximizing a utilitarian welfare function implicitly assumes that inter and intragenerational discount rates are equal thereby confounding the normative standard of intergenerational equity and the positive impatience of members of society within their own lifetimes.⁴ Burton incorporates both considerations into models of optimal resource harvesting by postulating two separate discount rates: a personal discount rate, designated in the subsequent development by β , which reflects the rate of pure time-preference of individuals; and a generational discount rate, ρ , which addresses society's degree of concern for

of a finite number of individuals or income groups. As noted by Rawls (1971), they do not apply to the case of an infinite number of generations.

⁴ Olson-Bailey (1981), Arrow (1999), and Dasgupta (2001) criticize zero utility discounting, albeit without recognizing this distinction.

intergenerational equity.⁵ We apply this approach below in order to explore the consequences of intergenerational neutrality, i.e., $\rho = 0$. In order to engage the sustainable growth literature, we extend the basic model to allow both produced capital and renewable resources.⁶

In summary, our objective is to extend the neoclassical approach to sustainable growth in order to distinguish between intragenerational impatience and intergenerational equity. We show that incorporating empirical information about intragenerational impatience into a model with intergenerational neutrality does not affect the golden rule level of aggregate consumption and that optimal trajectory of aggregate consumption is rising. This implies that a sustainability constraint, if imposed, would be non-binding in the model. We also show that sustainable income, defined as Green Net National Product (GNNP), is actually sustained along the optimal path, avoiding a paradox in the conventional approach.

The paper is organized as follows. In section 2, we extend the neoclassical approach to sustainable growth to allow separate intragenerational and intergenerational discount rates. The social planner respects consumer sovereignty regarding lifetime utility, thus incorporating the individual's personal rate of time preference via intragenerational discounting. The intergenerational discount rate is based on the planner's preferences regarding intergenerational equity. In section 3, we consider the special case of intergenerational neutrality ($\rho = 0$). Section 4 summarizes and concludes.

2. General model and results

⁵ Blanchard and Fisher (1989, ch 3) make similar distinction in developing the command optimum in a simple overlapping generations (OLG) model in which each generation lives for two periods and the planner applies generation weighting.

⁶ Burton's objective was to analyze the consequences of distinguishing the two concepts of discounting for optimal resource extraction. For this purpose, he confined his analysis to cake-eating models of a constant-population, resource-based economy.

Burton (1993) examined implications of distinguishing intra and intergenerational discounting for optimal resource extraction in a cake-eating economy. In what follows, we use a similar distinction to examine issues relating to sustainable growth albeit in a production economy with renewable resources. This allows the planner to respect consumer sovereignty regarding impatience within a generation while simultaneously imposing different weighting between generations. We assume that the production function is $Y = F(K, R)$, where K is the stock of capital that depreciates at a constant rate, δ , and R is the extraction from a renewable natural resource stock, X , that grows at rate $G(X)$, and is extracted at a constant unit cost, θ .

We assume that the economy is made up of overlapping generations of otherwise identical individuals. In the simplest representation, each generation contains one individual who lives to age N . At each time t , society is made of individuals who range in age from 0 to N , and no two individuals have the same age. An individual of age τ is allotted consumption good in amount $c(t, \tau)$ and enjoys utility $u(c(t, \tau))$. We assume identical utility functions across individuals and time periods.

An individual born at time T measures remaining lifetime utility, U_T , according to the formula,⁷

$$U_T = \begin{cases} \int_{\tau=0}^N u(c(T + \tau, \tau)) e^{-\beta\tau} d\tau & \text{for } T \geq 0 \\ \int_{\tau=-T}^N u(c(T + \tau, \tau)) e^{-\beta\tau} d\tau & \text{for } -N \leq T < 0 \end{cases} . \quad (3)$$

⁷ We replace Burton's discrete-time formulation with continuous time to facilitate optimal control. Similarly, Burton's discounting factors α and β are replaced with $e^{-\rho}$ and $e^{-\beta}$.

Social welfare is then a weighted sum of the lifetime utilities of individual members of society, where the weights are based on the generational discount rate, ρ .

$$W = \int_{T=-N}^{\infty} U_T e^{-\rho T} dT \quad (4)$$

Thus, the social planner's problem is to choose consumption for each individual at each time $c(t, \tau)$, $\forall t, \tau$ (a time path for each individual's consumption) subject to the equations of motion of the two stocks, i.e.,

$$\begin{aligned} \text{Max}_{c(t, \tau)} \quad & W, \quad W = \int_{T=-N}^{\infty} U_T e^{-\rho T} dT \\ \text{s.t.} \quad & \frac{dK}{dt} = F(K, R) - \delta K - \theta R - \int_0^N c(t, \tau) d\tau, \quad K(0) = K_0 \\ & \frac{dX}{dt} = G(X) - R, \quad X(0) = X_0 \end{aligned} \quad (5)$$

This can be solved as an optimal control problem. Since the two equations of motion are describing rates of change in terms of pure time, t , application of the Maximum Principle to this problem would be greatly simplified by reformulating the objective function in terms of time, t , instead of the generational index, T . For this purpose, we switch from separability of social welfare by individual to separability by time period, following Burton's welfare trade-off analysis, involving both generational and personal discount factors (Burton (1993), p. 122-125). We can then re-write the optimization problem (5) as:

$$\begin{aligned} \text{Max} \quad & W, \quad W = \int_0^{\infty} V(C) e^{-\rho t} dt \\ \text{st.} \quad & \frac{dK}{dt} = F(K, R) - \delta K - \theta R - C(t), \quad K(0) = K_0 \\ & \frac{dX}{dt} = G(X) - R, \quad X(0) = X_0 \end{aligned} \quad (6)$$

where

$$V(C(t)) = \text{Max} \int_0^N u(c(t, \tau)) e^{-(\beta-\rho)\tau} d\tau$$

$$\text{s.t.} \int_0^N c(t, \tau) d\tau \leq C(t)$$

where $C(t)$ representing the aggregate level of output available for consumption at time t , and V representing aggregate utility.

This involves two steps. First, we provide the first order conditions from the maximization problem that determines V .

To solve for $V(C)$, form the Lagrangian:

$$\mathcal{L} = \int_0^N u(c(t, \tau)) e^{-(\beta-\rho)\tau} d\tau + \lambda \left[C(t) - \int_0^N c(t, \tau) d\tau \right] \quad (7)$$

Inspection of the Lagrangian indicates that a necessary condition for maximization is that the weighted marginal utilities be equated, so that along the optimum consumption path, c^* , is

$$u'(c^*(t, \tau)) = u'(c^*(t, 0)) e^{(\beta-\rho)\tau}, \quad 0 \leq \tau \leq N \quad (8)$$

From equation (8), each generation's share of aggregate consumption is determined by the difference between the intra and intergenerational discount rates. Suppose $\beta > \rho$, i.e., the personal discount rate is greater than the generational discount rate. It then follows directly from equation (8) that $u'(c^*(t, \tau)) > u'(c^*(t, 0))$, i.e., that marginal utility is lower for younger individuals and $c^*(t, \tau) < c^*(t, 0)$. That is, at any time t , a younger individual consumes a larger share of society's aggregate consumption than an older one. On the other hand, if the intergenerational discount rate is higher than the intragenerational one (i.e., $\beta < \rho$), a younger individual will consume a smaller share of aggregate consumption. All generations share the same share when intra and intergenerational discount rates are equal ($\beta = \rho$).

Following the standard approach in neoclassical sustainable growth theory, we now assume that utility function is in the form of constant elasticity of marginal utility: i.e.,

$$u(c(t, \tau)) = -[c(t, \tau)]^{-(\eta-1)}, \quad \eta > 1 \quad (9)$$

Equation (9) then implies that at the optimum,

$$c^*(t, \tau) = c^*(t, 0)e^{-(\beta-\rho)\tau/\eta} \quad (10)$$

As shown in the Appendix, this functional specification permits the computation of the utility of aggregate consumption:

$$V(C) = -AC^{-(\eta-1)}, \quad (11)$$

where A represents the “aggregation coefficient”, which depends on the parameters β , ρ , and η .

A few observations are in order with respect to this result. First, consider the effect on aggregate marginal utility of an increase in the personal discount rate, β :

$$\frac{\partial V(C)}{\partial C} = A(\eta-1)C^{-\eta} \quad (12)$$

$$\frac{\partial^2 V(C)}{\partial \beta \partial C} = \frac{\partial A}{\partial \beta}(\eta-1)C^{-\eta} \quad (13)$$

A tedious but straight-forward computation (see Appendix) shows that:

$$\frac{\partial A}{\partial \beta} < 0, \quad \text{so that} \quad \frac{\partial^2 V}{\partial \beta \partial C} < 0 \quad (14)$$

This comparative static provides analytical validation of Burton’s (1993) remark that a higher personal discount rate results in lower marginal utility of aggregate consumption for society, i.e., aggregate consumption increases. Similar comparative static analysis shows that

$$\frac{\partial A}{\partial \rho} > 0, \quad \text{so that} \quad \frac{\partial^2 V}{\partial \rho \partial C} > 0 \quad (15)$$

Thus, an increase in the generational discount rate increases the marginal utility of society, i.e., i.e., aggregate consumption decreases.

We now form the current-value Hamiltonian for maximizing W :

$$H = V(C) + \lambda [F(K, R) - \theta R - C] + \psi [G(X) - R] \quad (16)$$

Application of the maximum principle yields the standard first order conditions, which can be manipulated to generate the Ramsey condition and generalized form of the Hotelling rule for renewable resources:

$$\frac{V''(C)}{V'(C)} \dot{C} = \rho - (F_k - \delta) \quad (17)$$

and

$$(F_R - \theta) = \frac{1}{(F_k - \delta)} \{ \dot{F}_R + (F_R - \theta) G'(X) - \theta'(X) G(X) \} \quad (18)$$

Further development of equation (17) is especially revealing. Computing derivatives and then forming the ratio V''/V' results in cancellation of the coefficient A . The Ramsey condition can then be written in the standard way

$$\eta \frac{\dot{C}}{C} = F_k - (\delta + \rho) \quad (19)$$

Arbitrage conditions (18) and (19) show that even in this overlapping generations model, the optimum trajectory of aggregate consumption is governed at each time by the relationships among aggregate quantities and the generational discount rate, ρ , but not the personal discount rate, β .

In steady-state:

$$\begin{aligned} \dot{C} = 0 &\Rightarrow \eta \frac{\dot{C}}{C} = F_k - (\delta + \rho) = 0 \\ &\Rightarrow F_k = (\delta + \rho) \end{aligned} \quad (20)$$

Thus, if the optimal trajectory involves capital accumulation over time, then before reaching the steady-state, we have $F_k > (\delta + \rho)$ and, therefore, $\dot{C} > 0$.

3. The case of intergenerational neutrality

We believe that this finding has important implications for modeling economic growth in a manner compatible with intergenerational equity. As suggested by the quotes in the introduction, stewardship for the future can be accommodated by setting the generational discount rate, ρ , equal to zero. The immediate problem with this approach is that the welfare maximand is infinite for any consumption path that does not converge to zero. This objection can be readily overcome, however, by transforming the objective function *ala* Koopmans (1965) as:

$$\text{Max}_{C(t)} W, \quad W = \int_{t=0}^{\infty} [V(C(t)) - V(\hat{C})] dt \quad (21)$$

where \hat{C} is the bliss point as suggested by Koopmans.

Now, the Hamiltonian becomes:

$$H = [V(C(t)) - V(\hat{C})] + \lambda[F(K, R) - \theta R - C] + \psi[G(X) - R] \quad (22)$$

The Hamiltonian (22) is an autonomous control problem. Following Chiang (1992) (equations 8.15 – 8.17), we have:

$$\begin{aligned} \frac{\partial H}{\partial t} &= 0, H = 0, \forall t \\ \Rightarrow V(\hat{C}) &= V(C(t)) + \lambda[F(K, R) - \theta R - C] + \psi[G(X) - R] \\ \Rightarrow V(\hat{C}) &= V(C(t)) + \lambda[\dot{K}] + \psi[\dot{X}] \end{aligned} \quad (23)$$

That is, sustainable income defined as the maximum value of social utility at any point in time is equal to the value of consumption and net investment, evaluated at the shadow prices of

produced and natural capital. The latter provides a definition of GNNP (see e.g. Weitzman, 1976, 1997). That is GNNP is sustained forever. This avoids the conundrum in models with positive intergenerational discounting that optimal consumption and sustainable income are eventually declining (see also Endress et al., 2005).

4. Conclusions

We develop a model of optimal and sustainable growth that accommodates distinct intragenerational and intergenerational discount rates. Under the assumption of constant elasticity of marginal utility, the optimum trajectory of aggregate consumption is governed at each time by the relationships among aggregate quantities and the generational discount rate but not the personal discount rate. As the personal discount rate increases, each generation allocates a higher share of its lifetime consumption to earlier stages of life. However, an increase in the personal discount rate does not affect the trajectory of aggregate consumption.

We also show that the optimal consumption trajectory is continually rising, if the society is approaching golden rule from a stock of physical capital lower than the steady state quantity. This implies that a sustainability constraint, if imposed, would be non-binding. We also show that sustainable income, defined as GNNP, is actually sustained along the optimal path, avoiding the paradox in the conventional approach.

In the general case, without restrictions on the elasticity, we find that, at any time t , consumption shares of each generation depend on the difference between intra and intergenerational discount rate. If the intragenerational discount rate is greater than the generational discount rate, the younger generation consumes a larger share of aggregate income than the older ones at any point in time. On the other hand, if the intergeneration

discount rate is higher, a younger generation consumes a smaller share than the older ones. All generations have the same share if the intra and intergenerational discount rates are equal.

For project evaluation of large investment projects such as climate-change mitigation, it may be informative to ask, to what extent intragenerational impatience would change such findings as in *Stern Review (2006)*, even in the presence of intergenerational neutrality. Just as we have distinguished between the intra and inter-generational utility discount rates, one could further distinguish between the planner's intergenerational inequality aversion and the intragenerational elasticity, η , as revealed by market transactions. We leave this exploration to further research.

BIBLIOGRAPHY

- Arrow, K., 1999, "Discounting, Morality, and Gaming," in P. Portney and J. Weyant (eds.), *Discounting and Intergenerational Equity*, RFF: Wash D.C.
- Blanchard, O. and S. Fisher, 1989, *Lectures in Macroeconomics*. MIT Press.
- Burton, P., 1993, "Intertemporal Preferences and Intergenerational Equity considerations in Optimal Resource Harvesting," *Journal of Environmental Economics and Management*, 24.
- Chiang, A.C., 1992, *Elements of Dynamic Optimization*, McGraw Hill.
- Dasgupta, P., 2001, *Human Well-Being and the Natural Environment*, Oxford U Press, Oxford.
- Dasgupta, P.S. and G.M. Heal, 1979, *Economic Theory and Exhaustible Resources*, Cambridge University of Press.
- Endress, L., T. Zhou, and J. Roumasset, 2005, "Sustainable Growth with Environmental Spillovers" *Journal of Economic Behavior and Organization*, 58(4):527-547.
- Harrod, R., 1948, *Towards a Dynamic Economy*, St. Martin 's Press. London, UK.
- Heal, G.M., 1993, "Optimal Use of Exhaustible Resources," in A.V.Kneese and J.L. Sweeney, *Handbook of Natural Resource and Energy Economics*, Vol. III, North Holland.
- Intergovernmental Panel on Climate Change (IPCC), 1995, *IPCC Second Assessment Report: Climate Change*, <http://www.ipcc.ch/ipccreports/assessments-reports.htm>, accessed on May 12, 2008.
- Koopmans, T.C., 1965, "On the Concept of Optimal Growth." *The Economic Approach to Development Planning*. Chicago: Rand McNally.
- Olson, M. and Bailey, M.J., 1981, "Positive Time Preference," *Journal of Political Economy*, 89(1):1-25.
- Pigou, A. 1920. "*The Economics of Welfare*," London: Macmillan and Co.
- Ramsey, F.P., 1928, "A Mathematical Theory of Saving," *Economic Journal*, 138: 543-59.
- Rawls, J., 1971, *A Theory of Justice*, Harvard University Press.
- Stern, N. 2006. "*Report on the Economics of Climate Change*," available at http://www.hm-treasury.gov.uk/independent_reviews/stern_review_economics_climate_change/stern_review_report.cfm

Solow, R. 1974, "The Economics of Resources or the Resources of Economics", *American Economic Review*, 64(2):1-14.

Solow, R.M., 1993, "Sustainability, An Economists Perspective," in Dorfman and Dorfman, *Economics of the Environment*, Third Edition, Norton.

Weitzman, M.L., 1976, "On the Welfare Significance of National Product in a Dynamic economy," *Quarterly Journal of Economics*, 90(1): 156-162.

Weitzman, M. L., and L. Karl-Gustaf, 1997, "On the Welfare Significance of Green Accounting as Taught by Parable," *Journal of Environmental Economics and Management*, 32(2):139-153.

APPENDIX

Utility of aggregate consumption in a model of overlapping generations.

Assume that individual utility of consumption has a functional form with constant elasticity of marginal utility:

$$u(c(t, \tau)) = -[c(t, \tau)]^{-(\eta-1)}, \eta > 1$$

$$\text{Then } u'[c(t, \tau)] = (\eta-1)[c(t, \tau)]^{-\eta}$$

Efficiency condition (9) can then be written:

$$(\eta-1)[c^*(t, \tau)]^{-\eta} = (\eta-1)[c^*(t, 0)]^{-\eta} e^{(\beta-\rho)\tau}$$

$$\text{or } c^*(t, \tau) = c^*(t, 0)e^{-(\beta-\rho)\tau/\eta}$$

Substitute in the consumption constraint of (7) to solve for the aggregate consumption, C(t):

$$\begin{aligned} C(t) &= \int_0^N c^*(t, \tau) d\tau \\ &= \int_0^N c^*(t, 0) e^{-(\beta-\rho)\tau/\eta} d\tau \\ &= \left\{ \frac{\eta(1 - e^{-(\beta-\rho)N/\eta})}{(\beta-\rho)} \right\} c^*(t, 0) \\ &= Kc^*(t, 0), \end{aligned}$$

$$\text{where the constant } K = \left\{ \frac{\eta(1 - e^{-(\beta-\rho)N/\eta})}{(\beta-\rho)} \right\}$$

$$\begin{aligned} \text{Now } V(C(t)) &= \text{Max} \int_0^N u(c(t, \tau)) e^{-(\beta-\rho)\tau} d\tau \\ &= \int_0^N u(c^*(t, \tau)) e^{-(\beta-\rho)\tau} d\tau \end{aligned}$$

$$\text{But, } u(c^*(t, \tau)) = -[c^*(t, \tau)]^{-(\eta-1)}$$

$$\begin{aligned}
&= -[c^*(t, 0)e^{-(\beta-\rho)\tau/\eta}]^{-(\eta-1)} \\
&= -[c^*(t, 0)]^{-(\eta-1)} e^{(\beta-\rho)(\eta-1)\tau/\eta} \\
&= -\left\{\frac{C(t)}{K}\right\}^{-(\eta-1)} e^{(\beta-\rho)(\eta-1)\tau/\eta}
\end{aligned}$$

$$\begin{aligned}
\text{So, } V(C(t)) &= -\left\{\frac{C(t)}{K}\right\}^{-(\eta-1)} \int_0^N e^{(\beta-\rho)(\eta-1)\tau/\eta} e^{-(\beta-\rho)\tau} d\tau \\
&= -\left\{\frac{C(t)}{K}\right\}^{-(\eta-1)} \int_0^N e^{-(\beta-\rho)\tau/\eta} d\tau \\
&= -\left\{\frac{C(t)}{K}\right\}^{-(\eta-1)} \left[\frac{\eta(1 - e^{-(\beta-\rho)N/\eta})}{(\beta-\rho)} \right] \\
&= -\left\{\frac{C(t)}{K}\right\}^{-(\eta-1)} \cdot K \\
&= -K^\eta [C(t)]^{-(\eta-1)} \\
&= -A[C(t)]^{-(\eta-1)}
\end{aligned}$$

where, $A = K^\eta$

Comparative statics:

We wish to show that $\frac{\partial A}{\partial B} < 0$ and $\frac{\partial A}{\partial \rho} > 0$. Since A is a monotonically increasing

function of K, it is sufficient to examine the expression: $\bar{A} = \frac{1 - e^{-bx}}{x}$,

where $x = (\beta - \rho)$ and $b = \frac{N}{\eta}$.

Now, $\frac{\partial \bar{A}}{\partial x} = \frac{(bx+1)e^{-bx} - 1}{x^2}$.

It is easy to show that the expression $(bx + 1)e^{-bx}$ attains a maximum of 1 for $x = 0$ and is

monotonically decreasing thereafter. Thus for $x \neq 0$, $\frac{\partial \bar{A}}{\partial x} < 0$. The desired result follows

immediately from the chain rule, since $\frac{\partial x}{\partial \beta} = 1$ and $\frac{\partial x}{\partial \rho} = -1$.