Renewable Resource Management with Alternative Sources: the Case of Multiple Aquifers and a "Backstop" Resource

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Abstract

While renewable resource economics is typically confined to one source and one aggregate demand, resource managers must often decide how to manage multiple sources of a resource simultaneously. In addition, studies of extraction sequencing are typically confined to non-renewable resources. We propose a dynamic optimization model to determine the efficient allocation of groundwater when two coastal aquifers are available for exploitation. We find that Herfindahl's least-cost-first result for nonrenewable resources does not necessarily apply to renewable resources, even when there is only one demand. Along the optimal trajectory extraction may switch from single to simultaneous use, depending on how the marginal opportunity cost of each resource evolves over time. A numerical simulation for the South Oahu aquifer system, which allows for differentiation of users by elevation and hence distribution costs, illustrates the switching behavior.

Keywords: Renewable resources, dynamic optimization, multiple resources

JEL codes: Q25, Q28, C61

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James Roumasset and Christopher Wada

1 Introduction

Efficient groundwater management is typically modeled as a single aquifer serving a single group of consumers. In many cases, however, the water manager must decide how fast to draw down or otherwise manage multiple aquifers. In Florida, for example, five water management districts overly a large system of interconnected aquifers (Florida DEP, 2009), while in California, over 500 distinct groundwater systems underlie approximately 40% of the state's surface area (California DWR, 2003). The Oahu case is similar, albeit with not nearly as many aquifer systems to potentially draw from. Previous studies of the Southern Oahu aquifer system, have estimated the optimal intertemporal water allocation for the Pearl Harbor aquifer (Krulce, Roumasset, and Wilson, 1997) and the optimal spatial and intertemporal allocation of water for the Honolulu aquifer system (Pitafi and Roumasset, 2009), under the assumption that each source serves separate water districts. In reality however, groundwater pumped from the Pearl Harbor aquifer is currently being transferred[1] to the Honolulu consumption region, suggesting that joint management is more appropriate.

The problem can be set up as one of a single demand supplied by multiple sources. In the theory of nonrenewable resources, Herfindahl's (Herfindahl, 1967) rule

[1] Primary wells in both Honolulu and Pearl Harbor are connected directly to a common pipeline. Pressure within the pipe is kept high enough that water flows at no additional pumping cost between regions toward wherever groundwater is drawn out of the system.

states that when facing a single demand, a resource manager should extract deposits of a resource in the order of unit extraction costs. When multiple demands exist, the least-unit-extraction-cost-first rule is replaced by a least-price rule (Chakravorty and Krulce, 1994), according to which the optimal shadow price is given by the sum of extraction cost, conversion cost, and the endogenous marginal user cost. Gaudet et al. (2001) generalize the result to spatially differentiated resource sites and users. They find that in the presence of setup costs, however, the least-price rule need not hold. If there is only one demand, the extraction profile over multiple nonrenewable resources is again determined exogenously, according to the sum of extraction and conversion costs for each resource.[2]

In what follows, we show that Herfindahl's least-cost-first principle does not extend to renewables, even in the presence of a single demand. A few recent studies consider similar problems involving multiple renewable resources. Zeitouni and Dinar (1997) construct a model with two adjacent sources of groundwater, but recharge is not allowed to vary with the quantity of groundwater in stock, and they do not derive a generalized rule to determine the optimal order of groundwater extraction. Horan and Shortle (1999) develop a theoretical framework for the optimal management of multiple Mink-Whale stocks but do not solve for the transitional dynamics. Costello and Polasky (2008) construct a more general multiple renewable resource model, which incorporates stochasticity in addition to space and time. They find that harvest closure is optimal whenever the stock of a particular spatial patch falls below the patch-specific escapement target for breeding. When that occurs, the expected biological returns from escapement exceed the returns from current harvest.

[2] See the discussions in Chakravorty et al. (2005) and Im et al. (2006).

However, their results are based on the assumption of state independent control, i.e. they assume that marginal returns to harvest are independent of the amount harvested. Although this may be a standard assumption in the fisheries literature, downward sloping demand curves may be more common in other natural resource contexts. The current study proposes an analytical model for the optimal joint management of two coastal aquifers, where the natural net recharge of each aquifer varies with its respective head level. Whereas most of the above-cited studies are analytical, we also provide a numerical illustration of how to apply the model.[3]

The rest of the paper is organized as follows. The next section extends the usual single-aquifer economic-hydrologic optimization model to allow for multiple groundwater resources. We derive a least-price rule for renewable resources analogous to Chakravorty and Krulce's rule for nonrenewables. We then contrast it with the Herfindahl least-cost rule for the optimal extraction of nonrenewable resources. The subsequent section utilizes data from the Honolulu and Pearl Harbor aquifers on the island of Oahu, Hawaii to provide a numerical illustration of a two-aquifer problem. The final section summarizes major analytical and empirical results and discusses general conclusions and potential research extensions.

2 The model

Coastal groundwater aquifers are usually characterized by a "Ghyben-Herzberg" lens (Mink, 1980) of freshwater sitting on an underlying layer of seawater. The upper surface of the freshwater lens is buoyed up above sea level due to the difference in density between the freshwater and the displaced saltwater (Figure 1). The head, or the

[3] Zeitouni and Dinar (1997) numerically estimate a simplified version of their two-aquifer model. However, they do not provide a description of the algorithm used for the estimation, and the aquifer net recharge functions are state-independent.

distance between the top of the lens and mean sea level, is denoted *h*, and is one of several measures of the freshwater stock in the aquifer. As the stock declines, i.e. the lens contracts, the head level falls, and water extraction becomes more costly for several reasons. Freshwater must be pumped longer distances, and that requires more energy. In addition, when the lens contracts to the point where the lower surface reaches the bottom of the deepest well, the remaining wells must take on larger shares of the pumping until physical limitations on the rate of pumping or capacity restrictions necessitate the construction of costly new wells. Thus, the extraction cost is a non-negative, decreasing, convex[4] function of head: $c(h) \geq 0$, $c'(h) < 0$, and $c''(h) \geq 0$.

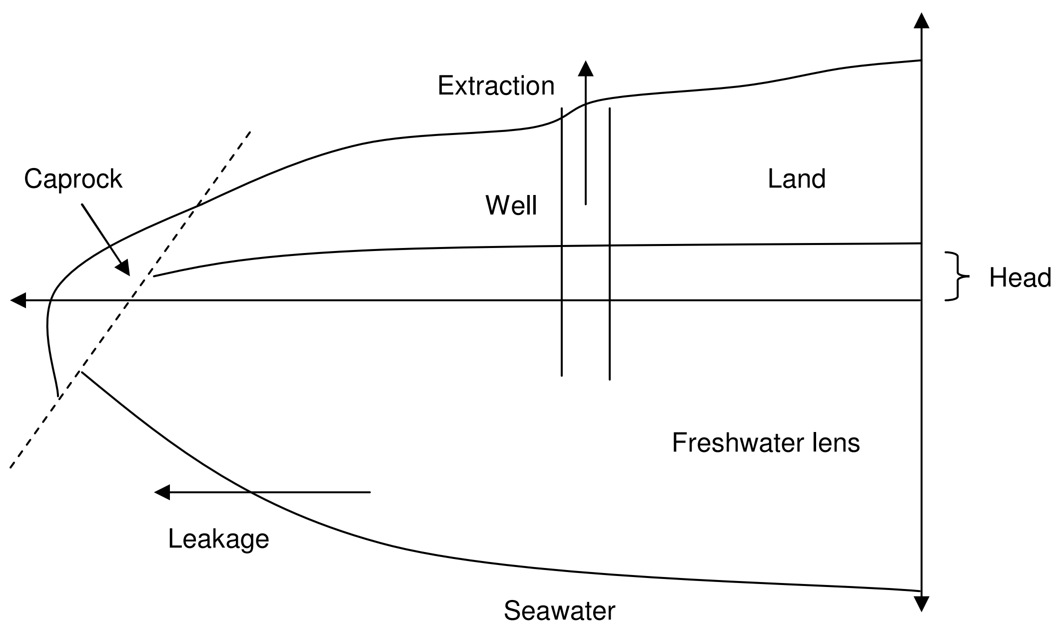

Figure 1: Coastal aquifer cross-section. Adopted from Mink (1980)

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[4] This characteristic of the cost function ensures that the necessary conditions for the maximization problem described below are also sufficient. In the application section, we assume a function that is linear in lift and therefore linear in head, thus satisfying the convexity requirement.

Leakage from a coastal aquifer is also a function of the head level. Low permeability caprock bounds the freshwater lens along the coast,[5] but pressure from the lens causes some freshwater to leak or discharge into the ocean as springflow and diffuse seepage through the caprock. As the head level declines, leakage decreases both because of the smaller surface area along the ocean boundary and because of the decrease in pressure due to the shrinking of the lens. Thus leakage is a positive, increasing, convex function of head: $l(h) \geq 0$, $l'(h) > 0$, and $l''(h) \geq 0$. Assuming that natural inflow from precipitation and adjacent water bodies is fixed at some rate *R*, net recharge is defined as $f(h) \equiv R - l(h)$. Since coastal groundwater is a renewable resource whose pre-extraction growth rate depends on the stock of the resource, the following model may prove useful for other renewables.

The cost of distributing water from the primary wells to consumers at spatially heterogeneous locations varies, so the unit cost of transporting water to users in category *j* is denoted c_d^j. In particular, distribution costs are higher for users at higher elevations because energy is required to boost the pumped water uphill. Consumption is also differentiated across spatial categories, so q_t^{ij} is the quantity extracted from aquifer *i* for consumption in category *j* at time *t*, and $\sum_j q_t^{ij}$ is the total quantity extracted for category *j* in period *t*.

Because coastal aquifers are located near the ocean, access to seawater is essentially unlimited. Desalination technology, although currently expensive, can

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[5] Caprock are coastal plain deposits (e.g. marine and terrestrial sediments, limestone, and reef deposits) that impede discharge of groundwater to the sea. Although caprock borders the coastline of Southern Oahu and is relevant to this study, it is not a general characteristic of coastal aquifers.

produce freshwater as an alternative to extraction. The unit cost of this backstop source is assumed constant at c_b, and the quantity of desalinated water produced for consumption in category *j* at time *t* is b_t^j.

The resource manager faces a non-autonomous optimal control problem with bounded controls and state-space constraints. The problem is non-autonomous because demand is allowed to grow over time. The control variables are restricted to be non-negative, and the state-space is constrained by minimum allowable head levels for each aquifer $h_{\min}^i$.[6] Given a discount rate $r > 0$, the planner chooses the rates of extraction and desalination over time to maximize the present value of net social benefits:[7]

$$(1) \qquad \underset{q_t^{ij}, b_t^j}{Max} \sum_{t=0}^{\infty} \rho^t \left\{ \sum_j \left(\int_0^{\sum_i q_t^{ij} + b_t^j} D_j^{-1}(x,t)dx - \sum_i \left(q_t^{ij}[c_i(h_t^i) + c_d^j] \right) - b_t^j [c_b + c_d^j] \right) \right\}$$

$$\text{subject to} \qquad \gamma_i [h_{t+1}^i - h_t^i] = f_i(h_t^i) - \sum_j q_t^{ij}$$

$$q_t^{ij} \geq 0, b_t^j \geq 0, h_t^i \geq h_{\min}^i \; \forall i, j$$

where ρ^t is the discount factor, $\rho \equiv (1+r)^{-1}$, $D_j^{-1}(x,t)$ is the inverse demand function for consumption category *j*, and γ_i is a height to volume conversion factor for aquifer *i*.

If the price for consumption category *j* is defined as $p_t^j \equiv D_j^{-1}(\sum_i q_t^{ij} + b_t^j, t)$ and the marginal opportunity cost (MOC)[8] of a unit of water extracted from aquifer *i* as

[6] Flow at the interface between the freshwater and underlying saltwater creates a thick transition zone comprised of brackish water that varies in salinity. The EPA standard for salinity of potable water is 2% of seawater salinity. As the aquifer is depleted, the transition zone eventually rises to the point where the well bottoms reach the minimum allowable salinity. This quality consideration is incorporated into the model as a head level constraint.

[7] See Pitafi and Roumasset (2009) for a similar setup in the case of a single aquifer.

[8] We use this terminology in honor of David Pearce (see e.g. Pearce et al., 1989).

$\pi_t^{ij} \equiv c_i(h_t^i) + c_d^j + \rho\gamma_i^{-1}\lambda_{t+1}^i$, then the following can be derived from the necessary conditions for the maximization problem (1):[9]

(2) $p_t^j \leq \pi_t^{ij}$, if <, then $q_t^{ij} = 0$

$p_t^j \leq c_b + c_d^j$, if <, then $b_t^j = 0$.

In other words, if the price for category *j* is less than the MOC of a unit of groundwater extracted from aquifer *i* for consumption in *j*, then no water is extracted for that purpose. Since it is never the case that zero water is consumed, it must be that $p_t^j = \min\left(\pi_t^{1j}, \pi_t^{2j}, ..., \pi_t^{Ij}, c_b + c_d^j\right)$.[10] Consequently $p_t^j = \pi_t^{mj}$ when aquifer *m* is being used $(q_t^{mj} > 0)$ and $p_t^j = \pi_t^{mj} = \pi^{nj}$, $\Delta p_t^j = \Delta\pi_t^{mj} = \Delta\pi_t^{nj}$ $(q_t^{mj} > 0, q_t^{nj} > 0)$ when aquifers *m* and *n* are being used simultaneously. This pricing and extraction rule accords with the least-price rule for nonrenewables. Herfindahl's least-cost rule for nonrenewables does not extend to renewables, however, even in the case of a single demand.

Although the optimal order of resource extraction appears to be governed by a fairly simple rule (2), solving for the MOC of each resource turns out to be a non-trivial task. In the case of non-renewable resources with constant extraction costs, the entire path of a given resource's shadow price is determined once the initial value is specified. When multiple renewable resources with stock-dependent extraction costs and growth are considered, however, each feasible MOC path must be solved for in conjunction with the associated feasible path of the aquifer head level. Forward-looking shooting algorithms encounter difficulties of iterating on multiple starting values of MOC. In the presence of

[9] The discrete-time Hamiltonian and its necessary conditions are in Appendix A.

[10] Without loss of generality, suppose that $\pi_t^{mj} < \pi_t^{nj}$ for $m < n$. Then let the price be determined by π_t^{kj}, where $k > 1$, i.e. $p_t^j = \pi_t^{kj} > \pi_t^{ij}$, $i = 1, \ldots, k-1$. But that violates one of the necessary conditions, which states that $p_t^j \leq \pi_t^{ij}$ for all *i*.

multiple non-linearities, gradient ascent algorithms take many iterations to converge or may not even converge at all. Because of these difficulties, it may be necessary to condition the search algorithm on different possible orders of extraction.

Head levels change over time as a result of extraction and/or natural recharge. Accordingly, the MOCs also change inasmuch as the extraction costs and shadow prices are dependent on the head levels. Thus it may be optimal to extract exclusively from a single aquifer for a finite period of time and then switch to pumping from more than one source in the periods that follows. A planner with many aquifers at her disposal will face multiple endogenously determined switching points as described by the optimal pricing condition (2). In what follows, we explore the two-aquifer case, which is also the subject of our subsequent numerical illustration. In the case of even two aquifers, many scenarios are conceivable, but we focus on three of interest. Without loss of generality, aquifer A is chosen as the aquifer to optimally extract from first in each scenario.[11] The actual extraction pattern calculated in the numerical illustration is described by scenario 1.

	Stage 1	Stage 2	Stage 3	Stage 4
Scenario 1	Extract from A	Extract only MSY from A Extract from B	Steady State	
Scenario 2	Extract from A	Extract from A Extract from B	Extract only MSY from A Extract from B	Steady State
Scenario 3	Extract from A	Extract from A Extract from B	Extract from A Extract only MSY from B	Steady State

Figure 2: Order of extraction in the case of two aquifers[12]

[11] We later establish a sufficient condition that determines which aquifer is optimally used first. Our application illustrates the case wherein the condition is satisfied and indicates the larger aquifer for early withdrawal.

[12] We use "extract" to mean any removal of groundwater from an aquifer, even when the extraction rate does not exceed net recharge, i.e. an aquifer's head level may be rising while extraction is occurring.

In the initial stage of scenario 1, aquifer A is used exclusively, while aquifer B is built up.[13] As the head level of aquifer B rises, the marginal extraction cost declines but the shadow price rises, so the MOC may increase or decrease.[14] At the same time, the MOC of aquifer A increases until the first switch-point occurs, whereupon aquifer A reaches its minimum allowable head constraint.

In stage 2, extraction from aquifer A is limited to net natural recharge, and the remaining optimal consumption is supplied by aquifer B. Price, now determined by the MOC of aquifer B, continues to increase, and the shadow price of groundwater for aquifer A rises apace. Although the extraction cost for aquifer A remains constant once MSY extraction is imposed, the MUC still rises as the multiplier on the head constraint becomes positive and rises (equation A.4).

The final stage of extraction is characterized by MSY extraction from both aquifers. A steady state is maintained,[15] in which extraction from each aquifer is limited to net recharge, and the price remains at the backstop cost. The stages of extraction for scenario 1 are depicted graphically in Figure 3 below.

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[13] By sheer coincidence $(\pi_0^{1j} = \pi_0^{2j})$ it would be optimal to simultaneously draw down both aquifers from the start.

[14] Recall that $\pi_t^{ij} = c_i(h_t^i) + \rho\lambda_{t+1}^i$. Since $c_i'(h_t^i) < 0$ by assumption, the first term on the left hand side is decreasing as the head is building. Condition (A.4) in Appendix A reduces to $\rho\lambda_{t+1}^i - \lambda_t^i = -\rho\gamma_i^{-1}\lambda_{t+1}^i f_i'(h_t^i)$ when extraction is zero. Rearranging yields $[\lambda_{t+1}^i - \lambda_t^i]/\lambda_t^i = r - \gamma_i^{-1}\lambda_{t+1}^i f_i'(h_t^i)/\lambda_t^i > 0$, i.e. the marginal user cost of aquifer *i* is always increasing when extraction from aquifer *i* is zero. Hence, MOC may be increasing or decreasing.

[15] Technically the system never reaches a steady state since demand is growing. However, it is meant that the price and head levels remain constant in the "steady state."

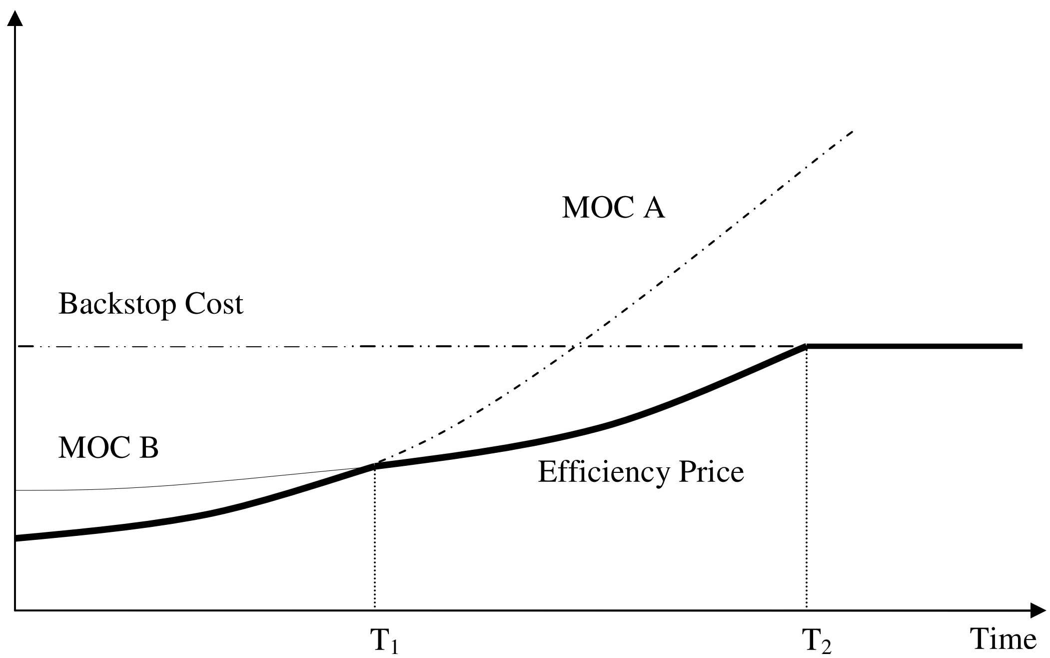

Figure 3: Hypothetical efficiency price path. In the scenario depicted, aquifer A reaches its head constraint at the first switch point.[16] The efficiency price is the lower envelope of the MOC paths.

In scenario 2, stage 1 remains the same; aquifer A is used exclusively while aquifer B is built up. At the first switch point, however, the MOCs of the two aquifers become equal. Since neither aquifer is at its constraint, stage 2 is characterized by simultaneous extraction of both aquifers, the rates of which are optimally chosen to keep the MOCs equal. Eventually, aquifer A reaches its head constraint, and extraction is limited to natural recharge. In the final stage, aquifer B is drawn down to its minimum allowable head level, and the system reaches a steady state, in which both aquifers are pumped at MSY. Scenario 3 is exactly the same as scenario 2 except that aquifer B reaches its head constraint before aquifer A does.

[16] The dotted portion of the MOC A curve represents what the MOC would be if the head constraint did not bind at T_1.

Generally for autonomous, single renewable resource problems, optimality calls for monotonic state paths (Kamien and Schwartz, 1991). If the stock starts below its steady state level, then it optimally builds monotonically, whereas if the stock starts above its steady state level, then it is optimally drawn down monotonically. With multiple resources, whether the problem is autonomous or not, optimality is not always characterized by monotonic state paths. The result can be attributed to the resource-specific extraction cost and growth functions, i.e. inter-resource comparisons must be made.

It is not necessarily the case that the exclusively used aquifer is drawn all the way down to its MSY level at the end of the first stage. We can verify that such an extraction path is optimal, however, by checking that the following sufficient condition holds:[17]

$$c_1(h_{\min}^1) + \frac{[p_t - c_1(h_{\min}^1)]f_1'(h_{\min}^1)\gamma_1^{-1}}{r} - \frac{\gamma_1^{-1}c_1'(h_{\min}^1)f_1(h_{\min}^1)}{r} <$$
$$c_2(h_{\max}^2) + \frac{[p_t - c_2(h_{\max}^2)]f_2'(h_{\max}^2)\gamma_2^{-1}}{r} - \frac{\gamma_2^{-1}c_2'(h_{\max}^2)f_2(h_{\max}^2)}{r}$$

for all $p_t \in [\min\{c_1(h_{\min}^1), c_2(h_{\max}^2)\}, c_b]$. In other words, if for all feasible head combinations and prices, the extraction cost plus marginal user cost of aquifer 1 is less than that of aquifer 2, then it is always optimal to draw aquifer 1 down to its MSY level in the initial stage. In the illustration that follows, the sufficiency condition is satisfied, thus ensuring that Pearl Harbor aquifer (PHA), the larger of the two, is optimally drawn down first, and no water is extracted from the Honolulu aquifer (HNA) until PHA reaches its minimum head level, where maximum sustainable yield is achieved.

[17] See Appendix B for a proof of this result.

3 Application: Honolulu and Pearl Harbor aquifers

3.1 Functional forms and parameters

As previously discussed, the volume of water stored in a coastal aquifer is a function of head, but it also depends on various hydrologic parameters such as the aquifer boundaries, lens geometry, and rock porosity (Mink, 1980). Although the surfaces of the lens are technically parabolic, the hydraulic gradient in southern Oahu is small enough that storage and head are approximately linearly related. Following Krulce, Roumasset and Wilson (1997), we suppose that 78.149 billion gallons of freshwater are stored per foot of head in PHA. The net recharge function for PHA is constructed using Mink's recharge estimate of 220 million gallons per day (mgd) in combination with the leakage function econometrically estimated by Krulce, Roumasset and Wilson (1997): $l(h_t) = 0.24972h_t^2 + 0.022023h_t$. In a similar manner, we use Pitafi and Roumasset's (2009) volume-head conversion factor of 61 billion gallons of water per foot of head for HNA and construct a net recharge function using Liu's (2006) 64 mgd estimate of natural inflow.

The extraction cost is specified as a convex (linear) function of lift: $c_i(h_t^i) = \xi_i(e_i - h_t^i)$, where lift is defined as the difference between the average ground surface elevation of the wells, e_i, and the head level. To simplify the discussion, we will focus primarily on functional forms in the text and refer the reader to Table 1 for all of the aquifer-specific parameter values. The energy-cost parameter, ξ_i, is calculated using the initial unit extraction cost $c_i(h_0^i)$, which is a volume-weighted average of unit extraction costs for all primary wells in the initial period, and the initial head level, h_0^i.

Distribution costs c_d^j are calculated for each elevation category *j* from booster station pumping data (Table 2). The unit cost c_b of desalinating water is estimated at $7.43/tg.[18]

Parameter	Honolulu	Pearl Harbor
e_i [ft]	50	272
ξ_i [$/(tg*ft)]	0.00786	0.00121
$c_i(h_0^i)$ [$/tg]	0.22	0.31
h_0^i [ft]	21.5	16
g	0.01	0.01
η	0.3	0.3
r	0.03	0.03
$h_{\min}^i$ [ft] [19]	21	15.125

Table 1: Parameter values for the Honolulu and Pearl Harbor aquifers

The demand for water is modeled as a constant elasticity function $D_j(x_t,t)=\alpha_j e^{gt}(x_t^j)^{-\eta}$. The coefficient α_j for each elevation category is calculated using actual pumping data and the retail price for the year 2006 (Table 2). Following Pitafi and Roumasset (2009), the exogenous rate of population growth *g* is assumed to be 1% in the baseline scenario. Demand elasticities vary considerably among studies, but recent estimates for increasing block price structures (Olmstead, Hanemann, and Stavins, 2007) tend to be high relative to the elasticities used in previous studies of the southern

[18] See Appendix C.
[19] Taking into account average well depth below mean sea level, upconing, and the thickness of the brackish transition zone, Liu (2006, 2007) estimates the minimum allowable head level required to avoid seawater intrusion of the wells.

Oahu aquifer system. In the current study, the baseline value is taken as η=0.3. Finally the interest rate *r*=3%.[20]

Category	Elevation (ft)	Distribution Cost ($/tg)	Qty (mgd)	Coefficient (α_j)
1	0	$1.81	70.34	89.48
2	500	$2.35	6.21	7.9
3	789	$3.21	1.17	1.49
4	1039	$4.37	0.65	0.83
5	1086	$5.62	0.17	0.21
6	1345	$6.90	0.12	0.15
7	0	$1.86	47.43	60.34
8	552	$2.37	3.7	4.7
9	887	$2.95	1.18	1.51

Table 2: Demand coefficients and distribution costs. Categories 1-6 represent the Honolulu consumption district and 7-9 the Pearl Harbor consumption district.

3.2 Computational strategy

The problem is solved using a forward-iterating algorithm. The initial and terminal conditions for the head level are known. Growing demand ensures the implementation of the backstop at some point since the size of the aquifer system is finite, and steady state calculations reveal that the head level constraints are binding. Thus the terminal price is the backstop cost, and the optimal path will be determined once we know the correct initial shadow prices.[21]

As described in section 2, the order of extraction will depend on which of the resources is “cheaper.” In our study, the aquifers satisfy the sufficiency condition such that Pearl Harbor aquifer is used first and is drawn down exclusively to its MSY head

[20] The fact that the discount rate is greater than the population growth rate ensures convergence of the objective functional.
[21] Along the optimal trajectory, the price rises to the backstop cost and the head constraint becomes binding at precisely the same time. See Appendix D for a proof of this result.

level, while Honolulu aquifer is allowed to build.[22] Once Pearl Harbor aquifer reaches its MSY level, extraction is limited to recharge and Honolulu aquifer is drawn down to its MSY level as the resources approach a steady state. Given the order and stages of extraction, we proceed to solve for the endogenously determined switch-points and the time at which the aquifers reach a steady state.

Following a "shooting" method,[23] trial values are assumed for the initial shadow prices, and condition (A.5) allows one to solve for the shadow prices in the following period. Once the period 2 shadow prices are determined, the price (A.2) and therefore the rates of extraction can be ascertained for the current period. The rates of extraction reveal the head levels in the next period via the equation of motion (A.6), and the whole process can be repeated, using the period 2 head levels and shadow prices as the new starting point. Eventually, one of the terminal conditions is reached. If at least one of the other terminal conditions is inconsistent, then the initial guesses for the shadow prices are revealed as incorrect. The guesses must be adjusted and the process repeated until all of the initial and terminal conditions are satisfied for the head of each aquifer and the price, so that the PV functional is maximized given the boundary conditions.

3.3 Results

We determine the optimal paths of price, extraction, and head for each of the aquifers. The efficiency price for consumption category 1 starts at $2.15 and rises relatively slowly until year 40, at which point PHA reaches its head level constraint. After year 40, the price path is determined by the MOC of HNA, and the price begins to

[22] If the sufficiency condition is not met, then the algorithm described will need to be adjusted, inasmuch as there will likely be a stage of simultaneous extraction before either aquifer reaches its MSY level.
[23] See e.g. Judd (1998).

rise more rapidly as HNA is depleted. During the second stage, any quantity demanded at the optimal price in excess of maximum sustainable yield (MSY) for PHA is supplied by extraction from HNA. Eventually, the price rises to the cost of the backstop as HNA approaches its minimum allowable head level. The steady state is reached after 100 years, and optimal consumption in excess of MSY for both aquifers from that point forward is met by desalination. For a graphical example, see Figures 4 and 5 below.

Figure 5 depicts the MOC curves for each aquifer. Since the extraction cost functions are linear and relatively flat in this application, the order of extraction is determined primarily by differences in the net recharge functions. Intuitively, PHA is drawn down first exclusively because the resulting decrease in leakage (increase in net recharge) is greater than the increase in leakage (decrease in net recharge) that occurs as HNA builds. It should also be noted that the PHA is building for an initial period even when it is being used exclusively (Figure 4). Because the problem is non-autonomous, non-monotonicity can result from the need to build the stock in anticipation of future scarcity.

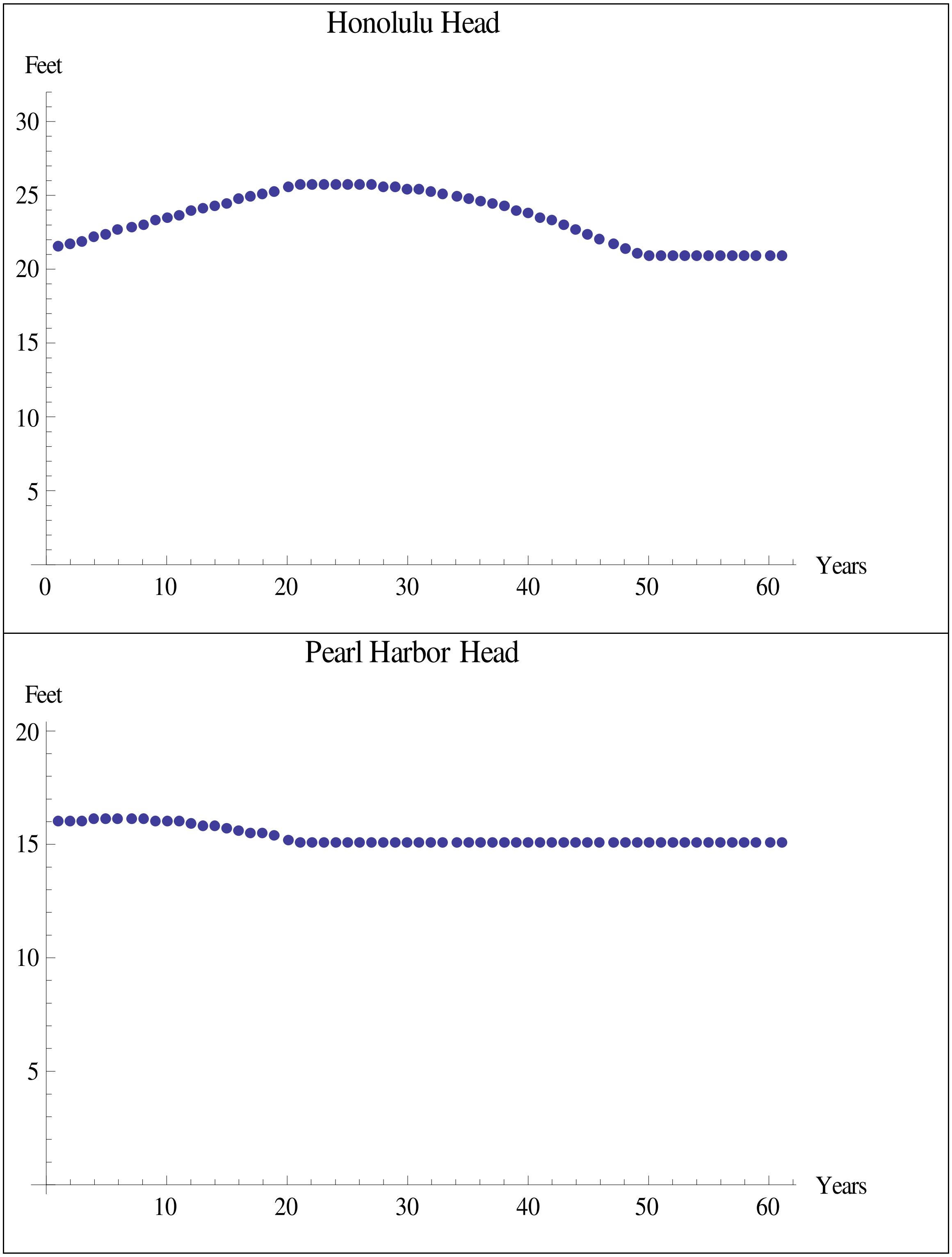

Figure 4: Head paths (g=0.02, 90%R)[24]

[24] A scenario other than the baseline is chosen for illustrative purposes inasmuch as the MOCs are more clearly differentiated.

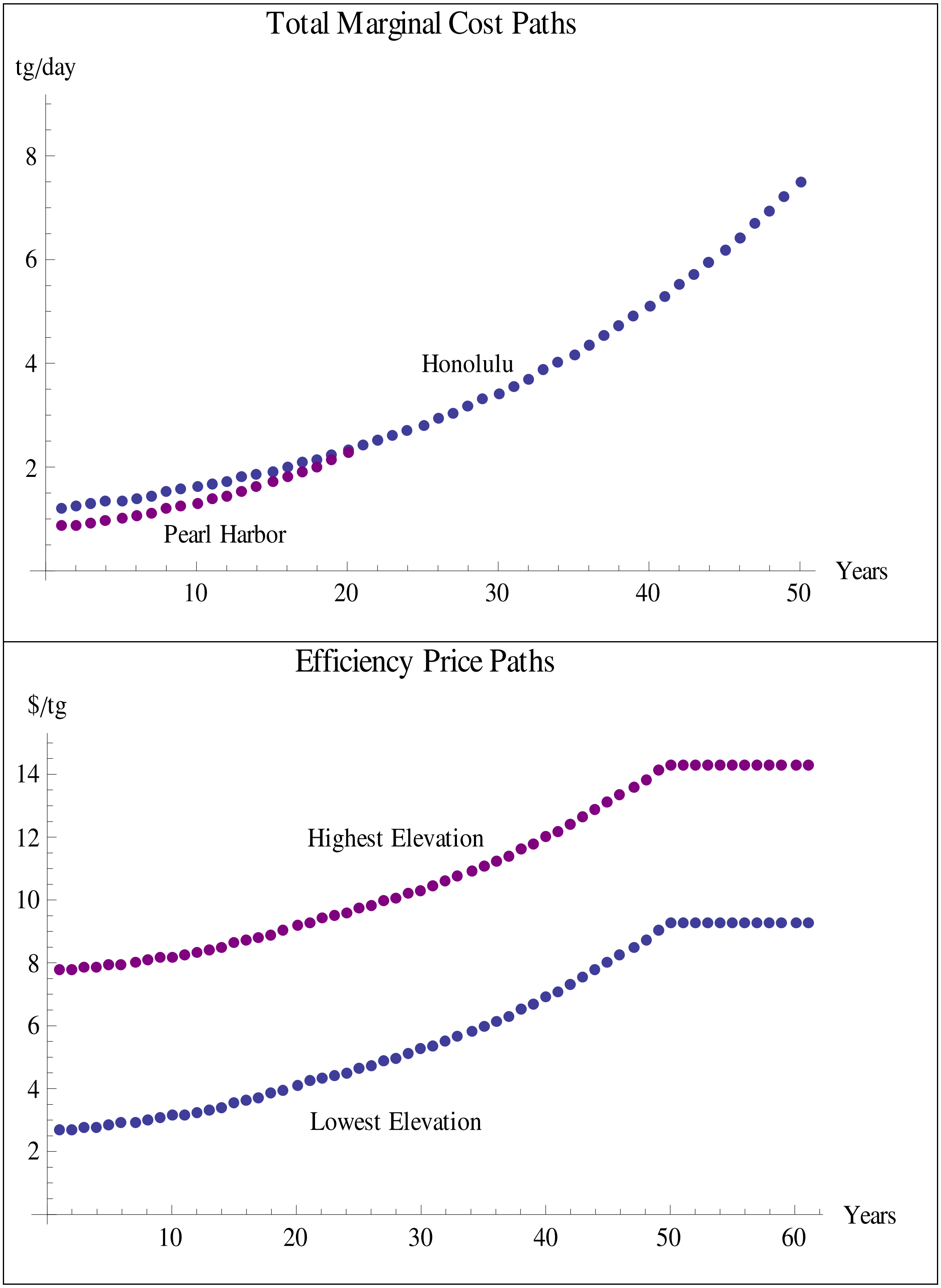

Figure 5: MOC and efficiency price paths (g=0.02, 90%R)

3.4 Sensitivity analysis

Sensitivity analysis is performed to not only test the sensitivity of the results to uncertain parameter values, but also to consider the possible impacts of climate change and watershed degradation, caused by, for example, invasive species.

The three parameters with the most uncertainty in this study are the growth rate of demand, the elasticity of demand for water, and the natural rate of infiltration to the aquifer. More accurate measurement of the first two parameters is feasible but beyond the scope of this study. The gains to such an endeavor are likely to be large, however. Increasing the 1% demand growth per year assumed in the baseline scenario to 2% results in the implementation of desalination 42 years closer to the present. Clearly, optimizing with a wrongly assumed growth value can lead to large welfare losses; underestimating growth would result in underpricing, and hence necessitate the implementation of a backstop even sooner.

Scenario	τ_1 (yrs)	T (yrs)	PV (million $)	PV lost recharge (million $)
Baseline	40	100	$12,047.80	-
$g = 2\%$	29	58	$15,912.90	-
90% R	24	83	$11,615.70	$432.10
80% R	12	63	$11,492.70	$555.10
70% R	8	46	$10,942.70	$1,105.10
$g = 2\%$ & 90% R	20	49	$16,713.70	$686.90
$\eta = 0.5$	40	112	$10,457.80	-

Table 3: Sensitivity analysis

The baseline value for demand elasticity is based on existing studies of water demand. Recent work however, (see e.g. Olmstead et al, 2007) suggests that elasticity under increasing block pricing structures may be larger than previous estimates suggest.

Increasing the demand elasticity to -0.5 delays the need for desalination by 12 years. This follows from the fact that the price need be increased by less to achieve any given desired level of demand-side conservation. Small changes in elasticity have a large impact on the optimal paths.

The third parameter of interest, natural infiltration to the aquifer or natural recharge, can be broken down into several components: precipitation, evapotranspiration, and runoff. In most economic studies of groundwater, natural recharge is taken as constant for the entire planning horizon. This may have been a good approximation for the past century, but with climate change looming on the horizon and threats of invasive species on the rise, historic measurements of recharge are likely no longer accurate approximations of future recharge. Ideally, a hydrologic-economic model would incorporate factors that influence recharge such as climactic conditions and the state of the watershed, inasmuch as global warming will certainly have an impact on recharge via changes in quantities and patterns of precipitation, changes in evapotranspiration, and changes in runoff; and increases in invasive species will affect runoff and evapotranspiration as a result of their impact on landcover. However, regional climate change and watershed models are still in a developmental stage, so incorporating the science into an economic model is currently not feasible. Instead, we look at various recharge scenarios that may result from the complicated physical interactions associated with climate change or degradation of the watershed.

The numerical simulations are re-run assuming a 10%, 20%, and 30% reduction in recharge. The reduction could be due to a decline in precipitation, increased runoff due to more extreme rainfall events or damage to the watershed from invasive species, or any combination of those factors. It is intuitive that lower recharge results in the need for desalination sooner (17 years in the 10% reduction case); the growth of the aquifer is slower so the resource is depleted more quickly. The $432.1 million present value of lost recharge is calculated by taking the difference between the PV of the baseline scenario and the 90%-recharge scenario. Although in reality the recharge would be declining gradually over time, the 10% reduction is an approximation and the PV cost of doing nothing is not trivial. The lost PV is even larger with higher reductions in recharge. Preventing climate change on a local scale is not feasible, but investment in natural capital within the watershed will help to mitigate the impact on recharge.

4 Conclusion

Although many real-world natural resource problems involve the use of multiple resources, most renewable resource economic models focus on a single resource supplying a single demand. When more than one source is available, optimal management involves how much to extract from each source. We extend the usual groundwater economics model to include multiple sources and find that at a given point in time, optimality may call for extraction from a single aquifer exclusively, extraction from both sources, or extraction from both aquifers and use of a backstop simultaneously.

Whereas Chakravorty and Krulce's (1994) *least-price-first* rule for non-renewables collapses to the Herfindahl least-cost-first rule for a single demand, optimal extraction from multiple renewable resources such as aquifers is governed by the more general least-price-first rule even when there is only one demand. Also contrary to the nonrenewable case (e.g. Gaudet et al., 2001), a single demand can optimally be supplied by more than one renewable resource in any given period. In the steady state, the renewable characteristic of renewable resources ensures that all resources are used simultaneously. Stages of simultaneous use prior to the steady state are also feasible.

While optimal extraction accords with *least-price-first,* those "prices" (which refer to marginal opportunity costs along the optimal path) are endogenous. Which resource has the lower initial marginal opportunity costs is not generally determinable without actually solving the problem at hand. There are two countervailing forces; extraction decreases leakage by lowering the head level, but at the same time it increases extractions costs. Intuitively, if extraction costs are approximately constant for example, optimality requires initially extracting exclusively from the aquifer for which the value of net gained recharge (including movement along the leakage function) exceeds the value of lost recharge from the unused aquifer. We find a sufficiency condition that not only determines the order of extraction but also ensures complete drawdown of the first resource to its MSY level before switching to the second resource.

In an application to the Southern Oahu aquifer system, we find that extraction occurs in three stages. In the first, Pearl Harbor aquifer is used exclusively[25] while Honolulu aquifer is built up. In this particular case, the order of extraction is determined primarily by differences in the net recharge functions inasmuch as extraction costs are linear and fairly flat. Intuitively, Pearl Harbor aquifer is drawn down first because the resulting decrease in leakage (increase in net recharge) is greater than the increase in leakage (decrease in net recharge) that occurs as Honolulu aquifer builds. Stage 2 begins once Pearl Harbor reaches its minimum allowable head level. Extraction from Pearl Harbor is limited to MSY and the remaining quantity demanded at the optimal price is

[25] For an initial period, Pearl Harbor aquifer's stock is actually increasing even when it is being used exclusively. That both state variable approach paths are non-monotonic is not a general result. For example, with a larger initial stock, Pearl Harbor would be optimally drawn down monotonically.

met by extraction from Honolulu. During stage 3, price rises to the backstop cost and Honolulu reaches its head constraint. Beyond stage 3, extraction is limited to recharge for each aquifer and the remaining consumption is met by desalination.

There are many possible research extensions to the current work, but a few in particular stand out. First, the multiple-aquifer hydrologic-economic framework should integrate watershed and regional climate models. As scientists gain a better understanding of the physical models, this type of integration will become a more feasible undertaking. Second while the simulation results suggest that there exist large potential welfare gains resulting from optimal management of multiple resources, the analysis does not indicate how such pricing could be achieved. Most water utilities are required to keep a balanced budget, and optimal pricing generates excess revenue since the price exceeds physical marginal costs. One possible solution is an increasing block pricing structure with a free first block (see e.g. Pitafi and Roumasset, 2009). Third, there may be cases in which multiple aquifers may be available to serve multiple consumption districts with non-zero transportation costs between districts. In that case, independent management may be optimal until scarcity increases enough to justify costly transportation of water between districts. The endogenous boundary between locales would adjust over time along the optimal trajectory. Finally, many natural resource stocks are directly linked and adjacent aquifers are no exception. In the case of Southern Oahu, subsurface inter-aquifer flow is relatively small so it is ignored in the analysis, but that may be an exception rather than a generality. In the general case, the equations of motion for the aquifer stock should be modified to include the state variable for each interconnected aquifer.

Although the numerical simulation addresses a specific coastal aquifer problem, the analytical framework is sufficiently general to have many other potential applications. For example, a regional fish market likely draws from many fisheries. Thus a single demand for fish channeled through that market is served by multiple fish stocks. Similarly, an integrated lumber market would draw trees from multiple locales.

Appendix A

The corresponding discrete-time current value Hamiltonian[26] is:

$$\text{(A.1)}\quad H = \sum_j \left(\int_0^{\sum_i q_t^{ij} + b_t^j} D_j^{-1}(x,t)dx - \sum_i \left(q_t^{ij}[c_i(h_t^i) + c_d^j] \right) - b_t^j [c_b + c_d^j] \right)$$

$$+ \rho \sum_i \left(\gamma_i^{-1} \lambda_{t+1}^i [f_i(h_t^i) - \sum_j q_t^{ij}] \right) + \sum_i \left(\mu_t^i [h_t^i - h_{\min}^i] \right)$$

and the Maximum Principle requires that $\forall i, j$:

$$\text{(A.2)}\quad \frac{\partial H}{\partial q_t^{ij}} = D_j^{-1}(\sum_i q_t^{ij} + b_t^j, t) - c_i(h_t^i) - c_d^j - \rho \gamma_i^{-1} \lambda_{t+1}^i \le 0, \qquad q_t^{ij} \ge 0 ,\ q_t^{ij} \frac{\partial H}{\partial q_t^{ij}} = 0$$

$$\text{(A.3)}\quad \frac{\partial H}{\partial b_t^j} = D_j^{-1}(\sum_i q_t^{ij} + b_t^j, t) - c_b - c_d^j \le 0, \qquad b_t^j \ge 0 ,\ b_t^j \frac{\partial H}{\partial b_t^j} = 0$$

$$\text{(A.4)}\quad \rho \lambda_{t+1}^i - \lambda_t^i = -\frac{\partial H}{\partial h_t^i} = \sum_j q_t^{ij} c_i'(h_t^i) - \rho \gamma_i^{-1} \lambda_{t+1}^i f_i'(h_t^i) - \mu_t^i$$

$$\text{(A.5)}\quad \frac{\partial H}{\partial \mu_t^i} = h_t^i - h_{\min}^i \ge 0 \qquad \mu_t^i \ge 0 ,\ \mu_t^i \frac{\partial H}{\partial \mu_t^i} = 0$$

$$\text{(A.6)}\quad \gamma_i [h_{t+1}^i - h_t^i] = \frac{\partial H}{\partial (\rho \lambda_{t+1}^i)} = f_i(h_t^i) - \sum_j q_t^{ij} .$$

[26] See, e.g. Conrad and Clark (1987).

Appendix B

Proposition: When

$$c_1(h_{\min}^1) + \frac{[p_t - c_1(h_{\min}^1)]f_1'(h_{\min}^1)\gamma_1^{-1}}{r} - \frac{\gamma_1^{-1}c_1'(h_{\min}^1)f_1(h_{\min}^1)}{r} <$$
$$c_2(h_{\max}^2) + \frac{[p_t - c_2(h_{\max}^2)]f_2'(h_{\max}^2)\gamma_2^{-1}}{r} - \frac{\gamma_2^{-1}c_2'(h_{\max}^2)f_2(h_{\max}^2)}{r}$$

for all $p_t \in [\min\{c_1(h_{\min}^1), c_2(h_{\max}^2)\}, c_b]$, it is optimal to draw aquifer 1 down to its MSY level before extracting from aquifer 2.

Proof: Since we do not know the price and head paths before solving the entire optimization problem, the proposition considers the most extreme head values, and hence is a sufficient but not a necessary condition.

Since the terms on the left and right hand side of the inequality above are the marginal opportunity costs for aquifer 1 and 2 respectively, we will show first that the left hand side is the largest feasible value. Since $c_1 > 0$ and $c_1' < 0$, $h_{\min}^1$ results in the largest possible value for the first term. By definition, $f_1' < 0$ and $f_1'' < 0$, so $h_{\min}^1$ ensures that the second term is at its highest feasible value. Finally, since c_1' is constant inasmuch as $c_1'' = 0$, $h_{\min}^1$ yields the largest possible value for f_1 and hence the largest possible value for the third term. By an analogous argument, the RHS yields the lowest feasible value for the marginal opportunity cost of aquifer 2. If the smallest feasible marginal opportunity cost for aquifer 2 still exceeds the largest feasible marginal opportunity cost for aquifer 1, then by the *least-price-first* rule, aquifer 1 should be drawn down exclusively to its MSY level in the first stage of extraction.□

Appendix C

The desalination cost estimate is based on reverse osmosis (RO) membrane technology. The following simple amortization formula is used:

$$C_K = \frac{P \cdot i \cdot (1+i)^t}{(1+i)^t - 1}$$

where C_K is the amortized cost, P is the original start up investment, i is the bond rate, and t is the expected plant life. Adding the amortized start up cost to the operation and maintenance costs yields the total annual cost of desalination. Dividing that cost by total output per year then gives the unit cost of desalination. Values for plant capacity (5 mgd), set up cost ($40 million) and operation and maintenance costs ($5.75 million) are obtained from a study by Oceanit (2003) and adjusted for inflation and increases in energy costs. Amortization rates and estimated average plant life varies among desalination studies, but for the current study i=7.5% and t=25 years. Finally, it is assumed that buildings and equipment have no salvage value and capital replacement costs are already incorporated in operation and maintenance costs. Thus the unit cost of desalination is $7.43/tg.

The unit cost of desalination is an important factor in the optimal allocation of groundwater. The current values of parameters that determine the desalination cost estimate may change over time or may need adjustment. Sensitivity analysis can be performed to address this issue.

Appendix D

Proposition: If $p_{T_1} = c_b$ and $h_{T_2} = h_{\min}$ in the optimal steady state, it must be $T_1 = T_2$.

Proof: Suppose $p_{T_1} = c_b$, $h_{T_2} = h_{\min}$, and $T_1 < T_2$. Then for $t \in (T_1, T_2)$, $p_t = c_b$, $h_t > h_{\min}$, and the head level must be drawn down further to satisfy its terminal condition. However, drawing the head down would raise the marginal opportunity cost of groundwater above c_b. If this were true, then the backstop would optimally be used to satisfy all of the quantity demanded at the optimal price over the interval. But if extraction is zero, then the head level rises and $h_T = h_{\min}$ is never achieved, which is a contradiction.

Suppose instead that $h_{T_1} = h_{\min}$, $p_{T_2} = c_b$, and $T_1 < T_2$. Then for $t \in (T_1, T_2)$, $p_t < c_b$ and $h_t = h_{\min}$. Once the head constraint is binding at T_1, extraction is limited to recharge and the price must be equal to c_b, since desalination must be used to satisfy any consumption in excess of natural recharge. But $p_{T_1} < c_b$, which is a contradiction. Thus, if $p_{T_1} = c_b$ and $h_{T_2} = h_{\min}$, it must be that $T_1 = T_2$. □

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