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AND SIGNALLING**

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Abstract

A signalling model is developed to investigate the consequences of corporate income taxation in the presence of adverse selection in the equity market. The model obtains a unique, informationally-constrained efficient equilibrium in which a better quality firm retains more inside-equity, and, as in the complete information case, only profitable firms are supported. The corporate income tax affects signalling costs as well as the profitability of projects. Numerical experiments with exponential utility functions find that the inside-equity position of a better quality firm increases as the corporate income tax rate rises. This reaction is, however, insensitive to the tax rate change due to the risk-sharing with the government, which leads to the interesting result that the corporate income tax only incurs a lower welfare cost than the lump sum tax with the same tax revenue.

Keywords and JEL Codes: Corporate Income Taxation(H25),
Excess Burden(H21), Signalling(D82).

I. Introduction

This paper investigates the effects of corporate income taxation on an entrepreneur's financial decision-making, using a simple model in which the quality of a new project is private information held by an entrepreneur who seeks external financing. In the presence of such asymmetric information, an adverse selection problem occurs with the risk-taking contracts of equity trade. The focus of this paper is on the financial market where the signals sent by the informed agents (entrepreneurs) reveal sufficient information to the uninformed agents (outside investors) in equilibrium. The signal employed in the model is the position of inside-equity retained by risk-averse entrepreneurs, which represents the extent to which they bear the risk of their own projects. With this implementation of signalling, we find a plethora of Nash equilibria among which a unique equilibrium is selected by using the Cho and Kreps Criterion. This unique equilibrium is Pareto-dominant among separating equilibria, hence informationally-constrained efficient, and equity is priced at its true value in this equilibrium.

The introduction of the corporate income tax affects the costs of signalling as well as the profitability of projects, both of which jointly determine the equilibrium inside-equity position. The tax consequences on entrepreneurial risk-bearing are analyzed, and the efficiency costs of the tax are examined, for various specifications of individual risk-preferences. It is found from numerical experiments that the risk-sharing with the government through tax obligations is strong enough to generate a negative marginal excess burden of the tax.

The objective of this paper is to explore the consequences of corporate income taxation in a richer model incorporating informational asymmetries. Most of the existing literature on the efficiency and incidence of taxes ignores information and incentive

problems. While the effect of corporate income taxation on financial decision-making under uncertainty has been the topic of many studies, only a few of them consider asymmetric information. Recent examples include John and Williams[1985], Bernheim[1990], Mackie-Mason[1990], and Innes[1992]. In contrast to these studies ¹ which are largely concerned with the dividend puzzle and impact of taxation, this paper highlights the risk-sharing induced by the incentive compatibility principle that determines the financial structure of a firm.

Signalling with inside equity position has been often employed in previous studies in the finance literature, such as Leland and Pyle[1977], Diamond[1984], and Grinblatt and Hwang[1989]. In these studies, the equity-holders bear unlimited liability; in our model, they are protected by limited liability. ² We do not, however, pursue the study of optimal taxation, and only consider an exogenous tax system in this paper.

This paper is organized as follows: Section II describes the model. Sections III-V are the main body of the paper, and they examine the tax consequences in the case where there are two types of equity-financed projects. First, a signalling game is defined and a unique separating equilibrium is constructed in Section III. Section IV analyzes the consequences of the corporate income tax, and Section V discusses the results of numerical experiments. Section VI discusses two meaningful directions to extend the model: multiple types and additional debt-financing. Section VII summarizes the implications of the paper.

¹ Mackie-Mason[1990] presents an empirical study of testing various hypotheses including taxes and signalling effects.

² Also, our model may be used to determine the optimal financial leverage even in the existence of tax shields provided by debt financing.

II. The Model

Every new investment project requires only a fixed amount of capital cost, K , and produces a gross return, $\tilde{y}$, realized out of the common outcome space, Y . Thus, the only difference among projects lies in their own probability distribution over Y . Furthermore, we assume that the return distribution of each project is uncorrelated with that of any other project, so that each project incurs idiosyncratic risk but no systematic risk. This enables a valuation scheme which depends only on the expected return of a project.³ For simplicity, we assume there are only two possible outcomes from each project; that is, $Y = \{y_1, y_2\}$, where $0 \leq y_1 < y_2$.⁴ With this specification, we can distinguish projects by the probability of outcome y_2 . Types of projects are defined with respect to this probability.

Outside investors are less informed about new projects in the sense that they cannot tell the type of a project, but they know the population frequency of every type in the market. In the presence of such informational asymmetries, an adverse selection problem occurs in the equity market where entrepreneurs seek external financing for their projects. We assume that the informed agents (entrepreneurs) move first by signalling the quality of their projects, and then, observing the signals, the uninformed agents (outside investors) evaluate the prices of projects. The signalling decision by entrepreneurs is based upon their own expectation on the market valuation scheme for their equity, and any self-fulfilling expectation leads to an equilibrium.⁵ Signalling

³ According to CAPM, idiosyncratic risk is not priced since it is diversified away in the market portfolio. The rationale in APT is to insure that any arbitrage portfolio earns no positive return. See, for example, Pflleiderer[1983] for more details.

⁴ The idea of non-negative gross returns is that the entrepreneur bears no responsibility on the outcome once a project is undertaken, which may not be plausible in the presence of moral hazard problems.

⁵ An alternative to this signalling approach is the screening approach, in which the

ensures that projects are not over-priced in equilibrium; it is also assumed that they are not under-priced either due to the competition among outside investors.

Entrepreneurs are risk averse, and endowed with the same initial wealth, w_0 , which defines their reservation utility level. Each of them has only one investment project which he wants to execute with the help of external financing. In the first period, he sets up a firm and issues the equity of his firm. The cash-flow from the equity sale, together with his own wealth, is spent in financing his project, and the remainder is invested in the risk-free security. In the next period, his firm pays the corporate income tax out of the output produced, and the rest of the output is distributed as dividends. All consumption takes place in this period, and there is no further activity as the capital becomes obsolete.

Let α and V denote respectively the fraction of a firm's equity retained by the entrepreneur and the market price of the equity of his firm. Then, the amount of money available to the entrepreneur in each period is given by

$$B = (1 - \alpha)V + w_0 - K$$

$$\tilde{C} = (1 + r)B + \alpha\tilde{y}$$

where $\tilde{C}$ is the consumption in the second period. Among these equations, the first may be called the budget constraint in the sense that it relates the equity price and the initial wealth.

Entrepreneurs maximize their expected utility using an identical state-independent utility function, $u(\tilde{C})$ where $u' > 0, u'' < 0$. For convenience, we redefine the utility

uninformed agents offer *menus* of their own pricing schemes to the informed, and any scheme agreed by the informed leads to an equilibrium. In reality, however, it seems hard to believe that individual investors compete with each other by offering prices for new equity.

function in terms of B and α , so that

$$\begin{aligned} U(B, \alpha) &= Eu(\tilde{C}) \\ &= P_1 u((1+r)B + \alpha y_1) + P_2 u((1+r)B + \alpha y_2) \end{aligned}$$

where P_i is the probability of outcome y_i from a given project, so that $P_1 + P_2 = 1$. From the assumptions on u , it follows that U is concave in α and B . Therefore, the indifference curves on the (α, B) plane are convex to the origin and the marginal rate of substitution between α and B is obtained as

$$-\frac{P_1 y_1 u'(\Gamma_1) + P_2 y_2 u'(\Gamma_2)}{P_1 (1+r) u'(\Gamma_1) + P_2 (1+r) u'(\Gamma_2)}$$

$$\Gamma_i = (1+r)B + \alpha y_i, \quad i = 1, 2.$$

It is easily seen that the indifference curves get flatter as P_1 increases, other things being equal. It implies that the return from risk-bearing increases with the project value; in other words, the so-called single crossing property is obtained.

If the true quality of a project were known to outside investors, the equity of a firm would be priced at the present value of its project, and α would be chosen at zero in every firm as the budget constraint and an indifference curve are tangent at $\alpha = 0$. In the complete information equilibrium, entrepreneurs would hold no (non-negligible) inside equity and all risk would be passed on to outside investors. In the presence of informational asymmetries, however, this is not possible since the true quality is not known unless a firm distinguishes itself from others. Given this adverse selection problem, a higher-quality entrepreneur tries to hold the least amount of inside equity that can deter the imitation of lower-quality entrepreneurs.

III. The Nature of the Game and the Equilibrium

We consider a simple model with two types of projects: ⁶ *High-type* and *Low-type*.

Types are denoted by the subscripts H and L respectively, as in the following:

Expected Return:

$$\bar{y}_H = P_1^H y_1 + P_2^H y_2$$

$$\bar{y}_L = P_1^L y_1 + P_2^L y_2$$

Present Value (= Equilibrium Price of Equity):

$$V_H = \frac{1}{1+r} \bar{y}_H$$

$$V_L = \frac{1}{1+r} \bar{y}_L$$

Market Average Value:

$$V_M = \lambda V_H + (1 - \lambda) V_L$$

λ = population frequency of High-type projects

The equity market is competitive in the sense that outside investors bid for equity in Bertrand fashion, so that the price of each firm's equity is determined at its expected value conditioned on α 's, which is the present value of the project in a separating equilibrium or the market average value in a pooling equilibrium. Therefore, the critical part of each entrepreneur's strategy is to decide on the strength of his signal, α . ⁷

An equilibrium in our model is described by a set of signals from the representative High-type and Low-type entrepreneurs, α_H and α_L respectively, and the corresponding equity-valuation scheme. It is rather easy to find an equilibrium due to the single

⁶ Extended models with more than two types are discussed in Section VI.

⁷ The key component of our model is the assumption that outside investors cannot tell whether it is sent by a High-type by looking at a signal. Instead, we could assume that each entrepreneur suggests the price of his equity as well as his position (α), and outside investors use only α in deciding whether or not to accept this offer, without changing any results in the paper.

crossing property.⁸ In general, the existence of equilibria is not a problem in signalling models, but the uniqueness of equilibrium is. Our model is not an exception, and it has a plethora of equilibria.

Figure 1 illustrates two different types of equilibria: pooling and separating. Every equilibrium is Nash as a result of competitive pricing of equity. It is also sequential as long as its valuation scheme is bounded by V_H and V_L .⁹ However, some of these equilibria are not satisfactory. For example, the separating equilibrium in Figure 1 is sequential but Pareto-dominated by many other signalling equilibria. Suppose outside investors believe that any stronger signal than α^* cannot be sent by a Low-type since sending such a signal only makes himself worse-off even in the case he is taken for a High-type. Then the evaluation scheme should assign V_H to the equity with such a signal, and the given equilibrium collapses as a result of High-types' deviation. This is the idea of the Cho and Kreps Intuitive Criterion. It is easy to see that nothing but the Pareto-dominant separating equilibrium, depicted in Figure 2-I, remains as we refine our Nash equilibria using the criterion. This unique equilibrium in our model is in fact the one that we would obtain if we adopted a Rothschild-Stiglitz type screening model.

In equilibrium, only High-types should hold a certain fraction of their own firms' equity. Their equilibrium utility level is lower than that under the complete information, and the difference measures the signalling costs. Low-types bear no risk as if

⁸ Without the single crossing property, we have to consider every possible position of indifference curves. See, for example, Bernheim[1990] who constructs signalling equilibria in a model without this property.

⁹ A sequential equilibrium requires the consistency and rationality on every information set. For example, any separating equilibrium with $\alpha_L^S > 0$ is not sequential because a Low-type prefers zero signal to α_L^S when the valuation scheme assigns a price no higher than V_L .

information were complete. Their risk-preferences, however, are important in determining the High-types' inside-equity position, α_H^S in equilibrium. The less risk-averse Low-types are, the more inside-equity High-types have to hold, since it is less costly for Low-types to imitate the High-types' behavior. Other things being equal, α_H^S gets higher as the value of Low-type projects gets lower. This is because the potential gains for a Low-type from pretending to be a High-type are larger. This relationship, however, does not hold if Low-type projects have a negative present value. Once High-types retain enough inside-equity to separate themselves from the projects with zero present value, the entrepreneurs holding projects with a negative present value cannot do any better than giving up their projects and enjoying their reservation utility. As a result, there would be only one type of firms, High-types, in the market, and these firms hold inside-equity to the extent that they would hold if Low-type projects have zero present value. Figure 2-II illustrates an equilibrium in this situation. Therefore, the equilibrium signal by High-types depends upon the quality of the projects with which they are competing. In any case, only projects with a positive present value are supported in equilibrium as in the case with no informational asymmetries.

The efficiency of equilibrium is doubtless in the context of signalling; it is Pareto-efficient given the informational constraint. The efficiency of equilibrium with the same characteristics is, however, often questioned in the context of screening and optimal contracts. It is worthwhile to explore a similar question in our signalling model since the answer suggests an insight into the role of financial intermediaries which would be included in an extended model of the financial market. Let us introduce a financial intermediary which offers two different prices directly to all firms, depending upon their inside equity positions. It announces that it will buy up the equity of the firms with no inside equity at a price (slightly) higher than V_L . It also announces that it

will pay a price (slightly) lower than V_H but higher than the price offered to the firms with no inside equity, for the outside-equity of the firms with a positive but (slightly) lower than α_H^S in the separating equilibrium. As illustrated in Figure 3, the financial intermediary can pick a set of prices that both types of firms would like to accept, and, as a result, their types are separated. In Figure 3, the financial intermediary pays extra ϵ dollars per each Low-type firm in addition to V_L , and purchases $(1 - \alpha_H^{S'})$ fraction of equity from every High-type at the price lower than V_H by the amount of $\frac{1 - \lambda}{\lambda} \epsilon$. Low-type firms do not want to imitate High-type firms, and they better off than in the separating equilibrium. High-type firms are also better off since the benefit from their reduced risk-bearing outweighs the loss from the decrease of their equity price. After purchasing equity from the issuing-firms, the financial intermediary publicly offers a new (secondary) security that is a composite of both type of equity at the fair price of $\lambda(1 - \alpha)V_H + (1 - \lambda)V_L$ which would be accepted by outside investors. The financial intermediary can always break even with this kind of trade, and it is also plausible that both types of firms would be willing to pay some amount of commission to the financial intermediary as they become better off. Therefore, the introduction of a financial intermediary brings about a Pareto-improvement. However, the likelihood of such a Pareto-improvement declines as the population frequency of High-type firms gets lower. An intuitive reason is that even an infinitesimal amount of over-pricing of Low-type equity requires a severe under-pricing of High-type equity when there are relatively too few High-type firms, so that High-type firms cannot benefit from the trade.¹⁰

¹⁰ See Rothschild and Stiglitz [1976] for a rigorous treatment in the case of an insurance market.

IV. The Consequences of Corporate Income Taxation

The corporate income tax is levied on the cash-flow from projects,¹¹ and it allows for full loss-offset. One may think of the full loss-offset provision as an abstraction of the loss carry-backward or carry-forward into our model which contains only a single production-period. With this provision, the expected tax revenue from a project is given by $t(\bar{y} - \delta K)$, where t denotes the tax rate, $\bar{y}$ the expected return from the project, and δ the parameter representing the capital deduction allowed in the tax system. For example, $\delta = 1$ describes the depreciation scheme when capital equipment bears no value after being used once, and $\delta = 1 + r$, the expensing scheme by which the capital costs are immediately written off, when the risk-free interest rate, r , serves as the opportunity cost of a dollar investment.

It should be noted that the tax base is greater than the net present value, $\bar{y} - (1 + r)K$ as long as $\delta < 1 + r$. To be precise, the tax base includes not only economic rents but also the opportunity cost which could be tax-free elsewhere. For the sake of simplicity, we will consider only the expensing scheme which would incur no efficiency loss without the informational asymmetries as it permits the deduction of the entire economic costs from the tax base.¹²

The introduction of the corporate income tax affects not only the profitability of projects but also the risk-taking behavior of entrepreneurs. The after-tax cash-flow

¹¹ The cash-flow is in fact equal to the gross return in this model, since there are no production costs other than the capital outlay K .

¹² With no loss-offset provisions, the non-distortionary property of the expensing scheme might disappear. Suppose $y_1 < K < y_2$, then the expected tax revenue from a project is given by $P_2 t(y_2 - K)$, where P_2 denotes the probability of outcome y_2 from the project. This amount is obviously greater than that with the full loss-offset provision, and even the projects with zero net present value are expected to make positive tax payments on average.

and present value of a project are respectively expressed as follows:

$$\begin{aligned} y_i^t &= y_i - t(y_i - (1+r)K), & i &= 1, 2 \\ V_j^t &= V_j - t(V_j - K), & j &= H, L. \end{aligned}$$

As noted earlier, the decrease of the equity price due to the tax is proportional to the net present value, $V - K$; thus, the allocation of capital goods is not affected by the tax imposition. The project that has zero net present value continues to have zero net present value after the tax imposition.¹³

The tax effects on the equilibrium inside-equity position held by High-types are not obvious in the following equilibrium condition:¹⁴

$$u((1+r)B_L) = P_1^L u((1+r)B_H + \alpha y_1^t) + P_2^L u((1+r)B_H + \alpha y_2^t)$$

$$B_L \equiv \max[w_0, V_L^t + w_0 - K]$$

$$B_H \equiv (1 - \alpha)V_H^t + w_0 - K,$$

where the superscript t denotes the after-tax values. Comparative static experiments do not determine the sign of the effects on the indifference curves without additional information on parameters.¹⁵

¹³ In contrast, some entrepreneurs might find their projects no more profitable if the tax were imposed with the depreciation scheme instead of the expensing scheme. If the tax rate with the depreciation scheme is so high that every project ends up with a negative net present value, the equity market disappears and every entrepreneur retains his reservation utility level. If only Low-type projects become unprofitable, the equity of High-type firms is still traded in the market and these firms hold the same inside-equity position as they would if Low-type projects had zero net present value. If the tax rate is as high as $(V_H - K)/(V_H - \frac{K}{1+r})$, then High-types earn nothing after their tax payments. Therefore, they will give up their projects unless they could hold no inside-equity ($\alpha_H = 0$). However, High-types' holding no inside-equity would be copied by Low-types at no costs, and, consequently, there will be no separating equilibrium.

¹⁴ In the case that both types remain profitable after the tax change and the utility function is exponential, High-types' inside equity position increases with the tax rate when V_L and V_H are moderately different.

¹⁵ One of the trivial results is that, as the tax rate rises, Low-types' indifference

Suppose the government were able to distinguish types and impose a lump-sum tax on each type separately. Let γ represent the fraction of the tax revenue collected from High-types, so that $\gamma = 1$ implies that only High-type firms pay taxes, and $\gamma = 0$ that only Low-type firms pay taxes. Given the incomplete information on types, this kind of type-contingent tax may not be feasible unless the tax treats all firms equally, that is, $\gamma = \lambda$.

The exercise with different γ 's, however, helps us understand the welfare effect of the corporate income tax in our model, which can be viewed as a tax on risky income. As long as the tax revenue, denoted by T , is sufficiently small, the single crossing property is preserved, and it is easily seen that

$$\text{Sign}\left(\frac{d\alpha}{dT}\right) = \text{Sign}[(1 - \gamma)u'(\Lambda_0) - \gamma(P_1^L u'(\Lambda_1) + P_2^L u'(\Lambda_2))]$$

$$\Lambda_0 \equiv \bar{y}_L + (1 + r)(w_0 - K)$$

$$\Lambda_i \equiv (1 - \alpha)\bar{y}_H + (1 + r)(w_0 - K) + \alpha y_i,$$

$$i = 1, 2.$$

Therefore, the equilibrium inside-equity position in High-type firms increases with the tax revenue when $\gamma = 0$, and decreases when $\gamma = 1$. When γ is between 0 and 1, the sign depends upon, intuitively speaking, whether the net-of-tax marginal utility of Low-types from their truth-telling becomes greater than their net-of-tax marginal utility from successfully pretending to be High-types. If the former exceeds the latter, the net-of-tax utility level of Low-types from truth-telling must be lower than their net-of-tax utility level from cheating since the utility function is increasing and concave. It then implies that Low-types have a stronger incentive to cheat, so that High-types have to hold more inside equity, than they should before the tax increase.

curves get steeper when their projects have zero present value prior to the tax with the full loss-offset provision.

There is a unique γ with which the tax revenue does not influence the equilibrium inside-equity position, and it turns to be λ in the case of exponential utility functions. In other words, a tax imposed equally on both types is non-distortionary and hence is supposed to incur the lowest welfare costs to the society when individual welfare is represented by such a utility function. This result will be compared with the findings from numerical experiments in the following section.

V. Numerical Experiments: Efficiency Cost Analysis

In numerical experiment of assessing the efficiency costs of the corporate income tax, we employ an exponential utility function such as

$$u(C) = -e^{-sC}, \quad s > 0$$

where s denotes the degree of absolute risk-aversion. One of the useful properties of this utility function is that it entails no wealth effect and therefore the equilibrium inside-equity position is independent of the initial wealth level. For a wide range of parameter values, we collect the same qualitative results, and Tables 1-3 summarize the results from the case in which $y_1 = 10, y_2 = 40, P_1^H = 0.2, P_2^H = 0.8, P_1^L = P_2^L = 0.5, K = 20, r = 0.05$ and $w_0 = 10$.

In Table 1, the equilibrium inside-equity position decreases with the degree of risk-aversion. It increases with the tax rate, which implies that the incentive to lower the position due to the reduction in the High-type project value is outweighed by the pressure to increase the position due to the reduction in the Low-type project value. However, the equilibrium inside-equity position is rather insensitive to the change of the tax rate. For example, the increase in the position is only about 16% when the new tax system seizes about 50% of cash-flow when $s = 0.3$. Table 2 suggests an explanation

of this phenomenon. The table shows the relative risk-aversion of Low-types computed at their expected income. It is Low-types' risk preferences that basically determine the High-types' inside-equity position in equilibrium, and their relative risk-aversion, which is approximately twice the risk-premium per unit of variance for proportional risk, ¹⁶ slowly decreases with the tax rate. This implies that their willingness to take risk is not very responsive to a tax increase, and thus High-types have only to adjust their inside equity position moderately. This finding then suggests that the income variations resulting from tax changes are not fully reflected in High-types' risk-bearing decisions. In other words, High-types income is partly *insured* ¹⁷ in the sense that the risk from income variations is shared by the government via the tax system and thus the individual risk-taking does not need to be adjusted to the full extent of income variations.

In general, the substitution of a lump-sum tax for an existing distortionary tax would raise social welfare, and the extent of welfare enhancement would measure the deadweight loss or excess burden of the initial tax. By the same token, we examine the efficiency of equilibrium in our model by calculating the marginal excess burden of an infinitesimal increase in the tax rate of an existing tax. First, individual utility costs are computed, normalized in dollars using the marginal utility of wealth, and then aggregated over all agents. Then, the marginal excess burden is obtained as the aggregate utility costs per each dollar of expected tax revenues, in excess of one dollar. It measures the deadweight loss per every dollar paid in taxes — that is, the degree of tax-distortion, and it is expected to be non-negative since the tax is distortionary and its rate is raised.

¹⁶ See Chapter 2 in Laffont[1989] for details.

¹⁷ It is not actuarially fair insurance due to the positive expected tax revenue.

Table 3 shows the marginal excess burden on a High-type entrepreneur when the tax rate is raised by 0.001 from the initial rates ranging from 0 to 50%. Since a Low-type entrepreneur bears no risk in equilibrium, the tax is virtually lump-sum to him; hence, his individual utility cost equals the increase in his tax payment, which amounts to \$0.004. In other words, the tax collected from Low-type firms does not incur any deadweight loss. On the other hand, the utility cost to a High-type entrepreneur is always lower than the increase in his tax payment, which is \$0.017. As a consequence, the marginal excess burden is negative in every single case in Table 3. If a lump-sum tax were imposed equally on both types with the government tax revenue kept constant, everyone would suffer the same utility cost of \$0.0085, and the marginal excess burden of this lump-sum tax would be zero. This rather *surprising* result that the excess burden of a distortionary tax is lower than that of a lump-sum tax may cast doubt on the efficiency of equilibrium. In fact, the expectation of a non-negative excess burden, which is legitimate in the certainty cases, are unwarranted here since it does not take into account the effect of government risk-sharing through the tax on risky income. In other words, the non-distortionary tax does not necessarily minimize the deadweight loss to the economy under uncertainty.

VI. Extensions of the Model

1. Multiple Types of Projects

As long as the number of types is finite, the analysis is straight-forward. We can easily extend the results from the two-type model and find a unique separating equilibrium of the same nature. In equilibrium, each entrepreneur's decision on risk-bearing depends only upon the risk-preferences of the entrepreneurs whose projects are inferior to his own project. Figure 4 illustrates the case of three types, and in this case, High- and Intermediate-types' inside-equity positions increase with the profitability of their own projects.

If there is a continuum of types, we need to derive explicitly the equilibrium valuation scheme that identifies a type or a value of P_2 from a given α . It is obtained as the solution to the first order condition of each entrepreneur's utility maximization in terms of α when P_2 is a function of α . Assuming the second order condition is satisfied, the relationship between P_2 and α in equilibrium is given by

$$\begin{aligned}\frac{dEu}{d\alpha} &= P'_1(\alpha)u(\Gamma_1(\alpha)) + P'_2(\alpha)u(\Gamma_2(\alpha)) \\ &\quad + P_1(\alpha)[y_1 - \bar{y}(\alpha) + (1 - \alpha)\bar{y}'(\alpha)]u'(\Gamma_1(\alpha)) \\ &\quad + P_2(\alpha)[y_2 - \bar{y}(\alpha) + (1 - \alpha)\bar{y}'(\alpha)]u'(\Gamma_2(\alpha)) \\ &= 0\end{aligned}$$

where Γ_i denotes the consumption in the state i , and $P_i(\alpha) = P_i$ for *true* valuation in a separating equilibrium, for $i = 1, 2$. This differential equation can only be solved numerically.¹⁸ Once we have the solution to this differential equation, an immediate consequence of the tax imposition is the change in the boundary condition as the set

¹⁸ Employing the same signal of inside-equity position, Leland and Pyle[1977], and Grinblatt and Hwang[1989] derive an equilibrium valuation scheme that is increasing and continuous in the quality of projects. In their models, the gross return, $\hat{y}$ follows a

of profitable projects shrinks. The details of the change will depend upon whether the tax system includes depreciation or expensing, and full loss-offset or no loss-offset.

2. Debt Issue by Firms

We relax the assumption that equity issue is the only means of external financing, and allow firms to issue debt as well. We begin with the case of risk-free debt, that is, $D \leq y_1$, where D denotes the face value of debt. No firm will default, and the value of debt, equity and total value of a firm are respectively given by

$$\begin{aligned} V_D &= \frac{P_1 D + P_2 D}{1 + r} = \frac{D}{1 + r} \\ V_E &= \frac{P_1(y_1 - D) + P_2(y_2 - D)}{1 + r} = \frac{\bar{y} - D}{1 + r} \\ V &= V_D + V_E = \frac{\bar{y}}{1 + r} \end{aligned}$$

Thus, the total value of a firm is independent of the firm's financial structure,¹⁹ which is consistent with the Modigliani-Miller Theorem. The level of risk-free debt issued by a firm carries no information on the quality of the firm since the value of debt does not depend upon the type of the issuing firm. Moreover, it has nothing to do with the

normal distribution, so that the exponential utility function reduces to a simple mean-variance type utility function. Given that the variances are revealed, α is a signal only for the expected return which is private information, and then the signalling differential equation has an analytic solution. However, their model is not realistic since negative and positive gross returns are equally possible due to the assumption of a normal distribution, and equity-holders are subject to unlimited liability in bankruptcy.

¹⁹ Scott[1977] discusses the possibility that the total value of a firm depends upon the level of risk-free debt. In his model, debt payment in bankruptcy is secured by the value of equity which is priced *ex ante* with the consideration of possible bankruptcy. Therefore, additional secured debt increases the firm's value since equity value is reduced only up to the non-bankruptcy probability.

risk-bearing decision because $\tilde{C}$ does not depend upon D as seen in the following:

$$B = (1 - \alpha)V_E + V_D + (w_0 - K)$$

$$\tilde{C} = (1 + r)B + \alpha(\tilde{y} - D)$$

$$= (1 - \alpha)\bar{y} + \alpha\tilde{y} + w_0 - K.$$

Therefore, issuing risk-free debt generates only the transactions that involves neither risk trades nor informational transfers.

When the corporate income tax is levied, the interest payment is deducted from the tax base. Hence, the value of a firm increases with the level of debt issued, other things being equal including the total external financing. Since debt conveys no information, every firm would like to utilize this tax shield to the maximum extent, and as a result, $D = y_1$ in every firm. Therefore, firms' decision on issuing debt is made without regard to their risk-bearing or inside-equity position, and the introduction of taxes only leads to the equalization of the debt level in every firm.²⁰

The introduction of risky debt is a different matter. Equity is protected by the limited liability in bankruptcy, and the debt level should not exceed the highest possible return; that is, $0 \leq y_1 \leq D \leq y_2$, and

$$V_D = \frac{P_1 y_1 + P_2 D}{1 + r}$$

$$V_E = \frac{P_2(y_2 - D)}{1 + r}$$

$$V = V_D + V_E = \frac{\bar{y}}{1 + r}$$

The Modigliani-Miller Irrelevance remains valid, but the value of debt depends upon the type of an issuing firm, which means that debt is no longer information-free. It

²⁰ Because firms benefit from the tax shield, the unused debt capacity, $y_1 - D$, may be considered as an additional signal for the profitability of a firm. However, this possibility is not explored in this paper.

is because issuing risky debt is an alternative way of firms' passing out risk toward outside investors as seen in the expression of expected utility:

$$Eu(\tilde{C}) = P_1 u[(1 - \alpha)P_2 y_2 + \alpha P_2 D + P_1 y_1 + (1 + r)(w_0 - K)] \\ + P_2 u[(1 - \alpha)P_2 y_2 + \alpha P_2 D + P_1 y_1 + (1 + r)(w_0 - K) + \alpha(y_2 - D)].$$

It is obvious that $\frac{dEu}{d\alpha} = \frac{dEu}{dD} = 0$ when either $\alpha = 0$ or $D = y_2$, and otherwise $\frac{dEu}{d\alpha} < 0$, $\frac{dEu}{dD} > 0$. Thus, the optimal choice by firms under complete information would be either $\alpha = 0$ or $D = y_2$. In other words, firms would like to either issue as much debt as possible or hold no inside-equity. Any financial structure including debt less than y_2 and non-zero inside-equity is inferior since there remains some risk inside firms.

Under informational asymmetries, none of the alternative optimal choices are feasible for High-types who need to separate themselves from Low-types. Thus, High-types are induced to issue less debt than y_2 as well as to hold a positive inside-equity position since both means of external financing serve as signals to outside investor. Finding a separating equilibrium in the case of such dual signals begins with constructing the following incentive compatibility condition:

$$u(\bar{y}_L + w_0 - K) = P_1^L u[(1 - \alpha)P_2^H y_2 + \alpha P_2^H D + P_1^H y_1 + (1 + r)(w_0 - K)] \\ + P_2^L u[(1 - \alpha)P_2^H y_2 + \alpha P_2^H D + P_1^H y_1 + (1 + r)(w_0 - K) + \alpha(y_2 - D)].$$

This condition is an indefinite equation for α and D that represents a continuum of different combinations of α and D , among which High-types choose the one that maximizes their utility. It would be an interesting quest to examine how this optimal combination is affected as the tax is imposed or the tax rate changes. It would necessarily involve a numerical analysis, and is beyond the scope of this paper.

VII. Concluding Remarks

The purpose of this paper is to contribute to better understanding of the consequences of corporate income taxation in the financial market where the informational asymmetries persist, which has been rarely considered in the literature.

Developing a simple-yet-useful signalling model, we examine the adverse selection problem caused by the existence of private information on the true quality of firms, and find a unique, informationally-constrained efficient equilibrium in which the willingness of an entrepreneur to bear the risk of his own project serves as a separating signal. In equilibrium, a better quality firm retains more inside-equity, and, as in the case with complete information, only profitable firms are supported in the market.

The corporate income tax affects signalling costs as well as the profitability of projects. Numerical experiments with exponential utility functions find that the inside-equity position of a better quality firm increases as the tax rate rises. This reaction in the risk-bearing is, however, not as responsive as one would expect, due to the risk-sharing with the government. This leads to the interesting result that the excess burden of the corporate income tax is negative, that is, the tax only incurs a lower welfare cost than the lump sum tax with the same tax revenue.

The model in this paper can be extended in many meaningful directions. Increasing the number of types and introducing debt-financing are discussed in detail. To add a few, we may consider endogenizing the size of capital investment, including more than one production periods, or introducing an additional tax such as an interest income tax.

Figure 1. Examples of Equilibria

Figure 2. The Separating Equilibrium

Figure 3. Financial Intermediation

$$B'_H \equiv V_H + w_0 - K - \frac{1 - \lambda}{\lambda} \epsilon$$

$$B'_L \equiv V_L + w_0 - K + \epsilon$$

$$\lambda B'_H + (1 - \lambda) B'_L = V_M + w_0 - K$$

Figure 4. Separating Equilibrium with Three Types

Numerical Experiments with Different Tax Rates(t) and Risk-Aversion Coefficients(s)

[$y_1 = 10, y_2 = 40, P_1^H = 0.2, P_2^H = 0.8, P_1^L = P_2^L = 0.5, K = 20, r = 0.05$ and $w_0 = 10$]

Table 1 Equilibrium Inside-Equity Position (%)

$t \setminus s$	0.01	0.1	0.3	0.5	0.7
0	89.9	60.0	46.9	43.3	41.6
0.1	90.8	61.5	47.9	43.9	42.1
0.2	91.6	63.3	49.0	44.7	42.7
0.3	92.5	65.3	50.4	45.7	43.4
0.4	93.5	67.6	52.2	46.9	44.3
0.5	94.4	70.4	54.5	48.6	45.7

Table 2 Relative Risk-Aversion of a Low-type at the Expected Income

$t \setminus s$	0.01	0.1	0.3	0.5	0.7
0	0.15	1.45	4.35	7.25	10.15
0.1	0.14	1.41	4.23	7.05	9.87
0.2	0.14	1.37	4.11	6.85	9.59
0.3	0.13	1.33	3.99	6.65	9.31
0.4	0.13	1.29	3.87	6.45	9.03
0.5	0.13	1.25	3.75	6.25	8.75

Table 3 Marginal Excess Burden for a High-type (%)

$t \setminus s$	0.01	0.1	0.3	0.5	0.7
0	-8.8	-42.3	-65.6	-68.8	-69.2
0.1	-8.1	-39.8	-64.2	-68.5	-69.1
0.2	-7.3	-37.2	-62.4	-68.0	-69.0
0.3	-6.5	-34.3	-59.9	-67.1	-68.8
0.4	-5.7	-31.2	-56.5	-65.6	-68.3
0.5	-4.9	-27.9	-52.2	-63.0	-67.1

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