CAN MEASUREMENT ERRORS OR SEASONAL ADJUSTMENTS EXPLAIN THE NEGATIVE AUTOCORRELATION OF U.S. MONTHLY CONSUMPTION CHANGES?

by

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ABSTRACT

The negative first-lag autocorrelation of U.S. monthly consumption changes rejects the continuous-time random walk model of consumption. This paper addresses the question of whether data distortions due to measurement errors or the application of the X-11 filter may explain this negative autocorrelation and thus possibly rescue the model from failure. Regarding measurement errors, the paper finds that the conditions under which plausible types of errors may explain it violate a cross-frequency restriction. Regarding seasonal adjustment, the paper finds that if the unknown seasonal component in consumption is stochastic with unit seasonal roots, the application of the X-11 filter makes the first-lag autocorrelation positive instead of negative; if the unknown seasonal component is stationary, it can make it positive or negative depending on the unknown parameters of the seasonal process. Overall, though far from proving an impossibility theorem, the paper provides some arguments against the continuous-time random walk model.

Keywords: temporal aggregation, measurement errors, seasonal adjustment, consumption

JEL Classification: no. C4, E2
Can Measurement Errors or Seasonal Adjustment Explain the Negative Autocorrelation of U.S. Monthly Consumption Changes?

1. Introduction

This paper is a contribution to the small but growing literature that aims at investigating explicitly the effects of data distortions (measurement errors, seasonal adjustment procedures, etc.) on inference about consumer behavior.

Recent literature has investigated the possibility that temporal aggregation - i.e. the possibility that consumers make decisions at intervals shorter than the interval of data observation - can explain the empirical failure of such models as the permanent income hypothesis or the consumption-based asset pricing model. See inter alia Christiano, Eichenbaum and Marshall [1991] (henceforth CEM), Ermini [1988, 1989, 1992, 1994a], and Heaton [1993] for the former case; Grossman, Melino and Shiller [1987] (henceforth GMS), Breeden, Gibbons and Litzenberger [1989] (henceforth BGL), Ermini [1991], and Longstaff [1989] for the latter case.

With some exceptions (namely, Ermini [1988, 1989, 1992]), all the cited studies assume that consumers make decisions in continuous time. As this assumption is as arbitrary as the assumption that consumers make decisions as frequently as data is observed (no temporal aggregation effects) typical of previous studies on consumer behavior, it is interesting to ask if the available data permits to discriminate whether the true but unknown interval of consumer decision is small (close to continuous-time decisions) or large (close to the interval of data observation).

The answer to this question affects the identification of the generating mechanism of consumption in the presence of a nuisance parameter and the estimation of parameters of the utility function like the coefficient of relative risk aversion (on this issue, compare for example GMS and Ermini [1991]). At a deeper level, the answer to this question relates to the fundamental issue of the existence of frictions in agents’ behavior and market mechanisms. Assuming that decisions are taken in continuous-time (or at very small intervals) presupposes the belief that agents respond effortlessly and costlessly to any small variation of their information set, whereas conjecturing that decisions are taken at larger intervals reflects the belief that agents’ behavior is characterized by inertia and slow reaction to new information, perhaps as a result of costs of decision making and implementation.

As pointed out in Ermini [1989], and briefly summarized in section 2, U.S. monthly consumption data provides some empirical evidence that permits to discriminate between ranges of decision intervals: the negative autocorrelation of monthly consumption changes rejects the continuous-time approach, and corroborates under durability of consumer expenditures the conjecture of decision intervals not too smaller than a month\(^1\). This
conclusion, however, may be weakened or reversed altogether if this negative autocorrelation is the result of measurement errors, seasonal adjustment procedures, or other phenomena of data distortion.

The purpose of this paper is to investigate whether measurement errors and the application of the X-11 filter can alter the rejection of the continuous-time hypothesis. Regarding measurement errors, economists know next to nothing about them - with the exception of the recent work of Bell and Hillmer [1990], Wilcox [1992], and Bell and Wilcox [1993] on estimating the sampling error of retail store recordings. The treatment of this case is thus necessarily conjectural. To establish which classes of measurement errors are plausible or not, the paper adopts a signal-to-noise criterion: monthly measurement errors should not eventually dominate the signal in variance, nor be dominated by it.

Under this criterion, the issue of which classes of measurement errors are plausible replicates the issue of trend-stationarity vs. difference-stationarity of consumption data. To satisfy the criterion, in fact, the errors must be stationary in variance in the former case, and must exhibit a unit root in the latter case. By endorsing a difference-stationary model of consumption and some of the suggestions advanced by Bell and Wilcox [1993] and Wilcox [1992] about the nature of sampling errors in consumption, the paper finds the conditions - indeed quite restrictive - under which unit-root errors may explain the negative autocorrelation of monthly consumption changes. These conditions, however, if satisfied, violate a cross-frequency restriction on the data generating process of observed consumption.

Regarding seasonal adjustment, the paper studies the effect of the application of the X-11 filter on the first-lag autocorrelation of monthly consumption both theoretically and empirically. As seasonally unadjusted monthly consumption data are unavailable 2, the analysis is again conjectural, and it is based on assuming two quite different models of stochastic seasonality (deterministic seasonality is not considered). One model describes the seasonal component as having unit roots at all the 11 seasonal frequencies; the effect of this model on the first-lag autocorrelation is studied theoretically. The other model describes the seasonal component as a stationary process; due to the complexity of the analytical treatment, its effect is studied empirically via Monte Carlo simulations based on the filter weights reported in Ghysels and Perron [1991]. In the former case, the paper finds that the application of the X-11 filter makes the first-lag autocorrelation of monthly consumption positive instead of negative; in the latter case, it can make it positive or negative depending on the parameters of the unknown stationary seasonal process.

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1. The same evidence and the same paper's conclusions hold if one consider log consumption and the autocorrelation of the growth rate of monthly consumption.
2. The Bureau of Economic Analysis publishes "retrospective" seasonally-unadjusted quarterly data in the Journal of Current Issues, but not monthly.
The main contribution of the paper - though far from proving an impossibility theorem - is to build a case against the continuous-time random walk model, by raising strong doubts that the negative first-lag autocorrelation of monthly consumption changes is convincingly explained by measurement errors and/or the application of the X-11 filter.

The paper is organized as follows. Section 2 summarizes some empirical evidence; section 3 discusses the measurement error issue; section 4 discusses the seasonal adjustment issue; finally, section 5 reports some concluding remarks.

2. Some Empirical Evidence

For a summary of the issue, consider figure 1 which reports the effect of temporal aggregation on the first-lag autocorrelation of a first-order integrated, first-order moving average process, or IMA(1,1) (which includes the random walk case); $m$ is the sampling ratio, the ratio between the interval of data observation and the consumer decision interval. When the process is observed as frequently as decisions are made, $m = 1$; if decisions are made more frequently than observed, $m > 1$; in the limit, when decisions are made in continuous time, $m = \infty$. Curve $a$ describes the effect of temporal aggregation on a random walk process (zero first-lag autocorrelation for $m = 1$): it becomes a positive IMA(1,1) for any $m > 1$, and its first-lag autocorrelation approaches the limit value of 0.25 as $m$ goes to infinity (Working [1960]). Curves $b$ and $c$ refer to processes generated as a negative and positive IMA(1,1) with a first-lag autocorrelation close to their lower and upper bounds of -0.5 and 0.5 respectively; as $m$ goes to infinity, these processes also converge to a positive IMA(1,1) with 0.25 first-lag autocorrelation (Ermini [1989]).

Quarterly data provides no indication about the true consumer decision interval. As quarterly consumption of non-durables and services is fitted by a positive IMA(1,1) with an estimated first-lag autocorrelation of 0.202 and 95% confidence interval [0.051, 0.326] (Ermini [1989]; for similar results, see also BGL [1989], Ermini [1988], and CEM [1991]), it follows from figure 1 that all the values $m > 1$ are admissible. In particular, the continuous-time version of the random walk model of consumption behavior - assumed, for example, by CEM to test the permanent income hypothesis and by GMS to test the consumption-based capital asset pricing model - is not rejected, as one cannot reject its transformation at quarterly frequency into a positive IMA(1,1) with first-lag autocorrelation of 0.25.

The continuous-time model of consumption has been well received in the literature, though modelling consumers decisions at infinitesimally small intervals makes even more compelling, from a theoretical point of view, the criticism against the additive-separability of the utility function and the non-durability of consumption goods. This criticism has recently received considerable attention in the literature, with a number of articles dealing with non-separable utility functions (see, among others, Dunn and Singleton [1988], Epstein and Zin [1989], Constantinides [1990]), and durability of consumer expenditures (Ermini [1992] and [1994b], Ferson and Constantinides [1991], Heaton [1993]).
In support to this criticism, the evidence provided by *monthly* data rejects the continuous-time random-walk model. The latter would be transformed into the same IMA(1,1) with 0.25 first-lag autocorrelation even when observed at monthly frequency - the sampling ratio would still be infinite; but monthly consumption is well fitted by a *negative* IMA(1,1) with a first-lag autocorrelation of -0.211 and 95% confidence interval [-0.31, -0.11] (Ermini [1989]; see also BGL [1989]). In fact, looking at figure 1, it is clear that this evidence rejects the random walk model for *any* value of the sampling ratio.

As anticipated, an interesting and quite plausible alternative is to introduce durability of consumer expenditures into the model, in which case consumption would be generated as a negative IMA(1,1) ([Ermini [1992]]. Under this alternative, the estimated 95% confidence interval of figure 1 would entail decision intervals not too much smaller than a month (say, between 1 and 4), thus corroborating the conjecture that decisions are taken, on average, at *larger* than smaller intervals.

These conclusions, however, may be weakened or reversed altogether by the effect of measurement errors and/or of data distortions resulting from seasonal adjustment procedures. Some researchers have argued in fact that these factors lower the first-lag autocorrelation of consumption data enough to make a positive IMA(1,1) with 0.25 autocorrelation appear as a negative IMA(1,1)\(^3\).

3. The effect of measurement errors

The rejection of the continuous-time random walk is based on the assumption that monthly data do not contain measurement errors. Most economists would consider this assumption unrealistic, though the majority of empirical works carried out with U.S. monthly data do not account for measurement errors. In fact, some economists even advocate that monthly data should never be used because of their noisiness; to do so, however, would amount to wasting potentially relevant information.

As economists know practically nothing about measurement errors in consumption - notwithstanding the recent work of Bell and Hillmer [1990], Wilcox [1992], and Bell and Wilcox [1993] on estimating the sampling error of retail sales - a sensible way to proceed is to investigate the effects of plausible conjectures about the statistical properties of such errors. To this aim, the paper adopts a *signal-to-noise criterion*: monthly measurement errors should not eventually dominate the signal in *variance*, nor be dominated by it. Though some economists may not agree with the propriety of this criterion, it is hard to believe that the technology of data collection, reporting, coding, etc. will progress to the extent that eventually

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3. Anonymous referees for previous author's papers, personal communications and scattered mentions in the literature (Backus, Gregory and Zin [1988], CEM [1991]).
measurement errors will be eliminated altogether.

Under this criterion, the choice of plausible classes of measurement errors replicates the controversy about trend-stationarity vs. difference-stationarity of consumption data: in the former case, the errors must be stationary in variance; in the latter case, they must exhibit a unit root. As this paper deals with unit-root models of consumption (the IMA(1,1) class), the investigation of measurement error effects will be confined to unit-root errors. Note that these errors by construction are highly autocorrelated, a property that Bell and Hillmer [1990] and Bell and Wilcox [1993] empirically find in actual sampling error (their model however is stationary).

3.1 Unit-Root Measurement Errors

Consider first the simple case of a random walk error, \( \Delta u_t = \alpha + \eta_t \), with variance \( \sigma^2_\eta \), uncorrelated with unobserved consumption. If true monthly consumption follows the IMA(1,1) process \( \Delta C_t = \delta + \varepsilon_t + h \varepsilon_{t-1} \) with variance \( \sigma^2_\varepsilon \), then observed consumption \( C_{t}^* = C_t + u_t \) is generated as\(^4\)

\[
\Delta C_{t}^* = (\delta + \alpha) + (1 + hB)\varepsilon_t + \eta_t ,
\]

which is also an IMA(1,1) with first-order autocorrelation \( \rho^* \)

\[
\rho^* = \frac{h \, SN}{(1+h^2)SN + 1} ;
\]

\( B \) is the backward operator, that is \( x_{t-k} = B^k x_t \); \( SN = \sigma^2_\varepsilon / \sigma^2_\eta \) is the signal-to-noise ratio.

As \( SN \) is a non-negative number, \( \rho^* \) will have the same sign of \( h \). It follows that the theoretical value \( h = 0.268 \) corresponding to the temporal aggregation to monthly frequency of the continuous-time random walk model - in fact, of any continuous-time model - cannot be transformed by this type of measurement errors into the observed negative moving average coefficient.

In addition to a high degree of autocorrelation, Bell and Wilcox [1993] find a significant moving-average component in sampling errors. Aside from seasonal components, they estimate a stationary ARMA model of the type

\[
(1 - \beta B)u_t = (1 + \theta B)\eta_t ,
\]

with \( \beta = 0.75 \) and \( \theta = 0.10 \) (see also Wilcox [1992]). Though this model is stationary, and thus it does not strictly belong to the class of unit-root errors we are considering here, we can take up the suggestion of a moving-average component, and explore the effect on observed

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4. If consumption and errors are expressed in log values, the errors are taken to be multiplicative rather than additive. The exposition that follows remains practically unchanged.
consumption of an IMA(1,1) measurement error, \( \Delta u_t = \alpha + (1 + \theta B) \eta_t \). In this case, observed consumption follows

\[ \Delta C_t^* = (\delta + \alpha) + (1 + hB)e_t + (1 + \theta B)\eta_t, \]

which is an IMA(1,1) process with first-order autocorrelation

\[ \rho^* = \frac{h^2 SN + \theta}{(1 + h^2)SN + 1 + \theta^2}. \]

From this, the signal-to-noise ratio necessary to transform the true \( h \) into the observed \( \rho^* \) is

\[ SN = \frac{\theta - \rho^* (1 + \theta^2)}{\rho^* (1 + h^2) - h}. \]

For the negative observed \( \rho^* \) and for positive \( h \), the condition that the signal-to-noise ratio cannot be negative implies the restriction \( \theta < \rho^* (1 + \theta^2) \), that is the restriction that the monthly changes of the error must be more negatively autocorrelated than the monthly changes of consumption. For example, the value \( \theta = -0.10 \) estimated by Bell and Wilcox [1993] would not satisfy this restriction, although, strictly speaking, this value may be inappropriate here as it is associated to the first-order autoregressive coefficient \( \beta \) being equal to 0.75 and not to one\(^5\).

In addition to the condition \( SN \geq 0 \), it is also reasonable to expect the variance of the measurement errors to be quite smaller than the variance of the signal, i.e. \( SN \gg 1 \). Moreover, it is easily seen from (6) that for \( \theta \) to be a real number, the signal-to-noise ratio must satisfy the restriction

\[ 4 \rho^* \left[ \rho^* + (\rho^* (1 + h^2) - h) SN \right] \leq 1; \]

note that this restriction does not depend on \( \theta \). For the given parameter values \( \rho^* = -0.211 \) and \( h = 0.268 \), one gets \( SN \leq 1.97 \), which is possibly too low to be plausible. Noting that with the given parameters, \( \text{var}(\Delta C_t)/\text{var}(\Delta u_t) = 1.06 SN \), this corresponds to a standard deviation of measurement errors equal to 71% of the standard deviation of consumption. The results improve slightly if one consider the extreme value -0.11 of the 95%-confidence interval for \( \rho^* \); in this case, the implied maximum value for \( SN \) becomes 5.6 - corresponding to a standard deviation of measurement errors equal to 42% of the standard deviation of consumption.

\(^5\) An additional reason for caution in applying to this paper's analysis the empirical regularities found by Bell and Hillmer [1990] and Bell and Wilcox [1993] is that their findings relate to nominal consumption as opposed to real consumption. Economists still know nothing on how errors in nominal consumption and errors in the consumer price index combine to yield errors in real consumption. Moreover, we need to know more about measurement errors of each single subcategory of the broad NIPA (National Income and Product Account) category of non-durables and services, before extrapolating to the latter the regularities found from retail stores.
If these conditions are not satisfied, measurement errors do not rescue the continuous-time random walk model from failure. If these conditions are satisfied, however, a cross-frequency restriction on the data generating process of observed consumption is violated - and thus the model again is not rescued - as explained below.

Incidentally, while the continuous-time random walk does not appear to be consistent with this class of errors, the negative IMA(1,1) does, thus corroborating a consumption model with durability of expenditures and decision intervals not too smaller than a month. Suppose, for example, that consumption is generated as a negative IMA(1,1) at intervals not too shorter than a month. Then, if by temporally aggregating this process at monthly frequency one gets a monthly random walk (h = 0), the signal-to-noise ratio that explains the observed $\rho^*$ is $SN = 4.6$ [1.6, 19.7]. Even more compelling, if by temporally aggregating the same process to monthly frequency one gets a negative IMA(1,1) (say, $h = -0.10$), then $SN = 8.9$ [2.3, 96.7].

3.2 A Cross-Frequency Restriction

Suppose that the IMA(1,1) measurement error meets the two conditions above stated about the value of $\theta$ and $SN$; that is, by adding this IMA(1,1) error to unobserved consumption (which follows by hypothesis an IMA(1,1) with $\rho = 0.25$), one gets observed monthly consumption. As quarterly observed consumption is the temporal aggregation of monthly observed consumption with $m = 3$, it must also be equal to the sum of the temporal aggregation to quarterly frequency of the IMA(1,1) error and of unobserved monthly consumption. From figure 1, the temporal aggregation of the IMA(1,1) error yields another IMA(1,1) process (with positive or negative first-order autocorrelation depending on the monthly value), while the temporal aggregation of the IMA(1,1) with $\rho = 0.25$ yields the same process; it follows that quarterly consumption must also follow an IMA(1,1) process. In fact, quarterly U.S. consumption of nondurables and services is best fitted by an ARIMA(2,1,1), and the IMA(1,1) is rejected against it with test statistic $F(2,148) = 13.28$ whose $P$-value is practically zero$^6$.

Suppose now, more generally, that one disputes that observed monthly consumption is an IMA(1,1), and endorses a richer parametrization. To rescue the continuous-time random walk, then, the error must also follow a richer parametrization than IMA(1,1), say, an ARIMA($p,1,q$). Through temporal aggregation to quarterly frequency, this becomes a more parsimonious ARIMA($p,1,q$) process, with $p \leq \overline{p}$ and $q \leq \overline{q}$ (Weiss [1984]). As the temporal

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6. The data is per-capita seasonally adjusted 82-dollars quarterly consumption from 1951(IV) to 1990(II), obtained (in Citibank notation of NIPA categories) as (GCN82 + GCS82)x(GYDPC8)/(GYD82). Fitting this data with ARIMA($p,1,q$) models with $p, q \leq 2$, the ARIMA(2,1) came out the best under mean-square and Schwarz criteria. Furthermore, the ARIMA(2,1,1) is not rejected against the ARIMA(2,1,2) with $F(1,147) = 0.38$ (P-value = 0.537), while all the more parsimonious models are rejected against the ARIMA(2,1,1).
aggregation of unobserved consumption leads to the same IMA(1,1) with $\rho = 0.25$, it follows from Granger's theorem on summing independent time series (Granger and Newbold [1986], p. 29) that quarterly observed consumption must follow a more parsimonious process than monthly observed consumption. Again, this restriction is violated.

3.3 Alternative Models of Measurement Errors

For the sake of completeness, the remaining of this section discusses three models of measurement errors often considered in the literature; neither of these models, however, satisfies the signal-to-noise criterion as in the first one the errors are stationary, and in the other two the error variance grows faster than the variance of consumption.

(i) *Additive stationary errors*. This is the model most frequently used in the literature; the model adopted by Wilcox [1992] and Bell and Wilcox [1993], for example, belongs to this class. Measurement errors $u_t$ are additive to the signal, are stationary, and are uncorrelated with the signal. Observed consumption becomes

$$\Delta C^*_t = \gamma + (1 + hB)e_t + (1 - B)u_t; \quad (8)$$

thus, for arbitrary stationary $u_t$, $C^*_t$ would follow an ARIMA($p$, 1, $q$) with an auto-regressive component identical to the error autoregressive component. Under Bell and Wilcox's [1993] parametrization of the sampling error - equation (3) - observed consumption would follow an ARIMA(1,1,2). Fitting this model to U.S. monthly data of non-durables and services, the estimates of the autoregressive coefficient and of the first- and second-order moving-average coefficients were 0.13, 0.41, and -0.15 respectively. At the 95% confidence level, the value 0.13 is significantly different from the value 0.75 of Bell and Wilcox. Furthermore, the ARIMA(1,1,2) model is rejected against more parsimonious parametrizations (see also Ermini [1994b]).

Although this model of measurement errors can rescue the continuous-time random walk for some values of the signal-to-noise ratio, as anticipated it does not satisfy the plausibility criterion: the assumption of stationarity errors in the presence of an I(1) signal implies that their importance relative to consumption levels becomes smaller and smaller, thus ultimately making the consumption series noise-free.

(ii) *Additive proportional errors*. An alternative conjecture often considered in the literature is to assume that the measurement errors are additive but proportional to the level of consumption, $u_t = k_t C_t$, with the factor of proportionality $k_t$ generated as a zero-mean stationary process uncorrelated with consumption. For $k_t$ a white noise with variance $\sigma_k^2$, and $C_t$ an IMA(1,1), some algebra shows that $u_t$ has zero mean, zero autocorrelation at all lags, and a second-order heteroskedastic variance, $\text{var}(u_t) = \delta^2 \sigma_k^2 + (1 + h^2 \sigma_k^2, \sigma_e^2 (t-1)) + \sigma_k^2 \sigma_e^2$, where $\delta$ is the drift of $C_t$.

Under this conjecture, $C^*_t$ also follows an IMA(1,1) process, with first-order autocorrelation of consumption changes:
\[ \rho^* = \frac{h \sigma_e^2 \cdot \text{var}(u_{t-1})}{(1+h^2) \sigma_e^2 + \text{var}(u_t) + \text{var}(u_{t-1})}. \]  \hspace{1cm} (9)

In this case, for large enough \( t \), \( \rho^* \) tends to the value -0.5 regardless of the strength of the signal. As this corresponds to a process with a unit root in the MA component, it follows that for large enough \( t \), \( C_t^* \) eventually appears as being generated as a heteroskedastic trend-stationary series, \( \delta_t + u_t \). Heuristically, the variance of the measurement error \( u_t \) grows quadratically, while the variance of unobserved consumption grows linearly; eventually the former will overshadow the latter, so that observed consumption simply appears as measurement errors added to a linear deterministic trend. This second conjecture also does not satisfy the plausibility requirement.

(iii) Growth rate errors. A third conjecture is to hypothesize that measurement errors affect the growth rate of consumption, i.e.

\[ \frac{\Delta C_t^*}{C_t^*} = \frac{\Delta C_t}{C_t} + k_t, \]  \hspace{1cm} (10)

which can be approximated as \( \Delta C_t^* = \Delta C_t + k_t C_t \). For \( k_t \) having the same properties of the previous case, the term \( k_t C_t \) has again zero mean, zero autocorrelation at all lags, and second-order heteroskedastic variance. Therefore, \( C_t^* \) is still generated as an IMA(1,1), with first-order autocorrelation of observed consumption changes:

\[ \rho^* = \frac{h \sigma_e^2}{(1+h^2) \sigma_e^2 + \text{var}(u_t)}. \]  \hspace{1cm} (11)

For \( t \) sufficiently large, \( \rho^* \) goes to zero, as a quadratically growing \( \text{var}(u_t) \) eventually will dominate the constant terms of (6). Heuristically, as the growing variance of \( k_t C_t \) eventually dominates the constant variance of the stationary process \( \Delta C_t \), observed consumption \( C_t^* \) will eventually follow a random walk driven only by the heteroskedastic innovations \( k_t C_t \). This third conjecture again does not satisfy the plausibility criterion.

4. The effects of the X-11 filter

In this section the effects of the X-11 filter on the first-lag autocorrelation of an IMA(1,1) process are investigated. As seasonally unadjusted monthly consumption is not available, the analysis is based on assuming two quite different models of stochastic seasonality (deterministic seasonality is not considered): a unit-root model and a stationary model.

4.1 Seasonal Unit-root Model

In this case, the seasonal component is assumed to have unit roots at all the 11 monthly seasonal frequencies; the seasonally unadjusted consumption process is thus assumed to be generated as
\[ S(B)(1-B)C_t = (1 + hB)e_t , \]  

(12)

where \( S(B) = \sum_{j=0}^{s-1} B^j \), with \( s = 12 \) for monthly data. Note that the seasonal component does not have a unit root at the zero frequency, as suggested \textit{inter alia} by Maravall [1993]. This and the lack of a seasonal moving average component make this model different from the "airline" model of Box and Jenkins [1976] adopted by Bell and Wilcox [1993] to analyze the effect of measurement errors on seasonally unadjusted data. Model (12) is adopted here instead of the airline model on the grounds that consumption is not an I(2) process (particularly, it does not exhibit a quadratic trend).

The effect of the X-11 filter on (12) is studied theoretically, by passing the consumption process \( C_t \) through a seasonal adjustment filter \( F(B) \) having the properties of being double-sided, finite, symmetric, and decomposable as

\[ F(B) = F^*(B,F)S(B)S(F) ; \]  

(13)

here \( F \) is the forward operator (\( = 1/B \)), and the polynomial \( F^*(B,F) \) is convergent (no unit roots). As in actual applications this polynomial is rather unimportant compared to the \( S(F)S(B) \) component of the filter, to simplify the analysis we set \( F^*(B,F) = 1 \). Then filtered consumption \( C^f_t = F(B)C_t \) follows the process

\[ (1-B)C^f_t = (1 + hB)S(F)e_t . \]  

(14)

Clearly, as the seasonal filter is double sided, its application adds the non-invertible component \( S(F) \) to the moving average of seasonally adjusted consumption. Some algebra shows that \( C^f_t \) follows an IMA(1,12) process, and that the first-lag autocorrelation of \( \Delta C^f_t \) is

\[ \rho_1 = \frac{(s - 1)(1 + h)^2}{s(1 + h^2) + (s - 1)(1 + h)^2} , \]  

(15)

which is always positive for any value of \( h \); for some positive values of \( h \) it is also greater than 0.5, suggesting some autoregressive distortion introduced by the filter. The filter in fact introduces additional distortions through \( F^*(B,F) \) and by lengthening the moving-average process to lag 12. However, for our purposes, the main result is that the first-lag autocorrelation \( \rho_1 \) cannot be made negative.

4.2 Seasonal Stationary Model

In this case, the seasonal component is assumed to have no unit root at any of the 11 seasonal frequencies. The seasonally unadjusted consumption process is thus assumed to be generated as

\[ A(B^s) (1-B)C_t = (1+h B)D(B^s) e_t ; \]  

(16)

in what follows the two seasonal polynomials \( A(B^s) \) and \( D(B^s) \) will be simplified to the first-order case only. Due to the complexity of studying the effect of the X-11 filter on (16) analytically, the study of this case is carried out empirically, via Monte Carlo simulations.
based on the filter weights reported in Ghysels and Perron [1991].

The Monte Carlo experiment consisted of 150 repetitions (obtained by generating 150 different zero-mean white noise series $e_i$ of 600 data points) of each combination of model (13) corresponding to five innovations variances $\sigma_i^2$ (1, 5, 10, 50, 100), to seven values of the nonseasonal moving average coefficient $h_i$ (-0.8, -0.3, -0.1, 0, 0.1, 0.3, 0.8), and to five values of both the first-order seasonal autoregressive parameter $\alpha$ in $A(B^4) = 1 - \alpha B^4$, and of the first-order seasonal moving-average parameter $\delta$ in $D(B^4) = 1 + \delta B^4$, (-0.7, -0.2, 0, 0.2, 0.7). The experiment thus includes both the case of autoregressive-only seasonal component and of moving average-only.

Each of the 150 repetitions of each model was passed through the linear X-11 filter described and discussed in Ghysels and Perron [1991], and the corresponding first-lag autocorrelation of the seasonally adjusted series in first differences, $\hat{\rho}$, calculated. The results turned out to be qualitative similar for all the combinations, and unaffected by the variance of the process innovations. Figure 2 reports the seasonally adjusted $\hat{\rho}$ vs. the seasonally unadjusted $\rho = h/(1+h^2)$ for the autoregressive-only seasonal model. It is seen that $\hat{\rho}$ can be higher or lower than the seasonally unadjusted $\rho$, depending on the seasonal autoregressive parameter $\alpha$: with a positive $\alpha$, the X-11 filter decreases the first-lag autocorrelation of consumption changes; with negative $\alpha$, it increases it. Figure 3 reports $\hat{\rho}$ vs. $\rho$ for the moving average-only case, obtaining similar results. Finally, figures 4 and 5 report $\hat{\rho}$ vs. $\rho$ for the seasonal ARMA(1,1) cases corresponding to a value of the moving-average seasonal parameter $\beta$ of -0.7 and 0.7 respectively. In the former case, the X-11 filter increases the first-lag autocorrelation of monthly consumption changes, regardless of the value of the seasonal autoregressive parameter $\alpha$; $\alpha$, however, affects the extent by which the autocorrelation is increased. In the latter case, the filter decreases the autocorrelation.

In actuality, the true seasonal component in monthly consumption may well be in between the two extreme cases considered here, with unit roots at some seasonal frequencies and stable roots at the others; the effects of these intermediate cases have not been studied. Once again, the above results do not unambiguously show that seasonal adjustment rescue the continuous-time random walk process. In fact, if one endorses the view of unit-root models of seasonality, the results seem to provide some corroboration against the continuous-time model. The ambiguity can only be removed by greater knowledge about seasonally unadjusted consumption. It should also be emphasized that the application of the X-11 filter of Ghysels and Perron [1991] does not correspond exactly to the complete procedure of deseasonalization adopted by the BEA. Moreover, given that an IMA(1,1) error added to an IMA(1,1) signal produces an IMA(1,1) corrupted signal, the conclusions of this section also apply to the deseasonalization of unadjusted consumption corrupted by measurement errors. Finally, it is worth recalling the conclusion of Bell and Wilcox [1983], who, studying a model-based deseasonalization of consumption series assumed to follow Box and Jenkins' [1976] airline model with an additional unseasonal moving average component, determined that measurement errors do not affect the latter.
5. Conclusions

The negative autocorrelation of monthly consumption changes rejects the continuous-time random walk model of consumer behavior, and corroborates under durability of consumer expenditures the conjecture that decision intervals are not too smaller than a month. This paper has sought to verify the extent to which data distortions induced by measurement errors and/or the application of the X-11 filter can alter these conclusion (and in particular rescue the continuous-time model from failure). Regarding measurement errors, the paper adopts a plausibility criterion based on the signal-to-noise ratio (whereby the signal cannot eventually dominate the measurement error, nor be dominated by it), and finds that the conditions - indeed quite restrictive - that plausible types of errors could explain this negative autocorrelation violate a cross-frequency restriction. Interestingly, these measurement errors are consistent with the alternative negative IMA(1,1) model of consumption.

Regarding seasonal adjustment, the paper finds that if the unknown seasonal component in consumption is stochastic with seasonal unit roots, the application of the X-11 filter makes the first-lag autocorrelation positive instead of negative; if the unknown seasonal component is stationary, it can make it positive or negative depending on the unknown parameters of the seasonal process. The consensus among economists, however, seems to be directed toward unit-root seasonality.

The main contribution of this paper to build a case against the continuous-time random walk model, by raising strong doubts that the negative first-lag autocorrelation of monthly consumption changes is explained by measurement errors and/or the application of the X-11 filter. At a broader level, the paper makes a contribution to the small but growing literature that aims at explicitly investigating the effects of data distortions (measurement errors, seasonal adjustment procedures, etc.) on inference about consumer behavior.
References


Figure 1 - Effect of temporal aggregation on IMA(1,1) processes

(1) 95% Confidence Interval for rho estimated with quarterly data

(2) 95% Confidence Interval for rho estimated with monthly data