THE EFFECT OF ERROR-CORRECTION ON TESTING
THE RATIONAL-EXPECTATIONS NEUTRALITY HYPOTHESIS

by

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Abstract

By adopting a VAR framework in first differences, recent literature has confirmed previous results in testing the macro rational-expectations hypotheses of rationality and neutrality: rationality is corroborated, neutrality is rejected. However, this paper shows that, by correctly incorporating a long-run cointegrating relationship between money, output and interest rate in the form of an error-correction term, the test results are reversed, in that neutrality is no longer rejected.

Keywords: rationality, neutrality, error-correction

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The Effect of Error-Correction on Testing the Rational-Expectations Neutrality Hypothesis

1. Introduction

Recently in this Journal, Bohara [1991] reported identical results to Mishkin's [1982] in testing the macro rational-expectations hypotheses of rationality and neutrality: rationality is corroborated, neutrality is rejected. Bohara used a bivariate VARX model in first differences, with money and output as endogenous variables and interest rate as exogenous. To motivate his model as an improvement over Mishkin's model, Bohara used a more parsimonious parametrization and, more importantly, incorporated the evidence that these three variables follow difference-stationary rather than trend-stationary processes, as instead assumed by Mishkin. The fact that Bohara obtained identical test results, despite a different and perhaps more appropriate parametrization, apparently makes an important contribution to the money neutrality debate.

However, on the basis of some recent advances in the theory of multivariate first-order integrated I(1) processes (see particularly Engle and Granger [1987], Johansen [1988], and Engle and Granger [1991]), this paper argues that Bohara's model is incorrect in that it completely ignores the possibility of cointegrating relationships between the variables of interest. More specifically, by taking first differences without allowing for the presence of error-correction terms, Bohara's model may be misspecified.

This paper shows that, by properly taking into account a long-run cointegrating relation between money, output and interest rate, money neutrality is no longer rejected (rationality is still not rejected, as in Mishkin and Bohara). As our investigation essentially endorses all other Bohara's improvements over Mishkin, the reversal of the test result can be safely attributed to the presence of this long-run cointegrating relation. Furthermore, our result seems to indicate that the money neutrality debate may be related to the debate of whether money supply, output and interest rates are difference-stationary or trend-stationary: in the latter case, Mishkin model would be more appropriate, and neutrality would be rejected; in the former case, our model (that is, Bohara's plus error-correction) would be more appropriate, and neutrality would not be rejected.

The result of this paper is relevant in two ways. Firstly, it reopens the debate about money neutrality in U.S. data. Secondly, it offers a good counterfactual example against some arguments recently appeared in the literature about the irrelevance for empirical work of distinguishing between difference-stationary and trend-stationary processes (for example, Christiano and Eichenbaum [1989]).

The paper is organized as follows. Section 2 describes the model, and the procedure followed to derive it. Section 3 describes the test results. Section 4 adds some concluding remarks.
2. The Model

The first step is to specify the forecasting equation for money, of the general form

$$A(B)M_t = C(B)Z_{t-1} + u_t$$ (1)

where $M_t$ is the monetary aggregate, $Z_t$ a vector of macro-variables that have predictive power for $M_t$, and $u_t$ is the one-step ahead forecasting error. $A(B)$ is a polynomial in the lag-operator $B$ (i.e., such that $M_{t+k} = B^k M_t$), and $C(B)$ is a matrix of polynomials in $B$. Both $A(B)$ and $C(B)$ may include unit roots.

In Bohara’s specification (equation (2) of his paper), $M_t$ is the natural log of the money measure M2; $Z_t$ is the natural log of the return on a three-month treasury bill, $r_t$; $A(B) = (1-B)(1-\gamma_1 B)$, and $C(B) = (1-B)(\gamma_2 + \gamma_3 B + \gamma_4 B^2)$. In choosing this specification, Bohara assumes difference-stationarity for the three variables \(^1\), and only unidirectional causality from the interest rate to money growth.

As we cannot rule out a priori a bidirectional causality between money and the interest rate, as well as a possible predictive power of output for money, we decided to approach the identification of the forecasting equation for money from a three-variable VAR system point of view. Following Johansen [1988], and using "\(\Delta\)" for first differences, we estimated the model \(^2\)

$$D(B) \Delta X_t = \Pi X_{t-1} + v_t$$ (2)

where $X_t$ is the vector $[M_t, r_t, Y_t]'$ with $Y_t$ the natural log of real GNP; $D(B)$ is a matrix polynomial of degree $k-1$, i.e. $D(B) = \sum_{j=0}^{k-1} D_j B^j$, with $D_0$ equal to the identity matrix. The 3x3 matrix $\Pi$ is decomposable into the product $\alpha \beta'$ of two 3xp matrices ($p < 3$), and the $p$ elements of $\beta' X_{t-k}$ are called the error-correction terms. These terms identify the long-run cointegrating relations among the three variables, and help forecast the vector $X_t$ in addition to its autoregressive component.

We first estimated (2) for several values of $k$, and chose $k = 4$ as the optimal lag length based on both AIC and FPE criteria. Then, we applied Johansen’s procedure to the case $k = 4$, to test for the number of significant cointegrating relations. We found one significant relation (i.e., the rank of $\beta$ is one). We also found that output has no predictive power for money, except for its presence in the error-correction term. For this reasons, we decided to

\(^1\) We tested the three series for unit roots, following Perron’s [1989] model C of structural break, and could not reject the hypothesis of unit root at the 5% level for all the three series.

\(^2\) We used the same type of Citibase quarterly data that Bohara used, and we tested our model both with the full available sample (1959:I-1990:IV) and Bohara’s sample (1959:I-1986:IV). We obtained identical test results in both cases. The discussion in the paper refers to the full sample estimates.
eliminate output from the list of predictors for money.

Next, we consider the output equation, of the general form

$$ F(B) \Delta Y_t = G(B) (\Delta M_t - P_{t-1} [\Delta M_t]) + H(B) P_{t-1} [\Delta M_t] + e_t, $$

where $P_{t-1} [\cdot]$ stands for forecast. Neutrality implies $H(B) \equiv 0$, and rationality implies the forecast to be identical to the mathematical expectation, so that $\Delta M_t - P_{t-1} [\Delta M_t] = v_{1,t}$, the innovation term of the first equation of (2) (under the hypothesis that (2) is correctly specified). In Bohara’s specification, $F(B)$ is of order two, and $G(B)$ and $H(B)$ are both of order one. For consistency, in our model we adopt these same orders.

By incorporating into the output equation a linear forecast for money, Bohara obtains an "unrestricted" VARX(4,4) with money and output as endogenous variables, and the interest rate as the exogenous variable ("unrestricted" with respect to the rationality and neutrality hypotheses; the model still exhibits many zero restrictions). Similarly, in our case, we replace $P_{t-1} [\Delta M_t]$ with the linear forecast:

$$ P_{t-1} [\Delta M_t] = (1 - D_{11}^* (B)) \Delta M_t - D_{12}^* (B) \Delta r_t + \alpha_1^* W_{t-4}, $$

where $D_{11}^* (B)$ and $D_{12}^* (B)$ are elements of a matrix $D^* (B)$ in general different from $D (B)$ of (2). Combining (4) with (2)-(3), we obtain a tri-variate "unrestricted" VAR model of the form

$$
\begin{bmatrix}
D_{11}(B) & D_{12}(B) & 0 \\
D_{21}(B) & D_{22}(B) & 0 \\
N_1^*(B) & N_2^*(B) & F(B)
\end{bmatrix}
\begin{bmatrix}
\Delta M_t \\
\Delta r_t \\
\Delta Y_t
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha^*(B)
\end{bmatrix}
W_{t-4} +
\begin{bmatrix}
v_{1,t} \\
v_{2,t} \\
e_t
\end{bmatrix},
$$

where

$$
\begin{align*}
N_1^*(B) &= (1 - D_{11}^* (B)) [G(B) - H(B)] - G(B) \\
N_2^*(B) &= -D_{12}^* (B) [G(B) - H(B)] \\
\alpha^*(B) &= \alpha_1^* [H(B) - G(B)].
\end{align*}
$$

Note that $N_1^*(B)$ and $N_2^*(B)$ are polynomials of degree 4, and $\alpha^*(B)$ is a polynomial of degree 1, so that the error-correction term in (5) appears as $W_{t-4}$ and $W_{t-5}$.

By comparing our model (5) with Bohara’s VARX model (equation (5) of his paper), it is readily seen that the latter is a special form of our model, obtained by eliminating the error-correction term $W_{t-4}$ and by making the interest rate exogenous, that is by restricting $\alpha_1$, $\alpha_2$, $\alpha^*(B)$, $D_{21}(B)$ and $D_{22}(B)$ to be zero.

3. The test of rationality and neutrality

As anticipated, neutrality implies $H(B) \equiv 0$, and rationality implies $D_{1j}^* (B) = D_{1j} (B)$ for both $j=1, 2$. Imposing this joint restriction to our model (5) implies the following set of seven interequation non-linear restrictions:
where $\phi_{ij}^k$ is the $(i,j)$-th element of the 3x3 matrix of coefficients at lag $k$ of the "unrestricted" model (5), and $\psi_j^s$ is the $j$-th element of the vector of coefficients associated to the error-correction terms $W_{s.4}$ ($s = 0$) and $W_{s.5}$ ($s = 1$).

As in Bohara, we tested these restrictions through the Wald test, and obtained a test statistic $W = 0.99$, which compares against a critical value (for seven degrees of freedom) of 14.07 at the 5% confidence level. Thus, we cannot reject the joint restriction of rationality and neutrality. With Bohara's sample size (1959:I-1986:IV) the test statistics was $W = 0.88$.

This paper's reversal of Bohara's rejection of the neutrality hypothesis can be related to his neglect of the error-correction term $W_{s.4}$ in the output equation (3). By neglecting this term, the one-step ahead forecast error for money growth is $\nu_{1,t} + \alpha W_{s.4}$, which is a stationary process by construction, but not necessarily white noise. Thus, in Bohara's case, the forecast of money growth is sub-optimal. It follows that, under the neutrality restriction, Bohara eliminates from the output equation only a component of anticipated money growth, leaving in the long-run component represented by the cointegrating relation. As this long-run component is significant, it is not surprising that anticipated money growth matters, which is precisely what Bohara's rejection of money neutrality in this context means.

4. Conclusions

In testing the rational-expectations joint hypothesis of rationality and money neutrality, this paper has shown that, by correctly incorporating a long-run cointegrating relationship between money, output and interest rate in the form of an error-correction term in a VAR framework in first differences, test results previously reported in the literature are substantially modified: neutrality is no longer rejected.

This result is relevant in two ways. First, it reopens the debate about money neutrality in U.S. data. Secondly, it offers a good counterfactual example against some arguments appeared in the literature about the irrelevance for empirical work of distinguishing between difference-stationary and trend-stationary processes.
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