EFFECTS OF TRANSITORY CONSUMPTION AND TEMPORAL AGGREGATION ON TESTS OF THE PERMANENT INCOME HYPOTHESIS

by

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ABSTRACT

This paper investigates the effects on the dynamic properties of the permanent income hypothesis (PIH) of different conjectures about transitory consumption and temporal aggregation, and identifies representations for the null hypothesis that are better - in some specified sense - than those considered in the literature, from both a theoretical and an empirical point of view. Specifically, the paper suggests that a more appropriate univariate representation for the null hypothesis is the integrated-moving average IMA(1,1) process with a negative MA coefficient, and that a more appropriate bivariate consumption-income representation is the integrated-moving average-distributed lag IMA(1,1,3) process. By providing better representations of the null, the robustness of such alternatives as, for example, the liquidity constraint hypothesis may be strengthened.
1. Introduction

Following repeated rejections of certain implications of the permanent income hypothesis, or PIH (mainly Hall [1978], Davidson and Hendry [1981], Flavin [1981], Hayashi [1982], Mankiw [1982], Muellbauer [1983]), a vast majority of economists seem to have accepted the conclusion that PIH is not consistent with aggregate consumption data. This negative consensus is well captured by Deaton's [1989] suggestion that "no more intellectual effort should be expended in the attempt to rescue a model that seems to be so blatantly at odds with the raw, untreated data". As a result, a number of researchers have attempted to explain PIH failures by bringing into the model liquidity constraints, agents' myopia, labor rationing, seasonal fluctuations, costly decision-making or different functional forms for consumers utility (see, among others, Flavin [1985], Hall [1985], Myron [1986], Deaton [1987, 1989], Campbell and Deaton [1989], Campbell and Mankiw [1987, 1989], Ermini [1989b]), Wickens and Molana [1984].

However, without necessarily detracting from these research efforts, a number of economists have recently taken the opposite tack of challenging the validity of some of the cited rejections. Critiques of reported tests focus on a number of issues, among which: econometric problems, as discussed in Nelson [1987], which showed that Flavin's [1981] rejection of PIH could be spurious due to her detrending of non-stationary first-order integrated series; temporal aggregation bias, as in Ermini [1988] and Christiano, Eichenbaum and Marshall [1991], which show that Hall's model is not rejected if agents are assumed to make decisions at shorter intervals than the usually assumed quarterly interval (however, see also Ermini [1989a] for a critique of Christiano, Eichenbaum and Marshall's approach); a different frequency of consumption data, as in Ermini [1989a], which combined temporal aggregation bias with the use of monthly data; the durability of non-durable goods and services (Ermini [1990] and Caballero [1990]); the presence of measurement errors (Wilcox [1990]).

Such factors as aggregation over time and over agents, durability of non-durable goods and services, the time-alignment of consumption decisions or the reaction time of consumers to new information can be particularly relevant, as they raise the

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important issue of how "hidden" auxiliary assumptions inevitably embedded in economic models may affect test results and ultimately the direction of further research. Unfortunately, consideration of these hidden factors usually escapes the economists' attention, as they are seen more as "technicalities" of economic modelling than issues of "economic significance". Yet, this alternative research agenda has already shown that modifying hidden assumptions - possibly in the direction of greater plausibility - can drastically modify the model to be tested under the null, to the extent that widely accepted results may be reversed.

The contribution of this alternative agenda to the current debate on PIH should not be misinterpreted: while the research program captured by Deaton's suggestion exhorts economists to focus on alternative hypotheses, this agenda exhorts economists to focus on the null hypothesis as well, in an effort to identify the most appropriate model to be tested against whichever alternative the researcher might choose. By providing a null which is better - in some specified sense - from both a theoretical and an empirical point of view, the robustness of such alternatives as, for example, the liquidity constraint hypothesis may be strengthened.

The purpose of this paper is to add a further strand to this research agenda on hidden assumptions, by taking up the issue of transitory consumption and measurement errors under temporal aggregation. This is done in both a univariate and a bivariate consumption-income framework. For the univariate case, the paper develops first various generating mechanisms of consumption corresponding to PIH under a broad spectrum of conjectures about the dynamic properties of transitory consumption and measurement errors. Then, taking into account that the best univariate model - in the likelihood-ratio sense - to fit monthly consumption data appears to be a first-order integrated moving average process, or IMA(1,1), with a negative MA coefficient (see also Ermini [1989a]), it shows that this alternative generating mechanism is in fact an implication of PIH under a very plausible subset of these conjectures, and thus argues that this mechanism is a more appropriate representation of the null hypothesis than the repeatedly rejected random walk process, or IMA(1,0). The fact that with this choice of the null PIH is not rejected in a simple univariate framework gives an indication of how proper consideration of hidden assumptions can substantially alter test results.

The paper also suggests a bivariate consumption-income representation of the null hypothesis, to account for the theoretically more compelling interaction between consumption and income. A significant number of tests have rejected PIH by showing that income - whether in level or in changes - affects consumption differently from
what is predicted by the restrictions under the null. For example, Nelson [1987] found that one-period lagged income changes are significant, whereas in his version of PIH only current income changes belong to the model. Building on the previous univariate representation of the null, this paper argues that in fact income changes lagged up to three periods do belong in the PIH model; furthermore, the moving average property of the regression residuals is preserved. More precisely, this paper suggests that an appropriate consumption-income bivariate representation of PIH takes the form of an IMAX(1,1,3) process

\[ \Delta C_t = d + \sum_{j=0}^{3} \beta_j \Delta Y_{t-j} + w_t + c \, w_{t-1}, \]

where \( w_t \) is a zero-mean white noise process, and where the coefficients \( \beta_j \) obey specific restrictions (see section 3 for details). Again, model (1) is not rejected against other bivariate consumption-income alternatives, thus providing another example of possible reversal of previously reported failures of PIH.

This paper is relevant in two ways. First, it argues that previously reported tests of PIH were based on inappropriate null hypotheses resulting from an insufficient understanding of the effects of transitory consumption and temporal aggregation, and it suggests null hypotheses for PIH that are better from both a theoretical and an empirical point of view. Secondly, it emphasizes the relevance for empirical work of focusing on such hidden assumptions as transitory consumption and temporal aggregation.

The paper is organized as follows. In section 2 the univariate case is considered, by developing first the generating mechanism of consumption corresponding to PIH under a broad spectrum of conjectures about transitory consumption (section 2.1) and measurement errors (section 2.2), and then by identifying the subset of conjectures that are consistent with monthly data (section 2.3) \(^2\). Among others, the conjecture consistent with rational expectations (a zero-mean process uncorrelated with its own past) belongs to this subset. Section 3 considers the bivariate consumption-income case. Finally, section 4 offers some concluding remarks.

2. The Univariate Case

2.1 The generating mechanism of consumption

\(^2\) In fact, quarterly data were also used, yielding results which are consistent, under temporal aggregation, with those obtained with monthly data.
Following Flavin [1981], the permanent income hypothesis, or PIH, can be shown to imply the following generating mechanism for consumption:

$$C_t = \alpha + [1+r(1-\beta)]C_{t-1} + \varepsilon_t + u_t - (1+r)u_{t-1},$$

(2)

where $u_t$ is transitory consumption, $r$ is the (constant) real interest rate, and $\varepsilon_t$ is related to the innovations of income. Under the assumption of rational expectations, $\varepsilon_t$ is a zero-mean orthogonal (white noise) process; moreover, under PIH $u_t$ and $\varepsilon_t$ are uncorrelated at all lags. A minor controversy surrounds the nature of the process $\varepsilon_t$. Some researchers argue that this process is the innovation of labor income only; others argue that, as agents also face unexpected asset returns, this process is the innovation of total disposable income. This controversy does not matter for the univariate generating mechanism of consumption derived in this section, but it does for the bivariate consumption-income derivation of section 5, where the disposable income interpretation will be adopted.

Equation (2) shows that the generating mechanism of consumption is "driven" by income innovations and by the transitory component of consumption $u_t$. By assuming $\beta = 1$ and $u_t = 0$, as in Falvin [1981] and others, (2) reduces to Hall's familiar random walk model of consumption. This paper relaxes the latter assumption, so that the model of consumption as generated by the representative agent is now:

$$\Delta C_t = \alpha + \varepsilon_t + u_t - (1+r)u_{t-1},$$

(3)

where $\Delta C_t = C_t - C_{t-1}$. In what follows, model (3) will be taken to represent the permanent income hypothesis.

This paper also relaxes the standard assumption of empirical economics that agents make decisions as often as economists observe them, by replacing it with the assumption that consumption decisions can be made at intervals shorter than the interval of observation. In this case, economists observe a temporally aggregated version of consumption. This assumption can be supported by the argument that, as an implication of life-cycle rational expectations models with no costs of decision-making, consumers make decisions as often as they receive income and that in organized economies income is received fairly regularly at a monthly frequency or less (for further discussion, see also Ermini [1989b]).

To derive a testable hypothesis about the properties of both permanent and transitory consumption under temporal aggregation, suppose first that $u_t$ follows a zero-mean white noise process of variance $\sigma^2_u$. Then $C_t$ in (3) is a first-order integrated first-order moving average process, IMA(1,1), with $\text{var}(\Delta C) = \sigma^2_\varepsilon + \bar{r}\sigma^2_u$, $\bar{r} = 1 + (1+r)^2$, and first-order autocorrelation
\[
\rho(\Delta C) = - \frac{(1+r)\sigma_u^2}{\sigma_e^2 + \bar{r}\sigma_u^2} = - \frac{1+r}{ST + \bar{r}} 
\]  

where \( ST = \sigma_e^2 / \sigma_u^2 \) is the signal-to-noise ratio between permanent and transitory consumption. When transitory consumption is identically zero \((\sigma_u^2 = 0)\), \( ST = \infty \), i.e., only signal and no noise; in this case, consumption is only related to the "permanent" component generated by income innovations. When consumption is only transitory, with no relation to income innovations \((\sigma_e^2 = 0)\), then \( ST = 0 \). Note that, as both \( r \) and \( ST \) are non-negative, \( \rho(\Delta C) \) is negative. That is, the permanent income hypothesis \( (3) \), combined with the assumption of white noise transitory consumption, implies that consumption generated by the agent follows an IMA(1,1) process with a negative moving average coefficient.

Now let \( \bar{C}_i \) be the monthly aggregation of the consumption series generated by the agent, that is

\[
\bar{C}_i = \sum_{j=0}^{m-1} C_{i-j}, 
\]  

where \( C_{i-j} \) is generated as in \( (3) \) at shorter intervals, and \( m \) is the sampling ratio, that is the ratio between a month and the interval of data generation (the index \( i \) refers now to months). Under temporal aggregation an IMA(1,1) process remains an IMA(1,1) process, with corresponding variance and first-order autocorrelation depending on the sampling ratio \( m \) (Ermini [1989a]). Specifically, the generating mechanism of monthly aggregated consumption becomes:

\[
\Delta \bar{C}_i = m \alpha + \bar{e}_i + h \bar{e}_{i-1}, 
\]  

with first-order autocorrelation of the difference \( \Delta \bar{C}_i \) given by:

\[
\rho_m(\Delta \bar{C}) = \frac{h}{1+h^2} = \frac{(m^2-1) + 2(m^2+2) \rho(\Delta C)}{2(2m^2+1) + 8(m^2-1) \rho(\Delta C)}.
\]  

For \( m = 1 \) (corresponding to \( C_i \) generated at monthly frequency), \( \rho_m(\Delta \bar{C}) = \rho(\Delta C) \). For \( \rho(\Delta C) < 0.25 \), \( \rho_m(\Delta \bar{C}) \) is monotonically increasing with \( m \). Therefore, in correspondence with a negative first-order autocorrelation \( \rho(\Delta C) \) of consumption generated by the agent, \( \rho_m(\Delta \bar{C}) \) can remain negative for low values of \( m \) and can become positive for high values of \( m \). Moreover, if \( \rho(\Delta C) = 0 \), \( (7) \) reduces to Working’s [1960] result that a random walk under temporal aggregation becomes an IMA(1,1) process with a positive MA coefficient, and thus the IMA(1,1) generating mechanism \( (6) \) still holds for monthly consumption, except for \( m = 1 \).

Consider now the case that \( u_t \) is not white noise, but follows a higher-order autoregressive-moving average ARMA\((p,q)\) process, with \( p,q > 0 \). Then consumption
generated by the agent would follow an ARIMA process of higher order than IMA(1,1). By temporally aggregating to monthly intervals, this higher order ARIMA process remains in general a higher order ARIMA process, and, as the sampling ratio \( m \) goes to infinity, tends towards an IMA(1,1) with positive MA coefficient (see, for example, Tiao [1972] and Weiss [1984]).

These results can be summarized in the following proposition:

**PROPOSITION 1**: If consumption is generated by the agent according to the permanent income hypothesis (3) at intervals not greater than a month, and if consumption is observed by the econometrician at monthly intervals without measurement errors, then monthly-aggregated consumption \( \bar{C} \), follows:

(i) an IMA(1,1) process with negative MA coefficient, if transitory consumption \( u_t \) is white noise, and the sampling ratio is small \(^3\);

(ii) an ARIMA process of higher order than IMA(1,1), if \( u_t \) follows an ARMA model of higher order than white noise and the sampling ratio \( m \) is not significantly big;

(iii) an IMA(1,1) process with positive MA coefficient, if \( u_t \) follows any ARMA process (including white noise) and the sampling ratio \( m \) is big; or, if \( u_t \) is identically zero and \( m \) is not equal to one (i.e. if the agents’ decision interval is not a month, but lower);

(iv) a random walk process if \( u_t \) is identically zero and \( m = 1 \).

Proposition 1 has a strong implication: making use of the empirical finding (section 4) that the best univariate model to fit monthly data is the IMA(1,1) process with a negative MA coefficient, we are able to identify the subset of conjectures about transitory consumption and temporal aggregation that make PIH consistent with the data: with this finding, in fact, the three possibilities (ii), (iii) and (iv) are necessarily ruled out.

2.2 The model of observed consumption

Consider now the case that monthly consumption is observed by the econometrician with an additive measurement error, \( e_t \), uncorrelated with both transitory consumption \( u_t \) and income innovations \( e_t \). That is, \( \bar{C}^*, = \bar{C} + e_t \). If (6) holds for the monthly aggregate of consumption generated by the agent, then observed monthly consumption follows

\(^3\) In practice, of the order of 4 (see also Ermini [1989a]). This corresponds, for example, to weekly decision intervals.
\[
\Delta \bar{c}^*_t = m \alpha + \bar{e}_t + h \bar{e}_{t-1} + e_t - e_{t-1}.
\] (8)

If the measurement error \(e_t\) is a zero-mean white noise process of constant variance \(\sigma_e^2\), from (8) \(\bar{c}^*_t\), still follows an IMA(1,1) process, with \(\text{var}(\Delta \bar{c}^*_t) = (1+h^2)\sigma_e^2 + 2\sigma_e^2\), and first-order autocorrelation of the differences

\[
\rho^*_m(\Delta \bar{c}^*_t) = \frac{h \sigma_e^2 - \sigma_e^2}{(1+h^2)\sigma_e^2 + 2\sigma_e^2} = \frac{h SN - 1}{(1+h^2)SN + 2},
\] (9)

where \(\sigma_e^2\) is the variance of \(e_t\) (the innovation of monthly consumption in (6)), \(h\) is the MA coefficient in (6), and \(SN^* = \sigma_i^2/\sigma_e^2\) is the signal-to-noise ratio. Otherwise, if the measurement error is generated by a higher-order ARMA(\(p\), \(q\)) process with \(p\), \(q > 0\), then observed monthly consumption \(\bar{c}^*_t\), follows an ARIMA process of higher order than IMA(1,1). Note that this occurs also if the monthly aggregated consumption generated by the agent, \(\bar{c}_t\), follows an ARIMA process of higher order than IMA(1,1), regardless of the stochastic properties of the measurement error.

The effect of additive measurement errors on the IMA(1,1) process (6) is to reduce the first-order autocorrelation of consumption changes in proportion to its variance. So, if \(\rho_m(\Delta \bar{c})\) is already negative, the presence of measurement errors will make it more negative; if it is positive, it may make it negative for large enough error variance \(\sigma_e^2\) (or equivalently for small enough signal-to-noise ratio). Therefore, observed monthly consumption \(\bar{c}^*_t\), can follow an IMA(1,1) process with a negative MA coefficient, and the corresponding monthly aggregated consumption generated by the agent before the addition of measurement errors, \(\bar{c}_t\), may follow, depending on the signal-to-noise ratio, an IMA(1,1) with a negative or positive MA coefficient.

These results can be summarized in the following proposition:

**Proposition 2:** If consumption is generated by the agent according to the permanent income hypothesis (3) at intervals not greater than a month, and if the monthly aggregate \(\bar{c}_t\) of this consumption is observed by the econometrician with additive measurement errors, \(e_t\), uncorrelated with both transitory consumption and income innovations, then observed monthly consumption \(\bar{c}^*_t = \bar{c}_t + e_t\), follows:

(i) an IMA(1,1) process with negative MA coefficient, if \(e_t\) is white noise and \(\bar{c}_t\) follows an IMA(1,1) process with negative MA coefficient; or, if \(e_t\) is white noise, \(\bar{c}_t\) follows an IMA(1,1) process with positive MA coefficient and the signal-to-noise ratio is low (large measurement error variance);

(ii) an IMA(1,1) process with positive MA coefficient, if \(e_t\) is white noise, \(\bar{c}_t\) follows an IMA(1,1) process with positive MA coefficient, and the signal-to-noise ratio is high;
(iii) an ARIMA process of higher order than IMA(1,1) if \( e_t \) follows an ARMA process of higher order than white noise; or, if \( 
abla e_t \) follows an ARIMA process of higher order than IMA(1,1), regardless of the dynamic structure of \( e_t \).

Proposition 2 also has a strong implication: as monthly observed consumption follows an IMA(1,1) process with a negative MA coefficient, under PIH the two possibilities (ii) and (iii) are necessarily ruled out.

2.3 The empirical evidence

The theoretical results of the previous sections are here compared with the following empirical result (see also Ermini [1989a]) \(^4\): the maximum likelihood estimate of the IMA(1,1) model for observed monthly consumption \( \nabla C^* \), (standard errors in parentheses):

\[
\begin{align*}
\Delta C^* & = 11.22 + w_t - 0.253 w_{t-1}, \\
& \quad (\text{standard errors} \quad (0.428) \\
\end{align*}
\]

is not rejected at the 5% level against the set of all nesting univariate ARIMA(\( p, 1, q+1 \)) alternatives, with \( p + q \leq 3 \). Moreover, the random walk model, or IMA(0,1), is rejected at the 5% level against the IMA(1,1) model. Thus, model (10) can be considered the best univariate process to fit monthly consumption. Indeed, as reported in the cited reference, monthly data exhibit a correlogram with a marked first-lag autocorrelation, and negligible values at higher lags. Moreover, the 95% confidence interval of the estimated MA coefficient, \((-0.353, -0.153)\), indicates that within the class of IMA(1,1) models for monthly consumption the null hypothesis of a negative MA coefficient cannot be rejected as well.

From the two propositions of the previous section, the failure to reject the IMA(1,1) model with negative MA coefficient leads to the following conclusions:

(a) without measurement errors, monthly data corroborate PIH under the assumption that transitory consumption is white noise and that the sampling ratio is small (from proposition 1). On the other hand, if the sampling ratio is significantly big, it follows from proposition 1 that PIH is rejected regardless of the dynamic properties of transitory consumption. This last point bears some interesting implications for consumption-based asset pricing models, usually built on the assumption of infinitesimally small decision periods (see Ermini [1991] for further discussion). Note

\(^4\) US monthly, percapita, seasonally adjusted consumption data of non-durables and services were used for this work. Qualitatively identical results were obtained when using log consumption. Model (10) is a reestimated version of the model reported in the cited reference.
also that, as the random walk model is rejected, so is the assumption of identically zero transitory consumption. Thus, the rejection of the random walk model should not be necessarily interpreted as rejection of PIH - as inferred in some literature - but rather as rejection of this specific conjecture about transitory consumption.

(b) with additive measurement errors, monthly data corroborate PIH in the following cases:

(i) both the measurement error \( e \), and the transitory consumption \( u \), are white noise, and the sampling ratio is small (from part (i) of proposition 2, combined with part (i) of proposition 1);

(ii) the measurement error is white noise with a large variance compared to monthly consumption innovations (e.g., small signal-to-noise ratio), transitory consumption follows any ARMA process of higher order than white noise, and the sampling ratio is significantly high (from part (i) of proposition 1, combined with part (iii) of proposition 2);

(iii) the measurement error is white noise with large variance, transitory consumption is identically zero, and the sampling ratio is strictly less than a month (from part (i) of proposition 2, combined with part (iii) of proposition 1).

Unlike case (a), we have in (b) three observationally equivalent cases. Some further conclusions are:

(1) monthly data reject PIH under the assumption that measurement errors and transitory consumption are both identically zero. This null hypothesis is identical, for example, to Flavin's [1981], whose results are thus confirmed. From what argued in this paper, then, the cause of Flavin's rejection may rest on this specific conjecture for \( u \), and \( e \), rather than on PIH itself.

(2) monthly data reject PIH under the assumption that measurement errors are of higher order than white noise.

(3) monthly data reject PIH under the assumption of no measurement errors and big sampling ratios. Particularly, this rejects PIH for continuously made consumption decisions (\( m = \infty \)) observed with no error (see also Ermini [1989a]).

(4) under PIH, transitory consumption can be of higher order than white noise only if the sampling ratio is big enough (technically, infinite) to transform any ARIMA process into an IMA(1,1). In this case, the first-order autocorrelation of this limiting IMA(1,1) process is equal to 0.25. With an estimated first-order autocorrelation of -0.253 and 95%-confidence interval (-0.353, -0.153), from (9) a signal-to-noise ratio \( SN^* \) between 0 and 7 is required for this case to be possible (e.g., the variance of consumption
innovations must be no larger than seven times the variance of measurement errors). On the other hand, the possibility that transitory consumption follows a higher order process than white noise is ruled out by the argument that the true sampling ratio should be between 0 and 4 because consumers make decisions as often as they receive income innovations, and because in organized economies income is regularly received on average at biweekly or monthly intervals.

3. The Bivariate Case

In the original derivation of PIH with transitory consumption (model (3)), \( e_t \) is a zero-mean white noise process proportional to labor income innovations (see Flavin [1981] for details). Adding the possibility that agents also face unexpected asset returns, \( e_t \) becomes proportional to the innovations \( \eta_t \) of disposable income, that is \( e_t = k \eta_t \), with \( k \) a constant of proportionality whose value depends on the dynamic structure of the generating mechanism of disposable income. Letting

\[ A(B) \Delta Y_t = \gamma + \eta_t, \tag{11} \]

be the generating mechanism of disposable income \(^5\), with \( B \) the backward operator (that is, \( \Delta Y_{t-4} = B^4 \Delta Y_t \)), \( \gamma \) a drift and \( A(B) \) an invertible polynomial in \( B \), and substituting in (3), we get the bivariate generating mechanism of consumption at the frequency of agents' decisions

\[ \Delta C_t = \alpha + k A(B) \Delta Y_t + u_t - (1+r) u_{t-1}, \tag{12} \]

where \( \alpha \) is a suitable constant. To aggregate (12) to monthly frequency, and to keep the contributions of income and transitory consumption separated, we make use of the property that temporal aggregation is a linear operator and thus can be applied to the right-hand-side terms of (12) separately. Specifically, the IMA(1,1) process (12) can be seen as composed of an IMA(1,1) process associated with transitory consumption, and of an IMA(1,0) process associated with the white noise \( \eta_t \). Then, following Ermini [1989a], the IMA(1) process associated with transitory consumption is transformed under temporal aggregation into another IMA(1) process, say, \( \bar{\eta}_t + c \bar{\eta}_{t-1} \), where the coefficient \( c \) is related to \( (1+r) \) through a formula similar to (7). And, following Working [1960], the IMA(1,0) process associated with \( \eta_t \) is transformed into a IMA(1) process, say, \( \bar{\eta}_t + b \bar{\eta}_{t-1} \), where the coefficient \( b \) tends to 0.268 as the sampling ratio \( m \) goes to infinity, and where \( \bar{\eta}_t \) is the innovation process of monthly-aggregated

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\(^5\) In this section we make use of the well known empirical fact that disposable income appears to be a first-order integrated process.
disposable income, \( \bar{Y} \). Letting

\[
\bar{A}(B) \Delta \bar{Y}_t = m \gamma + \bar{\eta}_t, \tag{13}
\]

be the monthly-aggregate of (11), then the monthly-aggregate of consumption under PIH becomes

\[
\Delta \bar{C}_t = \bar{a} + k \bar{A}(B) \Delta \bar{Y}_t + k b \bar{A}(B) \Delta \bar{Y}_{t-1} + \bar{u}_t + c \bar{u}_{t-1}, \tag{14}
\]

which is the equivalent of (6) for the bivariate case. Depending on the nature of the polynomial \( \bar{A}(B) \) and of aggregated transitory consumption \( \bar{u}_t \), (14) can be shown to correspond to the null representation (1) mentioned in the introduction.

Three comments are in order. First, the residuals \( \bar{u}_t \) follow a MA(1) process if transitory consumption \( u_t \) is generated as white noise (they would follow a higher-order ARMA process, if transitory consumption also were generated as a higher-order ARMA process). Thus, simply based on this property, (14) would be rejected if its residuals followed a white noise process, unless decisions are effectively taken by the agent at monthly frequency (no temporal aggregation bias). Secondly, due to the temporal aggregation of \( \bar{\eta}_t \), lagged income changes belong in PIH even if monthly income is a random walk (\( \bar{A}(B) = 1 \)). This second comment provides an explanation to Nelson's [1987] finding of a significant first-lag of income changes with quarterly data, since (14) would also hold at a quarterly interval aggregation and quarterly disposable income is indeed found to follow a random walk. Third, the addition of measurement errors for consumption would not change the qualitative structure of (14); as seen in previous sections, their effect would be simply to lower the first-order lag autocorrelation of \( \bar{u}_t \). However, the addition of measurement errors for income would significantly affect the dynamic structure of the residuals in (14). For example, if disposable income were to follow a random walk and transitory consumption a white noise, the residuals in (14) would follow an MA(2). Bearing on the fact that these moving average terms of order higher than one would have very small coefficients, (14) will still be taken to represent PIH also in the presence of measurement errors for both consumption and disposable income.

3.1 The bivariate generating mechanism of consumption

To derive from (14) a testable representation, the generating mechanism of disposable income must be estimated. The maximum likelihood estimate of the ARI(2,1) model (standard error in parenthesis)

\[
\Delta \bar{Y}_t = 15.6 - 0.15 \Delta \bar{Y}_{t-1} - 0.18 \Delta \bar{Y}_{t-2} + \bar{\eta}_t, \tag{15}
\]
is not rejected at the 5% level against higher-order ARI models (up to order 5) even without penalizing for the number of parameters; also, the ARI(1,1) and the random walk are not rejected against (15). Moreover, the maximum likelihood estimate of the IMA(1,2) model

$$\Delta \tilde{Y}_t = 15.6 + \tilde{\eta}_t - 0.14 \tilde{\eta}_{t-1} - 0.16 \tilde{\eta}_{t-2}$$

(16)

is not rejected at the 5% level against higher-order IMA models, and the IMA(1,1) is rejected against (16); furthermore, adding an autoregressive component does not improve the estimation. The two models (15) and (16) are quite similar. For the present analysis, and based on both Akaike’s AIC criterion and Shibata’s SBC criterion, the ARI(2,1) model (15) is chosen over the IMA(1,2) model (16).

Replacing in (14) the polynomial $\tilde{\alpha}(B) = 1 + \alpha_1 B + \alpha_2 B^2$, we obtain

$$\Delta \tilde{C}_t = \tilde{\alpha} + k \Delta \tilde{Y}_t + k (\alpha_1 + b) \Delta \tilde{Y}_{t-1} + k (\alpha_2 + \alpha_1 b) \Delta \tilde{Y}_{t-2} + k \alpha_2 b \Delta \tilde{Y}_{t-3} + \tilde{\eta}_t + c \tilde{\eta}_{t-1}.$$ The null representation suggested in this paper is based on the following two conjectures, not rejected in the univariate analysis of the previous section: (i) transitory consumption is white noise; (ii) temporal aggregation bias is present in the data, with a low sampling ratio (of the order of 4); this entails approximately a value of 0.2 for $b$, instead of the limiting value of 0.268. Then, the representation under the null becomes the IMAX(1,1,3) model already mentioned in the introduction

$$\Delta \tilde{C}_t = \tilde{\alpha} + \sum_{j=0}^{3} \beta_j \Delta \tilde{Y}_{t-j} + \tilde{\eta}_t + c \tilde{\eta}_{t-1},$$

(16)

with the following linear restrictions

$$\beta_1 = (\alpha_1 + b) \beta_o,$$

$$\beta_2 = (\alpha_2 + \alpha_1 b) \beta_o,$$

$$\beta_3 = \alpha_2 b \beta_o.$$

Specifically, as from (15) the 95% confidence intervals for $\alpha_1$ and $\alpha_2$ are (0.05, 0.25) and (0.08, 0.28), these linear restrictions become

$$\beta_1 = (0.25 \beta_o, 0.45 \beta_o)$$

(17)

$$\beta_2 = (0.09 \beta_o, 0.33 \beta_o)$$

$$\beta_3 = (0.02 \beta_o, 0.06 \beta_o).$$

The maximum likelihood estimate of (16) is

$$\Delta \tilde{C}_t = 8.26 + 0.134 \Delta \tilde{Y}_t + 0.029 \Delta \tilde{Y}_{t-1} + 0.014 \Delta \tilde{Y}_{t-2} + 0.010 \Delta \tilde{Y}_{t-3} + \tilde{\eta}_t - 0.33 \tilde{\eta}_{t-1},$$

(18)
from which, taking ratios with respect to the estimated $\beta_0$, we get

\[
\beta_1 = (-0.11 \beta_0, 0.52 \beta_0)
\]

\[
\beta_2 = (-0.22 \beta_0, 0.45 \beta_0)
\]

\[
\beta_3 = (-0.25 \beta_0, 0.40 \beta_0)
\]

As the restricted confidence intervals under the null are not rejected, the permanent income hypothesis with white-noise transitory consumption and temporal aggregation bias is consistent with monthly consumption-disposable income data.

To corroborate model (18) in this bivariate testing framework, consider first that the residuals from all the regressions of $\Delta \bar{c}$, on several polynomials of lagged $\Delta \bar{y}$, (of order from 0 to 5) exhibit a marked first-lag autocorrelation and negligible values at other lags. Moreover, based on 95% confidence-level $F$-tests, (18) is not rejected against the polynomials of lagged $\Delta \bar{y}$, of order greater than 3.

4. Conclusions

By explicitly investigating the dynamic properties of the permanent income hypothesis under different conjectures about transitory consumption, additive measurement errors and temporal aggregation bias, this paper suggests representations for PIH that are better - in some specified sense - than those considered in the literature, from both a theoretical and an empirical point of view. Moreover, using monthly data, the paper identifies a plausible set of conjectures that are consistent with the evidence. In particular, the paper shows that PIH is not rejected if transitory consumption is generated as white noise, and if consumers make decisions at intervals moderately shorter than a month. The paper's contribution, however, is not to rescue PIH from previously reported failures; rather, it is to put in a better prospective the relevance of alternatives like, for example, the liquidity constraint hypothesis. It also indicates that further research efforts are needed along such relatively unexplored directions as the use of monthly data and the consideration of temporal aggregation bias.
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<table>
<thead>
<tr>
<th>Paper Number</th>
<th>Author(s)</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>91-9</td>
<td>Calla Wiemer, Zhao Xinghan</td>
<td>Closing the Scissors Gap: Reform and the Interface Between Agriculture and Industry</td>
</tr>
<tr>
<td>91-8</td>
<td>Luigi Ermini</td>
<td>Effects of Transitory Consumption and Temporal Aggregation on Tests of the Permanent Income Hypothesis</td>
</tr>
<tr>
<td>91-7</td>
<td>Sumner La Croix</td>
<td>An Economic Analysis of Decentralization In China: The Coevolution of Political Order and Property Rights</td>
</tr>
<tr>
<td>91-6</td>
<td>Sumner La Croix, Carl Bonham</td>
<td>Forensic Forecasting: Fact or Fiction?</td>
</tr>
<tr>
<td>91-4</td>
<td>Carl Bonham</td>
<td>Correct Cointegration Tests of the Long Run Relationship Between Nominal Interest and Inflation</td>
</tr>
<tr>
<td>91-3</td>
<td>Edwin T. Fujii, Clifford B. Hawley</td>
<td>Empirical Aspects of Self-Employment</td>
</tr>
<tr>
<td>91-1</td>
<td>Luigi Ermini</td>
<td>On the Durability of Non-Durable Goods: Some Evidence from US Time-Series Data</td>
</tr>
<tr>
<td>90-30</td>
<td>James Mak, Marcia Sakai</td>
<td>Taxation of Foreign Real Property Investments in the U.S.: A State Perspective</td>
</tr>
<tr>
<td>90-29</td>
<td>Carl Bonham, Sumner La Croix</td>
<td>Discount Rates and Earnings Growth Rates in Personal Injury Cases: New Evidence from Time Series Analysis</td>
</tr>
<tr>
<td>90-28</td>
<td>Calla Wiemer</td>
<td>Grain Prices, Land Rent, and Food Self-Sufficiency in China</td>
</tr>
<tr>
<td>No.</td>
<td>Author(s)</td>
<td>Title</td>
</tr>
<tr>
<td>------</td>
<td>---------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>90-27</td>
<td>Lou Rose</td>
<td>Land Values and Housing Rents in Urban Japan</td>
</tr>
<tr>
<td>90-26</td>
<td>Carl Bonham</td>
<td>Correct Cointegration Tests of the Long Run Relationship Between Nominal Interest and Inflation</td>
</tr>
<tr>
<td>90-24</td>
<td>Gerard Russo</td>
<td>The Demand for Physicians' Services and the Price of Cigarettes</td>
</tr>
<tr>
<td>90-23</td>
<td>Eric Iksoon Im, Marcellus S. Snow</td>
<td>Linear Unbiased Estimates as a GLS Class and the Generalized Aitken Theorem as a Corollary</td>
</tr>
</tbody>
</table>