FORENSIC FORECASTING:
FACT OF FICTION?

by

Carl Bonham and Sumner J. La Croix

Working Paper No. 91-6
February 1991
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ABSTRACT: This paper evaluates the manner in which forensic economists forecast earnings growth rates and discount rates. Efficient choice of forecasting techniques requires that the economist determine the stochastic properties of the two time series. We find that discount rates and growth rates are nonstationary over the full sample and that the two series are not cointegrated. If very simple rules of thumb are desirable, the following appear to be useful: for stationary series, use an historical average; for nonstationary series, employ the current observation in the series.

*Assistant Professor and Professor, respectively, Department of Economics, University of Hawaii, Honolulu, HI 96822. La Croix Phone: 808-956-7061 or V722780@UHCCMVS.BITNET. Bonham Phone: 808-956-7296 or U222780@UHCCMVS.BITNET. We wish to thank Louis Rose and Luigi Ermini for helpful discussions. The authors have contributed equally to the paper.
1. INTRODUCTION

Lawyers regularly ask economists to estimate the lost earnings of individuals unable to continue working due to injury. A common approach to this problem involves determining the victim's worklife, specifying the future earnings stream, and calculating its present value. This paper evaluates the manner in which forensic economists forecast earnings growth rates and discount rates. A wide variety of sophisticated and simple forecasting methods is available to the forensic economist. Efficient choice of forecasting techniques requires that the economist determine the stochastic properties of the time series in question. Yet forensic economists have usually dispensed with such investigations, tossed sophisticated methodologies to the wind, and instead used a variety of simple ad hoc rules.1 The purpose of this paper is to inquire whether the methodologies of the forensic economist have any basis in the underlying stochastic properties of commonly used discount and earnings growth rate series.

The paper proceeds as follows. Section 2 provides a brief discussion of how an individual's lost earnings are estimated and shows why specification of earnings growth rates and discount rates is critical to this process. In Section 3 we discuss stochastic properties of time series that need to be determined before the economist can select a forecasting technique. Section 4 presents econometric analysis of several earnings growth rate and discount rate time series to determine their underlying stochastic properties. Section 5 uses the reserved portion of the time series to evaluate particular forecasting techniques. Section 6 provides a brief overview of the paper's conclusions and recommends forecasting rules for particular types of series.
2. THE USE OF GROWTH AND DISCOUNT RATES IN DAMAGE CALCULATIONS

Suppose that a 21 year-old male college graduate has become totally disabled in a traffic accident. An economist is asked to estimate the individual's lost earnings. The individual has no previous earnings history and was an average student at an average university. Most economists begin their analysis by locating an age-income profile that fits the individual's characteristics. Without more specific occupational data, economists often use the cross-section age-income profiles reported in the Census Bureau's annual P-60 publication. Separate age-income profiles are provided by sex, the number of years of education completed, and full-time work status. Most age-income profiles rise with age until the person reaches age 50-55 and then either plateau or begin to fall gradually. The rising portion of the profile stems from productivity gains accruing to individuals as they acquire additional experience and human capital, while the falling portion stems from a decrease in hours of work and depreciating human capital.

The cross-sectional view of the age-income profile differs greatly from the time profile of earnings for a particular individual. Each year the cross-sectional profile shifts because of general price inflation and overall productivity gains. In a competitive labor market the earnings growth rate reflects changes in the individual's real productivity and in the product's nominal price, as \( W = VMP - MPP \times P \), where \( W \) is the wage rate, \( VMP \) is the value of the marginal product, \( MPP \) is the marginal physical product, and \( P \) is the product's price. On average, the price of the product produced by the average worker increases by the rate of inflation, \( \pi \). Changes in marginal physical product (MPP) reflect both the effects of increased experience and overall productivity changes. Since the cross-sectional age-income profile accounts only
for the effects of increased experience, the economist must select growth rates which also account for price inflation and overall productivity gains.

Given earnings in each year of the n-year worklife \( E_j \) and the predicted earnings growth rate \( g_i \) and discount rate \( i \), the formula for calculating the present value of pre-tax earnings losses \( EL \) is straight-forward:

\[
EL = \sum_{j=1}^{n} \frac{E_j \prod_{i=1}^{j} (1+g_i)}{\prod_{i=1}^{j} (1+i_i)}
\]

The economist specifies the discount rate by selecting the yield on a financial instrument with features consistent with the case's legal and economic constraints. Since human capital is both risky and long-lived, the discount rate should ideally reflect both factors. In most federal and in many state actions, the economist is constrained to use shorter-term Treasury issues so that the victim does not face undue risks of default or capital losses.\(^5\)

3. AN INVENTORY OF STOCHASTIC PROPERTIES OF GROWTH AND DISCOUNT RATES

The number of stochastic properties that must be known before selecting a forecasting technique is small. First, the economist must determine whether the relevant series are stationary or nonstationary. Second, if a series is nonstationary, three additional properties must be investigated: (1) the order of integration; (2) whether or not the series is a random walk (and whether or not it has drift); and (3) when both discount and growth rate series are I(1), whether or not they are cointegrated. Our analysis below proceeds in two stages. In part A we discuss forecasting a single time series. In part B, we show how to modify the analysis when the two series share a common trend. The analysis
in Part A corresponds to the left and center branches of the decision tree while that in part B corresponds to the right branch.

A. **Forecasting a Single Time Series**

Investigation of the stochastic properties of a time series begins by determining whether the series is stationary. A time series is stationary when its distribution is independent of time, with finite variances and autocovariances which are a function of the lag between observations but not the point in time at which they are observed. Series which are stationary generally appear jagged when plotted over time, with little or no long-run trend and frequent crossings of the mean line. The expected length of time between returns to the mean value is finite for a stationary series whereas this expectation is infinite for a nonstationary series. Although the current observation may lie above the historical mean, in the long run the series will have a tendency to generate observations that preserve the population mean. Thus the analysis of a stationary series using samples over different time periods should provide equally useful information about the true population moments. On the other hand, for a nonstationary series, "it is not meaningful to discuss the 'long-run' mean or variance of the process." If a series is found to be stationary, a variety of univariate and multivariate forecasting models is available to the economist. For instance, stationary series may be modeled as autoregressive moving average processes with future values predicted by the selected model.

If the series is nonstationary, the next step is to determine its order of integration. A series is integrated of order d, denoted I(d), if it must be differenced d times to obtain a stationary series. Thus, a random walk process, \( z_t \), is the simplest example of an I(1) series:

\[ z_t = z_{t-1} + u_t + \delta \]  

(2)
two series are cointegrated, we can improve on separate forecasts by taking advantage of the knowledge that the two series will not drift too far apart. Two $I(1)$ series are said to be cointegrated if some linear combination of the two is stationary. This means that the two series share a common trend. Intuitively, suppose there exists a long-run equilibrium relationship between two $I(1)$ economic time series. Although the individual series may wander a great distance from their starting points, the equilibrium relationship could prevent them from drifting too far apart. Thus, some linear combination of the two will produce a stationary series. In our context the common trend restricts the difference between the two series to a stationary process.\footnote{This result is important since the value of equation (1) is primarily driven by the difference between $i$ and $g$ rather than by their absolute values.} Theory does lead us to believe that interest rate and growth rate series may be cointegrated. One possibility is that expected inflation is a determinant of both series. Under the Fisher Hypothesis, nominal interest rates will completely adjust to any changes in expected inflation.\footnote{Because the value of the marginal product of an average worker is tied to the rate of inflation, changes in wages should be strongly correlated (perhaps with a long lag) with inflation.} For two series to be cointegrated, they must both be integrated. Schwert [16] demonstrates that some interest rate series are integrated. Blanchard [2, 1150-51] is unable to reject the hypothesis that the growth rate of the hourly earnings index is integrated of order one.

If the two series are cointegrated, forecasts should be obtained from a vector error correction representation of the cointegrating relationship. The existence of an error correction representation is guaranteed by the Granger Representation Theorem [6] and forecasts from an error correction model are
discussed in Engle and Yoo [7]. The long run forecasts from a cointegrated system are tied together whereas individual forecasts from vector autoregressions (VAR) or univariate methods will diverge without bound. If the two series are found to be I(1) or I(0), they may still be forecast jointly using information about both series and other predetermined variables in a VAR or VARMA representation.

4. TIME SERIES PROPERTIES OF EARNINGS GROWTH & DISCOUNT RATES

In this section we examine the stochastic properties of the earnings growth rate and discount rate. We use a variety of series to measure earnings growth and discount rates because practicing forensic economists use different series, and controversy exists regarding the appropriate measure. Also, different cases may require the use of different measures of the two variables. We use monthly observations of a six month treasury bill rate, a six month commercial paper rate, the rate of growth of an hourly earnings index, and the rate of growth of gross average weekly earnings of production workers. All data on earnings are for the private nonfarm sector and are seasonally adjusted. We use quarterly observations of the growth rate of the hourly compensation of private nonfarm workers (obtained from the Bureau of Labor Statistics) and quarterly yields on the two interest rate series.11

The time period considered, 1964:1 to 1990:1, is constrained by the availability of earlier observations for the monthly earnings series. Also, the monthly hourly earnings growth series ends in 1988:12. In order to ensure a more robust evaluation of forecast techniques, forecasts are generated for two subperiods: 1975:1 to 1985:1 and 1980:1 to 1990:1. The sample used to select a forecasting model is consistent with real time forecasting; all tests are
conducted using data prior to the first forecast for each subperiod. Results from unit root tests are also presented for the full sample period.

Tests for a unit root (i.e., nonstationarity) in the autoregressive representation of each series are conducted using a procedure suggested by Schwert [16]. Table 1 presents the standard Augmented Dickey Fuller (ADF) statistics. The results of Table 1 are consistent with the conclusion that each of the discount rate measures is characterized by a nonstationary process, regardless of the subperiod considered. The results for the earnings growth rate measures are, however, not as clear cut. For both of the subperiods we find that the monthly series of weekly and hourly earnings growth rates are stationary. Over the full sample this conclusion is reversed—both monthly series are nonstationary. The quarterly series of hourly compensation growth rates is found to be nonstationary in all periods. This conclusion is tempered by the extremely small values for the first order autoregressive parameter ($\rho$) reported in Table 1. A series possessing a unit root should have a coefficient of unity; the hourly compensation growth rate earnings series has coefficients ranging from 0.778 for the full period to 0.167 for the last subperiod.

Our conclusion is that over the two subperiods available to the economist in 1975 and 1980, both discount rate series are nonstationary and the three earnings growth rate measures are stationary. After estimating higher order ADF regressions to ensure white noise residuals, we also reject the hypothesis that any of the above series are well specified by a random walk process, with or without drift.\textsuperscript{12}

Given that the discount and earnings growth rates possess such different stochastic properties prior to 1980, there is no reason to proceed with cointegration tests. We did, however, test for cointegration using the full
sample and did not reject the null hypothesis of no cointegration for any combination of earnings growth and discount rate measures.

5. SELECTING A FORECAST FOR EACH SERIES

Economists have used a wide variety of techniques to forecast the earnings growth and discount rates. This section discusses the most commonly used ad hoc forecasting rules and compares forecasts generated using these rules with those from univariate time series models.

The historical average rule uses a sample mean of past rates as an estimate of future rates. Economists using this rule implicitly assume that earnings growth and discount rates are stationary with time invariant first and second moments. The random walk rule uses the current observation of a time series as an estimate of future values. Economists using this rule implicitly assume that past observations in the time series contain no information about future values.

A large literature has arisen to critique the various methodologies. However, this literature has failed to pay adequate attention to the time series properties of growth and discount rate series. Given the stochastic properties of the data discussed in section 4, we expect that the historical average rule (HAR) provides an acceptable forecast for stationary series (such as earnings growth rates) but a poor forecast for nonstationary series (such as discount rates). In addition, simply choosing a method that produces conditional rather than unconditional forecasts should improve on the performance of the HAR. The random walk rule (RWR) is likely to perform poorly when used to forecast stationary earnings growth rate series yet may work reasonably well with the nonstationary discount rate series.

For comparison purposes we use a univariate model selected automatically with a procedure developed by Hannan and Rissanen [9]. This procedure is used
to select, estimate, and generate forecasts from ARIMA \((p,d,q)\) models of earnings growth and discount rates. The autoregressive and moving average orders \((p\text{ and } q)\) are determined using a model selection criterion. The series being forecast is differenced \(d\) (the order of integration determined in section 4) times to achieve stationarity prior to executing the model selection program. For each series, forecasts from ARIMA models are compared with forecasts from the HAR and RWR. Forecasts are produced for two separate ten-year periods beginning in 1975:1 and 1980:1. Each technique employs data beginning in 1964:1 and ending in the period immediately preceding the first forecast. For example, the HAR obtains forecasts for the first subperiod by averaging data from 1964:1 to 1974:12 and for the second subperiod by averaging data from 1964:1 to 1980:12.

To evaluate each forecast we compare their performance in the earnings loss function in equation (1). We begin by restricting the earnings stream, \(E_{j=1}\), for all \(j\). Next, the actual and predicted earnings loss are calculated. To evaluate forecasts of earnings growth rates, actual values of the six-month commercial paper rate are used in the denominator of the earnings loss function. To evaluate forecasts of discount rates, actual values of average weekly earnings growth rates are used in the numerator of the loss function. Finally, the ratio of predicted to actual loss is calculated. A ratio of less than unity indicates that a given forecast underpredicts the actual losses that would be experienced, and vice versa. A ratio of unity indicates a perfect forecast of the present value of earnings losses.

Table 2 presents the ratios of predicted to actual earnings loss functions for each series and forecasting technique discussed above. Two conclusions stand out. First, it would be a mistake to forecast a trending series, such as the discount rate, by averaging past levels of the series. Second, the random walk
rule should not be used in forecasting stationary series such as earnings growth rates.

Adding up the number of first, second, and third-place finishes is one way of comparing the three forecasting techniques. The ARIMA predictions are closest to the actual value of the lost earnings function six times, are second-closest three times, and come in last only once. If the ARIMA forecast does not produce the best forecast in absolute terms, it is usually a close second. The RWR wins only once, comes in second-place three times, and in third-place six times; all but one of its third-place finishes occur in forecasts of the earnings growth rate. The RWR's one win occurs when forecasting the discount rate over the 1975:1 - 1985:1 subperiod; the RWR produces the worst forecast of the earnings growth rate for every series in both subperiods. The HAR has three wins, four second-place finishes, and three third-place finishes. All but one of the HAR wins occur when forecasting earnings growth rates in the 1980:1 - 1990:1 subperiod, and all of the HAR third-place finishes occur when forecasting the discount rate.

Finally, Table 2 also presents values for the root mean squared error (RMS) of each forecast. The ranking of forecasts by RMS differs from the rankings by the earnings loss ratios only twice.

6. CONCLUSION

Our results indicate that an economist predicting earnings growth and discount rates should first evaluate the stochastic properties of the data. If very simple rules of thumb are desirable, the following appear to be useful: for stationary series, apply the historical average rule; for nonstationary series, employ the random walk rule.
The stochastic properties of the data reject two other rules. The common trend rule is based on the assumption of cointegration; it assumes the two series share a common trend and forecasts the stationary net discount rate using historical averages. This rule should not be used as (1) earnings growth and discount rate series are rarely both integrated, and (2) when they are both integrated, we do not find them to be cointegrated. The total offset rule is a variant of the common trend rule. It assumes that the two series share a common trend and that the stationary net discount rate is zero. Since the two series are not cointegrated, there is no evidence that the net discount rate is stationary, let alone equal to zero.

As Havrilesky [11] points out, there is more to forecasting earnings growth and discount rates than extrapolation of past data trends. The economist should be alert to policy changes that lead to short-run and long-run changes in both series. This advice is often difficult to translate into the numbers needed to produce an estimate of lost earnings and many economists are likely to continue to extrapolate past data trends. Perhaps the central lesson of our investigation is that if the economist is to continue to use black threads to connect the data points, it is important to discover the stochastic properties of the data and to use forecasting methods designed for data with those properties. As we have shown above, paying attention to these properties produces more accurate forecasts and more equitable compensation of plaintiffs in tort cases.
REFERENCES


NOTES

1. See Brookshire [5]; La Croix & Miller [12]; Parks [14]; and Pelaez [15].

2. The economist must also specify either the individual's worklife or a sequence of labor force participation rates. We extrapolate from these issues as they are peripheral to forecasting the earnings growth and discount rate series.


4. See Parks [14, 314-19] for an excellent discussion of shifting age-earnings profiles. Abraham [1] argues that use of both age-income profiles and earnings growth rates involves double counting. His analysis ignores, however, the effects of inflation and overall productivity increases.

5. Problems with reinvestment and default risk can be minimized with the use of a Treasury bond money-market fund.


7. We are assuming that the cointegrating parameter is equal to unity.

8. Small errors arise from specifying a net discount rate \((1-g)\) and using the equation

\[
EL = \sum_{j=1}^{n} \frac{E_j}{[1+(1-g)]^j}
\]

instead of equation (1) in the text.

9. Interest rate models incorporating the effects of taxes, depreciation, and movements in real saving and borrowing rates remove the one-to-one link with \(\pi\) but preserve a strong positive correlation between \(i\) and \(\pi\).

10. In a simple monetarist model of the economy, an increase in the rate of growth of money leads to higher inflation and nominal wages in the long run. In a world of sticky wages or inefficient capital markets, the higher rate of inflation and higher prices cause a decline in the real interest rates on nominal assets and the real wage paid to workers. However, in a world of monetary neutrality, the real wage rate and the real rate of interest remain unchanged, so that both the nominal interest rate and the nominal wage must rise.

11. Monthly observations on the Treasury notes were converted to quarterly by averaging.

12. Results for quarterly discount rate series do not change any of our conclusions. We also ran tests using 3-year Treasury notes and 30-year Treasury bonds and found the same results as for the shorter term assets.

14. See Bonham [3], Bonham and Dacy [4] or Granger and Newbold [8] for discussion and application of this model selection technique. All of the work for this paper was conducted using RATS (v3.0). All data and programs are available from the authors on request.
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient/Test Statistic</td>
<td>ρ</td>
<td>τ</td>
<td>ρ</td>
</tr>
<tr>
<td>Commercial Paper, 6 months</td>
<td>.941</td>
<td>-2.45</td>
<td>.957</td>
</tr>
<tr>
<td>U.S. Treasury Bill, 6 months</td>
<td>.916</td>
<td>-2.51</td>
<td>.956</td>
</tr>
<tr>
<td>Weekly Earnings Growth Rates</td>
<td>-.474</td>
<td>-5.22*</td>
<td>-.459</td>
</tr>
<tr>
<td>Hourly Earnings Growth Rates</td>
<td>.090</td>
<td>-3.79*</td>
<td>.077</td>
</tr>
<tr>
<td>Hourly Compensation Index</td>
<td>.226</td>
<td>-2.10</td>
<td>.167</td>
</tr>
</tbody>
</table>

Note: The ADF test is based on the following regression:

\[ Y_t = \alpha + \beta t + \rho Y_{t-1} + \sum_{i=1}^{\ell} \phi_i \Delta Y_{t-1} + \epsilon_t \]

\( \tau \) is the regression 't-statistic' for the null hypothesis that \( \rho \), the coefficient on the lagged level, is equal to unity. The number of lags, \( \ell \), used in the ADF regression is determined using the rules of thumb suggested in Schwert [19]. \( \ell \) is equal to 4 for the first two subperiods and 15 for the full sample period. Under the assumption of a pure autoregressive process, critical values are given by \( \tau_{14} = -3.41 \) and \( \tau_{12} = -3.36 \). The full sample for the hourly earnings growth rate ends in 88:12.
### Table 2

**Earnings Loss Ratios & Root Mean Squared Error of Forecasts**

<table>
<thead>
<tr>
<th></th>
<th>Hourly Comp. Index</th>
<th>Weekly Growth Rate</th>
<th>Hourly Growth Rate</th>
<th>T-Bill</th>
<th>Commercial Paper</th>
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<tbody>
<tr>
<td></td>
<td>ELR</td>
<td>RMS</td>
<td>ELR</td>
<td>RMS</td>
<td>ELR</td>
</tr>
<tr>
<td>Forecast Period: 1975:1 - 1985:1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARIMA</td>
<td>0.8609</td>
<td>2.4562</td>
<td>0.6488</td>
<td>6.652</td>
<td>0.702</td>
</tr>
<tr>
<td>HAR</td>
<td>0.8297</td>
<td>2.504</td>
<td>0.6432</td>
<td>6.6532</td>
<td>3.5998</td>
</tr>
<tr>
<td>CIR</td>
<td>1.3909</td>
<td>3.7035</td>
<td>1.389</td>
<td>6.785</td>
<td>3.599</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forecast Period: 1980:1 - 1990:1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARIMA</td>
<td>1.432</td>
<td>4.335</td>
<td>1.035</td>
<td>7.089</td>
<td>1.232</td>
</tr>
<tr>
<td>HAR</td>
<td>1.1938</td>
<td>3.596</td>
<td>1.0538</td>
<td>7.092</td>
<td>0.947</td>
</tr>
<tr>
<td>CIR</td>
<td>1.9253</td>
<td>5.666</td>
<td>15.284</td>
<td>10.024</td>
<td>6.073</td>
</tr>
</tbody>
</table>

**Note:** ELR = earnings loss ratio, defined as the ratio of the predicted to the actual earnings loss. RMS = root mean squared error. ARIMA = forecasts from an autoregressive integrated moving average model. HAR = historical average rule of forecasting. RWR = random walk rule of forecasting. All of the above concepts are discussed in the text. The sample for hourly earnings growth rates ends in 88:12 rather than 90:1.