A NOTE ON A REVISITED PERMANENT-INCOME HYPOTHESIS

by

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Abstract

A recent paper by Christiano, Eichenbaum and Marshall [1991] shows that the continuous-time random walk model of consumption may not be rejected if one takes into account the effects of temporally aggregating continuous-time decisions to quarterly observations. This note, however, shows that this result does not hold if monthly data are used instead of quarterly data, with or without taking into account the noisiness of monthly data.
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1. Introduction

Recently in this journal, Christiano, Eichenbaum and Marshall [1991] - henceforth CEM - have shown that the random walk model of consumption, corresponding to the permanent income hypothesis formulated in continuous time, may not be rejected if the effects of temporally aggregating continuous-time decisions to quarterly-intervals observations are properly taken into account. This same result was shown in Ermini [1988] to hold even if the decision intervals are finite and shorter than a quarter.

CEM's result can be criticized on both theoretical and empirical grounds. On theoretical grounds, the criticism mainly concerns the implausibility, in a continuous-time context, of CEM's assumptions of additive-separability of the utility of consumption and of non-durability of consumption goods. Regarding the former, it has been long emphasized (see, for example, Deaton and Muellbauer [1980]) that the independence of the marginal rate of substitution between consumption this period and consumption next period from consumption in any past period - implied by the additive-separability of the utility function - is the less plausible the smaller the decision interval. This argument has recently received considerable attention in the literature, with a number of important articles dealing with non-separable utility functions (see, among others, Dunn and Singleton [1988], and Epstein and Zin [1989]). Regarding the latter, as the non-durability of consumption goods implies that goods do not yield utility beyond the period in which they are purchased, the assumption that the expenditures corresponding to the National Income and Product Account category of non-durables are still non durable when the decision intervals become infinitesimally small may also be implausible (see, for example, Ermini [1991a]).

On empirical grounds, Ermini [1989] and Heaton [1989] show that the negative first-order autocorrelation exhibited by monthly consumption changes contradicts the continuous-time random walk model under temporal aggregation. This fact is carefully recognized
by CEM as problematic for their model. The purpose of this note is to address this issue more closely, by following up on results reported in Ermini [1989], and updated in Ermini [1991b].

2. **Monthly consumption data and measurement errors**

The estimated first-order autocorrelation of monthly consumption changes is $-0.252$, with a 95%-confidence interval $[-0.326, -0.166]$. Under the hypothesis of no measurement errors in monthly consumption, the continuous-time random walk is thus rejected, as its monthly-aggregated equivalent is an IMA(1,1) with a first-order autocorrelation of consumption changes of 0.25 (Working [1960]). Similarly, the random walk model generated at any finite decision interval shorter than a month is also rejected, as its monthly-aggregated equivalent is again an IMA(1,1), with first-order autocorrelation between 0 and 0.25, monotonically increasing with the sampling ratio (the ratio between a month and the finite decision interval).

Most economists, however, would not accept this unfavorable conclusion, as the hypothesis of no measurement errors in monthly consumption data is widely recognized as unrealistic (though cases exist of empirical work with monthly data based on this hypothesis. See, for example, Eichenbaum, Hansen and Singleton [1988]). Some economists, in fact, even advocate that monthly data should never be used because of their noisiness; to do so, however, would amount to wasting potentially relevant information. Unfortunately, economists know practically nothing about measurement errors. A sensible way to proceed, then, would be to investigate the effects on CEM's result of a series of plausible conjectures about measurement errors. Their plausibility will be judged on the basis of a number of "acceptable" requirements.

Two main requirements will be used here as a guidance: (a) as quarterly data are typically assumed to be noise-free, the dynamic properties of monthly measurement errors should be such to make these errors insignificant when aggregated to quarterly intervals; (b) monthly measurement errors should not eventually dominate the
signal, nor vice versa.\textsuperscript{1}

The conjecture most frequently proposed in the literature is that the measurement errors $u_t$ are additive, and are generated by a stationary process uncorrelated with consumption levels. Ermini [1989] shows that, under this conjecture, the presence of measurement errors in monthly consumption may rescue the random walk model from failure. Briefly summarizing the argument, if $\Delta C_t = d + e_t + h e_{t-1}$ is the IMA(1,1) process followed by monthly consumption under the permanent income hypothesis and temporal aggregation, then observed monthly consumption, $C_t^* = C_t + u_t$, follows the process

\begin{equation}
\Delta C_t^* = d + e_t + h e_{t-1} + u_t - u_{t-1}.
\end{equation}

Note, first, that as with monthly data the IMA(1,1) model is not rejected against any IMA(1,q) model with q greater than one, one must conclude that this conjecture is plausible only if $u_t$ is a zero-mean white noise process. In this case, the first-order autocorrelation of observed consumption changes is

\begin{equation}
\rho^* = \frac{h SN - 1}{(1+h^2)SN + 2},
\end{equation}

where $SN$ is the signal-to-error ratio, i.e. $SN = \sigma_e^2/\sigma_u^2$ with $\sigma_e^2$ and $\sigma_u^2$ the variance of consumption innovations and measurement errors respectively, and where $h = 0.268$ when consumption is generated in continuous time.

The effect of additive measurement errors is to reduce the first-order autocorrelation of consumption changes in proportion to their variance. Thus, for large enough error variance the theoretical value of 0.25 can be reduced to within the limits of the estimated confidence interval. Specifically, from (2), one

\textsuperscript{1} Some recent literature has also argued that, owing perhaps to repetition and habit formation in data collection, measurement errors exhibit some degree of autocorrelation (for example, Wilcox [1990]). While this feature will be object of discussion, it will not be taken to be a requirement.
finds that an error variance between 66.7\% and 177.4\% of the consumption innovation variance, corresponding to a signal-to-error ratio of 1.5 and 0.56 respectively, suffices to this purpose. In practice, as monthly consumption is of the order of magnitude of $700 per-capita, with standard error of about $7 (see Ermini [1991b]), a measurement error with s.e. between $4.7 and $12.4 per month per capita would suffice to rescue CEM's result from failure. Incidentally, the order of magnitude of these errors, as percentage of consumption, are in line with estimates reported elsewhere (Wilcox [1990]).

Unfortunately, this conjecture, supportive of CEM's result, satisfies requirement (a) - the error variance reduces to a third at quarterly intervals (Ermini [1989]) - but does not satisfy requirement (b): as consumption is an upward-trending series, apparently exhibiting the statistical properties of first-order-integrated processes, or I(1), the assumption of stationary measurement errors implies that their importance relative to consumption levels becomes smaller and smaller, thus ultimately making the series noise-free.

An alternative conjecture that naturally comes to mind, then, is to assume that the measurement errors are additive but proportional to the level of consumption, \( u_t = k_t C_t \), with the factor of proportionality \( k_t \) being generated by a stationary process with zero mean, uncorrelated with consumption innovations \( e_t \). For \( k_t \) a white noise with variance \( \sigma^2_k \), and \( C_t \) an IMA(1,1), some algebra shows that \( u_t \) has zero mean, zero autocorrelation at all lags, and a second-order heteroskedastic variance, \( \text{var}(u_t) = \sigma^2_k t^2 + (1+h^2)\sigma^2_k \sigma^2_e (t-1) + \sigma^2_k \sigma^2_e \).

Under this conjecture, \( C_t \) again follows an IMA(1,1) process, with first-order autocorrelation of consumption changes:

\[
\rho^* = \frac{h \sigma^2_e - \text{var}(u_{t-1})}{(1+h^2)\sigma^2_e + \text{var}(u_t) + \text{var}(u_{t-1})}.
\]

In this case, for large enough \( t \), \( \rho^* \) tends to the value -0.5, regardless of the strength of the signal. As this corresponds to a process with a unit root in the MA component, it follows that, for
large enough $t$, $C_t^*$ is eventually generated as a heteroskedastic "trend-stationary" series, $dt + u_t$. Heuristically, the variance of $u_t$ grows quadratically, while the variance of unobserved consumption $C_t$ grows linearly; eventually the former will overshadow the latter, so that observed consumption simply appears as measurement errors added to a linear deterministic trend. This second conjecture does not satisfy requirement (b); in fact, it does not even satisfy requirement (a). Furthermore, to be consistent with the observed IMA(1,1), $u_t$ must be uncorrelated at all lags.

A third conjecture is to hypothesize that the measurement errors affect the growth rate of consumption, i.e.

$$
\frac{\Delta C_t^*}{C_t^*} = \frac{\Delta C_t}{C_t} + k_t,
$$

which can be approximated as

$$
\Delta C_t^* = \Delta C_t + k_t C_t.
$$

For $k_t$ as before, the term $k_tC_t$ has again zero mean, zero autocorrelation at all lags and second-order heteroskedastic variance. Therefore, $\Delta C_t^*$ is still generated as IMA(1,1), with first-order autocorrelation of observed consumption changes:

$$
\rho^* = \frac{h \sigma_e^2}{(1+h^2)\sigma_e^2 + \text{var}(u_t)}.
$$

For $t$ sufficiently large, as $\rho^*$ goes to zero, this third conjecture again does not satisfy requirement (b). Heuristically, as the growing variance of $k_tC_t$ eventually dominates the constant variance of the stationary process $\Delta C_t$, $C_t^*$ will eventually follow a random walk driven only by the heteroskedastic innovations $k_tC_t$. This would occur even if the term $k_t$ is autocorrelated - in which case, however, the autocorrelation must be confined to a form that makes $k_tC_t$ at the most MA(1), to make (5) consistent with the observed IMA(1,1). Furthermore, this conjecture would not satisfy requirement (a). Finally, from integrating (5), one finds that, under this third conjecture, the measurement errors are generated
as a random walk with heteroskedastic innovations, and therefore exhibit strong autocorrelation, as opposed to the two previous conjectures.

On the basis of these results, it appears that for measurement errors not to be dominated by the signal, they must have similar non-stationary properties; and, for them not to dominate the signal, their variance must not increase at a higher rate than the signal variance. This assessment offers the case for a fourth conjecture: measurement errors generated as a random walk with homoskedastic innovation, i.e. \( \Delta u_t = k_t \), with \( k_t \) a white noise process of constant variance \( \sigma^2_k \). Then

\[
\Delta C^*_t = \Delta C_t + k_t ,
\]

with first-order autocorrelation

\[
\rho^* = \frac{h \, SN}{(1+h^2)SN + 1} .
\]

As \( SN \) is a non-negative number, for \( h = 0.268 \) this conjecture is inconsistent with the estimated confidence interval for \( \rho^* \); in fact, it is inconsistent for any non-negative \( h \). Thus, under this fourth conjecture — indeed, the only one to satisfy both requirements (a) and (b), and also the possibility of error autocorrelation — CEM's result still cannot be rescued from failure.

3. Conclusion

Under a conjecture about measurement errors that satisfies two quite sensible requirements, one must reject CEM's hypothesis that the first-lag autocorrelation of monthly consumption changes prior to measurement is 0.25. More generally, one must reject the hypothesis that the negative first-lag autocorrelation of observed consumption changes can be explained simply with data noisiness.

Both rejections have strong implications for economic theory.
Regarding the rejection of 0.25, it is known that, as the decision interval becomes smaller, any ARIMA process tends under temporal aggregation toward an IMA(1,1) with first-lag autocorrelation of changes equal to 0.25 (for example, Tiao [1972], Weiss [1984]). It follows that consumption cannot be generated in continuous time, nor, plausibly, even at finite but very small intervals.

Regarding the second rejection, the implication is that monthly consumption prior to measurement follows an IMA(1,1) with negative MA coefficient. As the only process that under temporal aggregation is transformed into an IMA(1,1) with a negative MA is indeed an IMA(1,1) with negative MA (Ermini [1989]), a very good candidate to replace the rejected continuous-time random walk model is the hypothesis that consumption is in fact generated at finite intervals as an IMA(1,1) with a negative MA.

Interestingly, this alternative hypothesis may be consistent with the objections against CEM's continuous-time permanent-income model reported in the introduction. Specifically, it may be consistent with the durability of non-durable goods (Mankiw [1982], Hayashi [1985], Heaton [1989], Ermini [1991a]), as well as with the non-separability of the utility function (Dunn and Singleton [1988], Constantinides [1990], Ermini [1991d]). On a different tack, this alternative hypothesis has been shown to be necessary to justify positive risk premia in the term structure (Backus, Gregory and Zin [1988]), and sufficient to solve Mehra and Prescott's [1985] puzzle about the equity risk premium (Ermini [1991c]).

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