Multiplant strategy under core-periphery structure*

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January, 2011

Abstract

This paper examines the stable outcomes of organization choice between single-plant and multi-plant under asymmetric two regions. A typical implicit assumption on monopolistic competition models for trade and economic geography is that firms can produce and sell only at one place. This paper allows endogenous determination of the number of plants in a new economic geography model. In particular, we explicitly considers the firms’ trade-off between larger economies of scale under single plant configuration and the saving in interregional transport costs under multi-plant configuration, together with its consequence on the interregional population distribution.

JEL Classification : D21, F12, L23, R12,

Keywords : Multi-plant firms, Transaction costs, New economic geography

*I’d like to thank Tomoya Mori for the discussions and insightful comments. I am grateful to Masahisa Fujita, Taiji Furusawa, Jota Ishikawa, Jing Li, Giordano Mion, Kaz Miyagiwa, Frédéric Robert-Nicoud, Takatoshi Tabuchi, Jacque Thisse, Dao-Zhi Zeng and the seminar participants at Hitotsubashi University, Tohoku University, Zhejiang University and University of Tokyo for their discussions. I am also grateful to le Commissariat Général aux Relations Internationales de la Communauté Française de Belgique for their financial support and CORE, Université Catholique de Louvain, Belgium for research supports in 2008-09, where most parts of this paper were conducted. The usual disclaimer applies.

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1 Introduction

As is confirmed by many studies, economic activities are unevenly distributed among countries and regions. For example, Combes and Overman (2004) in EU, and Fujita, Mori, Henderson and Kanemoto (2004) in Japan show geographical concentrations of particular activities. Krugman (1991) and Fujita, Krugman and Venables (1999) established a systematic framework to analyze endogenous agglomeration of workers and firms using a combination of increasing returns and transportation costs, called the New Economic Geography (henceforth, NEG).

While there are a number of model variants proposed, it is a typical assumption that each firm consists of a single plant. When transportation costs are very high, it would be rational to establish another plant in a distant market (Brainard (1997)). Firms face proximity-concentration trade-off in serving for foreign markets, i.e., depending on the degree of trade barriers, firms may want to build a new plant there, or export from the existing plant in their home market. While proximity to market enables firms to earn larger profit by reducing trade costs, firms can exploit scale economies by concentrating their production at one place. Thus, for a firm, the number of places for production must also be a choice variable as important as their location.

In fact, there is a substantial presence of multiplant (unit) firms in reality. In Japan, the share of multiplant firms is not negligible and varies across regions and among industries: agriculture (28.3%), manufacturing (36.3%), and retailing (60.2%).\textsuperscript{1} The share of multi establishments is lower in Agriculture, Fisheries, and Forestry. On the other hand, manufacturing is relatively higher and service sectors are much higher. In other words, primary products have relatively smaller supply chain and services have larger supply networks. Since most of the services needs face to face communication with customers, it is difficult to ship the services to the other regions.

The aim of this paper is to propose a simple modification to NEG models, which allows for firms to endogenize both the number and location of their plants in a two-region economy. It is shown that the option to be multiplant changes the location equilibria which have been

\textsuperscript{1}The data source is from “Establishment and enterprise census of Japan” in 2006 which covers all the firms in Japan except foreign affiliates. The definition of “multiplant (multi-unit) firm”, here, is that an establishment which is not independent and is either main or branch establishment.
obtained in the previous studies: if all the firms are multiplant, changes in transportation costs doesn’t induce migration of firms and workers.

There are some early attempts of modeling the spatial organization of multiplant firms. Markusen (1984) is the first to explain horizontal foreign direct investment (FDI) in trade, while Ota and Fujita (1993) was first to solve location problem of multiplant firms in the continuous urban space. In monopolistic competition framework, multiplant firms have first been introduced by Markusen and Venables (1998) in trade, although their results heavily rely on numerical examples.\(^2\) Ekholm and Forslid (2001) is the closest in spirit to the present paper as they introduced multiplant firms in the NEG framework. However, due to their model specification, formal results obtained are rather limited. To improve analytical tractability, our model employ the specification proposed by Forslid and Ottaviano (2003).\(^3\) Recent works by Fujita and Thisse (2006) and Fujita and Gokan (2005) also consider multiplant firms in the NEG framework. But, these are partial equilibrium models.

In the present paper, the conditions for equilibria and their stability under different plant organization of firms are fully analyzed in a general equilibrium. Moreover, we show non-monotonic organization changes of firms which match the histories of multiplant (multinational) firms.\(^4\) The rest of this paper is organized as follows; in section 2, a two-region single-plant model is presented as a benchmark. In section 3, multiplant case is allowed, and is compared with the previous results. Firstly, the comparison shows monotonic organization change from multiplant to single plant. In the second step, we extend and generalize our model in order to show the non-monotonic organization change. Possible caveats and future extensions are discussed in the final section.

\(^2\)Yeaple (2003) considers the optimal organization of multinationals in three country model across all possible configurations. However stability analysis is still left aside. Moreover, with the presence of heterogeneity like in Melitz (2003), since the incentives for changing organization are different for heterogeneous firms, the interaction through the change of organization by the other firms doesn’t influence on the others’ incentives.

\(^3\)Separation between mobile (entrepreneurs) and immobile (workers) factors makes the analytical solution for the rent to mobile factors much easier. However, since location decisions are in mobile entrepreneurs’ hands, the criteria for the relocation is their real wage which is composed of the Ricardian rent from the profit of monopolistic competition firms and the price index of the residential region. These specification enables the two forces, so called home market effect and price index effect. With the two effects, we could analyze the stability of symmetric equilibrium. If we adopt the alternative setup, like footloose capital model, then between symmetric regions, symmetric equilibrium never be unstable.

\(^4\)The common wisdom would insist that a firm concentration of production comes after exporting. However, to my best knowledge, all of the previous theoretical studies including models with heterogeneous firms propose the scenario that reduction in transportation costs always encourage multi-plant firms to become exporters for exploiting scale economies.
2 Location choice without organization choice

The economy is composed of symmetric two regions 1 and 2. There are two production factors: \( H \) units of entrepreneurs and \( L \) units of immobile workers. While immobile workers are equally distributed between regions and are immobile, entrepreneurs can choose the region to stay and the share of entrepreneurs in region 1 is expressed by \( \lambda \).

2.1 Consumers

We assume that preference is identical across all workers and is expressed by

\[
U = \frac{A^{1-\mu}Q^\mu}{\mu^\mu (1 - \mu)^{1-\mu}},
\]

where \( A \) stands for the consumption of agricultural good, \( q(i) \) is the consumption of manufactured good variety \( i \in [0, N] \) and \( Q \) is an index of manufactured good consumption

\[
Q = \left[ \int_0^N q(i)^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}}. \quad N \text{ indicates the mass of differentiated varieties of manufactured goods and } \sigma > 1 \text{ is the elasticity of substitution between any pair of varieties. The expenditure share of manufactured goods is } \mu \text{ and that of agricultural good is } 1 - \mu. \]

We posit \( p^A, p(i), \) and \( P_r \) as the price of agricultural good, the price of a differentiated manufactured good \( i \), and the price index of manufactured goods in region \( r \), respectively.

Then we could derive the demand function for a differentiated manufactured good and the indirect utility function as,

\[
q_r(i) = \mu \left( \frac{P_r}{p(i)} \right)^{\sigma} \frac{Y_r}{P_r},
\]

\[
V_r = \left( p^A \right)^{-(1-\mu)} P_r^{-\mu} w_r, \text{ for immobile worker}
\]

\[
V_r = \left( p^A \right)^{-(1-\mu)} P_r^{-\mu} W_r, \text{ for entrepreneur.}
\]

Wages for entrepreneurs and immobile worker are expressed by \( W_r \) and \( w_r \), respectively. Lower subscript indicates the location, \( r \in [1, 2] \). We set \( \lambda_r \) as the share of mobile entrepreneurs in region \( r \), where \( \sum_r \lambda_r = 1 \). Then we may write regional total income as

\[
Y_r = \frac{L}{2} w_r + W_r H \lambda_r
\]
While the distribution of entrepreneurs is endogenous, for the simplicity of analysis, we set the distribution of immobile workers as half and half and the population of immobile workers and mobile entrepreneurs as one, $L = H = 1$.

### 2.2 Agriculture

Agricultural sector produces a homogeneous good under perfect competition and constant returns to scale using labour input only. This good is traded costlessly. Thus we take agricultural good as numéraire and normalize the wage of an immobile worker to be one, $p_A = w_r = w_s = 1$.

### 2.3 Single-plant firm

In manufacturing sector, we assume that firms are imperfectly competitive à la Dixit-Stiglitz and produce differentiated goods. Production of a differentiated good incurs one unit of entrepreneurs as fixed costs and one unit of immobile workers as marginal labour requirement. Interregional trade of manufactured goods incurs “iceberg” transport costs and selling one unit in the other region requires $\tau \geq 1$ units to be shipped. For later reference, we posit $\phi = \tau^{1-\sigma} \in [0, 1]$ as alternative measure of transport costs. We may call $\phi$ as trade freeness. When transport costs are high (low), $\phi$ takes the value close to zero (one). Then increasing $\phi$ expresses the decreasing transport costs and no transport costs can be expressed by $\phi = 1$. When the single-plant firm locates in region $r$, it faces the demand from the same region, $(P_r Pr i)\sigma \mu Y_r$, and the demand from the other region to export, $(P_s P_s i)\sigma \mu Y_s \phi$. Then the total demand for a differentiated good can be written as,

$$q_r (i) = \left( \frac{P_r}{p_r (i)} \right) \frac{\mu Y_r}{P_r} + \left( \frac{P_s}{p_s (i)} \right) \frac{\mu Y_s}{P_s} \phi. \quad (6)$$

When single-plant firms export their products to the other region where they do not locate, they incur transport costs. Thus the price indices can be written as

$$P_1^{1-\sigma} = N \left[ \lambda \int_0^1 p(i)^{1-\sigma} d_i + (1 - \lambda) \phi \int_0^1 p(i) \phi^{1-\sigma} d_i \right], \quad (7)$$

$$P_2^{1-\sigma} = N \left[ \lambda \phi \int_0^1 p(i)^{1-\sigma} d_i + (1 - \lambda) \int_0^1 p(i)^{1-\sigma} d_i \right]. \quad (8)$$
where \( N \) expresses the total mass of firms. As is mentioned in the introduction, all imperfectly competitive firms are assumed to be exporters with single-plant, when the number of plants is not a choice variable of a firm. When a single-plant firm locates in one region and establishes only one plant there, entrepreneurs have to set up an exporting facility. Including these export-fixed cost, fixed costs for single-plant firms are normalized as one in terms of entrepreneurs. Thus we have the total mass of firms as \( N = H = 1 \), as long as all firms are single-plant. This specification of additional fixed cost could clarify the role of cost difference between single and multiplant firms. Moreover, marginal input is assumed to be one unit of immobile worker. The profit function of a differentiated good firm with single-plant in one region \( r \) can be written as

\[
\pi^S_r(i) = (p_r(i) - w_r) q_r(i) - W^S_r(i),
\]  

(9)

The single-plant firm producing variety \( i \) chooses its mill price to maximize profit \( \pi_r(i) \). The price resulting from the profit maximization is a markup over marginal costs:

\[
p_r(i) = \frac{\sigma}{\sigma - 1} w_r.
\]  

(10)

Using the optimal prices both in profit function and in price index, we could obtain the equilibrium profits as,

\[
\pi^S_r(i) = \frac{\mu}{\sigma N} \left[ \frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \phi \right] - W^S_r(i),
\]  

(11)

where \( \Delta_r \) is the bracket of the price index, (7), in region \( r \). We use \( \lambda \) as the share of firms in region 1. By normalizing the population of entrepreneurs into one, we have the total mass of firms as, \( N = 1 \). Then the share of entrepreneurs in a region is the same with the share of firms in a region, \( \lambda \). Imposing the free entry condition on this monopolistic sector with the equation in (11) and substituting the total mass of firms, we could find entrepreneurs’ reward of an exporting firm in region \( r \) with a single-plant as,

\[
W^S_r(i) = \frac{\mu}{\sigma} \left[ \frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \phi \right],
\]  

(12)

where \( \Delta_r = [\lambda + (1 - \lambda) \phi] \), and \( \Delta_s = [\lambda \phi + (1 - \lambda)] \).

(13)

Using (5) and (12), we could perform the analysis on location equilibria with single-
plant monopolistic firms. The location equilibria follow three cases: symmetric distribution, agglomeration in one region, and agglomeration in the other region. The latter two cases can be referred as core-periphery structure. Since firms are agglomerated in one region, the region is called core and the other periphery. The critical values for trade freeness at which a symmetric equilibrium becomes stable to unstable under decreasing transport costs is called the symmetry break point and given by

\[
\phi_{\text{sym}} = \frac{(1 - \frac{\mu}{\sigma})(1 - \frac{\mu}{\sigma} - \frac{1}{\sigma})}{(1 + \frac{\mu}{\sigma})(1 + \frac{\mu}{\sigma} - \frac{1}{\sigma})}.
\] (14)

On the other hand, the critical values for trade freeness at which the core-periphery equilibrium becomes unstable under increasing transport costs is called the sustain point which can be expressed by \(\phi_{\text{sus}}\), in the following implicit form: 5

\[
\phi_{\text{sus}}^{\frac{\mu}{\sigma} - 1} \left(1 + \frac{\mu}{\sigma} \phi_{\text{sus}}^2 + \phi_{\text{sus}}^2 - \frac{\mu}{\sigma}\right) - 2\phi_{\text{sus}} = 0.
\] (15)

3 Location and organization choice

In this section, we study the location and organization choice of firms. We only modify the assumption on the number of plants. Introduction of multiplant firms means an additional choice for entrepreneurs. The share of entrepreneurs in multiplant firms and that of single-plant firms in region \(r\) \((= 1, 2)\) are denoted by, \(m_r\) and \((1 - m_r)\), respectively. Nominal rewards to entrepreneurs in multiplant firms are assumed to be the same across regions. Following these specifications, we rewrite regional income in (5) as

\[
Y_r = \frac{L}{2} + ((1 - m_r) W_r^S + m_r (1 + \alpha) W^M) H\lambda_r,
\] (16)

While for single-plant firms they incur one unit of entrepreneurs, as is assumed in the previous section, for multiplant firms they incur additional fixed requirements of entrepreneurs. This additional fixed costs to be multiplant is captured by \(\alpha\), which is specified more later.

While single-plant firms export to the other region where they do not locate, multiplant

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5 Details can be found in Forslid and Ottaviano (2003) for this model. For Krugman model, see Fujita et al. (1999).
firms can serve both regions without incurring transport costs. Thus the price index of the varieties sold in region $r$, $P_r$, is expressed as

$$P_r^{1-\sigma} = N \left[ (\lambda_r (1 - m_r) + \phi \lambda_s (1 - m_s) + (\lambda_r m_r + \lambda_s m_s)) \int_0^1 p(i)^{1-\sigma} \, di \right]. \quad (17)$$

First term expresses the price index of firms locating in region 1 and the second term expresses the price index of firms locating in region 2. The last expression is the price index of multiplant firms.

### 3.1 Multiplant producer

Multiplant firms are also depicted by imperfectly competitive firms à la Dixit-Stiglitz and produce a differentiated good. The only modification from the single-plant exporter is that establishment of multiplant incurs additional fixed cost, $\alpha > 0$. This fixed costs, $\alpha$, include the costs for maintaining a subsidiary in the other region and the duplicate overhead production costs.\(^6\) For simplicity, we assume that all of the entrepreneurs in a multiplant firm reside in a region.\(^7\)

For the production, multiplant firms employ immobile workers in both regions as variable input. Contrast to the cost function of single-plant firms, since multiplant firms locate in each region, the shipment of products by multiplant firms doesn’t incur transport costs, $\phi_{rr} = 1$ for $(r = 1, 2)$. Thus they face the demand from each regions without transport costs. Namely, the trade-off in organization between single-multi plant lies between transportation costs (proximity) and the scale economies in single-plant (concentration). Taking each regional demand as given in (2), multiplant firms maximize their profit. Then the output and the

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\(^6\)Fujita and Gokan (2005) assume that the fixed cost of a multi-plant firm to build an additional plant is larger than the fixed costs of the single plant. Toulemonde (2008) explain that several factors affect the fixed costs of a multinational.

\(^7\)Since entrepreneurs obtain the same nominal wage under multilplant firms, they reside in the region with the smaller cost of living, i.e., the smaller price index for differentiated goods. The possible symmetric distribution of entrepreneurs may occur only when everything is symmetric and can be a knife-edge case. As we focus on core-periphery case only, we exclude this possible case.

While alternative specifications on this additional fixed costs induce the solutions highly nonlinear and it is unable to perform analytical results, it would be very interesting to allow the possibility that some entrepreneurs in a multi-plant firm reside in periphery region with compensated wage under core-periphery structure. This could be an extension of this paper and is ignored here.
profit function of a multiplant firm can be written as

\begin{align*}
q^M_{rr} (i) &= \left( \frac{P_r}{p_r (i)} \right) ^\sigma \frac{\mu Y_r}{P_r}, \\
q^M_r (i) &= q^M_{rr} (i) + q^M_{ss} (i),
\end{align*}

(18)

(19)

\[ \pi^M (i) = (p_r (i) - w_r) q^M_{rr} (i) - (p_s (i) - w_s) q^M_{ss} (i) - (1 + \alpha) W^M (i), \]

(20)

where superscript \( M \) indicates multiplant firms and \( W^M \) is a entrepreneurs’ reward in multiplant firms in region \( r \). Since there is no location choice for multiplant firms, their profit function and their wage for entrepreneur do not include region specific subscript. A multiplant firm producing variety \( i \) chooses its mill price to maximize profit \( \pi^M_r (i) \) respect to each region using discriminatory price. The optimal price is given by a markup over marginal costs as

\[ p_r (i) = \frac{\sigma}{\sigma - 1} w_r, \quad r = 1, 2. \]

(21)

Substituting the optimal prices into profit function and price index, normalization of labour wage in agricultural sector as one, \( w_r = w_s = 1 \), and a given distribution of firms, the equilibrium profits can be obtained as follows\(^8\)

\[ \pi^M (i) = \frac{\mu}{\sigma N} \left[ \frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \right] - (1 + \alpha) W^M (i), \]

(22)

where \( \Delta_r = \lambda_r (1 - m_r) + \lambda_s (1 - m_s) \phi + (\lambda_r m_r + \lambda_s m_s) \).

\( \Delta_r \) expresses the brackets of price indices in (17) of region \( r \); \( P_r = \Delta_r^{1/\sigma} \), which reflect the magnitudes of multiplant firms. Assuming the existence of potential entrants ensures that the operating profit of suppliers is set to zero, the wage of entrepreneurs are obtained from the zero profit condition of (22). Single-plant firms’ offer to entrepreneurs are obtained from the same procedure as in (11) except that the price index is not the same. The free-entry

\(^8\)Note that the regional subscript for multi-plant firms are dropped since the symmetric technology imply the profit of the multi-plant firms as the same, \( \pi^M_r (i) = \pi^M_s (i) \). There could be only the difference at the real wage for entrepreneur.
condition should be hold for any organization. Hence the condition becomes,

$$\max \{ \pi_r^S (i), \pi_s^S (i), \pi_M (i) \} = 0. \quad (23)$$

Then, we obtain the entrepreneurs’ reward for single-plant firm \(i\) and multiplant firm \(j\) in region \(r\) as,

$$W^S_r (i) = \frac{\mu}{\sigma} \left[ \frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \phi \right], \quad (24)$$

$$W^M (j) = \frac{\mu}{\sigma (1 + \alpha) N} \left[ \frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \right]. \quad (25)$$

Note that \(1/\sigma\) in (24) and (25) reflects the share of entrepreneur’s reward in profit. Among the same organization, assuming the symmetry of monopolistic firms, without the loss of generality, we drop the individual index of \(i\) and \(j\).³ Using four equations, (24), (25), and their symmetric expressions for the other region, we obtain \(W^S_r, W^S_s, W^M_r, Y_A, Y_B\) explicitly (See Appendix II). Then we obtain the relative real wages across regions and organizational patterns as follows:

$$\frac{\omega^S_r}{\omega^M_r} = \frac{W^S_r}{W^M_r}, \quad \frac{\omega^S_r}{\omega^M_s} = \frac{W^S_r}{W^M_r} \left( \frac{\Delta_r}{\Delta_s} \right)^{\frac{\mu}{\sigma}} \cdot \frac{\omega^S_r}{\omega^S_s} = \frac{W^S_r}{W^S_s} \left( \frac{\Delta_r}{\Delta_s} \right)^{\frac{\mu}{\sigma}}, \quad (26)$$

$$\frac{W^S_r}{W^M} = \frac{\phi \Delta_r + \Delta_s - \theta (1 - \phi) (\lambda_s (1 + \phi) - \lambda_r m_r - \lambda_s \phi m_s)}{\Delta_r + \phi \Delta_s - \theta (1 - \phi) (\lambda_r (1 + \phi) - \lambda_r m_r - \lambda_s \phi m_s)}, \quad (27)$$

$$\frac{W^S_r}{W^S_s} = \frac{\phi \Delta_r + \Delta_s - \theta (1 - \phi) (\lambda_r (1 + \phi) - \lambda_r m_r - \lambda_s \phi m_s)}{\Delta_r + \phi \Delta_s - \theta (1 - \phi) (\lambda_r (1 + \phi) - \lambda_r m_r - \lambda_s \phi m_s)}. \quad (28)$$

where \(\Gamma \equiv 1/(1 + \alpha), \theta \equiv \mu/\sigma N\) and \(r = 1, 2\). Note that the reward from multiplant firms is identical in terms of nominal term. In a region, if both types of organization can exist only when \(\omega^M_r = \omega^S_r\) (\(= W^S_r = W^M_r\)) holds. Since the price indices are identical in the same region, comparison of real wages from the same region is boiled into nominal wage differential. On the other hand, when all firms are either single-plant or multiplant in a region, equilibrium wage condition is \(W^S_r > W^M\), or \(W^M > W^S_r\).

³In each cases, the labour market clearing condition of entrepreneurs implies the total mass of firms as \(N = 1\) for the case of single-plant firms and \(N = 1/(1 + \alpha)\) for the case of multi-plant firms. Note that, obviously, the number of firms under all-multi-plant-firm is smaller than that under all-single-plant-firm. On the other hand, when there exists mixed case, the total mass of firms is given by \(N = \frac{1}{\lambda m_1 + \lambda m_2 + (1 + \alpha)(\lambda m_1 + \lambda m_2)}\).
3.2 Equilibrium

As is discussed in the introduction, multiplant firms among regions are observed widely. At the same time, there are many single-plant firms in the same industry. We examine the possible equilibrium of such a mixed-organization configuration.

For the comprehensive analysis, there are two choices for entrepreneurs to be considered; location and organization. The timing of the decisions follows in the two steps. Firstly, entrepreneur choose location of the firm. Secondly, for a given location, organization is determined. Using (26) to (28), the indirect utility differentials on locations or organizations are defined. Since we have three variables which determine the equilibrium, $\lambda$, $m_1$, and $m_2$, this system of equations is highly complicated. In the following analysis, we focus on the core-periphery structure. Thus our analysis abstract from the location choice by assuming core-periphery structure of entrepreneurs, and focus on the configuration of organizations, and how introduction of organization choice change the stable location equilibrium. Accordingly, we assume that all the values are under the ranges to keep the core-periphery structures, which is expressed in (15), $\phi_{sus} < \phi < 1$.

If the offered wage is less than the others’, the firm cannot enter or remain the market because of the lack of fixed requirement. This could be interpreted as a bidding process of Entrepreneurs’ reward. Since entrepreneurs maximize their indirect utility, besides the nominal reward they also consider price index. Organization is determined by this mechanism. When one organization is superior under a certain transport costs, real wage under this configuration is higher. On the other hand, when both organizations coexist, the real wages must be the same under both configurations.

Organization of firms is determined by the real wage of entrepreneurs. In our model, regional real wages depend on the distribution of firms across regions and organization. Clearly, when $\varpi_S > \varpi_M$, all entrepreneurs choose to be single-plant, and when $\varpi_S = \varpi_M$, single-plant and multiplant can be coexisted. A organization equilibrium is stable if, for any marginal deviation from the equilibrium, the ordering of real wages under different organization choices

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10 In typical NEG model of two regions without organization choice, core-periphery structure is the case that all the firms and entrepreneurs locate in one of the regions. In the setup of this paper, all entrepreneurs locate in one region and the location of firms is not the same with the entrepreneur.

11 When we relax this assumption on stability of regional structure, we could have the relocation of core-region. Since this paper doesn’t deal with the analysis between the symmetric regions, we doesn’t exclude such possibilities. However, these are not analyzed in this paper.
is unchanged.

### 3.3 Core-periphery structure

We consider asymmetric structures of regions. Precisely, we focus on the core-periphery structure, where all firms agglomerate in one region. Under core-periphery structure, the results are to be modified. Since the organization choice is determined within one region, the choice solely depends on the nominal wage differential in the region. Substituting the distribution condition of multiplant firms and entrepreneurs, $\lambda_1 = 1$, $\lambda_2 = 0$, $m_2 = 0$ and $m_1 = 1$ or $m_1 = 0$, into (27), we obtain the nominal wage differential against the wage for multiplant firms boiled down into as follows:\footnote{Focusing only on organization equilibrium under core-periphery structure, the real wage comparison in one region for organization is boiled down into the nominal wage comparison since the price index is the same and is canceled out.}

\[
\frac{W^S_1}{W^M_1} \bigg|_{m_1 = 1} - 1 = \frac{1}{2} \left( \left( 1 - \frac{\mu}{\sigma} + \alpha \right) \phi - \left( 1 - \frac{\mu}{\sigma} - \alpha \right) \right), \tag{29}
\]

\[
\frac{W^S_1}{W^M_1} \bigg|_{m_1 = 0} - 1 = \frac{2\alpha \phi - (1 - \phi) \left( 1 - \frac{\mu}{\sigma} \right)}{(1 + \frac{\mu}{\sigma}) \phi + \left( 1 - \frac{\mu}{\sigma} \right)}. \tag{30}
\]

The decision is evaluated by the nominal wage differential. Solving the nominal wage differential in (29) and (30) for transportation costs, we obtain the following critical values respectively;

\[
\phi^M = \frac{1 - \frac{\mu}{\sigma} - \alpha}{1 - \frac{\mu}{\sigma} + \alpha}, \tag{31}
\]

\[
\phi^S = \frac{1 - \frac{\mu}{\sigma}}{1 - \frac{\mu}{\sigma} + 2\alpha}. \tag{32}
\]

We define $\phi^M$ as the critical value for trade freeness below which all the firms are multiplant and $\phi^S$ as the one above which all firms are single-plant. For a given additional fixed costs, $\phi^M$ and $\phi^S$ determine the boundaries that all firms are the same organization or not. As is obvious from (31) and (32), if it is indifferent to be multiplant or single-plant, $\alpha = 0$, then we have $\phi^M = \phi^S$ and thus single-plant is dominant and the possibility of multiplant is measure zero at $\phi = 1$. Moreover, since we have $\frac{d}{d\alpha} \phi^M = -2 \frac{1 - \frac{\mu}{\sigma}}{(1 - \frac{\mu}{\sigma} + \alpha)} < 0$, $\frac{d}{d\alpha} \phi^S = -2 \frac{1 - \frac{\mu}{\sigma}}{(1 - \frac{\mu}{\sigma} + 2\alpha)} < 0$, \ldots
increase in the additional fixed costs induce both critical values smaller and makes the ranges for all-firms-multiplant shrinks. Then we obtain the following lemma.

**Lemma 1** The smaller the cost differential between single-plant and multiplant firms, the stronger firms have the incentive to be multiplant.

The possible organization configuration is described in Figure 1. When transport costs are high, firms have incentives to be multiplant. As the additional fixed costs becomes smaller, the ranges that all firms are multiplant expands. For \( \phi < \phi^S \), all firms choose to locate in core-region and be single-plant exporter. For higher additional fixed costs, \( \phi^M \) would take the negative values which means the corner solution. This expresses the case that at this value for additional fixed costs, all-multiplant never happens. On the other hand, even though \( \alpha \) takes the maximum value at one, \( \phi^S \) cannot take the minimum value zero. Since \( \phi^S \) can only take non-negative values, this shows that it never happens that all-single-plant always

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In stead of (31) and (32), solving for additional fixed costs, we have \( \alpha^M = (1 - \mu/\sigma)(1 - \phi) / (1 + \phi) \), and \( \alpha^S = (1 - \mu/\sigma)(1 - \phi) / 2\phi \). The parameters to be specified follows the average of the estimation results in Table 4 by Hanson (2005) as \( \sigma = 2.11 \), and \( \mu = 0.71 \). Under these specifications, we have the break point in (14) and sustain point (15) as 0.109 and 0.047.
dominates the other two cases. Then the organization configuration goes through mixed case to all-single case. Decreasing transport costs makes the home market effect larger and set the share of multiplant firms smaller. For the range of $\phi^M < \phi < \phi^S$, since some firms are single-plant, this is the mixed configuration of organizations. Above discussions could be summarized by the following proposition.

**Proposition 1** For a given additional fixed costs, firms choose their organization as,

i) if $\phi < \phi^M$, all firms are multiplant,

ii) if $\phi^M < \phi < \phi^S$, multiplant and single-plant firms are mixed,

iii) if $\phi^S < \phi$, all firms are single-plant.

Moreover, we have the derivatives of (29) and (30) as

$$\frac{d}{dm_1} W^S_{M} \bigg|_{m_1=1} = \frac{(1 - \phi)}{4(1 + \alpha)} \left( \left( 1 + \frac{\mu}{\sigma} + \alpha \right) \left( 1 - \phi \right) (1 - \phi) - 2\alpha \frac{\mu}{\sigma} (1 + \alpha) \right), \quad (33)$$

$$\frac{d}{dm_1} W^S_{1} \bigg|_{m_1=0} = \frac{(1 - \phi)}{(1 - \frac{\mu}{\sigma} + (1 + \frac{\mu}{\sigma}) \phi)^2} \left( \left( 1 - \frac{\mu}{\sigma} \right) (1 + \frac{\mu}{\sigma}) (1 - \phi) (1 + \alpha) - 2\alpha \frac{\mu}{\sigma} \phi \right). \quad (34)$$

The sign of the derivative in (33) is determined by the large parenthesis. As is obvious, when transport costs decreases, ($\phi$ becomes larger), the second composite in the parenthesis exceeds the first one. This is the point, $\phi^* = 1 - \frac{2\alpha \frac{\mu}{\sigma}(1+\alpha)}{(1+\frac{\mu}{\sigma}) (1-\frac{\mu}{\sigma}+\alpha)} \leq 1$. Thus when all firms are multiplant and face organization change as transportation costs take the range of $0 < \phi < \phi^*$, organization change from multiplant to single-plant is not stable. With some work,\(^{15}\) we have $\phi^M < \phi^*$ and $\phi^S < \phi^{**}$.

The sign of the derivative in (34) is determined by the large parenthesis. While the two composite in the parentheses are the function of transport costs, it is clear that the first one is decreasing and that the second one is increasing. The point that the sign changes is given by $\phi^{**} = 1 - \frac{2\alpha \frac{\mu}{\sigma}}{(1+\alpha)(1+\frac{\mu}{\sigma})(1-\frac{\mu}{\sigma}+2\alpha)}$. When organization is mixed and most of the firms become single and transportation costs take the range of $0 < \phi < \phi^{**}$, still the organization to be single is not stable.

\(^{14}\)The uniqueness of the solution is shown in Appendix III.

\(^{15}\) $\phi^M - \phi^* = -\frac{2\alpha(1+\alpha)(1-\frac{\mu}{\sigma})}{(1+\frac{\mu}{\sigma})(1-\frac{\mu}{\sigma}+\alpha)} < 0$ and $\phi^S - \phi^{**} = -\frac{2\alpha(1-\frac{\mu}{\sigma})(1+\alpha+2\alpha)}{(1-\frac{\mu}{\sigma}+2\alpha)(1+\alpha)(1+\frac{\mu}{\sigma})(1-\frac{\mu}{\sigma}+2\alpha)} < 0$.  

14
4 Generalization

Before dealing with generalization, let us summarize our main results. When we consider organization choice with location choice in two region model, as is shown in proposition 1 and 2, we could find several organization configuration under core-periphery structure; all-multiplant, mixed, and all-single-plant. From the equation (29), the magnitude of demand, home market effect, appears and show the proximity-concentration trade off.

We know that when the differential between the two cost structure is smaller (larger), the range of transportation costs where firms can choose to be multiplant becomes larger (smaller). In other words, since the cost differential comes in both fixed costs, it expresses the relative size of the scale economies in either of the two. Larger cost differential means the attractiveness to change their organization becomes larger, which induces firms to make their production concentrated into a single-plant from multiplant.

It would give us sufficient discussions to examine more on the assumption of the additional fixed costs for multiplant firms. Until previous sections, we assumed constant fixed costs on both types of organization. However, it might be reasonable to think the fact that high transportation costs could be applied not only for goods transportation but also for establishment of secondary plants, transfer of managers, and supplemental communication across borders. It means that when transportation costs is high, operation costs for the additional plant would also be high. In the other words, when the transportation costs decrease, the setup costs would decrease, also. In such circumstances, the transportation costs include communication costs as well. In order to capture this aspect, we simply reformulate the profit function of multiplant firms as,

\[
\pi^M (i) = (p_r (i) - w_r) q_{rr}^M (i) - (p_s (i) - w_s) q_{ss}^M (i) - \left(1 + \alpha \phi^{-b}\right) W^M (i),
\]

Compared to the constant additional fixed costs case, (20), the difference comes in the term multiplied to the additional fixed costs, \((1 + \alpha \phi^{-b})\). This could express the above discussed situation. Note that we still keep the assumption on the fixed costs differential of the two organization \((0 < \alpha \leq 1)\). The demand functions the same in the last section as, (18) and (19). Then the profit maximization yields the same price as in (21). Substituting this optimal
price into profit function and price index, normalization of labour wage in competitive sector as one, \( w_r = w_s = 1 \), and a given distribution of firms, the equilibrium profits can be obtained as follows

\[
\pi^M (i) = \frac{\mu}{\sigma N} \left[ \frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \right] - \left( 1 + \alpha \phi^{-b} \right) W^M (i) ,
\]

(36)

where \( \Delta_1 \) and \( \Delta_2 \) expresses the brackets of price indices in region 1 and 2, \( P_1 = \frac{\Delta_1}{1-\phi} \), which is the same as the last section. Applying the zero profit condition on this profit function, we obtain the wage of entrepreneurs as follows,

\[
W^M (j) = \frac{\mu}{\sigma (1 + \alpha \phi^{-b}) N} \left[ \frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \right] .
\]

(37)

Again, entrepreneurs seek not only for the region but also for the organization which offers highest rewards. Their decision of organization and location depends on the nominal wage differential of their wages. Instead of the previous fixed cost differential, we redefine this parameter as \( \Gamma \equiv \frac{1}{(1+\alpha \phi^{-b})} \). Since the reformulation in the additional fixed costs, \( \Gamma \), doesn’t affect the system of equations listed in (A1) which result in (A2), (27) and (28), we could fully utilize the previous results in Appendix II\(^{16}\) and we obtain the nominal wage differential as

\[
\frac{W^S_1}{W^M} \bigg|_{m_1=1} - 1 = \frac{1 + \phi + \Gamma \phi (1 - \phi)}{2 \Gamma} - 1 ,
\]

(38)

\[
\frac{W^S_1}{W^M} \bigg|_{m_1=0} - 1 = \frac{2 \phi}{\Gamma (1 - \frac{\mu}{\sigma} + (1 + \frac{\mu}{\sigma}) \phi)} - 1 .
\]

(39)

Here we substitute \( \Gamma \). While with respect to \( \phi \) the equation is highly nonlinear, with respect to \( \alpha \) it is still linear equation. The solutions for each equation induces

\[
\left( \phi^M \right)^b \frac{(1 - \phi^M) (1 - \frac{\mu}{\sigma})}{1 + \phi^M} = \alpha ,
\]

(40)

\[
\frac{1}{2} \left( \phi^S \right)^{b-1} (1 - \phi^S) \left( 1 - \frac{\mu}{\sigma} \right) = \alpha .
\]

(41)

\( \phi^M \) is the critical value for trade freeness which \textit{all firms are multiplant}. If there is \( \phi^M \) for

\(^{16}\)As is shown in appendix, the system of equations in section 3 and 4 is identical except the specification in the cost differential, \( \Gamma \). Thus when we could obtain the result by using (27). Note that \( \Gamma \in [0, 1] \), since \( \Gamma > 0 \) and \( \Gamma - 1 = -\alpha \phi^{-b} / (1 + \alpha \phi^{-b}) < 0 \).
a given fixed costs, there is at three organization configuration that all-multi, mixed and all-single. $\phi^S$ is the one which some firms are multiplant. If there is $\phi^S$ for a given fixed costs, there is at least two organization configuration that all-single and mixed organization.

Due to the nonlineality of the solution, the organization configuration depends on the parameter of transportation costs effects on fixed costs, $b$. In order to understand the results, we show the results in graphically. Suppose we have the additional fixed costs in the forms of $(1 + \alpha \phi^{-1})$, $b = 1$, then we could draw the possible organization ranges as in Figure 2. In this case, there is always one critical value of $\phi^S$ and it exhibits inverse proportion which doesn’t cross alpha-axis. So there are always the ranges for mixed organization configuration. On the other hand, if $\alpha < \phi (1 - \phi) (1 - \frac{b}{\phi}) / (1 + \phi)$ holds, we have two critical values of $\phi^M$. Between the two critical values, there is organization configuration that all firms are to be multiplant. As long as $\alpha < \phi (1 - \phi) (1 - \frac{b}{\phi}) / (1 + \phi)$ holds, for a given additional fixed costs, under decreasing transportation costs, organization configuration changes from mixed to all-multi, all-multi to mixed and mixed to all-single.

For further discussion, in Figure 3 we show the critical values for different degree of specification. The effect of transportation costs on additional fixed costs of multiplant may be
more than proportional. Such cases are described in this figure. When the parameter of additional fixed costs, $\alpha$, is large, the organization configuration that all firms are single plant is dominant and no possible organization change. When it is smaller, we could confirm that possible organization configuration of three types (all-multi, mixed, and all-single). The solid lines depict $\phi^M$ and the dashed lines depict $\phi^S$, respectively. When transportation costs are very high, firms cannot establish multiplant due to high additional fixed costs so they export. With decreasing transportation costs, firms don’t export but establish multiplant. With further decrease in transportation costs, the economies of scale in single-plant become sufficient and afford transportation costs so that firms choose to concentrate their production into single location. The results fully describe the proximity-concentration tradeoff of reality. In certain ranges of $\alpha$, we could confirm that organization change occurs as single-plant to multiplant and multiplant to single-plant. The possible organization configurations are from all-single to mixed, mixed to all-multi, and vice versa. If $\alpha < \phi^b \frac{(1-\phi)(1-\frac{\sigma}{\gamma})}{1+\phi}$ holds, there is at least one critical value of $\phi^M$ and three organization configurations. If $\alpha < \frac{1}{2} \phi^{b-1} (1 - \phi) \left(1 - \frac{\mu}{\tau}\right)$ holds, there is at least one critical value of $\phi^S$ and at least two organization configurations. Summarizing the above discussions leads the following proposition.
Proposition 2 Suppose that the additional fixed costs for multiplant is a function of transportation costs, \((1 + \alpha \phi^{-b})\).

i) if \(\alpha \leq \frac{1}{2} \phi^{b-1} (1 - \phi) \left(1 - \frac{\mu}{\sigma}\right)\) holds, there is mixed configuration of single plant and multiplant firms.

ii) if \(\alpha \leq \phi \cdot \frac{(1-\phi)(1-\frac{\mu}{\sigma})}{1+\phi}\) holds, there are three organization configurations; all-single, mixed and all-multiplant.

Compared to the results in the previous section, we find the possible organization changes from single-plant into multiplant and multiplant to single plant. If the cost differential between the two organizations is small enough, the range of multiplant firms is larger. On the other hand, when the cost differential is larger, single-plant is organizationally dominant. In other words, for smaller additional fixed costs, \(\alpha\), more likely firms become multiplant.

Differently from the previous studies in the literature, from the generalized analysis, we observe non-monotonic organization change as single-plant to multiplant and from multiplant to single-plant.

5 Conclusion

The globalization and the development of transportation and information technologies can be characterized by lower transport costs of factors. At the same time, it is not negligible that there is a certain presence of multiplant firms in international economies and regional economies. In order to focus on the behavior and organization of multiplant, we explicitly relax the implicit and typical assumption on the solitariness of firms’ organization in monopolistic competition. In particular, we focus on the case which starts from the core-periphery structure; all entrepreneurs locates in one region.

Firstly, we show that the decrease in transport costs induce firms agglomerate in one region, and also promotes firms to concentrate their production from multiplant into a single-plant in all cases. As Ekholm and Forslid (2001) pointed out, “the fact that trade costs and the degree of multiplant economies of scale may change simultaneously has important implications”. We show that the combination of transportation costs and the cost differential between the two organization crucially affects the organization change. This is consistent to the simulation results in Markusen and Venables (1998) but is not explained analytically in
detail. In the next step, we specify the additional fixed costs for multiplant as a function of transportation costs. Then we could observe the organization change not only from multiplant into single-plant but also single-plant to multiplant. Intuitively, most of the histories of multiplant (multinational) firms would be such that firstly a firm was established in a region, served the region domestically, gradually started exporting from there, subsequently established secondary plants in the other regions, and later concentrated some of the plants into a few. Our generalized results that firms change their organization non-monotonically could explain this intuitive history of multiplant (multinational) firms. In empirical analysis following Brainard (1997), it sometimes occurs that the coefficients of tariff and transportation costs are insignificant or wrong sign. These may capture the nonlinear effects of broadly defined trade costs on fixed requirements as shown in the previous section.

From our analysis, some analytical results are emphasized. Firstly, the difference between the establishment fixed costs and the export fixed costs determine the stability of multiplant organization, horizontal FDI. When establishment costs becomes lower, more firms choose multiplant. On the other hand, when fixed export costs decrease, more firms choose single-exporting. We could confirm that transaction costs unambiguously affects not only the location choice of firms but also affects their organization choice. Secondly, under core-periphery structure, we show that there is a range of transportation costs where there is mixed organization and that the cost differential between two organization and home market effect exhibit the proximity-concentration trade off.

More detailed analysis would show some more interesting possibilities. There would be other formulation on the differences of transaction costs in different organization. In particular, in our model, the role of establishment costs needs managers or entrepreneurs. They are assumed to consume in one region where is their residency, or say the place of headquarter. However, in the process of establishment of multiplant, many managers are sent to the region and sometimes they spend more than ten years. It might be one way to change the assumption on the location of consumption. We try to capture the structure and the complicated decision of multiplant firms. This is a modest attempt to capture the multinational entrepreneurs.
References


**Appendix I**

Using four equations, (5), (12), and the corresponding equations for the other region, we obtain $W_1^S, W_2^S, Y_1, Y_2$ explicitly.

\[
W_1^S = \frac{\mu}{\sigma} \left[ \frac{Y_1}{\Delta_1} + \frac{Y_2}{\Delta_2} \phi \right],
\]

\[
W_2^S = \frac{\mu}{\sigma} \left[ \frac{Y_1}{\Delta_1} \phi + \frac{Y_2}{\Delta_2} \right],
\]

\[
Y_1 = \frac{L_2}{\Delta_1} + \lambda W_1^S,
\]

\[
Y_2 = \frac{L_2}{\Delta_2} + (1 - \lambda) W_2^S,
\]

where $\Delta_1 = [\lambda + (1 - \lambda) \phi], \Delta_2 = [\lambda \phi + (1 - \lambda)]$. This yields a unique solution as,
\[
Y_1 = \frac{\Delta_1 (\Delta_2 - \frac{\mu}{\phi} ((1 - \lambda) - \lambda \phi))}{2 \Phi_S},
\]
\[
Y_2 = \frac{\Delta_2 (\Delta_1 - \frac{\mu}{\phi} (\lambda + (1 - \lambda) \phi))}{2 \Phi_S},
\]
\[
W_1^S = \frac{\mu \phi \Delta_1 + \Delta_2 - \frac{\mu}{\phi} (1 - \phi) (1 + \phi) (1 - \lambda)}{\sigma},
\]
\[
W_2^S = \frac{\mu \Delta_1 + \phi \Delta_2 - \frac{\mu}{\phi} \lambda (1 - \phi) (1 + \phi)}{\sigma},
\]

where \(\Phi_S = \Delta_1 \Delta_2 - \frac{\mu}{\phi} (1 - \lambda) \Delta_1 - \frac{\mu}{\phi} \lambda \Delta_2 + \left(\frac{\mu}{\phi}\right)^2 \lambda (1 - \phi) (1 + \phi) (1 - \lambda).\) Note that since \(H = L = 1\) and \(N = 1.\)

Appendix II

We set the share of single-plant firms and that of multiplant firms as \((1 - m_r)\) and \(m_r,\) \(r = 1, 2\) and put \(\theta \equiv \frac{\mu}{\sigma N}, \Gamma \equiv \frac{1}{[1 + \alpha]}.\) Note that, for simpler notation, we set the share of firms in each region as \(\lambda_r,\) where \(\sum_r \lambda_r = 1.\)

\[
\begin{align*}
W_r^S &= \theta \left[\frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s} \phi\right], \\
W^M &= \theta \Gamma \left[\frac{Y_r}{\Delta_r} + \frac{Y_s}{\Delta_s}\right], \\
Y_r &= \frac{L}{2} + \lambda_r (1 - m_r) W_r^S + \lambda_r m_r \Gamma^{-1} W^M,
\end{align*}
\]

where \(\Delta_r = \lambda_r (1 - m_r) + \lambda_s (1 - m_s) \phi + (\lambda_r m_r + \lambda_s m_s).\)

Since there are five unknown variables with five equations, we obtain a unique solution. Wages for each firms are as follows

\[
\begin{align*}
W_1^S &= \frac{\phi \Delta_1 + \Delta_2 - \theta (1 - \phi) (\lambda_2 (1 + \phi) - \lambda_1 m_1 - \lambda_2 m_2)}{2 \Phi_M} L \theta, \\
W_2^S &= \frac{\Delta_1 + \phi \Delta_2 - \theta (1 - \phi) (\lambda_1 (1 + \phi) - \lambda_2 m_2 - \lambda_1 m_1)}{2 \Phi_M} L \theta, \\
W^M &= \frac{\Delta_1 + \Delta_2 - \theta (1 - \phi) (\lambda_1 (1 - m_1) + \lambda_2 (1 - m_2))}{2 \Phi_M} \Gamma \theta L
\end{align*}
\]

where \(\Phi_M = \Delta_1 \Delta_2 - \theta \lambda_2 \Delta_1 \Delta_2 - \theta \lambda_1 \Delta_2 + \theta^2 \lambda_2 \lambda_1 (1 - \phi) (\phi (1 - m_2) (1 - m_1) + 1 - m_1 m_2).\) As is expected, when all firms are single plant, \(m_1 = m_2 = 0,\) this denominator is identical to the one in the previous section, \(\Phi_S,\) and wage equations as well.
Appendix III

5.1 The proof of Proposition 1

Proof. Solving for the nominal wage differential between single-plant and multiplant equal to one, and without evaluating $m_1$, we could obtain the following equation from (27),

$$F(m_1) \equiv \frac{W^S}{W^M} - 1 = \frac{\Omega_1}{\Omega_2}$$

where $\Omega_1 \equiv -\frac{\alpha}{2} (1 - \phi) m_1^2 + (1 - \phi) \left( \alpha + \frac{2}{\sigma} \alpha + 1 \right) m_1 + (1 + \alpha) \left( 2\alpha \phi - (1 - \phi) \left( 1 - \frac{\mu}{\sigma} \right) \right)$,

and $\Omega_2 \equiv -\frac{2}{\sigma} \alpha (1 - \phi) m_1^2 + (1 - \phi) \left( \frac{2}{\sigma} + \alpha + 2 \frac{\mu}{\sigma} \alpha + 1 \right) m_1 + (1 + \alpha) \left( 1 - \frac{\mu}{\sigma} + \phi \left( 1 + \frac{\mu}{\sigma} \right) \right)$.

The sign of $F(m_1)$ is of interest. If $F(m_1) > 0$, all firms are single plant and if $F(m_1) < 0$, all firms are multiplant. When there is $m_1 \in [0, 1]$ such that $F(m_1) = 0$, some firms are multiplant. A simulation of $F(m_1)$ is in Figure 4, which has the correspondence with Figure 1 and there is a range of $m_1 \in [0, 1]$. Since $\Omega_2|_{m_1=0} > 0$, $\Omega_2|_{m_1=1} > 0$ and $\frac{d}{dm_1} \Omega_2 = 0$ when $m_1 = 1 + \frac{\frac{\alpha}{2} + 1 + \alpha}{2 \frac{\mu}{\sigma}} > 1$, we have $\Omega_2 > 0$, $\forall m_1 \in [0, 1]$. So we could focus on the sign of numerator, $\Omega_1$. Solving $\Omega_1$ for $m_1$, then we have a solution for $m_1 \in [0, 1]$ as $m_1 = \frac{\left( \left( 1 - \phi \right) \left( \alpha + \frac{\mu}{\sigma} \alpha + 1 \right) - \sqrt{\left( \left( 1 - \phi \right) \left( \alpha \left( \frac{\mu}{\sigma} + 1 \right) + 1 \right) \right)^2 + 4 \frac{\mu}{\sigma} \left( 1 - \phi \right) \left( 1 + \alpha \right) \left( 2\alpha \phi - \left( 1 - \phi \right) \left( 1 - \frac{\mu}{\sigma} \right) \right)}} \right)}{2 \frac{\mu}{\sigma} \left( 1 - \phi \right)}$. In a different way, solving $\Omega_1$ for $\phi$, then we have the function of $\phi(m_1)$ as $\phi(m_1) = 1 - \frac{2\alpha(1+\alpha)}{\frac{\mu}{\sigma} m_1^2 - \alpha m_1 (\alpha + \frac{\mu}{\sigma} + 1) + (1 + \alpha) (2\alpha + 1 - \frac{\mu}{\sigma})}$. $\phi(0) = \frac{1 - \frac{\mu}{\sigma} - \alpha}{1 - \frac{\mu}{\sigma} + 2\alpha}$ and $\phi(1) = \frac{1 - \frac{\mu}{\sigma} - \alpha}{1 - \frac{\mu}{\sigma} + \alpha}$, which we have in (29) and (30) as the critical values for different organization configuration.

Figure 4: Intermediate value of share of multiplant

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17 We set the parameters as $\alpha = 0.2$. The other two parameters, $\sigma$ and $\mu$, are the same as before.
5.2 The proof of Proposition 2

Proof. Solving for the nominal wage differential between single-plant and multiplant equal to one, and without evaluating \( m_1 \), we could obtain the following equation from (27),

\[
G(m_1) \equiv \frac{W_S}{W_M} - 1 = \frac{\Omega_3}{\Omega_4}
\]

where \( \Omega_3 \equiv -\frac{\mu}{\sigma} (1 - \Gamma) (1 - \phi) m_1^2 + (1 - \phi) (1 - \Gamma) \left( \frac{\mu}{\sigma} (1 - \Gamma) + 1 \right) m_1 + 2\phi - \Gamma (1 - \frac{\mu}{\sigma}) - \Gamma \phi (1 + \frac{\mu}{\sigma}) \),

and \( \Omega_4 \equiv \Gamma (-\frac{\mu}{\sigma} (1 - \phi) (1 - \Gamma) m_1^2 + (1 - \phi) \left( \frac{\mu}{\sigma} (2 - \Gamma) \right) m_1 + 1 - \frac{\mu}{\sigma} + \phi (1 + \frac{\mu}{\sigma}) ) \). If \( G(m_1) > 0 \), all firms are single plant and if \( G(m_1) < 0 \), all firms are multiplant. Since \( \Omega_4|_{m_1=0} > 0 \), \( \Omega_4|_{m_1=1} > 0 \) and \( \frac{d}{dm_1} \Omega_4 = 0 \) when \( m_1 = \frac{\frac{\mu}{\sigma} + \Gamma}{2(1 - \Gamma)} + 1 > 1 \), we have \( \Omega_4 > 0 \), \( \forall m_1 \in [0, 1] \). From the equation, it is obvious that for given \( \Gamma \) and \( \phi \), \( G(m_1) \) is continuous. So we could focus on the sign of numerator, \( \Omega_3 \). Solving \( \Omega_3|_{m_1=0} \leq 0 \) for \( \alpha \) yields the equation in (28). If \( \alpha \leq \frac{1}{2} \phi^{b-1} (1 - \phi) \left( 1 - \frac{\mu}{\sigma} \right) \) holds, \( \Omega_3|_{m_1=0} \leq 0 \), otherwise \( \Omega_3|_{m_1=0} > 0 \) and there is no possible multiplant organization. A simulation of \( G(m_1) \) is depicted in Figure 5,\(^{18}\) which has the correspondence with Figure 3 and shows two separated ranges of \( m_1 \in [0, 1] \). Solving \( \Omega_3 \) for \( m_1 \), then we have a solution for \( m_1 \in [0, 1] \) as \( m_1 = \frac{(1 - \phi) \left( \frac{\mu}{\sigma} (1 - \Gamma) + 1 \right) \pm \sqrt{((1 - \phi) \left( \frac{\mu}{\sigma} (1 - \Gamma) + 1 \right))^2 + 4\frac{\mu}{\sigma} (1 - \phi) (2\phi - \Gamma (1 - \frac{\mu}{\sigma}) - \Gamma \phi (1 + \frac{\mu}{\sigma}))}}{2\frac{\mu}{\sigma} (1 - \phi) (1 - \Gamma)} \).\(^{\blacksquare}\)

Figure 5: Intermediate value of share of multiplant

\(^{18}\)We set the parameters as \( \alpha = 0.06 \) and \( b = 2 \). The other two parameters, \( \sigma \) and \( \mu \), are the same as before.