Innovation, Foreign Direct Investment, and Detection of Illegal Imitation *

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February 21, 2011

Abstract

We construct a North-South quality ladder model with endogenous innovation, imitation, and foreign direct investment (FDI). Southern government regulates the imitation and detects the imitating firms by employing Southern labor. In this model, we regard the detection rate as how intellectual property right (IPR) protection is recognized in the South and we investigate how stronger IPR protection affects innovation, imitation and FDI. When IPR protection is sufficiently weak, only Northern firms which were imitated before invest R&D and a stronger IPR protection decreases the level of R&D investment. When IPR protection is sufficiently strong, both Northern firms which were imitated before and Northern firms invest R&D. A stronger IPR protection decreases the rate of imitation and R&D investment conducted by Northern firms which were imitated before. When IPR protection is sufficiently strong, a stronger IPR protection increases the level of R&D investment conducted by Northern firms.

*We would like to express sincere gratitude to Koichi Futagami for his helpful comments. We also thank Hiroki Arato, Masako Ikefuji, Tatsuro Iwaisako, Noritaka Kudo, Daisuke Miyagawa, Yoshiyasu Ono, and Dan Sasaki, Hitoshi Tanaka, and seminar participants at Tokyo Metropolitan University and at the Summer Workshop on Economic Theory in Hokkaido University.

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1 Introduction

Recently, many countries have been affiliated with World Trade Organization (WTO). To become a member of WTO, the affiliated countries have to agree the Trade-Related Aspects of Intellectual Property Rights (TRIPS) which sets down minimum standards for many forms of intellectual property regulation and makes the affiliated countries to enact the strict intellectual property laws. For example, when China became the member of WTO in 2001, China revised its IPR protection. However, many developing countries are afraid to protect the intellectual property right more because they think a stronger IPR protection damages their own economies. Some researchers investigate the relationship between a stronger IPR protection and the welfare level. McCalman (2001) investigates the impact of international patent harmonization as implied by the TRIPS agreement. He concludes that patent harmonization has generate large transfer of income from developing countries to developed countries. Goldberg (2010) investigates the patent enforcement in developing countries in the case of pharmaceuticals in India. He concludes that because prices in developing countries are much lower than in developed countries, the multinational firms enter the market in developing countries with delay or do not enter. Then, because consumers in developing countries can not access to patented drag, the welfare level in the developing countries decreases.

In this paper, we construct a North-South quality ladder model with endogenous innovation, imitation, and foreign direct investment (FDI) to investigate the effects of a stronger IPR protection. In this model, innovation is conducted only in the North and imitation is conducted only in the South. When Northern firms succeed in innovating, they choose whether they produce the manufactured goods in the North or in the South to perform FDI. Southern imitating firms can imitate the quality frontier of Northern firms performing FDI. When Southern firms succeed in imitating, they can produce manufactured goods and FDI firms cannot produce the manufactured goods. In this model, the Southern government detects the illegal imitator in the South by employing Southern labor. When the Southern government detects the Southern imitating firms, Southern imitating firms stop producing the manufactured goods and FDI firms which were imitated before can produce the manufactured goods again. We define a strengthening IPR protection
as equivalent to a higher detection rate. When Northern firms performing FDI are imitated, they choose whether they invest R&D to recover monopoly power from the Southern imitating firms or wait for Southern government to detect the Southern imitating firms. In this model, there are two types of firms conducting R&D investment: Northern firms and Northern firms which were imitated before. This paper concludes that a stronger IPR protection decreases the rates of R&D investment conducted by Northern firms which were imitated before, imitation, and the wage rate in the North. When IPR protection is sufficiently weak, Northern firms do not conduct R&D investment. However, when IPR protection is sufficiently strong, Northern firms conduct R&D investment and a stronger IPR protection increases the rate of R&D investment conducted by Northern firms.


The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 establishes the steady-state equilibrium and provides a comparative steady-state analysis. In Section 4, we study the welfare analysis in the steady-state equilibrium. We conclude in Section 5.

## 2 The model

We develop a dynamic equilibrium model based on Grossman and Helpman (1991, ch.12). There are two countries, North and South, denoted by $N$ and $S$, respec-
tively. In the North (South), there are \( L_N \) (\( L_S \)) labor and the population size in the world is 1. We assume that labor is not mobile between the North and the South. Each individual lives forever and is endowed with one unit of labor services, which is inelastically supplied at each point of time. There are two goods, agricultural good and manufactured goods.

### 2.1 Individuals

Individuals in both countries have identical preferences:

\[
U^i = \int_0^\infty e^{-\rho t} u^i_t \, dt, \quad i \in \{N, S\},
\]

where \( \rho \) is the constant subjective discount rate. The sub-utility function \( u^i_t \) is assumed to take the form

\[
u^i_t = y^i_t + \int_0^1 \log \left[ \sum_k q_k(j)c^i_{k,t}(j) \right] dj,
\]

where \( y^i_t \) is the consumption of an agricultural good in country \( i \), \( q_k(j) = \lambda^k (\lambda > 1) \) is quality level \( k \) of product \( j \) at time \( t \), and \( c^i_{k,t}(j) \) is consumption of manufactured goods \( j \) of quality level \( k \) in country \( i \) at time \( t \). As described below, the process of increasing quality level requires R&D by firms. Individuals maximize their intertemporal utility of (1) under the following budget constraint:

\[
\int_0^\infty e^{-r_t} ds E^i_t \, dt = A^i_t(0) + \int_0^\infty e^{-r_t} ds w^i_t \, dt - \int_0^\infty e^{-r_t} ds T^i_t \, dt,
\]

where \( r_t \) is the market interest rate at time \( t \), \( A^i_t(0) \) is the initial asset holdings in country \( i \), \( w^i_t \) is the wage rate in country \( i \), and \( T^i_t \) denotes the lump-sum tax in country \( i \). The term \( E^i_t \) represents the expenditure at time \( t \) in country \( i \), namely:

\[
E^i_t = y^i_t + \int_0^1 \left[ \sum_k p_{k,t}(j)c^i_{k,t}(j) \right] dj,
\]

where the price of agricultural good is unity because the agricultural good is chosen to be numeraire, and \( p_{k,t}(j) \) is the price of quality level \( k \) of manufactured good \( j \) at time \( t \).

The utility maximization problem of the individual can be solved in two steps. For simplicity, We drop the time index \( t \). In the first step, we solve the following static optimization problem:

\[
\max_{y^i, c^i_k} u^i_t,
\]

4
subject to $E^i = y^i + \int_0^1 \left[ \sum_k p_k(j)c_k^i(j) \right] dj$.  \hspace{1cm} (6)

From the first order conditions, individuals consume the manufactured goods at each instant to the quality level $\tilde{k}$ offering the lowest quality-adjusted price, $\frac{p_k(j)}{q_k(j)}$. Then, the demand function of manufactured good in each country becomes

$$c_{\tilde{k}}(j) = c_N^i(j) = c_S^i(j) = \frac{1}{p_{\tilde{k}}(j)}.$$  \hspace{1cm} (7)

Substituting (7) into (6), we obtain the demand of agricultural good as follows:

$$y^i = E^i - 1.$$  \hspace{1cm} (8)

Then, substituting (7) and (8) into utility function, we can obtain the indirect utility function as follows:

$$u^i = E^i - 1 + \int_0^1 \log \frac{q_k(j)}{p_k(j)} dj.$$ \hspace{1cm} (9)

The second step is to solve the intertemporal optimization problem. As is clear from the indirect utility function (9), the marginal utility of expenditures is constant. The market interest rate at time $t$, $r(t)$, must be equal to the subjective discount rate, as follows:

$$r(t) = \rho \quad \text{for all } t.$$  \hspace{1cm} (10)

2.2 Agricultural good Sector

Agricultural good can be produced in the both countries. The production of one unit of the agricultural good requires one unit of labor. The market of agricultural good is perfectly competitive in the world. We assume that there are no transportation cost and tariff. Therefore, the wage rate in the country where the agricultural good is produced becomes unity. \footnote{We will show that in the steady state, the agricultural good is not produced in the North and the agricultural good is produced only in the South because the wage rate in the North is larger than unity, that is $w_N > w_S = 1$ in the steady state.}

2.3 Manufactured goods Sector

Manufactured goods can be produced in either country. The manufactured good firms in the North participate in innovation races to discover the technology of
production by using Northern labor. We assume that R&D investment is uncertain activity. When a Northern firm succeeds in R&D investment, the firm can earn the monopolistic profit and can choose whether they produce the manufactured goods in the North or in the South to perform FDI. Hereafter, we name a manufactured good firm producing in the North Firm N and producing in the South to perform FDI Firm F. We index Firm N, N and index Firm F, F. Firms are separated into two types, leaders and followers. Leaders are firms with the ability to produce goods at the state-of-the-art products, whereas all other firms are followers. For simplicity, we assume that once a higher quality product is invented by Northern firms, the technology of one step behind product can be produced both in the North and in the South freely. This means that, in the equilibrium, the nearest rivals of the leaders are Southern firms with the ability to produce goods of the second-highest-quality. In addition, we assume that a Southern firm cannot invent the state-of-the-art products and imitates the quality frontier of Firm F only by using Southern labor. Imitation is also uncertain activity. When Southern firms succeed in imitating, they can earn the monopolistic profit. However, when Southern firms are detected by the Southern government, they lose their monopolistic profit. We name the Southern firm which succeeds in the imitative activity, Firm S. We index Firm S.

The production of one unit of the manufactured goods by Firm N (S) requires one unit of Northern (Southern) labor. The production of one unit of the manufactured goods by Firm F requires \( \eta > 1 \) units of Southern labor.\(^2\)

Firm S competes with Firm F and charges the limit price \( p^S = \eta w_s \). Then, Firm S clears out the quality frontier of Firm F out of the market. Firm N and Firm F compete with firms producing the one step behind product in the South which price is \( w_s \). Therefore, Firm N and Firm F charge the limit price \( p^N = p^F = \lambda w_s \).

\(^2\)There are two explanations. First, when Southern firms imitate the quality frontier of FDI firms, Southern firms not only imitate but also improve the efficiency to produce the manufactured goods. Second, when FDI firms produce the manufactured goods in the foreign countries to hire foreigners, they have to incur the monitoring cost and communication cost. Therefore, the cost of FDI becomes more expensive than the Southern firms.
Then, using demand function of (7), the profits of Firm N, Firm F, and Firm S are:

\[ \pi^N = 1 - \frac{w_N}{\lambda}, \]  \hspace{1cm} (11)  
\[ \pi^F = 1 - \frac{\eta}{\lambda}, \]  \hspace{1cm} (12)  
\[ \pi^S = 1 - \frac{1}{\eta}. \]  \hspace{1cm} (13)  

We assume that \( \lambda > \eta \) because we focus on the case that the profit of Firm F is positive.

### 2.4 Innovation and Imitation

Next, we consider the free-entry conditions of innovation and imitation. Northern followers target their innovation at all range. Let \( v^N \) and \( v^F \) denote the stock market value of Firm N and Firm F, respectively. At a cost of \( w_N a_{fo} \int^o dt \), a Northern follower can purchase a lottery ticket that attains the value of \( v^X \), \( X = NorF \), with probability \( \int^o dt \). Then, we can obtain the following free-entry condition:

\[ a_{fo} w_N \geq v^X, \quad \text{with equality whenever } \int^o > 0. \]  \hspace{1cm} (14)  

Northern leaders undertake no research. When Northern leaders conduct R&D investment, the technology of one step behind product can be product freely. Then, the gains of Northern leaders succeeding in the R&D investment is zero. However, Northern imitated leader may undertake R&D investment. The value of imitated Firm F whose state-of-the-art products have been imitated by Southern firms does not become zero and that value is denoted \( v^F_I \). Imitated Firm F targets his innovation at Southern firms. When imitated Firm F invests \( a_L \int^L dt \) Northern labor for a time interval of length \( dt \), he or she will succeed in developing the next generation product with probability \( \int^L dt \). R&D cost of Northern leader becomes \( w_N a_L \int^L dt \).

We assume that the R&D productivity of leaders is larger than the R&D productivity of followers, that is \( a_L < a_{fo} \). Northern leaders can obtain an expected gain of \( v^X - v^F_I \). Then, we can obtain following free-entry condition:

\[ a_L w_N \geq v^X - v^F_I, \quad \text{with equality whenever } \int^L > 0. \]  \hspace{1cm} (15)  

If (14) and (15) does not hold, either \( \int^o \) or \( \int^L \) becomes infinity. Then, to satisfy the labor market equilibrium condition in the North, these two equations must hold. In
this paper, we focus on the equilibrium which both Northern firms and FDI firms exist. Then, the values of Northern firms are equals to the values of FDI firms, that is,

$$v^N = v^F.$$ \hfill (16)

Southern firms target their innovation at Firm F. Southern firms which invest $a_S\xi_S dt$ units of Southern labor for a time interval of length $dt$ will succeed in developing a copy of the targeted product with productivity $\xi^S dt$. $v^S$ denotes the value of Firm S. When Southern firms succeed in imitating, Southern firms can obtain an expected gain of $v^S$ at cost $a_S\xi^S dt$. Then, we can obtain the following free-entry condition:

$$a_S \geq v^S, \quad \text{with equality whenever } \xi^S > 0.$$ \hfill (17)

When (17) does not hold, $\xi^S$ becomes infinity. Then, to satisfy the labor market equilibrium condition in the South, this equation must hold.

Next, we consider the detection activities conducted by Southern government. Southern government which hires $a_\epsilon dt$ units of Southern labor for a time interval of length $dt$ will succeed in detecting Southern firms with productivity $\epsilon dt$. In this model, the value of $\epsilon$ is a degree of IPR protection and an increase in $\epsilon$ means a stronger IPR protection.

We consider the capital market equilibrium conditions. There exists a stock market that channels individual saving to firms. First, we consider the valuation of Firm N $v^N$. Shareholders of Firm N earn dividends $\pi_N dt$ and capital gains $\dot{v}^N dt$ over a time interval of length $dt$ if no followers succeed in innovating a new state-of-the-art product in the industry. However, the stock value of Firm N becomes zero if followers succeed in innovating a new state-of-the-art product during the interval $dt$. The probability that the stock value of Firm N becomes zero is equal to the probability that innovation succeeds in the industry during $dt$, $\omega^o dt$. Provided that each shareholders can completely diversify away risk by holding a diversified portfolio of stocks, the return from holding the stock of Firm N must be the same as the risk-free interest rate, $r(t)$. The no-arbitrage condition between the stock of Firm N and a riskless asset is given by

$$rv^N = \pi^N + \dot{v}^N - \omega^o v^N.$$ \hfill (18)
Then, we next consider a valuation of Firm S $v^S$. The shareholders of Firm S earn dividends $\pi_S dt$ and capital gains $\dot{v}^S dt$ over a time interval of length $dt$ if no innovation occurs in the industry and Firm S is not detected by the Southern government, while suffering a total capital loss of amount $v^S$ with probability of $(\epsilon + \iota^o + \iota^L) dt$. Therefore, we can obtain the no-arbitrage condition between the stock of Firm S and a riskless asset as follows:

$$rv^S = \pi^S + \dot{v}^S - (\iota^o + \iota^L + \epsilon)v^S. \quad (19)$$

We consider a valuation of Firm F $v^F$. The shareholders of Firm F earn dividends $\pi_F dt$ and capital gains $\dot{v}^F dt$ during $dt$ if no innovation occurs in the industry, while suffering a total capital loss of amount $v^F$ with probability of $\iota^o dt$. In addition, the stock value changes $v^F_I$ if the Southern firm succeeds in imitating during $dt$, with probability of $\iota^S dt$. The sum of risky returns becomes equal to the riskless asset. Therefore, we can obtain the no-arbitrage condition between the stock of Firm F and a riskless asset as follows:

$$rv^F = \pi^F + \dot{v}^F - \iota^o v^F - \iota^S (v^F - v^F_I). \quad (20)$$

At last, we investigate a valuation of an imitated Northern leader $v^F_I$. The shareholders of imitated leader pay capital loss $-w_N a_L \iota^L dt$ and earn capital gains $\dot{v}^F_I dt$ if no innovation occurs in the industry, while suffering a total capital loss of amount $v^F_I$ with the probability of $\iota^o dt$. In addition, the stock value changes $v^F$ if the imitated leaders succeed in innovating and the Southern government detects Firm S during $dt$, with probability of $(\epsilon + \iota^L) dt$. The sum of risky returns becomes equal to the riskless asset. Therefore, using (15), we can obtain the no-arbitrage condition as follows:

$$rv^F_I = -w_N a_L \iota^L + \dot{v}^F_I - \iota^o v^F_I + (\epsilon + \iota^L)(v^F - v^F_I),$$

$$= \dot{v}^F_I - \iota^o v^F_I + \epsilon(v^F - v^F_I). \quad (21)$$

2.5 Labor Markets and Agricultural good Market

Labor market equilibrium condition requires labor demand be equal to labor supply for each country. First, we consider labor market in the North. The demand for labor in the North comes from the production by Firm N and R&D investment
conducted by leaders and followers. Then, the labor market equilibrium condition in the North becomes as follows:

\[ n^N x^N + a_{fo} l^o + n^S a_{LL} L = L_N. \quad (22) \]

In the South, the demand for labor comes from agricultural good, production by Firm F and by Firm S, R&D investment enforced by Southern imitating firms, and detection activities conducted by Southern government. Then, the labor market equilibrium condition in the South is given by

\[ n^F x^F + n^S x^S + n^F a_{S} l^S + n^S a_\epsilon L_A^S = L_S, \quad (23) \]

where \( L_A^S \) denotes the labor demand for agricultural good.

Next, we examine the agricultural good market equilibrium condition. The supply of agricultural good is \( L_A^N \). The demand of agricultural good is derived from utility maximization problem of (8). Therefore, the agricultural good market equilibrium condition becomes as follows:

\[ L_A^S = L_N(E^N - 1) + L_S(E^S - 1) \equiv E - 1, \quad (24) \]

where \( E \equiv L_N E^N + L_S E^S \) denotes the total expenditure in the world.

### 3 Steady-State Equilibrium

In the steady-state, the numbers of Firm N, Firm F and Firm S are constant. The valuations of Firm N, Firm F, Firm S, and imitated leader are also constant. Then, we can obtain the following equation:

\[ \dot{n}^N = \dot{n}^F = \dot{n}^S = \dot{v}^N = \dot{v}^F = \dot{v}^S = \dot{v}_I^S = 0. \quad (25) \]

Southern firms capture the production from Firm F at a rate of \( v^S n^F \), whereas Southern government detects Firm S at a rate of \( \epsilon n^S \) and Northern leaders and followers capture the production from Firm S at a rate of \( l^L n^S \) and \( l^{fo} n^S \). In the steady-state, because the number of Firm S is constant, we can obtain the following equation:

\[ (\epsilon + l^{fo} + l^L) n^S = v^S n^F. \]
Then, \( n^* \) is given by

\[
n^S = \frac{\epsilon + \nu^o + \nu^L n^F}{\epsilon + \nu^o + \nu^L}.
\]  

(26)

From (10), (18), (19), and (25), we can obtain valuations of Firm N and of Firm S as follows:

\[
v^N = \frac{\pi^N}{\rho + \nu^o}, \quad v^S = \frac{\pi^S}{\rho + \epsilon + \nu^o + \nu^L}.
\]  

(27)  

(28)

From (10), (20), (21) and (25), we can obtain the following equations:

\[
v^F = \frac{\pi^F}{\rho + \nu^o + \nu^S + \nu^S v^F}, \quad v^F_I = \frac{\epsilon}{\rho + \epsilon + \nu^o v^F}.
\]  

(29)  

(30)

Then, from (29) and (30), valuations of Firm F and of imitated leaders are given by

\[
v^F = \frac{\rho + \nu^o + \epsilon}{(\rho + \nu^o)(\rho + \nu^o + \nu^S + \epsilon)} \pi^F, \quad v^F_I = \frac{\epsilon}{(\rho + \nu^o)(\rho + \nu^o + \nu^S + \epsilon)} \pi^F.
\]  

(31)  

(32)

Substituting (11), (12), (27) and (31) into (16), We can obtain the following wage rate in the North:

\[
w^N = \frac{\lambda \nu^S + \eta (\rho + \nu^o + \epsilon)}{\rho + \nu^o + \nu^S + \epsilon} > \frac{\eta (\rho + \nu^o + \nu^S + \epsilon)}{\rho + \nu^o + \nu^S + \epsilon} > 1,
\]  

(33)

where these inequalities come from \( \lambda > \eta > 1 \). Because the wage rate in the North is larger than unity, the agricultural good is not produced in the North and the agricultural good is produced only in the South. Then, the wage rate in the South becomes unity. Substituting (13), (31), (32), (33) into (14), (15), and (17), we can rewrite the free-entry conditions as follows:

\[
\frac{\rho + \nu^o + \epsilon}{(\rho + \nu^o)(\lambda \nu^S + \eta (\rho + \nu^o + \epsilon))} \pi^F \leq 1, \quad \text{with equality for } \nu^o \geq 0,
\]  

(34)

\[
\frac{\rho + \nu^o}{(\rho + \nu^o)(\lambda \nu^S + \eta (\rho + \nu^o + \epsilon))} \pi^F \leq 1, \quad \text{with equality for } \nu^L \geq 0.
\]  

(35)

\[
\frac{\eta - 1}{a_S \eta (\rho + \epsilon + \nu^o + \nu^L)} \leq 1, \quad \text{with equality for } \nu^S \geq 0.
\]  

(36)
In the steady-state equilibrium, (34), (35), and (36) hold. When the left hand side of these equations are smaller than the right hand side, firms do not conduct R&D investment and imitation. From (34), (35), and (36), we can obtain the following proposition (See the Appendix for proof).

**PROPOSITION 1.** 1. When \( \hat{\epsilon}_1 \leq \hat{\epsilon}_1 \equiv \frac{a}{a_L} - \rho \), Northern followers do not conduct R&D investment and Northern firms which were imitated before only conduct R&D investment.

2. When \( \hat{\epsilon}_1 > \hat{\epsilon}_1 \) and \( \epsilon_f = \bar{\epsilon}_f \equiv \frac{a}{a_L} - \rho \) hold, both Northern followers and Northern firms which were imitated before conduct R&D investment.

3. When \( \hat{\epsilon}_1 > \hat{\epsilon}_1 \) and \( \epsilon_f < \bar{\epsilon}_f \) hold, Northern followers conduct R&D investment and Northern firms which were imitated before do not conduct R&D investment.

We explain Proposition 1 intuitively. When the detection rate is sufficiently small, Northern followers and imitated leaders become less profitable. Then, because the productivity of R&D conducted by Northern imitated leaders are larger than the productivity of R&D conducted by Northern followers, R&D investment conducted by Northern imitated leaders becomes profitable. Individuals invest only to the Northern imitated leader and only Northern imitated leader conducts R&D investment. On the other hand, when the detection rate is sufficiently large, the incentives of Northern followers to conducting R&D investment increases and Northern followers start to invest R&D. For Northern imitated leaders, the probability that they can earn monopolistic profit again becomes larger because Southern government detects Firm S frequently. When Northern followers invest small amount of R&D investment, the probability of losing patent rights that Northern imitated leaders hold becomes small and the imitated leaders does not invest. Then, the Northern firms which were imitated before must succeed in innovating before Northern followers succeed in R&D investment. Therefore, when Northern followers invest small amount of R&D investment, the probability of losing patent rights that Northern imitated leaders hold becomes small. Then, Northern imitated leaders wait until Southern government detects the Southern firms. On the other hand, when Northern followers invest large amount of R&D investment, the prob-
ability of losing patent rights that Northern imitated leaders hold become larger
and Northern imitated leaders invest R&D investment. Next, we show that there
exists a unique equilibrium (See the Appendix for proof).

**PROPOSITION 2.** Suppose that $\frac{\eta - 1}{\alpha S \eta} > \frac{\lambda - \eta}{\alpha L \lambda \eta}$ and $a_{fo} < \frac{\lambda - \eta}{\lambda \eta}$ hold. When $\epsilon \leq \hat{\epsilon}_1$, there exists a unique equilibrium of R&D investment and imitation.

$$
\begin{align*}
\iota^{fo} &= 0, \\
\iota^L &= \frac{\eta - 1}{\alpha S \eta} - (\rho + \epsilon), \\
\iota^S &= \frac{\lambda - \eta}{\alpha L \lambda^2} - \frac{\eta}{\lambda}(\rho + \epsilon).
\end{align*}
$$

When $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2 \equiv \frac{\lambda - \eta}{\alpha L \lambda} \cdot \alpha_{fo} - a_L$, there exists a unique equilibrium of R&D investment and imitation.

$$
\begin{align*}
\iota^{fo} &= \iota^{fo} = \frac{a_L}{a_{fo} - a_L} \epsilon - \rho, \\
\iota^L &= \frac{\eta - 1}{\alpha S \eta} - \frac{a_{fo}}{a_{fo} - a_L} \epsilon, \\
\iota^S &= \frac{\lambda - \eta}{\alpha L \lambda^2} - \frac{\eta}{\lambda} \frac{a_{fo}}{a_{fo} - a_L} \epsilon.
\end{align*}
$$

Next, we derive the numbers of Firm N, Firm F, and Firm S. Substituting (26) into $n^N + n^F + n^S = 1$, the number of Northern firms is given by

$$
n^N = 1 - \frac{\epsilon + \iota^L + \iota^{fo} + \iota^S}{\iota^S} n^S.
$$

Substituting (43) into (22), we can obtain the number of Firm S as follows:

$$
n^S = \frac{\iota^S (1 + a_{fo} \lambda^{f^o} - \lambda L N)}{\epsilon + \iota^L + \iota^{fo} + \iota^S - a_L \lambda \iota^L \iota^S},
$$

where we assume that $\lambda L N < 1$. This assumption precludes that all manufactured goods can not be produced in the North. Then, we analyze the comparative statics with respect to IPR protection.

**PROPOSITION 3.** When $\epsilon < \hat{\epsilon}_1$, a stronger IPR protection decreases the rate of innovation conducted by imitated Northern leaders, the rate of imitation, the number of Firm S, the wage rate in the North, total volume of R&D investment, total volume of imitation, and increases the number of Firm N. On the other hand, when
\( \hat{\varepsilon}_1 < \varepsilon < \hat{\varepsilon}_2 \), a stronger IPR protection decreases the rate of innovation conducted by imitated Northern firms, the rate of imitation, and the wage rate in the North, and increases the innovation rate conducted by Northern followers.

We explain the above proposition intuitively. When \( \varepsilon \leq \hat{\varepsilon}_1 \) holds and the IPR protection is strengthened, the probability that Northern imitated leaders can earn monopolistic profit again increases. Then, the incentive that a Northern imitated leaders conducts R&D investment decreases. Therefore, a Northern imitated leader decreases R&D investment. On the other hand, when the IPR protection is strengthened, the probability that Southern government detects Firm S increases. Then, the valuation of a Southern firm decreases and Southern firms decreases their investment for imitation. When \( \hat{\varepsilon}_1 < \varepsilon < \hat{\varepsilon}_2 \) and IPR protection is strengthened, the valuation of a Northern follower increases, the valuation of a Northern imitated leader decreases, and the valuation of Southern firms decreases. Therefore, Northern followers increase their R&D investment, Northern imitated leaders decrease their R&D investment, and Southern firms decrease their investment for imitation.

In the previous researches, higher levels of \( a_S \) is viewed as representing a strengthening of IPR protection. Then, we can obtain the following proposition to analyze the comparative statics with respect to \( a_S \).

**PROPOSITION 4.** When \( \varepsilon \leq \hat{\varepsilon}_1 \), higher level of \( a_S \) decreases innovation rate conducted by a Northern imitated firm, total amount of R&D investment, and total amount of imitation but does not affect imitation level conducted by a Southern firm. An increase in \( a_S \) increases the number of Northern firms and Southern firms but decrease the number of FDI firms. There is no relationship between \( a_S \) and the wage rate in the North. On the other hand, \( \hat{\varepsilon}_1 < \varepsilon < \hat{\varepsilon}_2 \), higher levels of \( a_S \) decreases innovation rate conducted by Northern firms which were imitated before, total level of R&D investment, and total level of imitation but does not affect the imitation level conducted by a Southern firm and the innovation rate conducted by a Northern follower. An increase in \( a_S \) increases the number of Southern firms and Northern firms and decreases the number of FDI firms. There is no relationship between \( a_S \) and the wage rate in the North.

In this model, a higher level of \( a_S \) decreases total volumes of R&D investment and imitation. This result is same as Glass and Saggi (2003).
4 Welfare Analysis

In the steady state, we analyze the welfare level in the North and in the South. We focus on the case that the detection cost is borne by the Southern individual only, that is $T_{i}^{S} = 0$. We assume that the Southern government runs a balanced budget in which it finances its total outlay. Thus, the Southern government’s budget constraint is

$$a_{i} \epsilon = T^{S} L_{S}. \quad (45)$$

In the steady state, the budget constraint of individual in the country $i$ is

$$E_{i} = \rho A^{N}(0) + w_{i}^{t} - T_{i}^{S}. \quad (46)$$

We assume that Northern individuals hold the stock in a ratio of $\xi \quad (0 \leq \xi \leq 1)$ among the sum of asset holdings and Southern individuals hold the stock in a ratio of $(1 - \xi)$. Then, the asset holdings of Northern individuals are given by

$$A^{N}(0) = \frac{\xi}{L_{N}} (v^{N} n^{N} + v^{F} n^{F} + v^{S} n^{S}). \quad (47)$$

Substituting (47) into (46), we can obtain the expenditure of Northern individuals as follows:

$$E^{N} = \frac{\rho \xi}{L_{N}} (v^{N} n^{N} + v^{F} n^{F} + v^{S} n^{S}) + w_{i}^{N}. \quad (48)$$

In the same way, the expenditure of Southern individuals is given by

$$E^{S} = \frac{\rho (1 - \xi)}{L_{S}} (v^{N} n^{N} + v^{F} n^{F} + v^{S} n^{S}) + 1 - T_{i}^{S}. \quad (49)$$

Then, substituting (48) and (49) into (9), the instantaneous utility levels of individuals in both countries are given by

$$u_{i}^{N} = w_{i}^{N} + \frac{\rho \xi}{L_{N}} (v^{N} n^{N} + v^{F} n^{F} + v^{S} n^{S}) - 1 + \int_{0}^{1} \log q_{k}(j) dj - (n^{N} + n^{F}) \log \lambda, \quad (50)$$

$$u_{i}^{S} = \frac{\rho (1 - \xi)}{L_{S}} (v^{N} n^{N} + v^{F} n^{F} + v^{S} n^{S}) + \int_{0}^{1} \log q_{k}(j) dj - (n^{N} + n^{F}) \log \lambda - T_{i}^{S}. \quad (51)$$

where $k$ is the highest quality of manufactured good $j$. Let $f(m, t)$ denote the probability that a given product will take $m$ steps up the quality ladder in a time interval of length $t$. Then, because each manufactured good follows the same Poisson process of technological innovation, when $\epsilon \leq \epsilon_{1}, f(m, t)$ becomes as follows:

$$f(m, t) = \frac{(n^{S} L_{t})^{m} e^{-n^{S}_{i} \epsilon_{1} t}}{m!}. \quad (52)$$
On the other hand, when \( \hat{\epsilon}_1 \leq \epsilon \leq \hat{\epsilon}_2 \), \( f(m, t) \) becomes as follows:

\[
f(m, t) = \frac{(n^S t L)^m e^{-n^S t L}}{m!} + \frac{(t^{f_{0}})^m e^{-t^{f_{0}}}}{m!}.
\]

(53)
The second terms of (50) or (51) can be rewritten as follows:

\[
\int_0^1 \log q_k(j) dj = \int_0^1 \log \lambda^k(j) dj = \sum_{m=0}^{\infty} f(m, t) \log \lambda^m.
\]

Therefore, when \( \epsilon \leq \hat{\epsilon}_1 \), the second terms of (50) or (51) are given by

\[
\int_0^1 \log q_k(j) dj = n^S t L \log \lambda.
\]

(54)
On the other hand, when \( \hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2 \), the second terms of (50) or (51) are given by

\[
\int_0^1 \log q_k(j) dj = (n^S t L + t^{f_{0}}) \log \lambda.
\]

(55)
Therefore, \( \epsilon \leq \hat{\epsilon}_1 \), the welfare levels in the both countries become as follows:

\[
U^N = \frac{1}{\rho} \left[ w_N + \frac{\rho \xi}{L_N} (v^N n^N + v^F n^F + v^S n^S) - 1 + \frac{m}{\rho} \log \lambda \right], \quad (56)
\]

\[
U^S = \frac{1}{\rho} \left[ \frac{\rho (1 - \xi)}{L_S} (v^N n^N + v^F n^F + v^S n^S) + \frac{m}{\rho} \log \lambda - T^S \right]. \quad (57)
\]
Differentiating the Northern welfare with respect to \( \epsilon \), we can obtain the following equation:

\[
\rho \frac{\partial U^N}{\partial \epsilon} = \frac{\partial w_N}{\partial \epsilon} + \frac{\rho \xi}{L_N} \frac{\partial (v^N n^N + v^F n^F + v^S n^S)}{\partial \epsilon} - \frac{\partial (n^N + n^F)}{\partial \epsilon} \log \lambda + \frac{\partial (n^S t L)}{\partial \epsilon} \log \lambda \frac{1}{\rho}.
\]

(58)
When IPR protection is strengthened, there are four effects on the Northern welfare. The first term is the negative effect that a stronger IPR protection decreases the wage rate in the North. Second term is the effect that a stronger IPR protection changes the value of the asset holdings. The third term is negative effect that a decrease in the number of Firm S increases the average price level. The last term is also negative effect that a decrease in the total volume of R&D investment decreases the growth rate. Differentiating the Southern welfare with respect to \( \epsilon \) when \( \epsilon \leq \hat{\epsilon}_1 \), we can obtain the following equation:

\[
\rho \frac{\partial U^S}{\partial \epsilon} = \frac{\rho (1 - \xi)}{L_S} \frac{\partial (v^N n^N + v^F n^F + v^S n^S)}{\partial \epsilon} - \frac{\partial (n^N + n^F)}{\partial \epsilon} \log \lambda + \frac{\partial (n^S t L)}{\partial \epsilon} \log \lambda \frac{1}{\rho} - \frac{\partial T^S}{\partial \epsilon}.
\]

(59)
When IPR protection is strengthened, there are four effects on the Southern welfare. The first term is the effect that a stronger IPR protection changes the value of the asset holdings. The second term is negative effect that a decrease in the number of Firm S increases the average price level. The third term is also negative effect that a decrease in the total volume of R&D investment decreases the growth rate. The last term is the negative effect that a stronger IPR protection has to employ more Southern labor and the expenditure of Southern government increases.

On the other hand, when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$, the welfare levels in the both countries become as follows:

$$U^N = \frac{1}{\rho} \left[ w_N + \frac{\rho \xi}{L_N} (v^N n^N + v^F n^F + v^S n^S) - 1 + \left( \frac{n^S \eta_L + \eta_f}{\rho} - n^N - n^F \right) \log \lambda \right],$$

$$U^S = \frac{1}{\rho} \left[ \frac{(1-\xi)}{L_S} (v^N n^N + v^F n^F + v^S n^S) + \left( \frac{n^S \eta_L + \eta_f}{\rho} - n^N - n^F \right) \log \lambda - T^S \right].$$

Differentiating the Northern welfare with respect to $\epsilon$, we can obtain the following equation:

$$\frac{\partial U^N}{\partial \epsilon} = \frac{\partial w_N}{\partial \epsilon} + \frac{\rho \xi}{L_N} \frac{\partial (v^N n^N + v^F n^F + v^S n^S)}{\partial \epsilon} - \frac{\partial (n^N + n^F)}{\partial \epsilon} \log \lambda + \frac{\partial (n^S \eta_L + \eta_f)}{\partial \epsilon} \log \lambda - T^S.$$

The effects of stronger IPR protection on the welfare level when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$ are similar to the case of $\epsilon \leq \hat{\epsilon}_1$. However, the last term is different from the case of $\epsilon \leq \hat{\epsilon}_1$ and is ambiguous effect because the relationship between a stronger IPR protection and total volumes of R&D investment is ambiguous. Then, because the effect of IPR protection on the welfare level is complicated, we present a numerical example in Figure 3. The parameters used to construct Figure 3 are the quality increment $\lambda = 1.7$, subjective discount rate $\rho = 0.05$, resource requirement in innovation conducted by leaders $a_L = 1.5$, resource requirement in innovation conducted by followers $a_f = 2$, resource requirement in imitation $a_S = 1$, resource requirement in detection $a_c = 0.1$, the ratio of stock holding by Northern individual $\xi = 0.5$, Northern labor supply $L_N = 0.5$, and Southern labor supply $L_S = 0.5$. The parameter of the productivity of Firm F is $\eta = 1.3$. Then, from Figure 3, a stronger IPR protection decreases the level of Northern welfare when $\epsilon \leq \hat{\epsilon}_1$. However, when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$, a stronger IPR protection increases the level of Northern welfare. On the other hand, a stronger IPR protection increases the level of Southern welfare.
5 Conclusion

In this paper, we construct a North-South quality ladder model with endogenous innovation, imitation, and FDI to investigate the effects of a stronger IPR protection. In this model, the Southern government detects the illegal imitator in the South. Many previous studies define strengthening of IPR protection as equivalent to an exogenous increase in the cost of imitation. This paper concludes that a stronger IPR protection decreases the rates of R&D investment conducted by Northern firms which were imitated before, imitation, and the wage rate in the North. When IPR protection is sufficiently weak, Northern firms do not conduct R&D investment. However, when IPR protection is sufficiently strong, Northern firms conduct R&D investment and a stronger IPR protection increases the rate of R&D investment conducted by Northern firms.

A Appendix

A.1 Proof for Proposition 1

At first, we prove when \( \tilde{\varepsilon} \leq \hat{\varepsilon}_1 \). When \( \tilde{\varepsilon} \leq \hat{\varepsilon}_1 \), we compare the left hand side of (34) with that of (35). Then, we can obtain the following equation:

\[
\frac{\rho + \nu^o}{(\rho + \nu^o)(\lambda L + \eta(\rho + \nu^o + \varepsilon))} a_L \pi_F - \frac{\rho + \nu^o + \varepsilon}{(\rho + \nu^o)(\lambda L + \eta(\rho + \nu^o + \varepsilon))} a_f o \pi_F
\]

\[
= \frac{\rho + \nu^o}{\lambda L + \eta(\rho + \nu^o + \varepsilon)} \left( \frac{\rho + \nu^o}{a_L} - \frac{\rho + \nu^o + \varepsilon}{a_f o} \right). \tag{A.1}
\]

Then, we can rewrite the parenthesis of (A.1) as follows:

\[
\frac{\rho + \nu^o}{a_L} - \frac{\rho + \nu^o + \varepsilon}{a_f o} = \frac{(\rho + \nu^o)(a_f o - a_L)}{a_L a_f o} - \frac{\varepsilon}{a_f o}
\]

\[
\geq \frac{(\rho + \nu^o)(a_f o - a_L)}{a_L a_f o} - \frac{(a_f o - a_L) \rho}{a_f o a_L}
\]

\[
= \frac{(a_f o - a_L) \nu^o}{a_f o a_L} \geq 0, \tag{A.2}
\]

where the inequality comes from \( \varepsilon \leq \hat{\varepsilon}_1 \). When \( \nu^o > 0 \), (34) holds with equality and the following inequality holds:

\[
\frac{\rho + \nu^o}{(\rho + \nu^o)(\lambda L + \eta(\rho + \nu^o + \varepsilon))} a_L \pi_F \geq \frac{\rho + \nu^o + \varepsilon}{(\rho + \nu^o)(\lambda L + \eta(\rho + \nu^o + \varepsilon))} a_f o \pi_F = 1. \tag{A.3}
\]
However, the inequality of (A.3) contradicts (35). Therefore, \( \iota^{fo} = 0 \) must hold when \( \epsilon \leq \hat{\epsilon}_1 \).

Secondly, we prove when \( \hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2 \) and \( \iota^{fo} = \hat{\iota}^{fo} \). In the same way, we compare the left hand side of (34) with that of (35). From the parenthesis of (A.1), we can obtain the following equation:

\[
\frac{\rho + \hat{\iota}^{fo}}{a_L} - \frac{\rho + \hat{\iota}^{fo} + \epsilon}{a_fo} = \frac{\rho + \hat{\iota}^{fo} - \rho - \hat{\iota}^{fo}}{a_L a_fo} - \frac{\epsilon}{a_fo} = 0.
\]  

(A.4)

From \( \iota^{fo} = \hat{\iota}^{fo} > 0 \), (34) holds with equality. Therefore, the following equation holds:

\[
\frac{\rho + \iota^{fo}}{(\rho + \iota^{fo})(\lambda \iota^S + \eta(\rho + \iota^{fo} + \epsilon)) a_L} = \frac{\rho + \iota^{fo} + \epsilon}{(\rho + \iota^{fo})(\lambda \iota^S + \eta(\rho + \iota^{fo} + \epsilon)) a_fo} = 1.
\]  

(A.5)

Therefore, from (35), \( \iota^L \geq 0 \) holds.

At last, we focus on the case when \( \epsilon > \hat{\epsilon}_1 \) and \( \iota^{fo} < \hat{\iota}^{fo} \). In the same way, we compare the left hand side of (34) with that of (35). From the parenthesis of (A.1), we can obtain the following equation:

\[
\frac{\rho + \iota^{fo}}{a_L} - \frac{\rho + \iota^{fo} + \epsilon}{a_fo} = \frac{\rho + \iota^{fo} - \rho - \iota^{fo}}{a_L a_fo} - \frac{\epsilon}{a_fo} \leq \frac{\rho + \hat{\iota}^{fo}}{a_L} - \frac{\rho + \hat{\iota}^{fo} + \epsilon}{a_fo} = 0.
\]  

(A.6)

We suppose that \( \iota^L > 0 \). Then, (35) holds with equality. Therefore, we can obtain the following equation:

\[
1 = \frac{\rho + \iota^{fo}}{(\rho + \iota^{fo})(\lambda \iota^S + \eta(\rho + \iota^{fo} + \epsilon)) a_L} < \frac{\rho + \iota^{fo} + \epsilon}{(\rho + \iota^{fo})(\lambda \iota^S + \eta(\rho + \iota^{fo} + \epsilon)) a_fo}.
\]  

(A.7)

However, this inequality contradicts (34). Then, \( \iota^L = 0 \) holds.

### A.2 Proof for Proposition 2

To prove the case when \( \epsilon \leq \hat{\epsilon}_1 \), we separate two stages. We derive the reaction function of \( \iota^L \) for \( \iota^S \) at the first stage and the reaction function of \( \iota^S \) for \( \iota^L \) at the second stage. At first stage, from Proposition 1 and \( \epsilon \leq \hat{\epsilon}_1 \), \( \iota^{fo} = 0 \) and \( \iota^L \geq 0 \)
hold. Then, (35) can be rewritten as follows:
\[
i^S \geq \frac{\lambda - \eta}{a_S \lambda^2} - \frac{\eta}{\lambda} (\rho + \epsilon).
\] (A.8)

When (A.8) holds with equality, \(i^L \geq 0\). On the other hand, when (A.8) holds with inequality, \(i^L = 0\). The right hand side of (A.8) has a positive value when \(\epsilon \leq \hat{\epsilon}_1\). \(^3\) (A.8) means the reaction function of \(i^L\) for \(i^S\).

At second stage, we derive the reaction function of \(i^S\) for \(i^L\). From (36) and \(i^{fo} = 0\), we can derive the reaction function of \(i^S\) for \(i^L\).

\[
i^L \geq \eta - \frac{1}{a_S \eta} - (\rho + \epsilon).
\] (A.9)

When (A.9) holds with equality, \(i^S \geq 0\). On the other hand, when (A.9) holds with inequality, \(i^S = 0\). The right hand side of (A.9) has a positive value. \(^4\) Therefore, we can depict (A.8) and (A.9) in Figure 1. There exists a unique equilibrium and the values of R&D investment and imitation as follows:

\[
i^L = \eta - \frac{1}{a_S \eta} - (\rho + \epsilon),
\] (A.10)

\[
i^S = \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta}{\lambda} (\rho + \epsilon).
\] (A.11)

\(^3\)The right-hand side of (A.8) can be rewritten as follows:

\[
\frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta}{\lambda} (\rho + \epsilon) \geq \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta}{\lambda} (\rho + \hat{\epsilon}_1)
= \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta a_{fo}}{\lambda a_L \rho}
= \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta}{\lambda} \frac{\rho \lambda - \eta}{\lambda a_L \eta \rho} = 0,
\]

where the third inequality is derived from \(a_{fo} < \frac{\lambda \eta}{\lambda \eta \rho}\). Then, the right-hand side of (A.8) has a positive value.

\(^4\)The right-hand side of (A.9) can be rewritten as follows:

\[
\frac{\eta - 1}{a_S \eta} - (\rho + \epsilon) \geq \frac{\eta - 1}{a_S \eta} - (\rho + \hat{\epsilon}_1)
= \frac{\eta - 1}{a_S \eta} - \frac{a_{fo}}{a_L \rho}
> \frac{\eta - 1}{a_S \eta} - \frac{\rho \lambda - \eta}{a_L \lambda \eta \rho}
= \frac{\eta - 1}{a_S \eta} - \frac{\lambda - \eta}{a_L \lambda \eta}
> \frac{\lambda - \eta}{a_L \lambda \eta} - \frac{\lambda - \eta}{a_L \lambda \eta} = 0,
\]

where the third inequality from the bottom is derived from \(a_{fo} < \frac{\lambda \eta}{\lambda \eta \rho}\) and the last inequality is derived from \(\frac{\eta - 1}{a_S \eta} > \frac{\lambda - \eta}{a_L \lambda \eta}\). Then, the right-hand side of (A.9) has a positive value.
Next, to prove the case when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$, we separate three stages. At first stage, from Proposition 1, we can derive the reaction function of $i^L$ for $i^{fo}$. From Proposition 1, when $\iota = \bar{i}^{fo}$, $i^L \geq 0$ holds. From Proposition 1, when $i^{fo} < \bar{i}^{fo}$, $i^L = 0$.

At second stage, we derive the reaction function of $i^S$ for $i^L$. From (36), we can obtain the following equation:

$$i^L + i^{fo} \geq \eta - 1 - (\rho + \epsilon) > \bar{i}^{fo},$$

(A.12)

where the last inequality is derived from $\epsilon < \hat{\epsilon}_2$. Suppose that $i^{fo} < \bar{i}^{fo}$. Then, we can rewrite (A.12) as follows:

$$i^L > \bar{i}^{fo} - i^{fo} > 0.$$  

(A.14)

Therefore, when $i^{fo} < \bar{i}^{fo}$, $i^L > 0$ holds. However, from Proposition 1, when $i^{fo} < \bar{i}^{fo}$, $i^L = 0$ hold. This is contradiction. Therefore, $i^{fo} = \bar{i}^{fo}$ and $i^L > 0$ hold. Then, we will show that $\frac{\eta - 1}{aS\eta} - (\rho + \epsilon) > \bar{i}^{fo}$.

$$\frac{\eta - 1}{aS\eta} - (\rho + \epsilon) - \bar{i}^{fo} = \eta - 1 \frac{a_{fo}}{a_{fo} - a_L} \epsilon > \frac{\eta - 1}{aS\eta} - \frac{a_{fo}}{a_{fo} - a_L} \hat{\epsilon}_2$$

$$= \frac{\eta - 1}{aS\eta} - \frac{\lambda - \eta}{aL\lambda\eta} > 0,$$

(A.13)

where the last inequality comes from the assumption of $\frac{\eta - 1}{aS\eta} > \frac{\lambda - \eta}{aL\lambda\eta}$. Therefore, (A.12) holds.
substituting $\gamma^o = \bar{\gamma}^o$ into (A.12), we can obtain the reaction function of $\nu^S$ for $\nu^L$ as follows:

$$\nu^L \geq \frac{\eta - 1}{a_S \eta} - \frac{a_{fo}}{a_{fo} - a_L} \epsilon.$$  \hspace{1cm} (A.15)

When (A.15) holds with equality, $\nu^L \geq 0$ holds. On the other hand, when (A.15) holds with inequality, $\nu^L = 0$ holds.

At third stage, we derive the reaction function of $\nu^L$ for $\nu^S$. From (35) and $\nu^o = \bar{\nu}^o$, the reaction function of $\nu^L$ for $\nu^S$ becomes

$$\nu^S \geq \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta}{\lambda} \frac{a_{fo}}{a_L} \epsilon.$$  \hspace{1cm} (A.17)

When (A.17) holds with equality, $\nu^L \geq 0$. On the other hand, when (A.17) holds with inequality, $\nu^L = 0$. Then, we can depict the reaction functions of (A.15) and (A.17). Therefore, there exists a unique equilibrium of R&D investment and imitation investment.

---

6 We show that $\nu^L$ has a positive value when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$.

$$\nu^L \geq \frac{\eta - 1}{a_S \eta} - \frac{a_{fo}}{a_{fo} - a_L} \epsilon$$

$$> \frac{\eta - 1}{a_S \eta} - \frac{a_{fo}}{a_{fo} - a_L} \hat{\epsilon}_2$$

$$= \frac{1}{a_S \eta a_L \lambda} [a_L \lambda (\eta - 1) - a_S (\lambda - \eta)]$$

$$> \frac{1}{a_S \eta a_L \lambda} \left[ \lambda (\eta - 1) \frac{a_S}{\eta - 1} - a_S (\lambda - \eta) \right]$$

$$= \frac{1}{a_L \lambda} > 0,$$

where the inequality of the second line from the top is derived by $\epsilon < \hat{\epsilon}_2$ and the inequality of the second line from the bottom is derived by $\frac{a_S}{\eta - 1} < a_L$. Then, when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$, $\nu^L$ has a positive value.

7 We show that $\nu^S$ has a positive value. From $\epsilon < \hat{\epsilon}_2$, the value of $\nu^S$ becomes as follows:

$$\nu^S \geq \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta}{\lambda} \frac{a_{fo}}{a_L} \hat{\epsilon}_2$$

$$= \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\lambda - \eta}{a_L \lambda^2} = 0,$$

where the inequality is derived by $\epsilon < \hat{\epsilon}_2$. Therefore, when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$, $\nu^S > 0$ has a positive value.
follows:

\[
\begin{align*}
\iota^L &= \frac{\eta - 1}{a_S \eta} - \frac{a_{f_0}}{a_{f_0} - a_L} \epsilon, \\
\iota^S &= \frac{\lambda - \eta}{a_L \lambda^2} - \frac{\eta}{\lambda a_{f_0} - a_L} \epsilon.
\end{align*}
\] (A.19)

A.3 Proofs for Proposition 3

When \( \epsilon \leq \hat{\epsilon}_1 \), differentiating (38) and (39) with respect to \( \epsilon \), we can obtain the following equations:

\[
\begin{align*}
\frac{\partial \iota^L}{\partial \epsilon} &= -1 < 0, \\
\frac{\partial \iota^S}{\partial \epsilon} &= -\frac{\eta}{\lambda} < 0.
\end{align*}
\] (A.21)

When \( \epsilon \leq \hat{\epsilon}_1 \), from Proposition 2, \( \iota^{f_0} = 0 \). Then, the number of Firm S becomes

\[
\eta^S = \frac{\iota^S (1 - \lambda L_N)}{\epsilon + \iota^L + \iota^S - a_L \lambda L^2 S}.
\] (A.23)

Differentiating this with respect to \( \epsilon \), we can obtain the following equation:

\[
\frac{\partial \eta^S}{\partial \epsilon} = \frac{1 - \lambda L_N}{(\epsilon + \iota^L + \iota^S - a_L \lambda L^2 S)} \left[ \frac{\partial \iota^S}{\partial \epsilon} (\epsilon + \iota^L) - \iota^S (1 + \frac{\partial \iota^L}{\partial \epsilon} - a_L \lambda S \frac{\partial \iota^L}{\partial \epsilon}) \right].
\]

The sign of \( \frac{\partial \eta^S}{\partial \epsilon} \) depends on the sign of the parenthesis above equation. Focusing on the parenthesis above equation, we can rewrite it as follows:

\[
\frac{\partial \iota^S}{\partial \epsilon} (\epsilon + \iota^L) - \iota^S (1 + \frac{\partial \iota^L}{\partial \epsilon} - a_L \lambda S \frac{\partial \iota^L}{\partial \epsilon}) = -\frac{\eta}{\lambda} (\epsilon + \iota^L) - a_L \lambda (\iota^S)^2 < 0.
\] (A.24)
Therefore, the sign of $\frac{\partial n}{\partial \epsilon}$ is negative. A stronger IPR protection decreases the number of Firm S. Next, we investigate how a stronger IPR protection affects total volumes of R&D investment and imitation. The total volumes of R&D investment and imitation is $\iota L n^S$ and $\iota S n^F$. Then, differentiating these with respect to $\epsilon$, we can obtain the following equations:

$$\frac{\partial (\iota L n^S)}{\partial \epsilon} = \frac{\partial n^S}{\partial \epsilon} l^L + n^S \frac{\partial l^L}{\partial \epsilon} < 0,$$  \hspace{1cm} (A.25)

$$\frac{\partial (\iota S n^F)}{\partial \epsilon} = \frac{\partial (n^S (\epsilon + l^L))}{\partial \epsilon} l^L = (\epsilon + l^L) \frac{\partial n^S}{\partial \epsilon} < 0.$$  \hspace{1cm} (A.26)

Therefore, a stronger IPR protection decreases the total volumes of R&D investment and imitation.

Then, the wage rate in the North of (33) becomes

$$w_N = \frac{\lambda t^S + \eta (\rho + \epsilon)}{\rho + t^S + \epsilon}.$$  \hspace{1cm} (A.27)

Then, differentiating this with respect to $\epsilon$, we can obtain the following equation:

$$\frac{\partial w_N}{\partial \epsilon} = \lambda - \eta \left( \rho + t^S + \epsilon \right) \left( \rho + t^S + \epsilon \right) < 0,$$  \hspace{1cm} (A.28)

because $\frac{\partial w_N}{\partial \epsilon} < 0$. Then, a stronger IPR protection decreases the wage rate in the North.

Next, when $\hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2$, differentiating (40), (41), and (42) with respect to $\epsilon$, we can obtain the following equations:

$$\frac{\partial \iota_{fo}}{\partial \epsilon} = \frac{a_L}{a_{fo} - a_{fo}} > 0,$$  \hspace{1cm} (A.29)

$$\frac{\partial \iota^L}{\partial \epsilon} = \frac{-a_{fo}}{a_{fo} - a_{fo}} < 0,$$  \hspace{1cm} (A.30)

$$\frac{\partial \iota^S}{\partial \epsilon} = -\frac{\eta}{\lambda a_{fo} - a_{fo}} < 0.$$  \hspace{1cm} (A.31)

Then, a stronger IPR protection increases the innovation rate conducted by Northern followers and decreases the innovation rate conducted by Northern firms which were imitated before and imitation rate. Differentiating the wage rate in the North of (33) with respect to $\epsilon$, we can obtain the following equation:

$$\frac{\partial w_N}{\partial \epsilon} = -\lambda t^S + \eta (\rho + \iota_{fo} + \epsilon) \left( \lambda - \eta \right) a_{fo} (\rho + \iota_{fo} + t^S + \epsilon) < 0.$$  \hspace{1cm} (A.32)

Then, a stronger IPR protection decreases the wage rate in the North.
A.4 Proof for Proposition 4

When \( \epsilon \leq \hat{\epsilon}_1 \), differentiating (38) and (39) with respect to \( a_S \), we can obtain the following equations:

\[
\frac{\partial \nu^L}{\partial a_S} = -\frac{\eta}{\eta^2} < 0, \quad (A.33)
\]
\[
\frac{\partial \nu^S}{\partial a_S} = 0. \quad (A.34)
\]

Differentiating (44) and \( \iota^o = 0 \) with respect to \( a_S \), we can obtain the following equation:

\[
\frac{\partial n^S}{\partial a_S} = \frac{1 - \lambda L N}{(\epsilon + \nu^L + \nu^S - a_L \lambda \nu^L \nu^S)^2} \left[ -1 + a_L \lambda \nu^S \right] < 0,
\]
\[
\frac{\partial n^N}{\partial a_S} = -a_L \left( \frac{n^S \nu^L}{\eta^2} \right)^2 \left( 1 - a_L \lambda \nu^S \right) \left( \frac{n^S(\eta - 1)}{\eta^2} \right) > 0. \quad (A.35)
\]

Therefore, an increase in \( a_S \) increases the number of both the Northern firms and the Southern firms. Then, an increase in \( a_S \) decreases the number of FDI firms. Differentiating the total volume of R&D investment \( \nu^L n^S \) with respect to \( a_S \) is given by

\[
\frac{\partial (\nu^L n^S)}{\partial a_S} = \frac{\partial \nu^L}{\partial a_S} n^S + \nu^L \frac{\partial n^S}{\partial a_S} < 0, \quad (A.37)
\]

where \( \frac{\partial \nu^L}{\partial a_S} \) and \( \frac{\partial n^S}{\partial a_S} \) have negative values. Differentiating the total volume of imitation with respect to \( a_S \), we can obtain the following equation:

\[
\frac{\partial (\nu^S n^F)}{\partial a_S} = \frac{\partial \nu^S}{\partial a_S} n^F + \nu^S \frac{\partial n^F}{\partial a_S} < 0, \quad (A.38)
\]

where \( \frac{\partial \nu^S}{\partial a_S} = 0 \) and \( \frac{\partial n^F}{\partial a_S} \) has a negative value. Therefore, an increase in \( a_S \) decreases the total R&D investment and total imitation. Differentiating (33) with respect to
\( a_S \) is given by

\[
\frac{\partial w_N}{\partial a_S} = \frac{1}{(\rho + \epsilon + \delta)^2} \left[ \lambda(\rho + \epsilon + \delta) \frac{\partial \mu^s}{\partial a_S} - (\lambda \epsilon^s + \eta(\rho + \epsilon)) \frac{\partial v^s}{\partial a_S} \right] = 0, \quad (A.39)
\]

because \( \frac{\partial \epsilon^s}{\partial a_S} = 0 \).

On the other hand, when \( \hat{\epsilon}_1 < \epsilon < \hat{\epsilon}_2 \), differentiating (40), (41), and (42) with respect to \( a_S \), we can obtain the following equations:

\[
\frac{\partial \mu^o}{\partial a_S} = 0, \quad (A.40)
\]

\[
\frac{\partial \mu^L}{\partial a_S} = -\frac{\eta}{\alpha^S} < 0, \quad (A.41)
\]

\[
\frac{\partial \mu^s}{\partial a_S} = 0. \quad (A.42)
\]

Next, we investigate the relationship between \( a_S \) and the number of firms. Differentiating (26) with respect to \( a_S \) is given by

\[
\frac{\partial n^S}{\partial a_S} = -\frac{\epsilon(1 + a_{fo} \lambda \mu^o - \lambda L_N)}{(\epsilon + \mu^L + \mu^o + \mu^S - a_L \lambda \mu^L \mu^S)^2} (1 - a_L \lambda \mu^S) \frac{\partial \mu^L}{\partial a_S} = -\frac{\epsilon(1 + a_{fo} \lambda \mu^o - \lambda L_N)}{(\epsilon + \mu^L + \mu^o + \mu^S - a_L \lambda \mu^L \mu^S)} \frac{\eta}{\alpha^S} \frac{\alpha_{fo}}{a_{fo} - a_L} \frac{\partial \mu^L}{\partial a_S} > 0, \quad (A.43)
\]

because \( \frac{\partial \mu^L}{\partial a_S} \) is negative. From the labor market equilibrium condition in the North, the number of Northern firm is given by

\[
n^N = \lambda (L_N - a_{fo} \mu^o - n^S a_L \mu^L) \quad (A.44)
\]

Then, differentiating above equation with respect to \( a_S \), we can obtain the following equation:

\[
\frac{\partial n^N}{\partial a_S} = -a_L \lambda \left( \frac{\partial n^S}{\partial a_S} \mu^L + n^S \frac{\partial \mu^L}{\partial a_S} \right) = -\frac{a_L \lambda n^S \mu^S}{\epsilon + \mu^L + \mu^o + \mu^S - a_L \lambda \mu^L \mu^S} \frac{\partial \mu^L}{\partial a_S} > 0. \quad (A.45)
\]

Differentiating (26) with respect to \( a_S \) is given by

\[
\frac{\partial n^F}{\partial a_S} = \frac{1}{\mu^S} \left[ \frac{\partial \mu^L}{\partial a_S} n^S - (\epsilon + \mu^o + \mu^L) \frac{\partial n^S}{\partial a_S} \right] = \frac{n^S}{\mu^S} \left( \frac{\partial \mu^L}{\partial a_S} n^S - (\epsilon + \mu^o + \mu^L) \frac{\partial n^S}{\partial a_S} \right) = \frac{n^S (1 + a_L \lambda (\epsilon + \mu^o))}{\epsilon + \mu^L + \mu^o + \mu^S - a_L \lambda \mu^L \mu^S} \frac{\partial \mu^L}{\partial a_S} < 0. \quad (A.46)
\]
Therefore, an increase in $a_S$ increases the number of both Firm N and Firm S but decreases the number of Firm F. Next, we analyze how an increase in $a_S$ affects the total volume of R&D investment and total volume of imitation. Differentiating total volume of R&D investment of $(\ell^f + \ell^L n^F)$ with respect to $a_S$ is given by
\[
\frac{\partial(\ell^f + \ell^L n^F)}{\partial a_S} = \frac{\partial n^S}{\partial a_S} \ell^L + n^S \frac{\partial \ell^L}{\partial a_S} < 0, \tag{A.47}
\]
because $\frac{\partial n^N}{\partial a_S} > 0$. Differentiating total volume of imitation of $(n^F \ell^S)$ with respect to $a_S$ is given by
\[
\frac{\partial(n^F \ell^S)}{\partial a_S} = \frac{\partial n^F}{\partial a_S} \ell^S < 0. \tag{A.48}
\]
Therefore, an increase in $a_S$ decreases the total R&D investment and the total imitation. Finally, we investigate the relationship between $a_S$ and the wage rate in the North. Differentiating (33) with respect to $a_S$, we can obtain the following equation:
\[
\frac{\partial w_N}{\partial a_S} = \frac{\lambda - \eta}{(\rho + \ell^f + \ell^S + \epsilon)^2} \left[ - \frac{\partial \ell^f}{\partial a_S} \ell^S + \frac{\partial \ell^S}{\partial a_S} (\rho + \ell^f + \epsilon) \right] = 0. \tag{A.49}
\]
Therefore, there is no relationship between $a_S$ and the wage rate in the North.

References


Figure 3: Numerical example