Trade Policy-Making in a Model of Legislative Bargaining*

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PRELIMINARY DRAFT

Abstract

In democracies, trade policy is the result of interactions among many agents with different agendas. In accordance with this observation, we construct a model of legislative trade policy-making in the realm of distributive politics. An economy consists of different sectors, each of which is concentrated in one or more electoral districts. Each district is represented by a legislator in the legislature. The legislative process is modeled as a multilateral sequential bargaining game à la Baron and Ferejohn (1989). Our results indicate that expected welfare gains (or losses) accruing to each industry as a result of legislative bargaining depend on three key factors: (i) industry dispersion, (ii) the status quo tariff/subsidy each industry faces, and (iii) aggregate output of each industry. As a result, our model not only integrates the findings of the existing literature but also goes beyond that by analyzing the effects of legislative bargaining and status quo on trade policy formation.

Keywords: Trade Policy; Multilateral Legislative Bargaining; Political Economy, Distributive Politics.

JEL classification: C72, C78, D72, F13.

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1 Introduction

“Perhaps no other area of economics displays such a gap between what policy makers practice and what economists preach as does international trade.” – Dani Rodrik (1995).

Through the years, trade economists’ interest in understanding the determinants of trade policies and their effects have been steady if not growing. The influential studies include but are not limited to the electoral model of trade policy formation by Mayer (1984), the tariff lobbying model of Findlay and Wellisz (1982) and the influence-peddling model of tariff-setting by Grossman and Helpman (1994).¹

In this paper, we aim to contribute to this broad literature in three dimensions. First, in most studies of trade policy, the emphasis is given to demand-side determinants, namely, the characterization of individual preferences over policy choices and the channel through which these preferences are converted into political demands. Until recently, little attention has been paid to the effects of institutional settings. This is at the heart of our study. Second, most of the influential theoretical work analyzes trade policy as controlled by a single individual with authority (such as the President). A prominent example is Grossman and Helpman (1994). It is also possible to consider Mayer (1984) in the same category if one interprets his model as a contest between pre-committed candidates for office. Once a candidate is elected, she sets the tariff without any deliberation. This is a far-fetched assumption since in democracies trade policy is a product of multiple decision-makers with different interests. In many countries, it is set by a parliament and is therefore the outcome of a legislative bargaining process. Acknowledging this observation, we explicitly model trade policy as the outcome of a bargaining game within a legislature. Finally, most models in this literature are static, whereas our model is dynamic. We believe that this is a better representation of the reality; policy-making involves debate, deliberation, negotiation, and compromise, so it is dynamic by its nature.

In his seminal work on pluralist theory, Schattschneider (1935) explained the costly increase in protection by stating, “Benefits are concentrated while costs are distributed.”

¹There is an extensive literature on trade policy formation; see Rodrik (1995) and Nelson (1999) for a review.
Following this argument, we establish a legislative model of trade policy-making in the realm of distributive politics. We consider a small open economy that accommodates three industries, each operating in one or more electoral districts (constituencies). Individuals that reside in the same district are identical and each one is endowed with one unit of capital that is specific to the industry located in that district. As a result, there is a potential conflict among districts based on industry attachment.

Each district is represented by a legislator in the legislature. Legislators care only about the welfare of their own districts, and the welfare of a district is closely related to the industry located in it. In our model, trade policy implies any tariff or subsidy levied on any sector’s output. This setup is consistent with distributive politics since an increase in the price of a particular good (say, due to protection) will be beneficial only to those districts that produce it, but will be costly to the whole economy due to its negative effect on consumption.

We analyze the legislative game as a sequential model of multilateral bargaining with a simple majority rule à la Baron and Ferejohn (1989). Each period, a proposer is selected randomly among risk-neutral legislators to propose a trade policy. If the proposal receives a simple majority of the votes, the program goes into effect and the

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2 Constituencies are usually described in geographic terms in studies of the Congress; see Anderson and Baldwin (1987), Irwin and Kroszner (1999) and Lindsay (1990). This is intuitive since the electorate for the members of the Congress is defined geographically.

3 Magee (1978), in his study of testimony on trade legislation, finds strong evidence for sector-based political activity. Moreover, in other studies, capital and labor are found to be relatively immobile over politically plausible time horizons; see Nelson (2007), footnote 4.

4 We do not model the election of legislators in this paper. However, the results would not change as long as there is no commitment to party loyalty. The assumption of no party loyalty is used in Grossman and Helpman (2005) and Willmann (2004). Recent studies provide some empirical evidence for this assumption: Lebo, McGlynn and Koger (2007) show that an increase in party unity on voting has adverse electoral costs. In a more recent study, Carson, Koger, Lebo and Young (2010) find that vote share declines with party unity and legislators’ party loyalty decreases when they are electorally vulnerable.

5 This is in line with the empirical literature; see, among others, Baldwin (1976; 1985), Hiscox (2002), Fordam and McKeown (2003) and Ladewig (2006). The underlying assumption is that each legislator must calculate how voters in his district will respond to a particular trade policy in order to maximize his probability of re-election.

6 Here, we use tariffs as a measure of protection. In reality, non-tariff barriers (NTBs) are also used and very closely related to tariffs. Ray (1981) and Marvel and Ray (1983) show that in some industries NTBs and tariffs are complements and in others they are substitutes.

7 While random recognition does not mimic any actual procedures of a legislature, it is a useful device for capturing the inherent uncertainty that legislators face in building distributive coalitions. Random recognition is a way of modeling the fact that legislators do not know exactly which coalitions will form in the future if the current coalition fails to enact the legislation.
legislature adjourns. Otherwise, the *status quo* trade policy prevails and the process is repeated with a new legislator (possibly the same). In his voting, a legislator compares the benefits accruing to his district from the current proposal to the value of continuing to the next stage.

A new trade policy affects individuals’ welfare through three distinct channels. First, all individuals earn capital rent through the sector-specific capital they own. A higher tariff for a particular good, therefore, benefits those individuals who have a stake in that industry. Second, individuals derive utility from consumption. Thus, a higher tariff on any good lowers their consumer surplus. Finally, individuals share the revenue (positive or negative) from all taxes and subsidies imposed through trade policy.

We find that the change in an individual’s welfare (over his *status quo* welfare) due to a new trade policy depends on three key factors: (i) geographical (equivalently, political) dispersion of industries (i.e., the number of electoral districts an industry operates in), (ii) the *status quo* tariff/subsidy each industry faces, and (iii) aggregate output of each industry.

A closer look to our findings yields the following observations. First, we analyze the effect of industry dispersion on individuals’ welfare change. To focus on pure dispersion, we keep the total output of the industry constant while changing the number of districts the industry operates. We find that the welfare gain (which may be negative) of an individual that owns a unit of industry $i$ capital is non-decreasing (non-increasing) with respect to the dispersion of industry $i$ (industry $j \neq i$). This makes sense; if an industry is dispersed enough to have a majority representation in the legislature, then it will receive more protection due to its agenda setting power. In fact, many empirical studies are in line with this finding. Examples include Brock and Magee (1974) who argue that geographically dispersed industries are more influential since they can reach a larger number of elected representatives; Pincus (1975) who shows that industries with sales spread more evenly across states or with establishments in many states obtained higher duties; and Ray (1981) who finds that non-tariff trade restrictions in the U.S. are biased.

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8 Busch and Reinhardt (1999) argue that geographical concentration is different from political concentration. Our focus is on industry spread across political districts, hence more in line with the definition of political concentration. However, for simplicity, we assume that these two terms are the same, so we do not distinguish between them.
toward industries in which production is dispersed across different regions.\footnote{In their study on U.S. manufacturing industries in 1990, Busch and Reinhardt (1999) find that politically dispersed industries are likely receive protection. By using U.S. tariff data from 1993, Fredriksson, Matschke and Minier (2008) show that an industry receives positive protection if an industry’s capital share in districts controlled by the majority party (implying majority of districts in their model) exceeds these districts’ population share. They also find that in obtaining protection, the impact of operating in majority of districts is comparable in magnitude to being an organized industry (the lobbying effect).}

Second, the welfare gain of an individual that owns a unit of industry $i$ capital is non-increasing (non-decreasing) with respect to the status quo tariff on good $i$ (good $j \neq i$). For example, if the initial protection on industry $i$ was already high, then an agent that owns industry $i$ capital incurs a larger welfare loss (or smaller welfare gain) with the implementation of the new trade policy since, due to the necessity of concessions by competing interests, the new trade policy moves the sector away from its ideal protection level. The effect of historical patterns of protection on current protection is documented by Ray and Marvel (1984). Cheh (1974) shows that the percentage change in trade barriers is affected by initial protection. To the best of our knowledge, this is the first theoretical analysis that considers the effect of status quo level of protection on new trade policy formation.

Third, the effect of a change in aggregate output on welfare gain, on the other hand, depends on the source of the change. If the aggregate output of good $i$ increases due to a relative increase in the population share of industry $i$ compared to other industries, then its effect on the welfare gain of an individual that owns a unit of industry $i$ capital is unambiguously negative. The intuition is as follows. A higher output due to a higher population share of industry $i$ affects an individual’s welfare only through the tariff revenue (or subsidy cost) he receives. A higher output of good $i$, however, implies a weaker tariff revenue effect since, at a given price and the other parameters, it implies fewer imports, hence a lower marginal tariff revenue. On the other hand, if the rise in aggregate industry output is due to an economy-wide positive technology shock, the effect on welfare gain is positive if the industry is dispersed enough to control the majority in the legislature whereas if no industry has majority control, then its effect may be negative due to the necessity of making a compromise to coalition partners. The evidence on the relationship between productivity and protection can be found in Karacaoglu (2011).

In addition to the findings above, we can also make a cross industry comparison.
Accordingly, all else equal, individuals that own industry $i$ capital attain unambiguously larger welfare gains than those that own industry $j$ capital: (i) if industry $i$ operates in the majority of districts; otherwise, (ii) if industry $i$ is subject to a lower status quo import tariff/export subsidy, and/or (iii) if total output of industry $i$ is lower.

Our model is closely related to Willmann (2004), McLaren and Karabay (2004) and Grossman and Helpman (2005). In Willmann’s (2004) citizen-candidate model, heterogeneous districts behave strategically such that they have a tendency to choose candidates that are more protectionist than the district median. From a regional perspective, voters prefer a positive tariff, ignoring the tariff costs imposed on other districts. In contrast, from a national perspective, these tariff costs are internalized and, therefore, each district’s representative has to make a compromise. Knowing this compromise, voters in the first stage (namely, regional election stage) choose someone who is more protectionist than the median voter.

McLaren and Karabay (2004) extend the standard median-voter framework to a rudimentary model of a government by assembly where parties compete by making binding election promises. In their model, the equilibrium tariff turns out to be the optimal tariff of the median voter in the median congressional district. They show that import-competing interests are more likely to receive protection if they are moderately geographically concentrated (neither too concentrated nor too dispersed). They also conclude that majoritarian systems tend to be more protectionist than presidential systems.

Grossman and Helpman (2005) show the presence of a protectionist bias in trade policy by focusing on party discipline. They consider a three-stage game. First, parties announce their policy platforms, then elections take place, and in the final stage, a particular trade policy is adopted. In this model, the lack of full commitment to announced party policies create protectionist bias as long as districts differ in their capital endowments.

Willmann (2004) and Grossman and Helpman (2005) model the legislative process as a joint welfare maximization problem, and McLaren and Karabay (2004) employ an election framework in which trade policy is predetermined. The common property of all of these papers is that there is not much scope for legislative procedures. In contrast,
the legislative bargaining is the core force behind trade policy formation in our model.

The rest of the paper is organized as follows. In the next section, we describe the basic model. In section 3, equilibrium is characterized. We discuss possible extensions in section 4. Section 5 concludes the analysis. All proofs are relegated to the Appendix.

2 Model

Consider a small open economy populated with a unit measure of individuals living in \( N \) districts (where \( N \geq 3 \) and divisible by 3). There are \( M = 3 \) industries each of which supplies a homogenous manufacturing good that is produced with a sector-specific capital under Ricardian technology.\(^{10}\) In particular, we assume that the production technology for each manufacturing good takes the following form: \( f_i(K_i) = \theta K_i \), where \( K_i \) and \( \theta \) denote the amount of the sector-specific capital used in sector \( i \) and the economy-wide productivity parameter, respectively.

Each district has one industry in it.\(^{11,12}\) Let the number of districts producing good \( i \) be denoted by \( n_i \) such that \( n_1 + n_2 + n_3 = N \). Without loss of generality, we assume that \( n_1 \geq n_2 \geq n_3 \). Individuals residing in the same district are endowed with one unit of the same type of sector-specific capital and districts that produce the same good are populated by the same number of individuals. To save on notation, we let \( K_i \) denote the total number of individuals residing in a district that produces good \( i \). Note that \( \sum_{i=1}^{3} n_i K_i = 1. \(^{13}\) Let \( q_i \) denote the amount of good \( i \) produced in a district that hosts industry \( i \), and \( Q_i \) denote the total amount of good \( i \) produced in the economy. Therefore, we have \( q_i = \theta K_i \) and \( Q_i = n_i q_i \). In addition, let \( p_i^* \) and \( p_i \) represent, respectively, the

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\(^{10}\)We limit the number of manufacturing industries to three for simplicity, as in Grossman and Helpman (2005). Results remain qualitatively similar for \( M > 3 \). There are, however, many more possibilities (of coalition formation) to consider. See section 4 for further remarks.

\(^{11}\)Our results carry over even if more than one industry is allowed in each district as long as each resident still holds only one sector-specific capital and in every district there is one industry with majority representation. This is true since each legislator will follow the interests of the median voter, which belongs to a particular industry under the conditions assumed here.

\(^{12}\)We do not model the location choice of a particular industry, rather we take it as given. However, we acknowledge that this choice may depend on the political influence they can exert in each location.

\(^{13}\)We allow only those districts that produce different goods to differ in the number of citizens residing. This is done to simplify the notation. Alternatively, it is possible to allow each district (even the ones producing the same good) to be populated by different number of individuals. All of our results continue to hold.
exogenous world price of good \( i \) and its domestic price. Thus, the total rent that accrues to capital in district \( i \) is \( p_i q_i = \theta p_i K_i \).

Each individual has an identical additively separable utility function given by:

\[
    u = \sum_{i=1}^{3} u_i(c_i),
\]

where \( c_i \) represents the consumption of good \( i \), \( i = 1, 2, 3 \). We assume that \( u_i(c_i) = R_i c_i - (c_i^2/2) \), where \( R_i > 0 \) and assumed to be sufficiently large.\(^{14}\) With these preferences, the demand for good \( i \), implicitly defined by \( u'_i(d(p_i)) = p_i \), is given by \( d(p_i) = R_i - p_i \).

The linearity of demand is not crucial for the main results of our paper, but it simplifies the analysis and permits a closed-form solution. The indirect utility of an individual with income \( y \) is \( y + s(p) \), where \( p = (p_1, p_2, p_3) \) is the vector of domestic prices, and \( s(p) = \sum_{i=1}^{3} [u_i(d(p_i)) - p_i d(p_i)] \) is the resulting consumer surplus.

Each district is represented by a single legislator who is concerned only with the welfare of his own district. A district’s welfare is the aggregate utility of all individuals in that district, which is equal to the total income plus the district’s share in total consumer surplus and total tariff revenue (or subsidy cost) for each good. Hence, a district which produces good \( i \) has a welfare (for \( i \neq j \neq k \)):

\[
    W_i(p) = p_i \theta K_i + K_i \sum_{l=i,j,k} \frac{(R_l - p_l)^2}{2} + K_i \sum_{l=i,j,k} [(p_l - p^*_l)(R_l - p_l - Q_l)],
\]

where the first term is the capital rent, the second term is the consumer surplus captured by that district, and the last term is the share of tariff revenue (or subsidy cost).\(^{15}\) In addition, we denote \( w_i(p) \) as the welfare of an individual with a stake in industry \( i \), hence \( W_i(p) = K_i w_i(p) \).

We consider an infinite-horizon model. Every period, there is a set of prices at which individuals make their production and consumption decisions, and enjoy the resulting welfare. The legislature can change the prevailing status quo, \( p^* = (p^*_1, p^*_2, p^*_3) \), by changing the domestic price of any good via legislative bargaining. We restrict the set of policy

\(^{14}\)To be more precise, we require \( R_i > p^*_i + \theta - Q_i = p^*_i + \theta(1 - n_i K_i) \). This ensures that demand for good \( i \) is positive at all prices that may occur in legislative bargaining. We also require \( p^*_i > Q_i \) for each price to be positive. See section 3 for the determination of optimal prices.

\(^{15}\)We assume that tariff revenue (or subsidy cost) is distributed equally as a lump-sum transfer to each individual.
instruments available to the legislature and allow only for trade taxes and subsidies. A domestic price in excess of the world price implies an import tariff for an import good and an export subsidy for an export good. Domestic prices below world prices correspond to import subsidies and export taxes.

The timing of the trade-policy formation game in our model is based on the standard Baron-Ferejohn bargaining framework. This is a game of complete information. At the start of each period (before any production or consumption takes place), a legislator is selected randomly to propose a vector of domestic prices for each manufacturing good produced.\textsuperscript{16} We restrict each domestic price to be non-negative and below some finite upper bound.\textsuperscript{17} If the proposal receives a simple majority, it is immediately implemented and the legislature adjourns. Each district’s welfare thereafter is evaluated at the new prices.\textsuperscript{18} If the proposal does not receive a majority, the process is repeated with another legislator randomly selected to propose a program. Bargaining continues until a program is implemented.\textsuperscript{19} Districts continue to receive their status quo welfare in every period until an agreement is reached.

There are a couple of things to note. First, it is straightforward to show that the aggregate welfare, \( W(p) = \sum_{i=1}^{3} n_i W_i(p) \), is maximized at the free trade prices of the three goods. Hence, if the prices were set by a central authority (such as a President), free trade would prevail forever. In the current paper, however, we are interested in policy-making by a legislature. This naturally introduces a conflict of interest among legislators since each legislator is selfishly interested in maximizing his own district’s welfare. Put

\textsuperscript{16}To simplify, we assume that a period in the legislative game coincides with a production/consumption period.

\textsuperscript{17}To be more precise, we require each domestic price to satisfy: \( 0 \leq p_i < \bar{p}_i \), where \( \bar{p}_i = p_i^* + \frac{(R_i - p_i^*)^2 + (\theta - Q_i)^2}{2(\theta - Q_i)} \). These limits ensure that we get an interior solution in prices.

\textsuperscript{18}Note that once a proposal is accepted, the game ends so that there will be no future proposals. The underlying assumption is that the legislative bargaining process is too costly to be repeated. Indeed, it is possible to show that if a new proposal is allowed in every period even after an agreement is reached, then the expected welfare of each industry is lower. See the discussion section for more on this issue.

\textsuperscript{19}The legislative process here is modeled as an infinite horizon game. Osborne and Rubinstein argue that “a model with an infinite horizon is appropriate if after each period players believe that the game will continue for an additional period, while a model with a finite horizon is appropriate if the players clearly perceive a well-defined final period.” Since our model focuses on bargaining over the course of a legislative term, uncertainty about the exact date of adjournment precludes such a well-defined final period. Furthermore, even if the exact end of the legislative term is known, it may be so distant in the horizon that it does not factor explicitly into agents’ calculations (Osborne and Rubinstein, 1994).
another way, when proposing a program, each legislator weighs the marginal benefits and costs for his own district but ignores the negative externality imposed on others. Therefore, the resulting trade policy is inefficient for the economy as a whole.

Second, from (2), a manufacturing good affects (through its price) a district’s welfare via three channels. The first one, the rent that accrues to the specific factor, is present if that good is produced in that district. The second one is the consumer surplus attained from the consumption of that good. The last one is the tariff revenue (or subsidy cost) due to trade. The effect of price through the first channel is always positive whereas it is always negative through the second channel. Its effect through the third channel, on the other hand, can be positive or negative (in fact the third channel is concave in all three prices). This is true since good $i$’s price has two distinct effects on tariff revenue/subsidy cost: (1) the direct effect (changing price while keeping imports/exports constant), and (2) the indirect effect through demand. These two effects work in opposite directions. To see this, assume that good $i$ is an imported good. First, start from a price just above the world price. As we increase the price, the direct effect leads to an increase in the tariff revenue whereas the indirect effect leads to a decrease (since import demand goes down). Initially, the direct effect dominates, and therefore, it is optimal to increase the price. When the price reaches a certain value, the indirect effect starts dominating and the tariff revenue decreases if we further increase the price.

### 3 Characterization of equilibrium

In this section, we investigate the properties of the bargaining outcome. The equilibrium concept is subgame perfect Nash equilibrium (SPE). Hence, the equilibrium strategies must constitute a Nash equilibrium in every proper subgame. To make our results as clear as possible, we focus on the case when the discount factor (denoted by $\delta$) approaches 1 in the limit.\(^{20}\)

Let the **per-period** equilibrium continuation welfare of a district producing good $i$ be denoted as $V_i$. This is the per-period equilibrium welfare a district expects in the event that bargaining is carried over to the following period and everyone plays their

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\(^{20}\)This corresponds to the case in which the time length between any two offers (sessions) is infinitesimally short. See section 4 for the implications of relaxing this assumption.
equilibrium strategies so that an equilibrium is reached (assuming that an SPE exists). When faced with a proposal, a legislator votes yes only if the welfare his district gets in the current proposal is not less than its continuation welfare.\textsuperscript{21} Since a random proposer is selected every period, the outcome of legislative bargaining depends on the identity of the proposer. In this sense, the outcome is ex-ante uncertain. We use the expected per-person welfare change due to bargaining as the basis for comparison among individuals with stakes in different sectors. To this purpose, let $v_i$ denote the equilibrium continuation welfare of an individual with a stake in industry $i$, thus $V_i = K_i v_i$.

The welfare of a district producing good $i$, given in (2), can conveniently be decomposed into two parts: the welfare evaluated at status quo prices, and the change in welfare due to legislative bargaining. Let, for all $i$, $x_i = p_i - p_i^s$ denote the price change over status quo. Then, (2) can be rewritten as:

$$W_i = W_i(p^s) + K_i \left[ \theta x_i - \frac{1}{2} \left( \sum_{l=i,j,k} (x_l + a_l)^2 - (a_i^2 + a_j^2 + a_k^2) \right) \right].$$

This decomposition is helpful as it allows us to express the welfare change in each district as a function of change in prices, $(x_i, x_j, x_k)$, productivity parameter, $\theta$, and a linear combination of each industry’s total output and status quo tariff, $(a_i, a_j, a_k)$, for $i \neq j \neq k$. The first term in the brackets indicates the per-capita change in capital rent while the second term indicates the per-capita change in consumer surplus plus tariff revenue.

\textsuperscript{21}We assume that a legislator who is indifferent between accepting a proposal and continuing to the next session accepts the proposal. Therefore, weakly dominated strategies are ruled out.
The first-best for each legislator is to maximize his district’s welfare without any constraints. Let \( \mathbf{p}^U_i \) denote the vector of prices that the unconstrained maximization problem leads to, i.e., \( \mathbf{p}^U_i = \arg \max_p W_i(p) \). Maximizing (3) with respect to \( x_i, x_j \) and \( x_k \) yields

\[
\begin{align*}
x_i &= \theta - a_i, \\
x_j &= -a_j, \\
x_k &= -a_k.
\end{align*}
\]

Inserting the values of \( (x_i, x_j, x_k) \) and \( (a_i, a_j, a_k) \), we have the following lemma.

**Lemma 1** Unconstrained maximization of \( W_i(p) \), \( i = 1, 2, 3 \), yields (for \( i \neq j \neq k \)) \( \mathbf{p}^U_i = (p^U_{i}, p^U_{j}, p^U_{k}) \), where

\[
\begin{align*}
p^U_{i} &= p^*_i + \theta(1 - n_i K_i), \\
p^U_{j} &= p^*_j - \theta n_j K_j, \\
p^U_{k} &= p^*_k - \theta n_k K_k.
\end{align*}
\]

Thus, a recognized (selected) legislator would ideally want to raise the price of the good his district produces over the world price (thereby protecting industry \( i \)), while lower the price of the other goods below their respective world prices. It is natural to assume that the status quo prices are in the range defined by the unconstrained maximization problem. For example, a legislator representing a district that produces good \( i \) has no reason to set the price of good \( i \) above \( p^*_i + \theta(1 - n_i K_i) \). Similarly, he has no reason to set the price of good \( j \neq i \) below \( p^*_j - \theta n_j K_j \). Hence, we make the following assumption.

**Assumption 1.** The status quo price for good \( i \), \( i = 1, 2, 3 \), satisfies

\[
p^*_i - \theta n_i K_i \leq p^s_i \leq p^*_i + \theta(1 - n_i K_i).
\]

Note that this assumption implies that

\[
0 \leq a_i \leq \theta. \tag{4}
\]
Hence, a value of \( a_i = \theta \) corresponds to the case in which the status-quo price of good \( i \) is at its optimum for the districts that produce good \( i \), while \( a_i = 0 \) corresponds to the case in which it is at its optimum for the districts that produce good \( j \neq i \). Accordingly, the status quo corresponds to the optimal price vector for the districts that produce good \( i \) when \((a_i, a_j, a_k) = (\theta, 0, 0)\).

Total industry output in any sector \( i \), \( Q_i = \theta n_i K_i \), can change via two means: (i) change in productivity, \( \theta \), (ii) change in the population share of districts that produce good \( i \), \( n_i K_i \). If the increase in \( Q_i \) occurs via the latter, then Lemma 1 shows that a producer in a sector that produces higher aggregate output \( Q_i \) will prefer a lower tariff (or export subsidy) for his own product than a producer in a sector that produces lower aggregate output. The reason is as follows. Focus for now on the case of an imported good. Recall the three channels discussed above through which the tariff affects the welfare of a producer in industry \( i \). Aggregate output, \( Q_i \), in this case does not affect the first two channels (the rent and consumer surplus channels). What it does affect is the third channel, tariff revenue. A higher value for \( Q_i \) implies a weaker tariff revenue effect since, at a given price and the other parameters, a higher value of \( Q_i \) implies fewer imports, hence a lower marginal tariff revenue for a given increase in tariff.\(^{22}\) Therefore, a higher value of \( Q_i \) implies a lower marginal benefit of the tariff, and a lower optimal tariff, from the point of view of a sector-\( i \) producer. Parallel reasoning holds for an exported good. On the other hand, if the increase in \( Q_i \) is the result of an increase in productivity, then according to Lemma 1, a producer in a sector that produces higher aggregate output, \( Q_i \), will prefer a higher tariff (or export subsidy) for his own product than a producer in a sector that produces lower aggregate output because, the productivity not only affects the third channel (tariff revenue) through its effect on aggregate output but also affects the first channel (the rent channel). The effect through the first channel is positive and it dominates the effect through the third channel.

\(^{22}\) The same conclusion holds for a comparison between two industries \( i \) and \( j \) even if, although \( Q_i > Q_j \), the demand parameter \( R_i \) is sufficiently higher than \( R_j \) that at a common tariff, imports of good \( i \) exceed those of good \( j \). The reason is that an increase in \( R_i \), holding all prices and other parameters constant, raises industry \( i \) imports, increasing the marginal tariff revenue from the tariff on good \( i \), but at the same time raises domestic consumption of good \( i \), raising the marginal consumer surplus loss from the tariff on good \( i \). The two effects cancel each other out, with the result that the demand parameters \( R_i \) have no effect on tariff preferences.
As mentioned earlier, a proposal will be accepted when majority support is obtained in the legislature. As a result, there are two possible cases to be considered. We first analyze the situation when one of the manufacturing goods is sufficiently dispersed so that the districts producing it have a majority representation in the legislature ($\frac{n_j}{N} > \frac{1}{2}$). We next turn attention to a more even distribution of industries in which no manufacturing good has a majority representation in the legislature.

3.1 Case 1: $\frac{n_3}{N} \leq \frac{n_2}{N} < \frac{1}{2} < \frac{n_1}{N}$

This is the case where one of the manufacturing goods has a majority representation in the legislature. The solution here is rather trivial; since we focus on the case when all players are very patient ($\delta \rightarrow 1$), legislators of districts producing good 1 would delay bargaining until they receive their first-best payoffs. In other words, districts that produce good 1 have the ultimate power in setting the prices of all three goods. When a legislator representing a district producing good 2 or good 3 is selected as the proposer, depending on the status quo welfare his district enjoys, he may choose to make a proposal that would be rejected by the legislature. However, in any subgame perfect equilibrium, the accepted price vector must be $p^{U_1}$.

Proposition 1 The unique SPE is described as follows. When a legislator representing a district that produces good 1 is selected to make a proposal, he proposes $p = p^{U_1}$, and it is accepted immediately. When a legislator representing a district that produces good $j = 2, 3$ is selected to make a proposal, he proposes $p = p^{U_1}$ if $W_j(p^*) \leq W_j(p^{U_1})$; otherwise he makes a proposal that would be rejected by the legislature. The equilibrium continuation payoffs are $V_i = W_i(p^{U_1})$.\[23\]

Here, industry 1 is sufficiently large (i.e., it has the necessary number of seats in the legislature) to set trade policy independently. As a result, the problem becomes an unconstrained maximization problem. By (3), the expected per-person welfare change

\[23\text{Note that for each status quo price vector, there is a critical value of } \delta, \delta^*, \text{ below which an agreement is always reached in the first period independent of the identity of the proposer. In contrast, the accepted price vector in this case depends on the identity of the proposer.}\]

\[24\text{These are the continuation payoffs in case of an agreement. Since an agreement can be reached only when the proposed prices are } p^{U_1}, \text{ it follows that } V_i = W_i(p^{U_1}).\]
can be expressed as

\begin{align*}
v_1 - w_1(p^*) &= \theta(\theta - a_1) - \frac{\theta^2 - (a_1^2 + a_2^2 + a_3^2)}{2}, \\
v_2 - w_2(p^*) &= -\theta a_2 - \frac{\theta^2 - (a_1^2 + a_2^2 + a_3^2)}{2}, \\
v_3 - w_3(p^*) &= -\theta a_3 - \frac{\theta^2 - (a_1^2 + a_2^2 + a_3^2)}{2},
\end{align*}

From (5), it is clear that for a given value of \( \theta \), \( v_i - w_i(p^*) \) is increasing in \( a_{j\neq i} \) and, given (4), decreasing in \( a_i \). For an individual that has a stake in industry \( i \), a low value of \( a_i \) corresponds to the case when the status quo price for good \( i \) is significantly different than its optimum value, and hence, there is room for welfare improvement. As \( a_i \) increases, the potential welfare gain gradually diminishes and reaches zero when \( a_i = \theta \). This is the case when the status quo price for good \( i \) is already at its optimum. A parallel argument can be made for \( a_{j\neq i} \). For an individual that has a stake in industry \( i \), a high value of \( a_{j\neq i} \) means that there is a big room for welfare improvement by lowering the price of good \( j \). As \( a_{j\neq i} \) goes down, the potential improvement becomes lower and reaches zero when \( a_{j\neq i} = 0 \). Again, this is the case when the status quo price for good \( j \) is already at its optimum.

From (5) and given that \( a_i = \theta n_i K_i + p_i^* - p_i^* \), we can conclude that the welfare gain of an individual who owns industry \( i \) capital is decreasing in the status quo tariff/subsidy, \( p_i^* - p_i^* \), and the population share of industry \( i \), \( n_i K_i \). A positive economy-wide technology shock, on the other hand, benefits only industry 1. This is mainly driven by the effect of \( \theta \) on the prices. Recall that industry 1 owners set prices such that good 1’s price is above its world price and good 2’s and good 3’s prices are below their world prices. As \( \theta \) increases, the rent effect benefits industry 1 even more whereas it hurts industry 2 and industry 3 (in other words for industry 2 and industry 3, as \( \theta \) increases, the negative rent effect is magnified). This is the main divergence between industry 1 and others since they all share the resulting consumer surplus and tariff revenue/subsidy cost. As a result, good 1 earns an even higher mark-up over the status quo (since the rent effect dominates the consumer surplus and tariff revenue effects), while the prices of goods 2 and 3 go further below their status quo prices. So, capital-owners of sector 1 earn higher...
rents while those of sectors 2 and 3 earn less. A higher $\theta$ reduces the welfare gain an individual expects through consumption and tariff revenue (the magnitude of change is $\sum_{i=1}^{3} n_iK_i a_i - \theta \leq 0$), but the reduction is much smaller than what capital-owners in industry 1 gain.

Moreover, in light of the assumed range for status quo prices, it is possible to rank the welfare change from the best to worst across individuals with stakes in different industries. Note that, for a given $a_2$ and $a_3$, $v_1 - w_1(p^*)$ attains its minimum at $a_1 = \theta$. Evaluated at this value, $v_1 - w_1(p^*) = \frac{a_2^2 + a_3^2}{2} \geq 0$, so there is always a welfare gain for individuals who have a stake in industry 1.\footnote{When $a_1 = \theta$ and $a_2 = a_3 = 0$, status quo prices coincide with the optimal prices; i.e., $p^* = p^{U_1}$. In this case, the welfare change will be zero.} On the other hand, for individuals associated with industries 2 and 3, whether there is a welfare gain or loss depends on the values of $a_i$. Given that $0 \leq a_i \leq \theta$, it is easy to see that the maximum value of $[v_2 - w_2(p^*) + v_3 - w_3(p^*)]$ is zero, implying that at least one of the two must be negative (except for when $(a_1, a_2, a_3) = (\theta, \theta, \theta)$ or $(\theta, 0, 0)$, in which case both of them are zero). Moreover, if $a_2 > a_3$, then $v_2 - w_2(p^*) < v_3 - w_3(p^*)$ (and vice versa). However, per-capita welfare gain accruing to industry 1 is at least as much as any possible welfare gain accruing to other industries. This is due to the agenda-setting power of the legislators representing industry 1. That is to say, due to their majority power, they can be considered as veto-players.

On the other hand, if no manufacturing industry can control the majority of seats in the legislature, then a recognized legislator needs the support of the legislators representing at least one other industry. To get their votes, he has to offer them a relatively more favorable price compared to the unconstrained case, which moves the final outcome away from his first-best. This is what we analyze next.

### 3.2 Case 2: $\frac{n_3}{N} \leq \frac{n_2}{N} \leq \frac{n_1}{N} \leq \frac{1}{2}$

Here, we analyze the bargaining outcome when no industry is highly dispersed throughout the economy. In this case, unlike in Case 1, a recognized legislator may not be able to achieve the first-best outcome for his district since he has to compromise with at least one other industry in order to attain simple majority (i.e., more than 1/2) of the votes.
refer to this situation as the proposer selecting a ‘coalition partner’ (or simply ‘forming a coalition’).

As common in multi-person bargaining problems, there are many SPE in this case.\textsuperscript{26} We focus on stationary equilibria whereby the continuation payoffs for each structurally equivalent subgame are the same.\textsuperscript{27} In a stationary equilibrium, a legislator who is recognized to make a proposal in any two different sessions makes the same proposal in both sessions. Hence, stationary equilibria are history-independent.

When a legislator is recognized to make a proposal, he has an incentive to propose a program that will be accepted, since if rejected, he faces the risk that his district will be worse off in the program adopted in the future. In equilibrium, in accordance with the “Riker’s size principle”,\textsuperscript{28} any proposal will be accepted with the minimal number of industries that constitute a quorum of districts. This is true since increasing the number of industries in the coalition would increase the costs without increasing the benefits. Hence, the recognized legislator will make a proposal that will be accepted by not only the representatives of the districts producing the same manufacturing good but also the representatives of the districts producing another good. In order to get the acceptance of the latter, the proposal should provide their districts an ex-post welfare not less than their discounted continuation payoffs.

Suppose a legislator representing a district which produces good $i$ is recognized to make a proposal. If he chooses to reward the districts which produce good $j \neq i$, his maximization problem becomes:

$$\max_{p} W_i(p) \text{ s.t. } \frac{W_j(p)}{1 - \delta} \geq W_j(p^*) + \frac{\delta V_j}{1 - \delta}. \quad (6)$$

The left-hand side of the inequality indicates the discounted total welfare at the proposed prices, whereas the right-hand side is what a legislator representing a district that produces good $j$ expects in case he does not vote in favor of the proposal (his district

\textsuperscript{26} Baron and Ferejohn (1989) show that any outcome (in their game that means any division of a cake) can be supported as an SPE using infinitely nested punishment strategies as long as there are at least five players and the discount factor is sufficiently high. Li (2009) shows that even with three players, there is a vast multiplicity of equilibria.

\textsuperscript{27} Baron and Kalai (1993) argue that stationarity is an attractive restriction since it is the “simplest” equilibrium such that it requires the fewest computations by agents.

\textsuperscript{28} See Riker (1962).
gets its status quo welfare for the current period and the expected continuation welfare thereafter).\textsuperscript{29} The recognized legislator will choose $p$ such that the constraint is satisfied with equality. Thus, the constraint can be rewritten as:

$$W_j(p) = (1 - \delta)W_j(p^*) + \delta V_j.$$

Whenever possible, a selected legislator employs a mixed strategy in choosing which other industry to reward. This is optimal (as in Baron and Ferejohn (1989)) since choosing an industry as a coalition partner with pure strategy strengthens the bargaining position of that industry and makes it relatively expensive to include in the coalition. The districts who are offered a favorable price vote in favor of a proposal because of the fear of possibly being excluded from the majority in the following round of the bargaining game.

When $\delta \rightarrow 1$, we first show that an SSPE exists in which all legislators randomize between the other two industries, and then prove that all SPPE are payoff equivalent. We present the main result in Proposition 2.

**Proposition 2** When $\frac{n_i}{N} \leq \frac{1}{2}$, an SSPE exists in which a selected legislator representing a district which produces good $i$ proposes a price $p_i^* + \theta \left(\frac{2}{3} - n_iK_i \right)$ for the good his district produces, a price $p_j^* + \theta \left(\frac{1}{3} - n_jK_j \right)$ for good $j \neq i$ where $j$ is selected randomly, and a price $p_k^* - \theta n_kK_k$ for the remaining good. The first proposal receives a majority vote, so the legislature adjourns after the first session. The continuation payoffs are $V_i = W_i(p^*) + K_i [\theta(\frac{2}{3} - a_i) - \frac{5\theta^2 - (a_i^2 + a_j^2 + a_k^2)}{2}]$, where $a_i$ is as defined before, $a_i = \theta n_iK_i + p_i^* - p_i^*$.\textsuperscript{30}

Compared to Case 1, since all legislators have to get the approval of one other industry, they compromise by proposing a lower price for their own industry and a higher price for the industry selected as the coalition partner. Below, we summarize the important properties of this SSPE.

1. For given values of $a_i$, the continuation payoff of any district (or expected welfare change of any individual) is in between the highest and the lowest continuation

\textsuperscript{29}Note that districts that accommodate the same industry are identical, so if this inequality holds for one, then it also holds for all.
payoffs obtained under Case 1. This makes sense since no industry is dispersed enough to single-handedly control legislature. Therefore, no industry is either very strong or very weak. Thus, compared to Case 1, districts producing goods 2 and 3 have significantly higher bargaining power, which, in turn, reduces the welfare gain (may even result in welfare loss) districts that produce good 1 expect.

2. Per capita expected welfare change of each individual with a stake in industry \( i \) is increasing in \( a_{j \neq i} \) and decreasing in \( a_i \) as in Case 1 for exactly the same reasons stated before. Particularly, it is decreasing (increasing) in the status quo tariff/subsidy of industry \( i \) (industry \( j \neq i \)) and the population share of industry \( i \) (industry \( j \neq i \)). Moreover, depending on the values of \( a_i \) (namely, output of each industry and status quo tariff/subsidy on each good), the welfare change can be positive or negative for each industry.\(^{30}\) In contrast, in Case 1, industry 1 always observes a welfare gain, and, independent of the values of \( a_i \), there is always one industry (must be either industry 2 or 3) which experiences a welfare loss.

3. Welfare ranking of individuals with stakes in different industries depends only on the values of \( a_i \) (negatively) and are independent of industry dispersion (as long as \( \frac{a_i}{N} \leq \frac{1}{2} \) for all \( i \)).

4. The welfare gain of an individual with a stake in industry \( i \) may increase or decrease as a result of a positive economy-wide productivity shock. The expected mark-up over the status quo price for good \( i \) is \( \theta - a_i \), hence the expected per-capita change in capital rent is \( \theta \left( \frac{\theta}{3} - a_i \right) \). Unlike industry 1 in Case 1, legislative bargaining may hurt capital-owners. There are basically two effects working in opposite ways. First, industry 1 cannot obtain a mark-up as high as in Case 1, therefore the effect of an increase in \( \theta \) through the rent effect is lower (may even be negative). Second, since a coalition with another industry is required to pass a proposal, one another industry’s output price will be higher than what industry 1 prefers it to be. This in turn, compared to Case 1, will increase the welfare gain due to consumption and tariff revenue for industry 1. In particular, the change in welfare gain due

\(^{30}\)For instance, when \( a_1 = a_2 = a_3 = \theta \), \( v_i - w_i(p^*) = \frac{3}{2} \theta^2 > 0 \) for all \( i \). Similarly, when \( a_1 = a_2 = a_3 = \frac{\theta}{3} \), \( v_i - w_i(p^*) = -\frac{1}{2} \theta^2 < 0 \) for all \( i \).
to consumption and tariff revenue may be negative or positive (the magnitude of change is \( \sum_{i=1}^{3} n_i K_i a_i - \frac{56}{9} < 0 \)) but still higher than under Case 1. Here, rent effect dominates such that welfare gain is lower compared to Case 1, but we cannot determine whether the net effect of positive productivity shock on welfare change is positive or not. However, we can say that for industry \( i \), the lower (higher) is the population share \( n_i K_i (n_j K_j, j \neq i) \) and \( a_i (a_j, j \neq i) \), the more positive is the effect of productivity shock on welfare change.

As claimed before, a recognized legislator always employs mixed strategies. The reason for this result is as follows. In equilibrium, one of the industries is always left outside the winning coalition and receives a price which is below its free-trade price by its size (i.e., \( p_k = p^*_k - \theta n_k K_k \)). A larger excluded industry means a bigger price distortion in that good’s price, which, in turn, allows the proposer to appropriate a larger welfare gain for his district. Therefore, as an industry becomes larger, the probability that it is selected as a coalition partner decreases. However, legislators still employ mixed strategies in selecting their coalition partners, because selecting a particular industry with pure strategy simply raises the bargaining power of the representatives of that industry. We should note that in the mixed strategy equilibria, randomization probabilities are not unique although they all lead to the same set of payoffs as stated below.\textsuperscript{31}

**Proposition 3** All SSPE are payoff-equivalent.

From the analysis of Case 1 and Case 2, we can conclude that the welfare of an individual with a stake in industry \( i \) is:

(i) non-decreasing with respect to industry \( i \)’s dispersion (where we keep the total output of industry \( i \) the same) and non-increasing with respect to the dispersion of industry \( j \neq i \).

(ii) decreasing with respect to the status quo tariff/subsidy and population share of industry \( i \) and increasing with respect to the status quo tariff/subsidy and population share of industry \( j \neq i \).

\textsuperscript{31}The same multiplicity is also present in the standard Baron-Ferejohn game. Eraslan (2002) shows that all SSPE in the Baron-Ferejohn game are payoff equivalent when the recognition probabilities are asymmetric.
(iii) in Case 1, for the industry with agenda setting power (for the other industries),
it is increasing (decreasing) with respect to a positive economy-wide productivity shock.
On the other hand, in Case 2, for any industry, it may increase or decrease with respect
to a positive economy-wide productivity shock.

4 Discussion

In this section, we discuss three points related to possible extensions of our model. The
first one regards the number of industries. We have, for simplicity, considered only three
(manufacturing) industries. It is possible to generalize this to a larger number industries.
The main intuition still holds. If one industry has majority representation, then that
industry gains the most. On the other hand, if none of the industries have majority, then
it is the total production and status quo tariff/subsidy that determine the gains for each
industry. Again, the industry with the lowest output and the lowest tariff/subsidy gains
the most. For example, consider four industries with the following distribution: \( \frac{n_1}{N} = 0.4, \)
\( \frac{n_2}{N} = 0.3, \frac{n_3}{N} = 0.25, \frac{n_4}{N} = 0.05. \) In this example, industry 4 is too small to be valuable
as a partner in any coalition. However, it is still possible for industry 4 to be the biggest
winner as long as \( a_4 \) is small enough and \( a_i \)'s for \( i = 1, 2, 3 \) are large enough.

The second point is about the bargaining procedure. We have assumed that once an
agreement is reached, bargaining ends. Instead, assume that legislators bargain every
period and that if an agreement is reached in the previous period, it constitutes status quo
for the current period. In the context of a fixed-pie game, Kalandrakis (2004) shows
that there is a Markov equilibrium in which, irrespective of the initial status quo payoffs,
the proposer is eventually able to achieve his first-best (i.e., take the whole pie). The
intuition is as follows. In every period, players reach an agreement in which there is
at least one player who receives nothing. Since this constitutes the status quo for the
following period, the proposer in the following period selects the player with zero payoff
as the coalition partner, and is thus able to take the whole pie for himself. The same
logic is also at work in our model. In every period, one industry, say industry \( i \), will be
left out of the winning coalition, getting a price \( p_i = p_i^* - \theta n_i K_i \), and having the lowest
status quo payoff in the following period. Thus, if a legislator representing industry
\( j \neq i \) becomes the proposer in the following period, he chooses industry \( i \) as his coalition partner, and is able to appropriate higher gains. After some time in the game, whoever is the proposer (say industry \( i \)) will propose \( p^{UI} \) (unconstrained maximization prices) and it will be accepted. This result is true irrespective of the discount factor and the status quo prices. However, although an industry is able to achieve its first-best when a legislator representing it becomes the proposer, it receives the worst possible payoff in the remaining scenarios. On average, it actually does worse relative to when bargaining ends once an agreement is reached.\(^{32}\) Hence, if we add an initial stage to our model where players can decide whether to play the game once or continuously, they will choose to play once. When \( \frac{n_i}{N} > \frac{1}{2} \), on the other hand, legislators will agree on \( p^{UI} \) after a few periods. In this case, the expected payoffs remain the same as in our model.

The final point is about the discount factor. For analytical convenience, we have considered the limiting case in which \( \delta \) approaches 1. In the context of a Baron-Ferejohn fixed-pie game with asymmetric recognition probabilities (as in our paper), Eraslan (2002) shows that an SSPE with fully mixed strategies does not exist when \( \delta \) is below a certain threshold. This is also true in our game. When \( \delta < 1 \), depending on the values of \( (\frac{n_1}{N}, \frac{n_2}{N}, \frac{n_3}{N}) \) and \((a_1, a_2, a_3)\), one or more industries may use pure strategies in choosing their coalition partners. For instance, when \( \frac{n_1}{N} = \frac{n_2}{N} = \frac{n_3}{N} \), the industry with the highest \( a_i \) may never be chosen as a coalition partner if \( \delta \) is sufficiently low. Similarly, when \( a_1 = a_2 = a_3 \), the industry with the highest \( \frac{n_i}{N} \) may never be chosen as a coalition partner if \( \delta \) lies below a threshold. However, our qualitative results would remain true. In particular, the ranking of welfare gains remains the same; i.e., the industry with the lowest \( a_i \) does the best while the one with the highest \( a_i \) does the worst. When \( a_1 = a_2 = a_3 \), all industries are equally well off unless one industry is sufficiently dispersed.

\(^{32}\)Once the game converges to a stationary stage in which the proposer is able to achieve its first-best, industry \( i \)'s per-period continuation payoff becomes:

\[
V_i = W_i(p^*) + L_i \left[ \theta \left( \frac{n_i}{N} \theta - a_i \right) - \frac{\theta^2 - (a_1^2 + a_2^2 + a_3^2)}{2} \right].
\]

Since \( \frac{n_i}{N} \leq \frac{1}{2} \), this is less than what industry \( i \) expects in our game:

\[
V_i = W_i(p^*) + L_i \left[ \theta \left( \frac{5}{9} \theta - a_i \right) - \frac{\theta^2 - (a_1^2 + a_2^2 + a_3^2)}{2} \right].
\]
and $\delta$ is sufficiently low so that it is never chosen as a coalition partner. In this case, that industry does better than others. If the expected welfare gains are positive (negative), then they would decrease (increase) with a lower $\delta$, eventually converging to zero as $\delta$ goes to zero.

5 Conclusion

We have developed a model of legislative trade-policy making in a setting of distributive politics. A small open economy has many districts each one of which is associated with a particular industry. Thus, there is a conflict among districts hinged on industry attachment. Trade policy is determined collectively in the legislature as a result of bargaining among legislators, each of whom seeks to serve the interests of the district he represents. The legislative process is modeled as a multilateral sequential bargaining game à la Baron and Ferejohn (1989).

Our analysis indicates that the expected welfare gains (which may be negative) accruing to each industry due to a new trade policy depends on three main factors: (i) industry dispersion, (ii) initial tariff/subsidy level, and (iii) aggregate output of each industry. In particular, if an industry is politically dispersed enough to hold the majority of seats in the legislature, then, independent of the remaining two factors, it guarantees to benefit the most from the new trade policy due to its agenda-setting power. On the other hand, if no industry has majority representation in the legislature, then industry dispersion does not matter. Rather, it is the aggregate output and the initial tariff/subsidy levels industries have that determine the welfare gains.

In short, our model is dynamic and considers a parliamentary setting that stresses the importance of institutional structure on trade-policy formation. Furthermore, it is rich enough to encompass the findings of the existing literature as well as to incorporate new elements to them by analyzing the effects of legislative bargaining and status quo on trade policy formation.

In our model, we have considered the legislature as an ultimate decision-making body without any outside interference. One might alternatively consider an executive with veto power, such as a President. In such a situation, even though the President does not
have a decision-making authority, she can veto some proposals that are not in agreement with her own agenda. For example, in the United States, legislators come from plurality elections in small districts whereas the President is elected in national elections. The difference in constituents of legislatures and the Presidency could plausibly affect the preferences and goals that each brings to legislative bargaining. This extension is outside the scope of our paper, however.\textsuperscript{33}
Appendix

Proof of Proposition 2. We will express the prices in terms of mark-ups. When a legislator representing industry \(i\) is selected as the proposer and chooses industry \(j \neq i\) as the coalition partner, we denote the chosen mark-ups as follows: \(x_{ij}\) is the mark-up industry \(i\) (the proposer) gets, \(y_{ij}\) is the mark-up industry \(j\) (the coalition partner) gets and \(z_{ij}\) is the mark-up industry \(k \neq i, j\) (the industry outside the coalition) gets.

Now, suppose a legislator representing industry \(i\) is selected as the proposer and he chooses industry \(j \neq i\) as the coalition partner. His maximization problem is

\[
\max_{x_{ij}, y_{ij}, z_{ij}} W_i(x_{ij}, y_{ij}, z_{ij}) \text{ s.t. } W_j(x_{ij}, y_{ij}, z_{ij}) \geq V_j,
\]

where

\[
W_i(x_{ij}, y_{ij}, z_{ij}) = W_i(p^*) + K_i \left[ \theta x_{ij} - \frac{1}{2} [(x_{ij} + a_i)^2 + (y_{ij} + a_j)^2 + (z_{ij} + a_k)^2 - c] \right],
\]

\[
W_j(x_{ij}, y_{ij}, z_{ij}) = W_j(p^*) + K_j \left[ \theta y_{ij} - \frac{1}{2} [(x_{ij} + a_i)^2 + (y_{ij} + a_j)^2 + (z_{ij} + a_k)^2 - c] \right].
\]

and \(c = a_1^2 + a_2^2 + a_3^2\).

The Lagrangian can be expressed as

\[
L(x_{ij}, y_{ij}, z_{ij}) = W_i(x_{ij}, y_{ij}, z_{ij}) + \lambda_{ij}(W_j(x_{ij}, y_{ij}, z_{ij}) - V_j),
\]

where \(\lambda_{ij}\) is the Lagrange multiplier when a legislator representing industry \(i\) is selected as the proposer and he chooses industry \(j \neq i\) as the coalition partner. It represents the cost to the proposing legislator of obtaining the additional votes needed to pass the proposal.

The first-order conditions, after simplification, are

\[
x_{ij} = \frac{\theta}{1 + \frac{K_i}{K_j} \lambda_{ij}} - a_i,
\]

\[
y_{ij} = \frac{K_i}{K_j} \lambda_{ij} \theta - a_j,
\]

\[
z_{ij} = -a_k.
\]

\footnote{See Celik et al. (2010).
We first show that, in an SSPE in which all proposers employ mixed strategies in choosing their coalition partners, the value of \( \frac{K_j}{K_i} \lambda_{ij} \) is independent of the identity of the proposer and of the coalition partner, i.e., \( \frac{K_j}{K_i} \lambda_{ij} = \lambda \) for all \( i \neq j, i, j = 1, 2, 3 \). This follows from the following two observations. First, a legislator would employ a mixed strategy in choosing a coalition partner only when the ex-post payoff his district enjoys is the same under each alternative. In other words, when a legislator representing industry \( i \) is selected as the proposer, he randomly picks an industry as a coalition partner if, for all \( i \neq j \neq k \),

\[
W_i(x_{ij}, y_{ij}, z_{ij}) = W_i(x_{ik}, y_{ik}, z_{ik})
\]

\[
\Leftrightarrow \quad \theta x_{ij} = \frac{1}{2} \left[ (x_{ij} + a_i)^2 + (y_{ij} + a_j)^2 + (z_{ij} + a_k)^2 - c \right]
\]

\[
= \quad \theta x_{ik} = \frac{1}{2} \left[ (x_{ik} + a_i)^2 + (y_{ik} + a_j)^2 + (z_{ik} + a_k)^2 - c \right].
\]

Using the equilibrium values of \( (x_{ij}, y_{ij}, z_{ij}) \) and \( (x_{kj}, y_{kj}, z_{kj}) \), we have

\[
\frac{\theta^2}{1 + \frac{K_j}{K_i} \lambda_{ij}} - \frac{1}{2} \left[ \left( 1 + \left( \frac{K_j}{K_i} \lambda_{ij} \right)^2 \right) \theta^2 \right] = \frac{\theta^2}{1 + \frac{K_k}{K_i} \lambda_{ik}} - \frac{1}{2} \left[ \left( 1 + \left( \frac{K_k}{K_i} \lambda_{ik} \right)^2 \right) \theta^2 \right].
\]

It is easy to see that this is possible only if \( \frac{K_j}{K_i} \lambda_{ij} = \frac{K_k}{K_i} \lambda_{ik} \). Second, when industry \( j \) is chosen as a coalition partner, the ex-post welfare it is offered would be independent of the identity of the proposer, because whoever is the proposer always offers an ex-post welfare of \( V_j \) to this industry, otherwise the proposal is rejected. Thus, for any \( i \neq j \neq k \),

\[
W_j(x_{ij}, y_{ij}, z_{ij}) = W_j(x_{kj}, y_{kj}, z_{kj})
\]

\[
\Leftrightarrow \quad \theta y_{ij} = \frac{1}{2} \left[ (x_{ij} + a_i)^2 + (y_{ij} + a_j)^2 + (z_{ij} + a_k)^2 - c \right]
\]

\[
= \quad \theta y_{kj} = \frac{1}{2} \left[ (x_{kj} + a_k)^2 + (y_{kj} + a_j)^2 + (z_{kj} + a_i)^2 - c \right].
\]

Using the equilibrium values of \( (x_{ij}, y_{ij}, z_{ij}) \) and \( (x_{kj}, y_{kj}, z_{kj}) \), we have

\[
\frac{K_j}{K_i} \lambda_{ij} \theta^2 - \frac{1}{2} \left[ \left( 1 + \left( \frac{K_j}{K_i} \lambda_{ij} \right)^2 \right) \theta^2 \right] = \frac{K_k}{K_i} \lambda_{kj} \theta^2 - \frac{1}{2} \left[ \left( 1 + \left( \frac{K_k}{K_i} \lambda_{kj} \right)^2 \right) \theta^2 \right].
\]
Again, this is possible only if $\frac{K_i^j \lambda_{ij}}{K_i^k} = \frac{K_i^j \lambda_{kj}}{K_i^k}$, Together with the earlier observation, $\frac{K_i^j \lambda_{ij}}{K_i^k} = \frac{K_i^k \lambda_{ik}}{K_i^k}$, which implies that $\frac{K_i^j \lambda_{ij}}{K_i^k} = \lambda$ for all $i \neq j, i, j = 1, 2, 3$. Next, we find the equilibrium value of $\lambda$ in an SSPE in which all proposers employ mixed strategies in choosing their coalition partners. We first write down the equilibrium net ex-post welfare a district gets in three distinct cases.

(i) when a legislator representing a district that produces good $j$ is selected as the proposer:

$$W_j^{\text{proposer}} = W_j(p^*) + K_j \left[ \frac{\theta^2}{1 + \lambda} - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda^2) - c} \right) \right].$$

(ii) when the districts that produce good $j$ are selected as a coalition partner:

$$W_j^{\text{partner}} = W_j(p^*) + K_j \left[ \frac{\lambda \theta^2}{1 + \lambda} - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda^2) - c} \right) \right].$$

(iii) when the districts that produce good $j$ are left outside the coalition:

$$W_j^{\text{outside}} = W_j(p^*) + K_j \left[ -\theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda^2) - c} \right) \right].$$

We next express the equilibrium continuation welfare of a district. To do so, we need to introduce randomization probabilities. Let $s_{ij}$ denote the probability that a legislator representing a district that produces good $i$ chooses the districts producing good $j$ as a coalition partner. Then, $V_j$ can be expressed as

$$V_j = \frac{n_j}{N} [s_{ij} W_j^{\text{proposer}} + (1 - s_{ij}) W_j^{\text{partner}}] + \frac{n_k}{N} [s_{kj} W_j^{\text{partner}} + (1 - s_{kj}) W_j^{\text{outside}}].$$

((A.1))

After simplification, this becomes

$$V_j = W_j(p^*) + K_j \left[ \frac{\theta^2}{1 + \lambda} \left( \frac{n_j}{N} + \left( s_{ij} \frac{n_i}{N} + s_{kj} \frac{n_k}{N} \right) \lambda \right) - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda^2) - c} \right) \right].$$

Next, observe that the maximization problem implies $W_j^{\text{partner}} = V_j$ (since the constraint is binding in equilibrium). Hence, it must be true that

$$\sum_{j=1}^{3} \frac{W_j^{\text{partner}} - W_j(p^*)}{K_j} = \sum_{j=1}^{3} \frac{V_j - W_j(p^*)}{K_j}.\)
Also note that
\[
\sum_{j=1 \atop i \neq k \neq j}^{3} \left( s_{ij} \frac{n_i}{N} + s_{kj} \frac{n_k}{N} \right) = \left( s_{12} \frac{n_1}{N} + s_{23} \frac{n_3}{N} \right) + \left( s_{13} \frac{n_1}{N} + s_{23} \frac{n_2}{N} \right) + \left( s_{21} \frac{n_2}{N} + s_{31} \frac{n_3}{N} \right)
\]
\[
= (s_{12} + s_{13}) \frac{n_1}{N} + (s_{21} + s_{23}) \frac{n_2}{N} + (s_{31} + s_{32}) \frac{n_3}{N}
\]
\[
= \frac{n_1 + n_2 + n_3}{N}
\]
\[
= 1.
\]

The condition \[
\sum_{j=1}^{3} \frac{W_{\text{partner},j} - W_j(p^*)}{K_j} = \sum_{k=1}^{3} \frac{V_k - W_j(p^*)}{K_j}
\]
can now be expressed as
\[
\frac{3\lambda \theta^2}{1 + \lambda} - \theta \sum_{j=1}^{3} a_j \frac{3}{2} \left( \frac{1 + \lambda^2}{1 + \lambda} \theta^2 - c \right) = \frac{\theta^2}{1 + \lambda} (1 + \lambda) - \sum_{j=1}^{3} \theta a_j \frac{3}{2} \left( \frac{(1 + \lambda^2)}{(1 + \lambda)^2} \theta^2 - c \right)
\]
\[
\iff \lambda = \frac{1}{2}.
\]

So, the value of \( \lambda \) can be determined without the knowledge of the randomization probabilities. Plugging the equilibrium value of \( \lambda \) into the mark-ups we found earlier gives
\[
x_{ij} = \frac{2\theta}{3} - a_i,
\]
\[
y_{ij} = \frac{\theta}{3} - a_j,
\]
\[
z_{ij} = -a_k.
\]

These mark-ups lead to the set of proposed prices in Proposition 2. The final step of the proof is to show that there is an interior solution to all of the randomization probabilities (this is what we assumed at the beginning of the proof). Since the continuation per-period welfare of a district is equal to its ex-post welfare when chosen as a coalition partner (by the maximization problem), i.e., \( V_j = W_{j, \text{partner}} \), we have
\[
\frac{\theta^2}{1 + \lambda} \left( \frac{n_j}{N} + \left( s_{ij} \frac{n_i}{N} + s_{kj} \frac{n_k}{N} \right) \lambda \right) = \frac{\lambda \theta^2}{1 + \lambda}.
\]
\[
\iff s_{ij} \frac{n_i}{N} + s_{kj} \frac{n_k}{N} = 1 - 2 \frac{n_j}{N}.
\]

For simplicity, let \( s_{12} = s_1, s_{23} = s_2 \) and \( s_{31} = s_3 \). Then,
\[
s_1 \frac{n_1}{N} + (1 - s_3) \frac{n_3}{N} = 1 - 2 \frac{n_2}{N},
\]
\[
s_2 \frac{n_2}{N} + (1 - s_1) \frac{n_1}{N} = 1 - 2 \frac{n_3}{N},
\]
\[
s_3 \frac{n_3}{N} + (1 - s_2) \frac{n_2}{N} = 1 - 2 \frac{n_1}{N}.
\]
Note that these equations are linearly dependent (two of them imply the third), so we lose one degree of freedom. It is easy to check that, when \( \frac{n_1}{N} \leq \frac{n_2}{N} \leq \frac{n_3}{N} \leq \frac{1}{3} \), there is an interior solution in which \( s_i \in [0,1] \) for all \( i \). To see this, fix \( s_3 \) and express \( s_1 \) and \( s_2 \) in terms of \( s_3 \):

\[
\begin{align*}
s_1 &= 1 - 2 \frac{n_2}{N} - (1 - s_3) \frac{n_3}{N}, \\
s_2 &= 1 - \frac{1 - n_3}{N} - s_3 \frac{n_3}{N}.
\end{align*}
\]

Any value of \( s_3 \in \left[0, \frac{1-2n_2}{n_3}\right] \) yields \( s_1, s_2 \in [0,1] \).

It is important to note that an industry may select its coalition partner with pure strategy. However, there are limitations. Feasible solutions (i.e., the solutions that satisfy \( s_i \in [0,1] \) for all \( i \)) are

\[
(s_1, s_2, s_3) = \left( \frac{1 - 2 \frac{n_3}{N} - \frac{n_3}{N}}{\frac{n_1}{N}}, 1 - \frac{1 - 2 \frac{n_3}{N}}{\frac{n_2}{N}}, 0 \right),
\]

\[
(s_1, s_2, s_3) = \left( \frac{1 - \frac{n_3}{N} - \frac{n_2}{N}}{\frac{n_1}{N}}, 1, \frac{1 - 2 \frac{n_3}{N}}{\frac{n_2}{N}} \right).
\]

All three industries may use pure strategies only when \( \frac{n_1}{N} = \frac{n_2}{N} = \frac{n_3}{N} = \frac{1}{3} \). In this case, \( s_1 = s_2 = s_3 \) in all SSPE, so \( (s_1, s_2, s_3) = (1, 1, 1) \) and \( (s_1, s_2, s_3) = (0, 0, 0) \) are both possible. Similarly, when \( \frac{n_1}{N} = \frac{1}{2} \), industries 2 and 3 may use pure strategies. In fact, \( (s_1, s_2, s_3) = (1 - 2 \frac{n_2}{N}, 1, 0) \) is the unique SSPE in this case. Other than these two special cases, only industry 2 or industry 3 may select its coalition partner with pure strategy.

Finally, the continuation per-period welfare of a district can be expressed, for all \( j = 1, 2, 3 \), as

\[
V_j = W_j(p^*) + K_j \left[ \frac{\lambda \theta^2}{1 + \lambda} - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right]
\]

\[
= W_j(p^*) + K_j \left[ \theta \left( \frac{\theta}{3} - a_j \right) - \frac{5 \theta^2 - c}{2} \right].
\]

**Proof of Proposition 3.** We will prove payoff uniqueness by showing that the randomization strategies that are not already accounted for in the proof of Proposition 2 cannot arise in an SSPE. Possible scenarios include all three industries or only two industries using pure strategies. We will eliminate all possibilities step by step. When only one industry uses pure strategy, the proof of Proposition 2 is perfectly applicable, so possible
SSPE involve industry 2 choosing industry 3 with pure strategy, or vice versa. All other possibilities can be ruled out because they lead to non-interior solutions (i.e., \( s_i \notin [0, 1] \) for some \( i \)).

Before proceeding with the proof, we would like make an important observation that will prove very helpful. The equilibrium welfare function of the proposer,

\[
W_{j}^{\text{propoer}} = W_j(p^*) + K_j \left[ \frac{\theta^2}{1 + \lambda} - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2)}{(1 + \lambda)^2} - c \right) \right],
\]

is strictly increasing in \( \frac{1}{1+\lambda} \) for all \( \lambda > 0 \). This means that if \( \frac{K_i}{K_k} \lambda_{ik} > \frac{K_j}{K_k} \lambda_{kj} \), then we must have \( s_{ik} = 0 \) because, otherwise, industry \( i \) can profitably deviate by mimicking industry \( k \) and selecting industry \( j \) as a partner rather than industry \( k \).

**Observation 1.** The following configuration cannot arise in an SSPE: \( s_{ik} = s_{jk} = 1 \) for \( i \neq j \neq k \).

This is the case when one industry is selected as a coalition member with pure strategy by each one of the other two industries. Intuitively, this puts industry \( k \) in a veto-player position which enables it to achieve its first-best. This, in turn, leads to a profitable deviation by each one of the other two industries. To see this, we follow the same steps as in the proof of Proposition 2. Under the strategies \( s_{ik} = s_{jk} = 1 \), both industry \( i \) and \( j \) give industry \( k \) its continuation payoff, so,

\[
\frac{K_k}{K_i} \lambda_{ik} = \frac{K_k}{K_j} \lambda_{kj}.
\]

For industry \( k \) to randomize between the other two industries according to \( s_{ki} \in [0, 1] \), it must also be true that

\[
\frac{K_i}{K_k} \lambda_{ki} = \frac{K_j}{K_k} \lambda_{kj}.
\]

Let \( \frac{K_i}{K_k} \lambda_{ik} = \frac{K_k}{K_j} \lambda_{jk} = \mu \) and \( \frac{K_i}{K_k} \lambda_{ki} = \frac{K_k}{K_j} \lambda_{kj} = \lambda \). Then, for industries \( i \) and \( j \) not to deviate from \( s_{ik} = s_{jk} = 1 \), we must have \( \mu \leq \lambda \). The equilibrium ex-post welfare of industry \( k \) as the proposer and as a partner can be expressed as

\[
W_{k}^{\text{propoer}} = W_k(p^*) + K_k \left[ \frac{\theta^2}{1 + \lambda} - \theta a_k - \frac{1}{2} \left( \frac{(1 + \lambda^2)}{(1 + \lambda)^2} - c \right) \right],
\]
\[ W_k^{\text{partner}} = W_k(p^*) + K_k \left[ \frac{\mu \theta^2}{1 + \mu} - \theta a_k - \frac{1}{2} \left( \frac{(1 + \mu^2) \theta^2}{(1 + \mu)^2} - c \right) \right]. \]

By ((A.1)), then,

\[ V_k = W_k(p^*) + K_k \left[ \frac{n_k \theta^2}{N} + \frac{(n_i + n_j) \mu \theta^2}{1 + \mu} - \theta a_k - \frac{1}{2} \left( \frac{n_k (1 + \lambda^2) \theta^2}{N} + \frac{(n_i + n_j) (1 + \mu^2) \theta^2}{N} - c \right) \right]. \]

This leads, by the constraint \( \frac{W_k^{\text{partner}} - W_k(p^*)}{K_k} = \frac{V_k - W_k(p^*)}{K_k} \), to

\[ \frac{\mu \theta^2}{1 + \mu} - \frac{1}{2} \left( \frac{(1 + \mu^2) \theta^2}{(1 + \mu)^2} \right) = \frac{\theta^2}{1 + \lambda} - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} \right). \]

This is possible only when \( \mu = 1/\lambda \). Together with the earlier condition that \( \mu \leq \lambda \), this requires \( \lambda \geq 1 \). Repeating the same steps for industry \( i \), we have

\[ W_i^{\text{partner}} = W_i(p^*) + K_i \left[ \frac{\lambda \theta^2}{1 + \lambda} - \theta a_i - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right], \]

\[ V_i = W_i(p^*) + K_i \left[ \frac{n_i \theta^2}{1 + \mu} + \frac{n_i \theta^2 s_{ki}}{1 + \lambda} - \theta a_i - \frac{1}{2} \left( \frac{(n_i + n_j) (1 + \mu^2) \theta^2}{N} + \frac{n_k (1 + \lambda^2) \theta^2}{N} - c \right) \right] = W_i(p^*) + K_i \left[ \frac{n_i \lambda \theta^2}{1 + \lambda} + \frac{n_i \theta^2 s_{ki}}{1 + \lambda} - \theta a_i - \frac{1}{2} \left( (n_i + n_j + n_k) (1 + \lambda^2) \theta^2 - c \right) \right], \]

where the last line uses the fact that \( \mu = 1/\lambda \). By the constraint \( \frac{W_i^{\text{partner}} - W_i(p^*)}{K_i} = \frac{V_i - W_i(p^*)}{K_i} \), and \( \frac{n_i + n_j + n_k}{N} = 1 \), then,

\[ \frac{\lambda \theta^2}{1 + \lambda} - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} \right) = \left( \frac{n_i}{N} + \frac{n_k}{N s_{ki}} \right) \frac{\lambda \theta^2}{1 + \lambda} - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} \right), \]

which can be satisfied only when \( \lambda = 0 \). Thus, we reach a contradiction.

**Observation 2.** Unless \( \frac{n_1}{N} = \frac{n_2}{N} = \frac{n_3}{N} \), the following configuration cannot arise in an SSPE: \( s_{ij} = s_{jk} = s_{ki} = 1 \) for \( i \neq j \neq k \).

Under this configuration, each industry is selected as a coalition member by one of the other two industries. As in the proof of Observation 1, for each industry to play according to \( s_{ij} = s_{jk} = s_{ki} = 1 \) and not to deviate to choosing another partner, we must
have

\[
\frac{K_i}{K_j} \lambda_{ij} \leq \frac{K_k}{K_j} \lambda_{jk}, \\
\frac{K_k}{K_j} \lambda_{jk} \leq \frac{K_i}{K_j} \lambda_{ki}, \\
\frac{K_i}{K_k} \lambda_{ki} \leq \frac{K_j}{K_i} \lambda_{ij}.
\]

These three inequalities can be satisfied only when \(\frac{K_i}{K_j} \lambda_{ij} = \frac{K_k}{K_j} \lambda_{jk} = \frac{K_i}{K_k} \lambda_{ki} = \lambda\).

Next, using \(((A.1))\), we express the continuation payoff of each industry:

\[
V_i = W_i(p^*) + K_i \left[ \frac{\left( \frac{n_i}{N} + \frac{n_k}{N} \lambda \right) \theta^2}{1 + \lambda} - \theta a_i - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right],
\]

\[
V_j = W_j(p^*) + K_j \left[ \frac{\left( \frac{n_j}{N} + \frac{n_i}{N} \lambda \right) \theta^2}{1 + \lambda} - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right],
\]

\[
V_k = W_k(p^*) + K_k \left[ \frac{\left( \frac{n_k}{N} + \frac{n_j}{N} \lambda \right) \theta^2}{1 + \lambda} - \theta a_k - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right].
\]

By the constraint \(\frac{W_l(p^*) - W_l(p^*)}{K_l} = \frac{V_l - W_l(p^*)}{K_l}\) for each \(l = i, j, k\), then,

\[
\lambda = \frac{n_i}{N} + \frac{n_k}{N} \lambda, \\
\lambda = \frac{n_j}{N} + \frac{n_i}{N} \lambda, \\
\lambda = \frac{n_k}{N} + \frac{n_j}{N} \lambda.
\]

Summing each side and using \(\frac{n_i + n_j + n_k}{N} = 1\) gives

\[
\lambda = \frac{1}{2}.
\]

Plugging the value of \(\lambda\) into the above equations leads to

\[
\frac{n_i}{N} + \frac{n_k}{2} = \frac{n_j}{N} + \frac{n_i}{2} = \frac{n_k}{N} + \frac{n_j}{2} = \frac{1}{2}.
\]

But this is possible only when \(\frac{n_i}{N} = \frac{n_k}{N} = \frac{n_j}{N} = \frac{1}{3}\).

**Observation 3.** The following configuration cannot arise in an SSPE: \(s_{ij} = s_{jk} = 1\) and \(s_{ki} \in (0, 1)\) for \(i \neq j \neq k\).

Under this configuration, industry \(i\) is selected as a coalition member less often than the others, which lowers its equilibrium continuation payoff. This, in turn, induces industry \(k\) to choose industry \(i\) with pure strategy. Note that we have assumed \(s_{ki} \in (0, 1)\)
since $s_{ki} = 0$ can be ruled out by Observation 1 while $s_{ki} = 1$ can be ruled out by Observation 2.

For each industry to play according to $s_{ij} = s_{jk} = 1$ and $s_{ki} \in (0, 1)$, we must have

\[
\frac{K_j}{K_i} \lambda_{ij} \leq \frac{K_k}{K_j} \lambda_{jk}, \\
\frac{K_k}{K_j} \lambda_{jk} \leq \frac{K_i}{K_k} \lambda_{ki}, \\
\frac{K_i}{K_k} \lambda_{ki} = \frac{K_j}{K_i} \lambda_{ij}, \\
\frac{K_j}{K_k} \lambda_{kj} = \frac{K_i}{K_j} \lambda_{ij}.
\]

These four inequalities can again be satisfied only when $\frac{K_j}{K_i} \lambda_{ij} = \frac{K_k}{K_j} \lambda_{jk} = \frac{K_i}{K_k} \lambda_{ki} = \frac{K_j}{K_k} \lambda_{kj} = \lambda$.

Next, using ((A.1)), we express the continuation payoff of each industry:

\[
V_i = W_i(p^*) + K_i \left[ \left( \frac{n_i + n_k s_{ki} \lambda}{N} \right) \frac{\theta^2}{1 + \lambda} - \theta a_i - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right], \\
V_j = W_j(p^*) + K_j \left[ \left( \frac{n_j + n_i s_{kj} \lambda}{N} \right) \frac{\theta^2}{1 + \lambda} - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right], \\
V_k = W_k(p^*) + K_k \left[ \left( \frac{n_k + n_j \lambda}{N} \right) \frac{\theta^2}{1 + \lambda} - \theta a_k - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right].
\]

By the constraint $\frac{W_{l_{\text{partner}}} - W_l(p^*)}{K_l} = \frac{V_l - W_l(p^*)}{K_l}$ for each $l = i, j, k$, then,

\[
\lambda = \frac{n_i}{N} + \frac{n_k}{N} s_{ki} \lambda, \\
\lambda = \frac{n_j}{N} + \left( \frac{n_i}{N} + \frac{n_k}{N} s_{kj} \right) \lambda, \\
\lambda = \frac{n_k}{N} + \frac{n_j}{N} \lambda.
\]

Summing each side and using $\frac{n_i + n_j + n_k}{N} = 1$ gives

\[
\lambda = \frac{1}{2}.
\]

Using $\lambda = \frac{1}{2}$, $s_{ki} = \frac{1 - 2 \frac{n_i}{N}}{N}$ from the first of the above equations. Plugging this expression into the second equation and using $\frac{n_i + n_j + n_k}{N} = 1$ gives

\[
\frac{n_j}{N} + 2 \frac{n_i}{N} = 1.
\]
However, the third equation above implies

\[
\frac{2n_k}{N} + \frac{n_j}{N} = 1,
\]

so it must be that \( \frac{n_i}{N} = \frac{n_j}{N} = \frac{n_k}{N} = \frac{1}{3}. \) But then \( s_{ki} = \frac{1-2n_i}{N} = 1, \) which is a contradiction.

**Observation 4.** Unless \( \frac{n_i}{N} = 1/2, \) the following configuration cannot arise in an SSPE: \( s_{ij} = s_{ji} = 1 \) and \( s_{ki} \in (0,1) \) for \( i \neq j \neq k. \)

Under this configuration, industry \( k \) is never selected as a coalition member, which lowers its equilibrium continuation payoff. This, in turn, leads to a deviation by the other two industries. Note that we have assumed \( s_{ki} \in (0,1) \) since both \( s_{ki} = 0 \) and \( s_{ki} = 1 \) can be ruled out by Observation 1.

For each industry to play according to \( s_{ij} = s_{ji} = 1 \) and \( s_{ki} \in (0,1) \), we must have

\[
\frac{K_i}{K_k} \lambda_{ki} = \frac{K_j}{K_k} \lambda_{kj},
\]

\[
\frac{K_i}{K_k} \lambda_{ki} = \frac{K_j}{K_i} \lambda_{ij},
\]

\[
\frac{K_j}{K_k} \lambda_{kj} = \frac{K_j}{K_i} \lambda_{ij}.
\]

Thus, \( \frac{K_i}{K_k} \lambda_{ij} = \frac{K_j}{K_k} \lambda_{ji} = \frac{K_i}{K_k} \lambda_{ki} = \frac{K_j}{K_k} \lambda_{kj} = \lambda. \)

Next, using \(((A.1))\), we express the continuation payoff of industries \( i \) and \( j \):

\[
V_i = W_i(p^*) + K_i \left[ \frac{\left( \frac{n_i}{N} + \frac{n_j}{N} + \frac{n_k}{N} s_{ki} \right) \lambda}{1 + \lambda} \theta^2 - \theta a_i - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right],
\]

\[
V_j = W_j(p^*) + K_j \left[ \frac{\left( \frac{n_i}{N} + \frac{n_j}{N} + \frac{n_k}{N} s_{kj} \right) \lambda}{1 + \lambda} \theta^2 - \theta a_j - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right].
\]

By the constraints \( \frac{W_{l \text{partner}} - W_l(p^*)}{K_l} = \frac{V_l - W_l(p^*)}{K_l} \) for \( l = i,j \) (note that \( \frac{W_{k \text{partner}} - W_k(p^*)}{K_k} \) cannot be used here since industry \( k \) is never selected as a coalition member), then,

\[
\lambda = \frac{n_i}{N} + \left( \frac{n_j}{N} + \frac{n_k}{N} s_{ki} \right) \lambda,
\]

\[
\lambda = \frac{n_i}{N} + \left( \frac{n_i}{N} + \frac{n_k}{N} s_{kj} \right) \lambda.
\]
Summing each side and using \( \frac{n_i + n_j + n_k}{N} = 1 \) gives
\[
\lambda = \frac{n_i + n_j}{N}.
\]

Now, let us write down industry \( k \)'s continuation payoff:

\[
V_k = W_k(p^*) + K_k \left[ \frac{n_k \theta^2}{1 + \lambda} - \theta a_k - \frac{1}{2} \left( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} - c \right) \right].
\]

But then, industry \( i \) (same applies for industry \( j \), too) can deviate by selecting industry \( k \) as a coalition partner and offering a mark-up vector:
\[
(x_{ik}, y_{ik}, z_{ik}) = \left( \frac{\theta}{1 + \frac{n_k}{N}} - a_i, \frac{n_k \theta}{1 + \frac{n_k}{N}} - a_k, -a_j \right),
\]
in which case,
\[
W_i^{\text{proposer}} = W_i(p^*) + K_i \left[ \frac{\theta^2}{1 + \frac{n_k}{N}} - \theta a_i - \frac{1}{2} \left( \frac{(1 + \frac{n_k}{N})^2 \theta^2}{(1 + \frac{n_k}{N})^2} - c \right) \right]
\]
\[
W_k^{\text{partner}} = W_k(p^*) + K_k \left[ \frac{n_k \theta^2}{1 + \frac{n_k}{N}} - \theta a_k - \frac{1}{2} \left( \frac{(1 + \frac{n_k}{N})^2 \theta^2}{(1 + \frac{n_k}{N})^2} - c \right) \right].
\]

Note that, unless \( \frac{n_k}{N} = \frac{1}{2} \), we have \( \frac{n_k}{N} < \lambda = \frac{n_i + n_j}{N} \). Also note that \( \frac{(1 + \lambda^2) \theta^2}{(1 + \lambda)^2} \) is increasing in \( \lambda \) for \( \lambda < 1 \). Hence, both industry \( i \) and \( k \) benefit. This is a contradiction to the initial configuration. \( \blacksquare \)
References


