Organizing the Global Value Chain

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Abstract

Existing incomplete contracting models of the boundaries of the firm have largely focused on production functions in which all tasks are executed simultaneously: All suppliers provide their customized inputs and these are combined at the same time to generate the final good. In practice, however, production processes often take on a sequential nature, with a distinction between upstream stages (such as the procurement or manufacture of basic components) and downstream stages (such as complex assembly or distribution). In this setting, the relationship-specific investments made by suppliers in upstream stages can affect the incentives of parties involved in later downstream stages.

To capture these interactions, we develop a model of firm production with a continuum of uniquely sequenced production stages. In each stage, the firm contracts with a distinct supplier for the procurement of a customized stage-specific input. This setup yields predictions regarding the organizational choices of the firm along its production line. We find that when production-stage inputs are sufficiently close complements, the optimal choice involves the outsourcing of upstream stages and the integration of downstream stages. Intuitively, outsourcing elicits high levels of investment from upstream suppliers. The complementarity of upstream with downstream inputs in turn alleviates the under-investment problem for downstream suppliers, and the firm instead chooses integration downstream to retain a larger share of the realized output. On the other hand, when production stage inputs are sufficiently strong substitutes, the optimal choice involves integration of early production stages and possibly outsourcing of the later ones.

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1 Introduction

1.1 Overview

The existing literature on incomplete contracting models of the boundaries of the firm has focused on production functions in which all tasks are executed simultaneously: All suppliers provide their customized inputs at the same time, and these are immediately combined to generate the output of the firm. In practice, however, production processes often take on a sequential nature, with a distinction between upstream tasks (such as the procurement or manufacture of basic components) and downstream tasks (such as complex assembly stages, distribution, and retail). In these settings, the relationship-specific investments made by contracting parties in earlier stages of production can affect the incentives of parties involved in later production stages. Since the make-or-buy decision of the firm at each production stage affects the quantum of relationship-specific investments that suppliers undertake, this suggests a potentially rich pattern in firm’s make-or-buy decisions as one moves along the production line.

This note builds a model based on Acemoglu, Antràs and Helpman (2007) that systematically analyzes these firm organization issues in a setting where production stages are sequential in nature. In particular, what is the optimal mode of organization for firms vis-à-vis whether to integrate or outsource its upstream versus downstream stages of production? The model offered below is very stylized in that it treats production stages as symmetric, the only difference being in the order in which they must be performed. (This order of production is an exogenous technological requirement.) For now, supplier tasks are fully non-contractible.\footnote{We thus abstract from the complication of partial contractibility introduced in Acemoglu, Antràs and Helpman (2007) and Antràs and Helpman (2008).} This note focuses first on a one-country setting, so that the only decision problem the firm faces is over whether to integrate or outsource each stage of production. Towards the end, I point to some directions in which the model could be extended to account for location decisions in a two-country setting wherein international outsourcing becomes relevant.

In contrast to an earlier note, this model does not allow for \textit{ex ante} transfers between the firm and its suppliers, presumably because agents are credit-constrained. The make-or-buy decision of the firm will therefore affect the share of revenues that it obtains through bilateral bargaining with suppliers at each stage: Integration allows the firm to obtain a larger share of the revenue pie, but will dis-incentivize suppliers to undertake relationship-specific investments.

It turns out that the optimal integration-outsourcing pattern along the production line depends critically on whether production stages are more complements or substitutes, in a way that will be made formal below. If production stages are sufficiently complementary, then there exists a unique cutoff production stage such that all production prior to this cutoff stage is outsourced, while all downstream stages are integrated. On the other hand, if production stages are sufficiently substitutable, then the optimal organizational mode calls for integration along the entire production line. I elaborate on the intuition later below when I present these results.
2 A Model of Sequential Production with Incomplete Contracts

2.1 Benchmark Model

We start off with a simple benchmark model of firm behavior in the tradition of Antràs (2003, 2005), Antràs and Helpman (2004) and Acemoglu et al. (2007). In particular, we consider the problem of a final-good producer deciding on how to organize production of a particular good taking as given the behavior of other producers.

**Production:** The production function for the final good is given by:

\[
q = \theta \left( \int_0^1 x(j)^\alpha dj \right)^{\frac{1}{\alpha}}
\]  

(1)

where \( \theta \) is a productivity parameter and \( j \in [0, 1] \) indexes production stages, with a larger \( j \) corresponding to stages further downstream (closer to the final end product). Labor is the only factor of production. Supplier \( j \)’s labor effort is converted one-for-one into units of a stage-\( j \) intermediate input or component. I thus use \( x(j) \) interchangeably to refer to the labor effort of the supplier at stage \( j \), as well as to the quantity of intermediate inputs that that supplier delivers to the firm. The parameter \( \alpha \in (0, 1) \) captures how substitutable the different production stages are; the elasticity of substitution between production stages is \( 1/(1 - \alpha) \).

The role of the firm is to serve as the essential provider of the technological blueprint, and as the facilitator that brings together the suppliers and organizes the production chain. The production technology is inherently sequential in that downstream stages cannot commence until the components / services from upstream stages have been delivered.

Final good demand is iso-elastic, and is given by the familiar inverse demand function, \( Ap^{-1/(1-\rho)} \), where \( \rho \in (0, 1) \). (A is a constant that shifts the level of demand.)

**Suppliers’ considerations:** There is a large mass of potential suppliers that the firm can contract with for each stage of production. To take a concrete example, this means that a given car manufacturer has a large number of potential suppliers who could provide him with tyre components. We assume that the competitive market price for a unit of labor effort is \( c \).\(^2\) That said, once the firm and a particular supplier are locked into a bilateral relationship, the standard holdup problem kicks in (Williamson 1975, 1985).

The supplier at stage \( j \) chooses her discretionary effort level \( x(j) \). Tasks are fully non-contractible. All effort is relationship-specific in that the outcome from each stage \( j \) is an input/service that is customized to the firm’s needs towards producing the final good. This opens the door for holdup problems, and suppliers choose their effort levels taking into account the organizational form (integration versus outsourcing) that the firm has chosen. Since production stages are sequential, the supplier chooses \( x(j) \) conditional on observing the effort levels put in by all suppliers prior to stage

\(^2\)This is assumed to be the same for all stages of production. Once we move to a multi-country setting, with the possibility of integration or outsourcing abroad, we may have to allow \( c \) to vary with production stages.
j. However, she cannot condition on the effort levels of suppliers who come after stage j.

At the j-th stage of production, the supplier j and firm engage in bilateral bargaining over the division of revenue. The specific pie that the two parties bargain over is the incremental contribution to total revenue made by supplier j, net of the firm’s reservation position if it does not transact with the supplier. (The mathematical expression for this incremental contribution is derived below.) This bargaining framework bears some discussion. Supplier j does not receive any share of the incremental contribution of suppliers from prior to stage j, as we assume that the firm’s reservation option is to terminate the production chain and sell whatever output it has generated prior to stage j on the open market, leaving supplier j with nothing. Similarly, supplier j does not receive any of the incremental contributions to total revenue from stages downstream of j, since she cannot force the firm to hand over a share of these downstream revenues once she has delivered her input/component to the firm.3 We adopt a standard Nash bargaining solution to this bilateral bargaining problem between each supplier and firm, with the share of the ex-post gains going to the firm being a constant $\beta \in [0,1]$. We rule out side transfers between suppliers at different stages, either because firms can easily discontinue suppliers who collude and rehire a supplier from the open market, or because suppliers cannot credibly commit to such transfers.

**Firm’s considerations:** The firm does not contribute direct labor input in the production stages, but can nevertheless affect suppliers’ labor effort through its make-or-buy decisions. The choice of integration versus outsourcing affects the reservation option of the firm in the event of a breakdown in the bilateral relationship with the supplier. Concretely, we assume that the firm is able to lay claim and recover a fraction $\delta(j)$ of the relationship-specific investment, $x(j)$, made by supplier j, where $\delta(j) = 0$ if the firm chooses to outsource at stage j, and $\delta(j) = \delta \in (0,1)$ if the stage j is integrated within the boundaries of the firm. Integration thus ensures the firm a better fall-back position in terms of input recovery in the event of a breakdown in the bilateral relationship; however, this comes at the cost of lowering the incentives for suppliers to invest discretionary effort given that the revenue pie over which bargaining takes place is now smaller under integration.

**Timing of events:** We summarize the timeline of the game played between the firm headquarters and the continuum of suppliers ($j \in [0,1]$) as follows:

1. The firm posts contracts for suppliers for each stage $j \in [0,1]$ of the production process. The contract stipulates the organizational form – full integration within the boundaries of the firm or arms-length outsourcing – that the potential supplier will have to sign on to.

2. Suppliers apply for each contract, and the firm chooses one supplier for each production stage $j$.

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3This is somewhat incomplete. Why are suppliers not forward looking? They realize that their ultimate marginal contribution to final output is different (actually larger). Why not push harder in the negotiations? One way out is to appeal to credit constraints. The final-good producer and the supplier only meet at one point. The maximum amount of money that the final-good producer could possibly pay the supplier is given by the value of output at that point (nobody is willing to lend $F$ more than that). The (incomplete) good thus serves as collateral. It is not plausible that the supplier is able to capture all this; rather only a fraction $\beta$ of the ex-post gains.
3. Production takes place sequentially. For each stage $j$, the supplier chooses her effort level $x(j)$, taking the effort levels of all production stages prior to $j$ as given. At the end of each stage $j$, supplier $j$ and the firm bargain over the addition to total revenue that supplier $j$ has contributed. The firm pays the supplier (potentially using the partially produced good as collateral to borrow).

4. Output of the final good is realized once the final stage is completed. This output is given by (1). The total revenue, $A^{1-\rho}q^\rho$, from the sale of the final good is split up between the final-good producer and the lenders, with the firm effectively becoming the residual claimant of the revenue remaining after paying all suppliers receiving the amount agreed upon during the respective bilateral bargaining processes.

Note that the level of effort undertaken by suppliers prior to stage $j$ is observable to supplier $j$, so that $j$’s effort levels can be conditioned on these prior effort levels. However, these cannot be independently verified by an objective third party, so that the incomplete contracting problem persists.

### 2.2 Equilibrium Firm Behavior

#### A. Suppliers’ Choice of Effort Levels

We solve for a subgame perfect equilibrium of this game. We start by first solving for each supplier’s effort level, conditional on the effort levels observed from prior stages. In order to solve for $x(j)$, we need first to derive the expression for the incremental contribution to total revenue made by supplier $j$. Define:

$$r(m) = A^{1-\rho}q^\rho \left[ \int_{j=0}^{m} x(j)^\alpha dz \right]^{\rho/\alpha}$$

which is the revenue that has been generated by production up to stage $m \in [0, 1]$. One can think of this as the revenue that would be obtained if the firm decided to terminate production at stage $m$, and sell the output on the open market at that point.

Let $r'(m)$ denote the incremental revenues over which the supplier at stage $m$ and the firm bargain over. Simple differentiation delivers:

$$r'(m) = \frac{\partial r(m)}{\partial m} = A^{1-\rho}q^\rho \frac{\rho}{\alpha} \left[ \int_{j=0}^{m} x(j)^\alpha dj \right]^{\rho-\alpha/\alpha} x(m)^\alpha$$

We denote by $\beta(m)$ the share of this incremental revenues that accrues to the firm in the bargain. Taking into account each party’s outside options, it is straightforward to show that $\beta(m)$ is equal to

$$\beta(m) = \begin{cases} 
\beta_O = \beta & \text{if the firm outsources stage } m \\
\beta_V = \beta + (1 - \beta) \delta^\alpha & \text{if the firm integrates stage } m
\end{cases}$$
We can now derive the optimal effort levels of each supplier. The \( m \)-th supplier obtains a share \( 1 - \beta(m) \in [0,1] \) of the incremental contribution to total revenue of her input, \( r'(m) \). Effort levels are chosen to maximize this share of revenue net of supplier \( m \)'s effort cost:

\[
\max_{x(m)} (1 - \beta(m)) r'(m) - cx(m) \tag{4}
\]

It is straightforward to solve this problem to obtain supplier \( m \)'s labor effort:

\[
x(m) = \left( (1 - \beta(m)) \frac{\rho A^{1-\rho} \theta^\alpha}{c} \right)^{-a} \frac{1}{1-a} \left[ \int_0^m x(j)^{\alpha} dj \right]^{\frac{\rho-a}{\alpha (1-a)}} \tag{5}
\]

Intuitively, the labor effort put in by supplier \( m \) is increasing in the demand level, \( A \), and in the supplier’s bargaining share, \( 1 - \beta(m) \), but decreasing in the unit labor cost, \( c \). As expected, an outsourcing arrangement, with a lower \( \beta(m) \), promotes more effort as it allows the supplier to bargain over a larger revenue pie. The effect of the labor effort invested in prior stages, \( \{x(j)\}_{j=0}^m \), is more subtle: If \( \rho < \alpha \), namely if the inputs from different stages are sufficiently strong substitutes in production, then a higher level of effort invested by prior suppliers actually lowers the marginal return on supplier \( m \)’s labor effort, and thus reduces \( m \)'s incentives to invest in \( x(m) \). Conversely, if \( \rho > \alpha \), the inputs are sufficiently complementary, which raises supplier \( m \)'s optimal effort choice. Throughout the paper, we shall refer to \( \rho < \alpha \) as the low-complementarity case and to \( \rho > \alpha \) as the high-complementarity case.

Equation (5) gives us each supplier’s effort level as a function of all prior suppliers’ own effort levels. To solve for \( x(m) \) solely as a function of the model’s primitives, observe first that:

\[
\int_0^m x(j)^{\alpha} dj = \left( \frac{r(m)}{A^{1-\rho} \theta^\alpha} \right)^{\frac{1}{\alpha}}. \tag{7}
\]

We thus turn our attention to obtaining a closed-form expression for the revenue function, \( r(m) \). Differentiating (2) via Leibnitz’s Rule, and plugging (5) we can express:

\[
r'(m) = \frac{\rho}{\alpha} \left( (1 - \beta(m)) \frac{\rho \theta^\alpha}{c} \right)^{\frac{\alpha}{1-\rho}} A^{\frac{\alpha(1-\rho)}{\alpha (1-a)}} (r(m))^{\frac{\rho-a}{\alpha (1-a)}}. \tag{6}
\]

The differential equation (6) is separable in \( r(m) \) and \( \beta(m) \), which together with the initial condition \( r(0) = 0 \), yields the following solution:

\[
r(m) = A \left( \frac{1-\rho}{1-\alpha} \right)^{\frac{\alpha(1-\alpha)}{\alpha (1-\rho)}} \left( \frac{\rho \theta^\alpha}{c} \right)^{\frac{\rho-a}{\alpha (1-a)}} \left[ \int_0^m (1 - \beta(j))^{\alpha} x(j)^{\alpha} dj \right]^{\frac{\rho-a}{\alpha (1-a)}}. \tag{7}
\]

This last equation gives us revenues up to stage \( m \) as a function of the organizational form chosen by the firm. These revenues tend to increase when outsourcing (a low \( \beta \)) is chosen, since outsourcing of a particular stage encourages the supplier of that stage to put in more labor effort. To see this formally, use \( \int_0^m x(j)^{\alpha} dj = \left( \frac{r(m)}{A^{1-\rho} \theta^\alpha} \right)^{\frac{1}{\alpha}} \) and plug the solution for \( r(m) \) in (7) into (5),
to obtain

\[ x(m) = A \left( \frac{1 - \rho}{1 - \alpha} \right) \left( \frac{\rho}{c} \right) \frac{1}{1 - \rho} \theta \frac{\rho}{1 - \rho} \left( 1 - \beta(m) \right) \frac{1}{1 - \rho} \left[ \int_{j=0}^{m} \left( 1 - \beta(j) \right) \frac{1}{1 - \rho} \, dj \right] \frac{\rho - \alpha}{\alpha (1 - \rho)} \). \] (8)

From this expression, it is clear that \( x(m) \) is decreasing in \( \beta(m) \) (and hence higher when supplier \( m \) is not integrated). On the other hand, the dependence of \( x(m) \) on the previous organizational choices of the firm is less clear cut. When \( \rho > \alpha \), i.e., in the high-complementary case, outsourcing upstream enhances investments downstream, while the converse is true in the low-complementarity case \( (\rho < \alpha) \). In choosing its optimal organizational structure, the firm will weigh these considerations together with the fact that outsourcing of any stage is associated with a lower share of surplus accruing to the final-good producer. We next turn to a study of this optimal organizational structure.

**B. The Firm’s Choice of Organizational Structure**

The firm seeks to maximize the revenues that it obtains aggregated over all the production stages. As a first step towards deriving the firm’s objective function, consider first the incremental revenue that accrues to the firm at each production stage \( m \). As argued above, the firm captures a fraction \( \beta(m) \) of the incremental revenues \( r'(m) \), which using (6) and (7) simplifies to:

\[ \beta(m) r'(m) = \beta(m) A \left( \frac{1 - \rho}{1 - \alpha} \right) \left( \frac{\rho}{c} \right) \frac{1}{1 - \rho} \theta \frac{\rho}{1 - \rho} \left( 1 - \beta(m) \right) \frac{1}{1 - \rho} \left[ \int_{j=0}^{m} \left( 1 - \beta(j) \right) \frac{1}{1 - \rho} \, dj \right] \frac{\rho - \alpha}{\alpha (1 - \rho)} \). \]

Integrating over all stages, we thus obtain that the profits obtained by the firm are given by:\(^4\)

\[ \Pi = \int_{j=0}^{1} \beta(j) r'(j) = A \left( \frac{1 - \rho}{1 - \alpha} \right) \left( \frac{\rho}{c} \right) \frac{1}{1 - \rho} \theta \frac{\rho}{1 - \rho} \left[ \int_{j=0}^{j} \left( 1 - \beta(j) \right) \frac{1}{1 - \rho} \, dj \right] \frac{\rho - \alpha}{\alpha (1 - \rho)} \]

\[ \times \int_{j=0}^{1} \beta(j) (1 - \beta(j)) \frac{1}{1 - \rho} \left[ \int_{k=0}^{j} (1 - \beta(k)) \frac{1}{1 - \rho} \, dk \right] \frac{\rho - \alpha}{\alpha (1 - \rho)} \, dj. \] (9)

The firm’s decision problem is then to choose the values of \( \beta(j) \in \{\beta_V, \beta_O\} \) for \( j \in [0, 1] \) that maximize \( \Pi \). For a given stage \( m \), in order to determine if integration or outsourcing is optimal, it proves useful to follow the approach in Antràs and Helpman (2004, 2008) and consider the hypothetical case in which the firm could freely choose \( \beta(m) \) from the continuum of values in \([0, 1]\). After some simplification, the partial derivative of the maximand with respect to \( \beta(m) \) is:

\[ \frac{\partial \Pi}{\partial \beta(m)} = A \left( \frac{1 - \rho}{1 - \alpha} \right) \left( \frac{\rho}{c} \right) \frac{1}{1 - \rho} \theta \frac{\rho}{1 - \rho} \left( 1 - \beta(m) \right) \frac{1}{1 - \rho} \left[ \int_{j=0}^{m} (1 - \beta(j)) \frac{1}{1 - \rho} \, dj \right] \frac{\rho - \alpha}{\alpha (1 - \rho)} \Phi(m) \]

\(^4\)It is easily verified that \( \Pi = r(1) - \int_{j=0}^{1} (1 - \beta(j)) r'(j) \, dj \).
where:

$$\Phi(m) = (1 - \beta(m)) \left[ \int_{k=0}^{m} (1 - \beta(k)) \frac{\alpha}{\alpha(1 - \rho)} dk \right]^{\frac{\rho - \alpha}{\alpha(1 - \rho)}} - \frac{\alpha}{1 - \alpha} \beta(m) \left[ \int_{k=0}^{m} (1 - \beta(k)) \frac{\alpha}{\alpha(1 - \rho)} dk \right]^{\frac{\rho - \alpha}{\alpha(1 - \rho)}} \int_{i=m}^{1} \beta(i)(1 - \beta(i)) \frac{i^{\alpha}}{i^{\alpha}} \left[ \int_{j=0}^{i} (1 - \beta(j)) \frac{\alpha}{\alpha(1 - \rho)} dj \right]^{\frac{\rho - \alpha}{\alpha(1 - \rho)}} - \frac{\rho - \alpha}{(1 - \alpha)(1 - \rho)} \int_{i=m}^{1} \beta(i) \frac{1}{i^{\alpha}} \left[ \int_{j=0}^{i} (1 - \beta(j)) \frac{\alpha}{\alpha(1 - \rho)} dj \right]^{\frac{\rho - \alpha}{\alpha(1 - \rho)}} di. \quad (10)$$

The function \( \Phi(m) \) captures how the firm’s incentives to raise or lower the bargaining share \( \beta(m) \) depends on the particular production stage \( m \in [0, 1] \) under consideration. It consists of three terms. The first term captures a rent extraction effect: other things equal, a higher bargaining power for the firm will naturally increase firm profits and the effect interacts with \( m \) because the surplus over which the firm and the supplier bargain over varies along the value chain. In the high-complementarity case \( (\rho > \alpha) \), the marginal contribution of downstream stages is particularly important, while the converse is true in the low-complementarity case \( (\rho < \alpha) \). The second term relates to incentive effects of an increase in \( \beta(m) \) on the effort level of the supplier providing stage \( m \). The effect is obviously negative and again the size of this effect depends on \( m \) again because the marginal contribution of that stage is a function of \( m \). Finally, the third term captures the effect of the increase in the bargaining share \( m \) on the incentives to invest of all suppliers that are downstream relative to \( m \). As explained in the discussion of equation (8), the effect is negative in the high-complementarity case, while it is negative in the low-complementarity case. Importantly, this last effect also naturally interacts with \( m \) as the absolute value of the effect will naturally be higher when \( m \) is low, i.e., when a given stage precedes a relatively large number of other production stages.

Setting \( \Phi(m) = 0 \), we then have that the optimal \( \beta^*(m) \) satisfies

$$\beta^*(m) = (1 - \alpha) \left( 1 - \frac{\rho - \alpha}{(1 - \alpha)(1 - \rho)} \int_{i=m}^{1} \beta(i)(1 - \beta(i)) \frac{i^{\alpha}}{i^{\alpha}} \left[ \int_{j=0}^{i} (1 - \beta(j)) \frac{\alpha}{\alpha(1 - \rho)} dj \right]^{\frac{\rho - \alpha}{\alpha(1 - \rho)}} - \frac{\rho - \alpha}{(1 - \alpha)(1 - \rho)} \int_{i=m}^{1} \beta(i) \frac{1}{i^{\alpha}} \left[ \int_{j=0}^{i} (1 - \beta(j)) \frac{\alpha}{\alpha(1 - \rho)} dj \right]^{\frac{\rho - \alpha}{\alpha(1 - \rho)}} \right). \quad (11)$$

This expression may seem complicated, but straightforward differentiation indicates that (see Appendix for a proof):

**Lemma 1** The (unconstrained) optimal bargaining share \( \beta^*(m) \) is an increasing function of \( m \) whenever \( \rho > \alpha \) (high-complementarity case) while it is a decreasing function of \( m \) when \( \rho < \alpha \) (low-complementarity case).

In words, whether the incentive for the firm to retain a larger surplus share increases or decreases along the value chain crucially depends on the relative size of the parameters \( \rho \) and \( \alpha \), which in turn govern the three effects discussed above. Intuitively, although when \( \rho > \alpha \) is low,

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5 An alternative way to see this is that how \( r'(m) \) in (3) varies with \( m \) crucially depends on the relative size of \( \rho \) and \( \alpha \).
integrating early stages of production would lead to relatively low underinvestment of these early suppliers (the second effect above), this effect is dominated by the fact that when inputs are high complementary, integration decisions early in the value chain are particularly costly because they reduce the incentives to invest not only of these early suppliers but also of all suppliers downstream (the third effect above), while the firm does not extract too much surplus by integrating these early stages (the first effect above). On the other hand, when $\alpha$ is high and inputs are substitutable, outsourcing is particularly costly in upstream stages because high investments in the early stages of the value chain lead to reduced incentives to invest for downstream suppliers, while the firm captures a disproportionate amount of surplus by integrating these early stages.

We have framed the intuition above in terms of the degree of complementarity $\alpha$. What explains the effect of $\rho$? Notice that this parameter governs the concavity of sale revenues with respect to output. When $\rho$ is low, this function is particularly concave, so the contribution of upstream suppliers to revenue is particularly high, while that of downstream suppliers is particularly low. In order to compensate for this, the firm optimally decides to grant more bargaining power to downstream suppliers (i.e., outsourcing is more and more appealing as $m$ increases).

Evaluating the function $\beta^*(m)$ at both extremes of the value chain, we obtain that $\beta^*(0) \to -\infty$ when $\rho > \alpha$ and $\beta^*(0) = 1$ when $\rho < \alpha$, while $\beta^*(1) = 1 - \alpha$ regardless of the relative magnitude of $\alpha$ and $\rho$ (see the proof of Proposition 1 in the Appendix). Hence, the decision of whether to integrate or not stage 0 again crucially depends on the relative size of $\rho$ and $\alpha$. In the high-complementarity case ($\rho > \alpha$), the firm would like to choose the minimum possible value of $\beta(m)$, which corresponds to choosing outsourcing in this initial stage (and, by continuity, a measurable set of the early stages). Conversely, in the low-complementarity case, the firm necessarily chooses to integrate this initial stage (and again, by continuity, a measurable set of the early stages). As for the latest stages of the value chain, the decision is less clear-cut. If $1 - \alpha > \beta_V$, then it is clear that that last stage will be integrated, while it will necessarily be outsourced if $1 - \alpha < \beta_O$. When $\beta_V > 1 - \alpha > \beta_O$, whether that stage is integrated or not depends on other parameter values. In sum, when $\rho > \alpha$ the firm will necessarily outsource relatively upstream inputs, while it may (depending on parameter values) find it optimal to integrate sufficiently downstream inputs. Conversely, when $\rho < \alpha$ the firm will necessarily integrate relatively upstream inputs, outsourcing may only prove profitable for sufficiently downstream inputs. A more formal statement of these results is as follows (see Appendix for a proof):

**Proposition 1** In the high-complementarity case ($\rho > \alpha$), there exists a unique $m^*_C \in (0, 1]$, such that: (i) all production stages $m \in [0, m^*_C)$ are outsourced; and (ii) all stages $m \in [m^*_C, 1]$ are integrated within firm boundaries. In the low-complementarity case ($\rho < \alpha$), there exists a unique $m^*_S \in (0, 1]$, such that: (i) all production stages $m \in [0, m^*_S)$ are integrated; and (ii) all stages $m \in [m^*_S, 1]$ are outsourced within firm boundaries.

We close this section by pointing out that one can actually derive a closed-form expression for the cut-off stages, $m^*_C$ and $m^*_S$ (and hence verify whether they are strictly lower than 1 or
not). For each case, this is achieved by plugging the optimal values \( \beta^* (m) \) for all \( m \in [0, 1] \) in the firm’s maximand in (9) and (after evaluating the integrals), solving for the value of \( m^*_C \) or \( m^*_S \) that maximizes the simplified maximand. This produces the following optimal threshold levels:

\[
m^*_C = \left[ 1 + \left( \frac{1 - \beta}{1 - \beta V} \right)^{\frac{\alpha}{1 - \alpha}} \left[ \left( \frac{1 - \beta}{1 - \beta V} \right)^{-\frac{\alpha}{1 - \alpha}} - 1 \right] \right]^{-1} \tag{12}
\]

and

\[
m^*_S = \left[ 1 + \left( \frac{1 - \beta V}{1 - \beta} \right)^{\frac{\alpha}{1 - \alpha}} \left[ \left( \frac{\beta V}{1 - \beta V} - 1 \right)^{-\frac{\alpha}{1 - \alpha}} - 1 \right] \right]^{-1}. \tag{13}
\]

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Note: This follows from solving

\[
\begin{align*}
m^*_C &= \arg \max \left\{ \beta (1 - \beta)^{\frac{\alpha}{1 - \alpha}} \int_{j=0}^{m} \frac{\alpha}{1 - \alpha} \, dj \right. + \beta V (1 - \beta V)^{\frac{\alpha}{1 - \alpha}} \int_{j=m}^{1} \left[ (1 - \beta)^{\frac{\alpha}{1 - \alpha}} m + (1 - \beta V)^{\frac{\alpha}{1 - \alpha}} (j - m) \right] \frac{\alpha}{1 - \alpha} \, dj \left\} ; \\
m^*_S &= \arg \max \left\{ \beta V (1 - \beta V)^{\frac{\alpha}{1 - \alpha}} \int_{j=0}^{m} \frac{\alpha}{1 - \alpha} \, dj \right. + \beta (1 - \beta)^{\frac{\alpha}{1 - \alpha}} \int_{j=m}^{1} \left[ (1 - \beta V)^{\frac{\alpha}{1 - \alpha}} m + (1 - \beta)^{\frac{\alpha}{1 - \alpha}} (j - m) \right] \frac{\alpha}{1 - \alpha} \, dj \left\} .
\end{align*}
\]
3 References


Appendix

Proof of Lemma 1: The key is how the ratio in the expression for $\beta^*(m)$ in (11) depends on $m$. When $\rho > \alpha$, it is clear that the numerator is decreasing in $m$, while the denominator is increasing in $m$. Due to the negative sign in front of the ratio, it is clear that $\beta^*(m)$ is increasing in $m$. The case $\rho < \alpha$ is a bit more cumbersome to study. It proves useful to write $\beta^*(m)$ as

$$ \frac{\beta^*(m) - 1}{(1-\alpha)(1-\rho)} = \left( \int_{i=m}^{1} \beta(i)(1-\beta(i))^{\frac{\alpha}{1-\alpha}} \left[ \int_{j=0}^{i} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right] \frac{\theta-\alpha}{\alpha(1-\rho)} \right) \left[ \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right] \frac{\theta-\alpha}{\alpha(1-\rho)} - 1.$$ 

Differentiation of the right hand side delivers

$$ (1-\beta(m))^{\frac{\alpha}{1-\alpha}} \left[ \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right]^{-1} \times \left( -\beta(m) + \frac{\alpha - \rho}{\alpha(1-\rho)} \left[ \int_{j=0}^{m} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right] \left( \int_{i=m}^{1} \beta(i)(1-\beta(i))^{\frac{\alpha}{1-\alpha}} \left[ \int_{j=0}^{i} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right] \frac{\theta-\alpha}{\alpha(1-\rho)} \right) \left[ \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right] \frac{\theta-\alpha}{\alpha(1-\rho)} - 1 \right) $$

which, using the above expression for $\beta^*(m)$, simplifies to

$$ (1-\beta(m))^{\frac{\alpha}{1-\alpha}} \left[ \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right]^{-1} \left( -\frac{1-\alpha}{\alpha} (1-\beta^*(m)) \right) < 0. $$

QED.

Proof of Proposition 1 (Incomplete): For now let us just look at the limits (later we might need to add a continuity argument using Lemma 1, along the lines of your previous proof). First take the case $\rho > \alpha$:

$$ \lim_{m \to 0} \beta^*(m) = (1-\alpha) \lim_{m \to 0} \left( 1 - \frac{\rho - \alpha}{(1-\alpha)(1-\rho)} \int_{i=m}^{1} \beta(i)(1-\beta(i))^{\frac{\alpha}{1-\alpha}} \left[ \int_{j=0}^{i} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right] \frac{\theta-\alpha}{\alpha(1-\rho)} \right) \left[ \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right] \frac{\theta-\alpha}{\alpha(1-\rho)} - 1 \right) $$

$$ = (1-\alpha) \left( 1 - \frac{0}{0} \right) = -\infty $$

$$ \lim_{m \to 1} \beta^*(m) = (1-\alpha) \lim_{m \to 1} \left( 1 - \frac{\rho - \alpha}{(1-\alpha)(1-\rho)} \int_{i=1}^{\infty} \beta(i)(1-\beta(i))^{\frac{\alpha}{1-\alpha}} \left[ \int_{j=0}^{i} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right] \frac{\theta-\alpha}{\alpha(1-\rho)} \right) \left[ \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right] \frac{\theta-\alpha}{\alpha(1-\rho)} - 1 \right) $$

$$ = (1-\alpha) \left( 1 - \frac{0}{0} \right) = 1-\alpha $$
Now take $\rho < \alpha$:
\[
\lim_{m \to 0} \frac{\beta^*(m) - 1}{\frac{\alpha - \rho}{(1-\alpha)(1-\rho)}} = \lim_{m \to 0} \left( \frac{1}{\alpha - \rho} \right) \left( \int_{i=m}^{1} \beta(i)(1-\beta(i))^{\frac{\alpha}{1-\alpha}} \left[ \int_{j=0}^{i} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right]^{\frac{\alpha - \rho}{\alpha(1-\rho) - 1}} di \right) \left( \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right)^{\frac{\alpha - \rho}{\alpha(1-\rho) - 1}}
\]
\[
= ((\infty) \times 0) = ?
\]

So we need to use l'Hopital
\[
\lim_{m \to 0} \frac{\beta^*(m) - 1}{\frac{\alpha - \rho}{(1-\alpha)(1-\rho)}} = \lim_{m \to 0} \left\{ \frac{-\beta(m)(1-\beta(m))^{\frac{\alpha}{1-\alpha}} \left[ \int_{j=0}^{m} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right]^{\frac{\alpha - \rho}{\alpha(1-\rho) - 1}}}{\left( -\frac{\alpha - \rho}{\alpha(1-\rho)} \right) \left( \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right)^{\frac{\alpha - \rho}{\alpha(1-\rho) - 1}} (1-\beta(m))^{\frac{\alpha}{1-\alpha}}} \right\} = \lim_{m \to 0} \left\{ \frac{\beta(0)}{\frac{\alpha - \rho}{\alpha(1-\rho)}} \right\},
\]
and thus
\[
\frac{\beta^*(0) - 1}{\frac{\alpha - \rho}{(1-\alpha)(1-\rho)}} = \frac{\beta^*(0)}{\frac{\alpha - \rho}{\alpha(1-\rho)}} \to \beta^*(0) = 1
\]

Finally
\[
\lim_{m \to 1} \frac{\beta^*(m) - 1}{\frac{\alpha - \rho}{(1-\alpha)(1-\rho)}} = \lim_{m \to 1} \left( \frac{1}{\alpha - \rho} \right) \left( \int_{i=m}^{1} \beta(i)(1-\beta(i))^{\frac{\alpha}{1-\alpha}} \left[ \int_{j=0}^{i} (1-\beta(j))^{\frac{\alpha}{1-\alpha}} dj \right]^{\frac{\alpha - \rho}{\alpha(1-\rho) - 1}} di \right) \left( \int_{k=0}^{m} (1-\beta(k))^{\frac{\alpha}{1-\alpha}} dk \right)^{\frac{\alpha - \rho}{\alpha(1-\rho) - 1}}
\]
\[
= 0 \times + = 0
\]

which implies $\beta^*(1) = 1 - \alpha$. 

14